Power rule

At this point, we understand the idea of the derivative, and we know how to find it using the definition.

While the definition of the derivative can always be used to find the derivative of a function, it's not usually the most efficient way of finding the derivative.

It'll be faster for us to use the derivative rules we're about to learn. In this lesson, we'll look at the first of those derivative rules, which is the power rule.

The power rule

The **power rule** lets us take the derivative of power functions. Power functions are things like x^2, $3x^4$, $6x^5$, etc.

Power rule tells us that, to take the derivative of a function like these ones, we just multiply the exponent by the coefficient, and then subtract 1 from the exponent.

Formally, power rule says that, for any function of the form ax^n, the derivative will be

$$\frac{d}{dx}(ax^n) = (a \cdot n)x^{n-1}$$

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For instance, to find the derivative of $3x^4$, we'll bring down the exponent of 4 to multiply it by the coefficient of 3, and we'll subtract 1 from the exponent of 4. So the derivative would be

$$3(4)x^{4-1}$$

$$12x^3$$

We can also use power rule to find the derivative of polynomials, which are combinations of power functions.

Example

Find the derivative of the function.

$$f(x) = 7x^3 + 2x^2 - 3x$$

We can use power rule to take the derivative of the function one term at a time. We'll apply the power rule to each term.

$$f'(x) = 7(3)x^{3-1} + 2(2)x^{2-1} - 3(1)x^{1-1}$$

$$f'(x) = 21x^{3-1} + 4x^{2-1} - 3x^{1-1}$$

$$f'(x) = 21x^2 + 4x^1 - 3x^0$$

$$f'(x) = 21x^2 + 4x - 3(1)$$

$$f'(x) = 21x^2 + 4x - 3$$

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We want to notice a couple of things about this last example. First, after applying power rule, we ended up with $4x^1$ for the second term of the derivative. It's not necessary to write an exponent when the exponent is 1; it's implied. So $4x^1$ can be written more simply as just $4x$.

Second, we used power rule to take the derivative of the third term, $-3x$. To apply power rule, we had to realize that $-3x$ is equivalent to $-3x^1$, so that we could use the exponent of 1. After applying power rule, like normal, to $-3x^1$, we got $-3x^0$. Anything raised to the 0 power is equal to 1, so x^0 turns into 1.

The takeaway here is that the derivative of any term where the exponent is 1, will be equal to the coefficient. So the derivative of $-3x$ is -3, the derivative of $7x$ is 7, and the derivative of x is 1.

The derivative of a constant

Similarly, power rule tells us that the derivative of any constant will always be 0. In other words, the derivative of -3 is 0, the derivative of 7 is 0, and the derivative of 1 is 0.

As an example, take the constant -7. We can rewrite -7 as $-7x^0$, since x^0 is equivalent to 1, and multiplying 1 doesn't change the value of the constant. If we then use the power rule to take the derivative of the constant $-7x^0$, we get $-7(0)x^{0-1}$. But because we now have 0 multiplying the constant, we get 0 for the value of the derivative.

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Let's do one more example where we apply the power rule to terms with different exponents.

Example

Find the derivative of the function.

$$f(x) = -2x^3 - 3x^2 + 6x - 5$$

We'll take the derivative one term at a time. We already know the derivative of $6x$ will be 6, and that the derivative of the constant -5 will be 0.

$$f'(x) = -2(3)x^{3-1} - 3(2)x^{2-1} + 6 - 0$$

$$f'(x) = -6x^{3-1} - 6x^{2-1} + 6$$

$$f'(x) = -6x^2 - 6x^1 + 6$$

$$f'(x) = -6x^2 - 6x + 6$$

Derivatives of combinations

Now that we've defined the power rule, and the rule for the derivative of a constant, let's summarize the set of basic derivative rules.

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Constant rule	$\frac{d}{dx}(a) = 0$
Constant multiple rule	$\frac{d}{dx}(af(x)) = af'(x)$
Power rule	$\frac{d}{dx}(ax^n) = (a \cdot n)x^{n-1}$
Sum rule	$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
Difference rule	$\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$

We actually see all five of these rules used in the last example, where we differentiated $f(x) = -2x^3 - 3x^2 + 6x - 5$. We use the sum and difference rules to take the derivative of the entire polynomial, one term at a time. We use the constant multiple rule and power rule to differentiate the first three terms, $-2x^3 - 3x^2 + 6x$, and the constant rule to differentiate the last term, -5.

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