Topic: Definition of the derivative

Question: Use the definition of the derivative to find the simplified form of the limit.

$$f(x) = x^3 - 2x$$

Answer choices:

A $\quad f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - 2(x + \Delta x)}{\Delta x}$

B $\quad f'(x) = \lim_{\Delta x \to 0} (3x^2 + 3x\Delta x + \Delta x^2 - 2)$

C $\quad f'(x) = \lim_{\Delta x \to 0} (\Delta x^2 - 2)$

D $\quad f'(x) = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - (x^3 - 2x)}{\Delta x}$

Solution: B

After replacing x with $(x+\Delta x)$ in $f(x)$,

$$f(x)=(x+\Delta x)^3-2(x+\Delta x)$$

we'll substitute for $f(x+\Delta x)$.

$$f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^3-2(x+\Delta x)-f(x)}{\Delta x}$$

Then plug $f(x)$ into the definition.

$$f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^3-2(x+\Delta x)-(x^3-2x)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{x^3+x^2\Delta x+2x^2\Delta x+2x\Delta x^2+x\Delta x^2+\Delta x^3-2x-2\Delta x-x^3+2x}{\Delta x}$$

Collect like terms,

$$f'(x)=\lim_{\Delta x\to 0}\frac{x^2\Delta x+2x^2\Delta x+2x\Delta x^2+x\Delta x^2+\Delta x^3-2x-2\Delta x+2x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{x^2\Delta x+2x^2\Delta x+2x\Delta x^2+x\Delta x^2+\Delta x^3-2\Delta x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{3x^2\Delta x+3x\Delta x^2+\Delta x^3-2\Delta x}{\Delta x}$$

then factor Δx out of the numerator and cancel out that common factor from the numerator and denominator.

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta x(3x^2+3x\Delta x+\Delta x^2-2)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} (3x^2 + 3x\Delta x + \Delta x^2 - 2)$$

Topic: Definition of the derivative

Question: Use the definition of the derivative to find the derivative of the function.

$$f(x) = x^2$$

Answer choices:

A $f'(x) = 0$

B $f'(x) = 2$

C $f'(x) = 2x$

D $f'(x) = x^2 + 2x$

Solution: C

After replacing x with $(x+\Delta x)$ in $f(x)$,

$$f(x)=(x+\Delta x)^2$$

we'll substitute for $f(x+\Delta x)$.

$$f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-f(x)}{\Delta x}$$

Then plug $f(x)$ into the definition.

$$f'(x)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-x^2}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{x^2+x\Delta x+x\Delta x+\Delta x^2-x^2}{\Delta x}$$

Collect like terms,

$$f'(x)=\lim_{\Delta x\to 0}\frac{x\Delta x+x\Delta x+\Delta x^2}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{2x\Delta x+\Delta x^2}{\Delta x}$$

then factor Δx out of the numerator and cancel out that common factor from the numerator and denominator.

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta x(2x+\Delta x)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}(2x+\Delta x)$$

Now we evaluate the limit using substitution, which means we'll substitute $\Delta x = 0$.

$$f'(x) = 2x + 0$$

$$f'(x) = 2x$$

Topic: Definition of the derivative

Question: Use the definition of the derivative to find the derivative of the function.

$$f(x) = 2 - x^2 + x$$

Answer choices:

A $f'(x) = 2$

B $f'(x) = 2x$

C $f'(x) = -2x$

D $f'(x) = -2x + 1$

Solution: D

After replacing x with $(x+\Delta x)$ in $f(x)$,

$$f(x)=2-(x+\Delta x)^2+(x+\Delta x)$$

we'll substitute for $f(x+\Delta x)$.

$$f'(x)=\lim_{\Delta x\to 0}\frac{2-(x+\Delta x)^2+(x+\Delta x)-f(x)}{\Delta x}$$

Then plug $f(x)$ into the definition.

$$f'(x)=\lim_{\Delta x\to 0}\frac{2-(x+\Delta x)^2+(x+\Delta x)-(2-x^2+x)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{2-(x^2+x\Delta x+x\Delta x+\Delta x^2)+x+\Delta x-2+x^2-x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{2-x^2-x\Delta x-x\Delta x-\Delta x^2+x+\Delta x-2+x^2-x}{\Delta x}$$

Collect like terms,

$$f'(x)=\lim_{\Delta x\to 0}\frac{2-x\Delta x-x\Delta x-\Delta x^2+x+\Delta x-2-x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{-x\Delta x-x\Delta x-\Delta x^2+x+\Delta x-x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{-x\Delta x-x\Delta x-\Delta x^2+\Delta x}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{-2x\Delta x-\Delta x^2+\Delta x}{\Delta x}$$

then factor Δx out of the numerator and cancel out that common factor from the numerator and denominator.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta x(-2x - \Delta x + 1)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} (-2x - \Delta x + 1)$$

Now we evaluate the limit using substitution, which means we'll substitute $\Delta x = 0$.

$$f'(x) = -2x - 0 + 1$$

$$f'(x) = -2x + 1$$