

Calculus 1 Workbook

Solving limits



SOLVING WITH SUBSTITUTION

■ 1. What is the value of the limit?

$$\lim_{x \to 3} (-x^4 + x^3 + 2x^2)$$

■ 2. What is the value of the limit?

$$\lim_{x \to 7} \frac{x^2 - 5}{x^2 + 5}$$

■ 3. What is the value of the limit.

$$\lim_{x \to -2} \frac{x^3 - 5x^2 + 4x - 6}{x^2 + 7x + 6}$$

■ 4. Evaluate the limit.

$$\lim_{y \to -2} \frac{|y-5|}{y+1}$$

■ 5. Evaluate the limit.

$$\lim_{x \to 2} \left(\sin \left(\frac{\pi x}{4} \right) + \ln \left(\frac{2e}{x} \right) \right)$$

■ 6. Evaluate the limits $\lim_{x\to -1} f(x)$ and $\lim_{x\to 2} f(x)$.

$$f(x) = \begin{cases} -3x + 5 & x < -1 \\ \frac{1}{2}x^2 - 3x + 1 & x \ge -1 \end{cases}$$



SOLVING WITH FACTORING

■ 1. What is the value of the limit?

$$\lim_{x \to -7} \frac{6x^3 + 42x^2}{2x^2 + 26x + 84}$$

■ 2. What is the value of the limit?

$$\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{x - 1}$$

■ 3. What is the value of the limit?

$$\lim_{x \to 0} \frac{(x+3)^2 - 9}{x^2}$$

■ 4. What is the value of the limit?

$$\lim_{x \to 7} \frac{x^3 - x^2 - 42x}{2x^2 - 20x + 42}$$

$$\lim_{x \to 8} \frac{x^2 + 2x - 80}{2x^3 - 24x^2 + 64x}$$

$$\lim_{x \to 0} \frac{1}{x} \left(1 - \frac{16}{(x-4)^2} \right)$$



SOLVING WITH CONJUGATE METHOD

■ 1. Use conjugate method to evaluate the limit.

$$\lim_{x \to 16} \frac{3(x - 16)}{\sqrt{x} - 4}$$

■ 2. What is the value of the limit?

$$\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$$

■ 3. What is the value of the limit?

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 16} - 4}{x^2}$$

■ 4. Use conjugate method to evaluate the limit.

$$\lim_{x \to 49} \frac{x - 49}{3(\sqrt{x} - 7)}$$

$$\lim_{x \to 1} \frac{4 - \sqrt{x + 15}}{2(x - 1)}$$

$$\lim_{x \to 2} \frac{\sqrt{11 - x} - 3}{\sqrt{6 - x} - 2}$$



INFINITE LIMITS AND VERTICAL ASYMPTOTES

■ 1. What is the value of the limit?

$$\lim_{x \to 2} \frac{x^2 - x - 6}{-3x^2 - 3x + 18}$$

■ 2. What is the value of the limit?

$$\lim_{x \to -1} \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

■ 3. What is the value of the limit?

$$\lim_{x \to 3} \frac{1}{|x - 3|}$$

■ 4. What is the value of the limit?

$$\lim_{x \to -4} \frac{\sqrt{x^2 - 1}}{x + 4}$$

$$\lim_{x \to 3} \frac{x^2 - 4x}{x^2 - 2x - 3}$$

$$\lim_{x \to -2} \frac{x^2 - 16}{-x^2 + x + 6}$$



LIMITS AT INFINITY AND HORIZONTAL ASYMPTOTES

■ 1. What is the value of the limit?

$$\lim_{x \to \infty} \frac{3x^3 - 5x + 2}{9x^3 + 7x^2 - x}$$

■ 2. What is the value of the limit?

$$\lim_{x \to \infty} \frac{\sqrt[3]{x} + 1}{\sqrt{x} - 1}$$

■ 3. What is the value of the limit?

$$\lim_{x \to \infty} \frac{x^3 + x^2 + 1}{1 + 2x}$$

■ 4. What is the value of the limit?

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 + 5}}{x + 4}$$

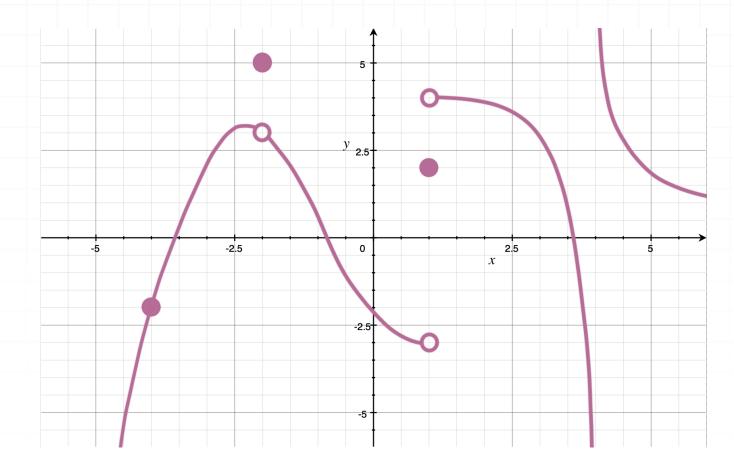
$$\lim_{x \to -\infty} \frac{19x + 21}{x^3 + 15x + 11}$$

$$\lim_{x \to \infty} (\sqrt{x^2 + 2x} - x)$$



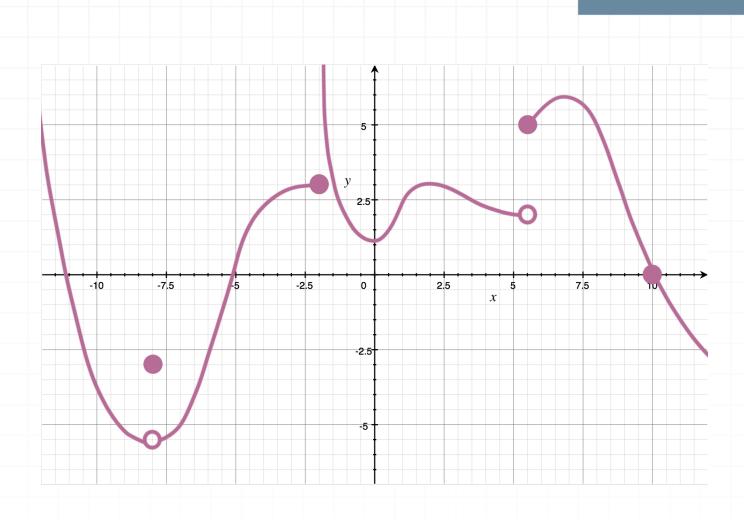
CRAZY GRAPHS

■ 1. Use the graph to find the value of $\lim_{x\to 1} f(x)$.

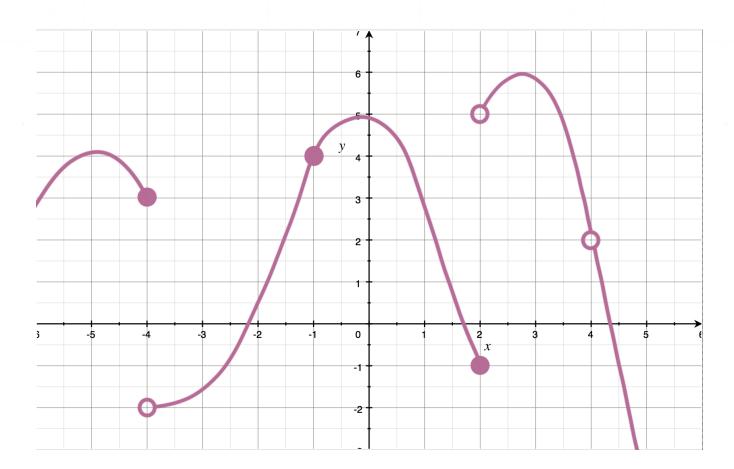


■ 2. Use the graph to find the value of $\lim_{x\to 5.5} g(x)$.

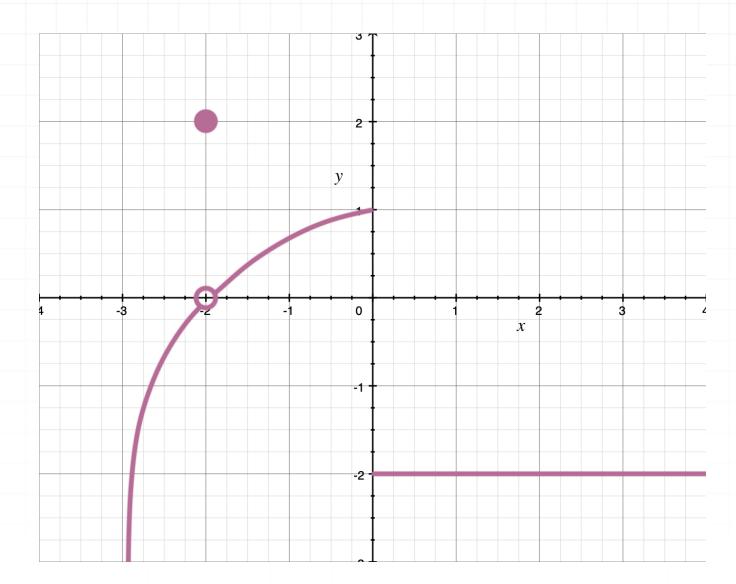




■ 3. Use the graph to find the value of $\lim_{x\to 4} h(x)$.



 \blacksquare 4. Use the graph to determine whether or not the limit exists at x=0.



■ 5. Sketch the graph of a function that satisfies each of the following conditions.

$$\lim_{x \to -1^{-}} f(x) = 2 \qquad \qquad \lim_{x \to -1^{+}} f(x) = -1 \qquad \qquad f(-1) = 0$$

■ 6. Sketch the graph of a function that satisfies each of the following conditions.

$$\lim_{x \to 0} f(x) = -5 \qquad f(0) \text{ does not exist}$$

$$\lim_{x \to 3^{-}} f(x) = -2$$

$$\lim_{x \to 3^+} f(x) = -4$$

$$f(3) = 0$$



TRIGONOMETRIC LIMITS

- 1. Find $\lim_{x \to \pi} f(x)$ if $f(x) = 3 \cos x 2$.
- 2. Evaluate the limit.

$$\lim_{x \to 0} \frac{\sin(8x)}{x}$$

■ 3. Evaluate the limit.

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \cot x}{\cos x - \sin x}$$

■ 4. Evaluate the limit.

$$\lim_{x \to 0} \frac{\tan(4x)}{\sin(2x)}$$

■ 5. Evaluate the limit.

$$\lim_{x \to 0} \frac{x}{\sin \frac{x}{3}}$$





$$\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos x}$$



MAKING THE FUNCTION CONTINUOUS

■ 1. What value of c makes the function's pieces meet each other at x = 4?

$$h(x) = \begin{cases} x^2 & x \le 4\\ 3x + c & x > 4 \end{cases}$$

■ 2. What value of k makes the function's pieces meet each other at x = 3?

$$f(x) = \begin{cases} kx^2 - 2x + 1 & x \le 3 \\ kx + 1 & x > 3 \end{cases}$$

■ 3. What values of a and b make the function's pieces meet each other at x = -2 and x = 2?

$$g(x) = \begin{cases} 3 & x \le -2 \\ ax - b & -2 < x < 2 \\ -2 & x \ge 2 \end{cases}$$

■ 4. What value of c makes the function's pieces meet each other at x = 1?

$$f(x) = \begin{cases} 2x^3 - 6x^2 + 8x + 3 & x \le 1\\ cx + 9 & x > 1 \end{cases}$$

■ 5. What value of c makes the function g(x) continuous?

$$g(x) = \begin{cases} \sqrt{x} + 18 & x \le 16 \\ x - 2c & x > 16 \end{cases}$$

■ 6. What values of a and b make the function h(x) continuous?

$$h(x) = \begin{cases} ax^2 & x \le -1\\ ax + b & -1 < x < 3\\ bx + 2 & x \ge 3 \end{cases}$$





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