Power rule for fractional powers

Power rule also applies in the same way when the exponent is a fraction. In other words, we bring the exponent down to multiply it by the coefficient, and subtract 1 from the exponent.

Example

Find the derivative of the function.

$$y = x^{\frac{3}{2}} + 6x^{-\frac{1}{4}}$$

Apply power rule to differentiate the function.

$$y' = \frac{3}{2}x^{\frac{3}{2}-1} + 6\left(-\frac{1}{4}\right)x^{-\frac{1}{4}-1}$$

$$y' = \frac{3}{2}x^{\frac{3}{2}-\frac{2}{2}} - \frac{3}{2}x^{-\frac{1}{4}-\frac{4}{4}}$$

$$y' = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{5}{4}}$$

We could leave the derivative this way, or we could rewrite it so that all the exponents are positive.

$$y' = \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2x^{\frac{5}{4}}}$$

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This example brings up the other point we want to make in this lesson, which is about the connection between fractional exponents and roots.

Every fractional exponent can be broken apart as a root. The numerator of the fraction becomes the power on the radicand (the value beneath the root), and the denominator of the fraction becomes the root. Here are a few examples:

$$x^{\frac{5}{4}} = \sqrt[4]{x^5}$$

$$x^{\frac{2}{3}} = \sqrt[3]{x^2}$$

$$x^{\frac{2}{5}} = \sqrt[5]{x^2}$$

In the first example we're taking the "fourth root of x^5," in the second example we're taking the "third root of x^2," and in the third example we're taking the "fifth root of x^2."

When the denominator of the fractional exponent is 2, it means we're taking the second root of something, which is the same as taking the square root, so we don't include the 2 with the root. Having the square root symbol on its own implies the 2. Therefore,

$$x^{\frac{1}{2}} = \sqrt{x}$$

This example follows the same pattern, because the numerator of 1 becomes the exponent on x, but an exponent of 1 is implied, so we don't need to write it in the radicand, and the denominator of 2 becomes the root, but a root of 2 is implied, so we don't need to write it with the root.

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The takeaway here is that, when we're asked to differentiate something with a root, we want to convert that root into a power function with a fractional exponent, and then use power rule to take its derivative.

Here's an example of how that works.

Example

Find the derivative of the function.

$$y = 3\sqrt[4]{x^5} - 4\sqrt[3]{x^7}$$

Before we take the derivative, we'll need to convert both roots into power functions with fractional exponents.

$$y = 3x^{\frac{5}{4}} - 4x^{\frac{7}{3}}$$

Now we'll apply power rule to differentiate.

$$y' = 3\left(\frac{5}{4}\right)x^{\frac{5}{4}-1} - 4\left(\frac{7}{3}\right)x^{\frac{7}{3}-1}$$

$$y' = \frac{15}{4}x^{\frac{5}{4}-\frac{4}{4}} - \frac{28}{3}x^{\frac{7}{3}-\frac{3}{3}}$$

$$y' = \frac{15}{4}x^{\frac{1}{4}} - \frac{28}{3}x^{\frac{4}{3}}$$

We can leave the derivative this way, or we can rewrite it by converting the fractional exponents back into roots.

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$$y' = \frac{15}{4}\sqrt[4]{x} - \frac{28}{3}\sqrt[3]{x^4}$$

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