Topic: Water level in the tank

Question: Water is being pumped into a rectangular tank at a rate of 0.8 cubic feet per minute. How fast is the water level rising if the base of the tank is a rectangle with dimensions 4×5 feet?

Answer choices:

A 25 ft/m

B 0.4 ft/m

C 0.04 ft/m

D 0.8 ft/m

Solution: C

The formula for the volume of a rectangular prism is

$$V = lwh$$

From the question, we know that $w = 4$ and $l = 5$, so we'll plug that in.

$$V = 5(4)h$$

$$V = 20h$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = 20(1)\frac{dh}{dt}$$

$$\frac{dV}{dt} = 20\frac{dh}{dt}$$

From the question, we know that $dV/dt = 0.8$, so make that substitution.

$$0.8 = 20\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{0.8}{20}$$

$$\frac{dh}{dt} = 0.04$$

Topic: Water level in the tank

Question: Water is being pumped from a cylindrical tank with a radius of 3 ft at a rate of 18 cubic feet per minute. How fast is the water level falling when the water is 2 ft deep?

Answer choices:

A $-\frac{2}{\pi}$ ft/min

B -2 ft/min

C $-\pi$ ft/min

D $-\frac{\pi}{2}$ ft/min

Solution: A

The formula for the volume of a cylinder is

$$V = \pi r^2 h$$

From the question, we know that $r = 3$, so plug that in.

$$V = \pi(3)^2 h$$

$$V = 9\pi h$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = 9\pi(1)\frac{dh}{dt}$$

$$\frac{dV}{dt} = 9\pi\frac{dh}{dt}$$

From the question, we know that $dV/dt = -18$, so make that substitution.

$$-18 = 9\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{18}{9\pi}$$

$$\frac{dh}{dt} = -\frac{2}{\pi}$$

Topic: Water level in the tank

Question: An inverted pyramid is standing on its tip. The base of the pyramid is 4×4 meters, and the depth is 9 meters. If oil is flowing into the vat at 8 m^3/min, how fast is the oil level rising when the depth of the oil is 7 meters?

Answer choices:

A $\frac{3}{2}$ m/min

B $\frac{81}{98}$ m/min

C $\frac{16}{81}$ m/min

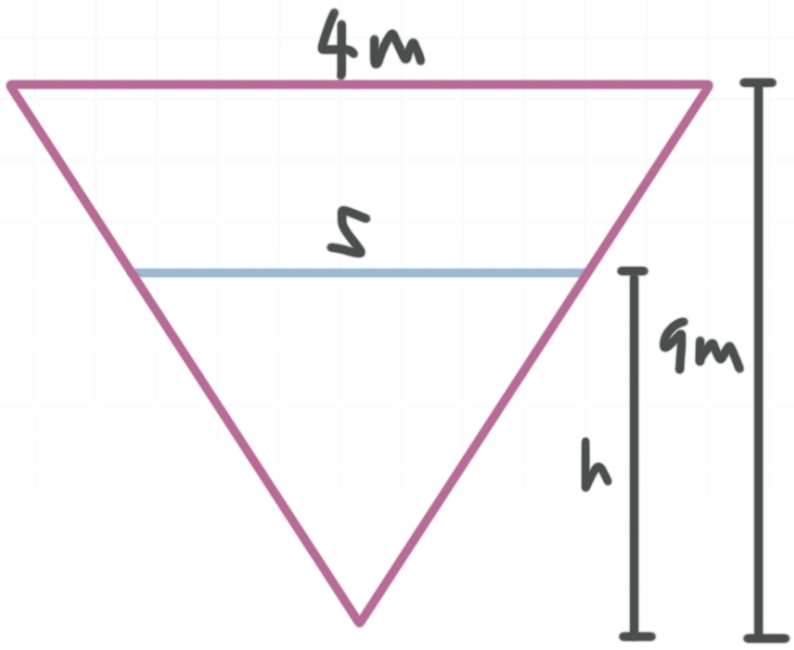
D $\frac{81}{16}$ m/min

Solution: B

The formula for the volume of a pyramid is

$$V = \frac{1}{3}s^2h$$

We want to express volume as a function of h only. Using the diagram of a cross-section of the pyramid,



and similar triangles, we see that

$$\frac{h}{s} = \frac{9}{4}$$

$$s = \frac{4}{9}h$$

Then the volume of the water is given by

$$V = \frac{1}{3}\left(\frac{4}{9}h\right)^2 h$$

$$V = \frac{1}{3}\left(\frac{16}{81}\right)h^3$$

$$V = \frac{16}{243}h^3$$

Use implicit differentiation to take the derivative of both sides.

$$(1)\frac{dV}{dt} = \frac{16}{243}(3h^2)\frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{16}{81}h^2\frac{dh}{dt}$$

From the question, we know that $dV/dt = 8$ and $h = 7$, so make those substitutions.

$$8 = \frac{16}{81}(7)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8(81)}{16(49)}$$

$$\frac{dh}{dt} = \frac{81}{98}$$