Topic: Estimating a root

Question: Use linear approximation to estimate $\sqrt[3]{9}$.

Answer choices:

A $\dfrac{41}{12}$

B $\dfrac{23}{12}$

C $\dfrac{7}{12}$

D $\dfrac{25}{12}$

Solution: D

We don't know the value of $\sqrt[3]{9}$, but we know that $\sqrt[3]{8} = 2$. So instead of trying to calculate $\sqrt[3]{9}$ directly, let's use the function $f(x) = \sqrt[3]{x}$.

Differentiate the function

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

$$f'(x) = \frac{1}{3x^{\frac{2}{3}}}$$

and then evaluate it at $a = 8$.

$$f'(8) = \frac{1}{3(8)^{\frac{2}{3}}}$$

$$f'(8) = \frac{1}{3(8^{\frac{1}{3}})^2}$$

$$f'(8) = \frac{1}{3(2)^2}$$

$$f'(8) = \frac{1}{3(4)}$$

$$f'(8) = \frac{1}{12}$$

So along the function $f(x) = \sqrt[3]{x}$, we have the point of tangency $(8,2)$ and the slope $m = 1/12$. Substitute these into the linear approximation equation.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 2 + \frac{1}{12}(x-8)$$

$$L(x) = 2 + \frac{1}{12}x - \frac{8}{12}$$

$$L(x) = \frac{1}{12}x - \frac{8}{12} + \frac{24}{12}$$

$$L(x) = \frac{1}{12}x + \frac{16}{12}$$

Now that we have the linear approximation equation, we can use it to estimate $\sqrt[3]{9}$. Substitute $x = 9$.

$$L(9) = \frac{1}{12}(9) + \frac{16}{12}$$

$$L(9) = \frac{9}{12} + \frac{16}{12}$$

$$L(9) = \frac{25}{12}$$

Topic: Estimating a root

Question: Use linear approximation to estimate $\sqrt[3]{29}$.

Answer choices:

A $\frac{83}{27}$

B $\frac{25}{27}$

C $\frac{137}{27}$

D $\frac{79}{27}$

Solution: A

We don't know the value of $\sqrt[3]{29}$, but we know that $\sqrt[3]{27} = 3$. So instead of trying to calculate $\sqrt[3]{29}$ directly, let's use the function $f(x) = \sqrt[3]{x}$.

Differentiate the function

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

$$f'(x) = \frac{1}{3x^{\frac{2}{3}}}$$

and then evaluate it at $a = 27$.

$$f'(27) = \frac{1}{3(27)^{\frac{2}{3}}}$$

$$f'(27) = \frac{1}{3(27^{\frac{1}{3}})^2}$$

$$f'(27) = \frac{1}{3(3)^2}$$

$$f'(27) = \frac{1}{3(9)}$$

$$f'(27) = \frac{1}{27}$$

So along the function $f(x) = \sqrt[3]{x}$, we have the point of tangency $(27,3)$ and the slope $m = 1/27$. Substitute these into the linear approximation equation.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 3 + \frac{1}{27}(x - 27)$$

$$L(x) = 3 + \frac{1}{27}x - 1$$

$$L(x) = \frac{1}{27}x + 2$$

Now that we have the linear approximation equation, we can use it to estimate $\sqrt[3]{29}$. Substitute $x = 29$.

$$L(29) = \frac{1}{27}(29) + 2$$

$$L(29) = \frac{29}{27} + \frac{54}{27}$$

$$L(29) = \frac{83}{27}$$

Topic: Estimating a root

Question: Use linear approximation to estimate $\sqrt[4]{79}$.

Answer choices:

A $\dfrac{322}{54}$

B $\dfrac{164}{54}$

C $\dfrac{161}{54}$

D $\dfrac{82}{54}$

Solution: C

We don't know the value of $\sqrt[4]{79}$, but we know that $\sqrt[4]{81} = 3$. So instead of trying to calculate $\sqrt[4]{79}$ directly, let's use the function $f(x) = \sqrt[4]{x}$.

Differentiate the function

$$f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$$

$$f'(x) = \frac{1}{4x^{\frac{3}{4}}}$$

and then evaluate it at $a = 81$.

$$f'(81) = \frac{1}{4(81)^{\frac{3}{4}}}$$

$$f'(81) = \frac{1}{4(81^{\frac{1}{4}})^3}$$

$$f'(81) = \frac{1}{4(3)^3}$$

$$f'(81) = \frac{1}{4(27)}$$

$$f'(81) = \frac{1}{108}$$

So along the function $f(x) = \sqrt[4]{x}$, we have the point of tangency $(81,3)$ and the slope $m = 1/108$. Substitute these into the linear approximation equation.

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = 3 + \frac{1}{108}(x - 81)$$

$$L(x) = 3 + \frac{1}{108}x - \frac{81}{108}$$

$$L(x) = \frac{1}{108}x - \frac{81}{108} + \frac{324}{108}$$

$$L(x) = \frac{1}{108}x + \frac{243}{108}$$

Now that we have the linear approximation equation, we can use it to estimate $\sqrt[4]{79}$. Substitute $x = 79$.

$$L(79) = \frac{1}{108}(79) + \frac{243}{108}$$

$$L(79) = \frac{79}{108} + \frac{243}{108}$$

$$L(79) = \frac{322}{108}$$

$$L(79) = \frac{161}{54}$$