Position, velocity, and acceleration

The relationship between position, velocity, and acceleration is a common application of derivatives.

That's because velocity is the derivative of position, and acceleration is the derivative of velocity. So if say that the position function is defined as $x(t)$, velocity by $v(t)$, and acceleration by $a(t)$, then we can describe the relationship between these functions in a table.

Position	$x(t)$
Velocity	$v(t) = x'(t)$
Acceleration	$a(t) = v'(t) = x''(t)$

In these kinds of problems, we're often given the position of some object, like a particle or a car, and then asked to calculate all different kinds of values from that position function.

Some of the most common values we'll compute for these three functions are given in the following table.

Speed (always positive, has no direction)

$$s(t) = |v(t)|$$

Speed is **increasing** when velocity and acceleration have the same sign: $v(t), a(t) > 0$ or $v(t), a(t) < 0$

Speed is **decreasing** when velocity and acceleration have opposite signs: $v(t) > 0$ with $a(t) < 0$, or $v(t) < 0$ with $a(t) > 0$

Velocity (positive or negative, has a direction)

$v(t) = x'(t)$

Object is **moving forward** (to the right) when $v(t) > 0$

Object is **moving backward** (to the left) when $v(t) < 0$

Object is **at rest** (not moving) when $v(t) = 0$

Velocity is **increasing** when $a(t) > 0$

Velocity is **decreasing** when $a(t) < 0$

Acceleration (positive or negative)

$a(t) = v'(t) = x''(t)$

Notice how we said that speed is always positive. That's because speed is given by the absolute value of velocity. When we take the absolute value of something, it means we'll turn that value positive, which is why speed will always be positive. And speed has no direction. It's simply a positive rate, for example, 40 km/h or 12 inches/second.

Velocity, unlike speed, indicates direction. So if velocity is positive, it means the object is moving forward; but if velocity is negative, it means the object is moving backward.

For these kinds of problems, we also want to remember the difference between "instantaneous" and "average." For instance, instantaneous velocity at $t = a$ is the velocity of the object at the exact moment $t = a$. On the other hand, average velocity on $t = [a, b]$ is the average velocity of the object over the entire interval from $t = a$ to $t = b$.

To find instantaneous velocity, we simply evaluate the velocity function $v(t)$ at $t = a$. But to find average velocity, we'll use

$$\Delta v(a, b) = \frac{x(b) - x(a)}{b - a}$$

Let's work through a full example to see how some of these values get calculated for a particular position function.

Example

From the position function given for a particle, find velocity and acceleration as functions of t. Find the direction in which the particle is moving when $t = 1$, and say whether its velocity and speed are increasing or decreasing at that same point.

$$x(t) = 3t^2 + 8t - 2t^{\frac{5}{2}}$$

To find velocity, take the derivative of the position function, and then to find acceleration, take the derivative of the velocity function.

$$v(t) = x'(t) = 6t + 8 - 5t^{\frac{3}{2}}$$

$$a(t) = v'(t) = x''(t) = 6 - \frac{15}{2}t^{\frac{1}{2}}$$

To find direction when $t = 1$, substitute $t = 1$ into the velocity function.

$$v(1) = 6(1) + 8 - 5(1)^{\frac{3}{2}}$$

$$v(1) = 9$$

Because $v(1)$ is positive, the particle is moving forward.

To determine whether velocity is increasing or decreasing, substitute $t = 1$ into the acceleration function.

$$a(1) = 6 - \frac{15}{2}(1)^{\frac{1}{2}}$$

$$a(1) = -\frac{3}{2}$$

Because acceleration is negative at $t = 1$, velocity is decreasing at that point. And since the velocity is positive and decreasing at $t = 1$, that means that speed is also decreasing at that point.