Topic: Inverse trigonometric derivatives

Question: Find the derivative of the inverse trig function.

$$f(x) = \tan^{-1}(x^2 - 1)$$

Answer choices:

A $\quad f'(x) = \dfrac{2x}{x^4 - 2x^2 + 2}$

B $\quad f'(x) = \dfrac{1}{x^4 - 2x^2 + 2}$

C $\quad f'(x) = \dfrac{2x}{1 + 4x^2}$

D $\quad f'(x) = \dfrac{1}{1 + 4x^2}$

Solution: A

Apply the formula for the derivative of inverse tangent, with $g(x) = x^2 - 1$, in order to differentiate the function.

$$y' = \frac{g'(x)}{1 + [g(x)]^2}$$

$$y' = \frac{2x}{1 + (x^2 - 1)^2}$$

Simplify the derivative.

$$y' = \frac{2x}{1 + (x^4 - 2x^2 + 1)}$$

$$y' = \frac{2x}{x^4 - 2x^2 + 2}$$

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Question: Find the derivative of the inverse trig function.

$$y = \frac{1}{\cos^{-1} x}$$

Answer choices:

A $\quad y' = \dfrac{1}{1 - x^2}$

B $\quad y' = \dfrac{1}{(\cos^{-1} x)^2 \sqrt{1 - x^2}}$

C $\quad y' = -\sqrt{1 - x^2}$

D $\quad y' = \dfrac{\sqrt{1 - x^2}}{(\cos^{-1} x)^2}$

Solution: B

Rewrite the function.

$$y = \frac{1}{\arccos x}$$

$$y = (\arccos x)^{-1}$$

Use substitution with $u = \arccos x$ and

$$u' = -\frac{1}{\sqrt{1 - x^2}}$$

Then the function is

$$y = u^{-1}$$

The derivative is

$$y' = -u^{-2}u'$$

$$y' = -(\arccos x)^{-2}\left(-\frac{1}{\sqrt{1 - x^2}}\right)$$

$$y' = \frac{1}{(\arccos x)^2}\left(\frac{1}{\sqrt{1 - x^2}}\right)$$

$$y' = \frac{1}{(\cos^{-1} x)^2\sqrt{1 - x^2}}$$

Topic: Inverse trigonometric derivatives

Question: Find the derivative of the inverse trig function.

$$y = x \sin^{-1} \sqrt{x}$$

Answer choices:

A $\quad y' = x \sin^{-1} \sqrt{x} + \dfrac{1}{\sqrt{1-x}}$

B $\quad y' = \sin^{-1} \sqrt{x} + \dfrac{x}{\sqrt{1-x}}$

C $\quad y' = x \sin^{-1} \sqrt{x} + \dfrac{1}{2\sqrt{x(1-x)}}$

D $\quad y' = \dfrac{\sqrt{x}}{2\sqrt{1-x}} + \sin^{-1} \sqrt{x}$

Solution: D

We need to apply product rule, with

$$f(x) = x$$

$$f'(x) = 1$$

and

$$g(x) = \sin^{-1}\sqrt{x}$$

$$g'(x) = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

Take the derivative using product rule.

$$y' = f(x)g'(x) + f'(x)g(x)$$

$$y' = (x)\left[\frac{1}{\sqrt{1-(\sqrt{x})^2}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)\right] + (1)(\sin^{-1}\sqrt{x})$$

$$y' = \frac{1}{2}x\left(\frac{1}{x^{\frac{1}{2}}\sqrt{1-x}}\right) + \sin^{-1}\sqrt{x}$$

$$y' = \frac{x}{2\sqrt{x}\sqrt{1-x}} + \sin^{-1}\sqrt{x}$$

$$y' = \frac{\sqrt{x}\sqrt{x}}{2\sqrt{x}\sqrt{1-x}} + \sin^{-1}\sqrt{x}$$

$$y' = \frac{\sqrt{x}}{2\sqrt{1-x}} + \sin^{-1}\sqrt{x}$$