Topic: Inflection points and the second derivative test

Question: Find the function's inflection points.

$$f(x) = x^4 - 6x^2 - 3x + 2$$

# **Answer choices:**

A (1,0) and (-1,-6)

B (-1,0) and (1,6)

C (-1,0) and (1,-6)

D (-1, -6) and (0,1)

## Solution: C

Find the second derivative of the function.

$$f(x) = x^4 - 6x^2 - 3x + 2$$

$$f'(x) = 4x^3 - 12x - 3$$

$$f''(x) = 12x^2 - 12$$

Set the second derivative equal to 0 and solve for x.

$$12x^2 - 12 = 0$$

$$12x^2 = 12$$

$$x^2 = 1$$

$$x = \pm 1$$

There are two possible inflection points at x = -1 and x = 1. Investigate x = -1 by testing x = -2 and x = 0 in the second derivative.

$$f''(-2) = 12(-2)^2 - 12$$

$$f''(-2) = 36$$

and

$$f''(0) = 12(0)^2 - 12$$

$$f''(0) = -12$$

Since f''(-2) = 36 > 0, the function is concave up to the left of x = -1, and since f''(0) = -12 < 0, the function is concave down to the right of x = -1.

Because the function changes concavity at x = -1 and f''(x) is continuous, there's an inflection point there. We'll get the y-coordinate of the inflection point by substituting x = -1 into f(x).

$$f(-1) = (-1)^4 - 6(-1)^2 - 3(-1) + 2$$

$$f(-1) = 0$$

The function has an inflection point at (-1,0).

Investigate x = 1 by testing x = 0 and x = 2 into the second derivative.

$$f''(0) = 12(0)^2 - 12$$

$$f''(0) = -12$$

and

$$f''(2) = 12(2)^2 - 12$$

$$f''(2) = 36$$

Since f''(0) = -12 < 0, the function is concave down to the left of x = 1, and since f''(2) = 36 > 0, the function is concave up to the right of x = 1.

Because the function changes concavity at x = 1 and f''(x) is continuous, there's an inflection point there. We'll get the y-coordinate of the inflection point by substituting x = 1 into f(x).

$$f(1) = (1)^4 - 6(1)^2 - 3(1) + 2$$



$$f(1) = -6$$

The function has a second inflection point at (1, -6).



Topic: Inflection points and the second derivative test

**Question**: Use the Second Derivative Test to classify the critical points at x = 0 and x = 2.

$$f''(x) = -6x + 6$$

### **Answer choices:**

- A Relative minimum at x = 0; Relative maximum at x = 2
- B Relative minimum at x = 2; Relative maximum at x = 0
- C Relative minima at x = 0 and x = 2
- D Relative maxima at x = 0 and x = 2

### Solution: A

The second derivative is positive at x = 0,

$$f''(0) = -6(0) + 6 = 6 > 0$$

so the function is concave up at that critical point, which means there's a relative minimum there.

The second derivative is negative at x = 2,

$$f''(2) = -6(2) + 6 = -6 < 0$$

so the function is concave down at that critical point, which means there's a relative maximum there.



Topic: Inflection points and the second derivative test

Question: Use the second derivative test to find the function's extrema?

$$f(x) = x^2 + x + 4$$

### **Answer choices:**

- A The function has a local minimum at x = -1/2.
- B The function has a local maximum at x = -1/2.
- C The function has a local minimum at x = 1/2.
- D The function has a local maximum at x = 1/2.



#### Solution: A

Take the first derivative.

$$f(x) = x^2 + x + 4$$

$$f'(x) = 2x + 1$$

Set the derivative equal to 0 and solve for x.

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

There's one critical point at x = -1/2. Take the second derivative.

$$f'(x) = 2x + 1$$

$$f''(x) = 2$$

Substitute the critical point x = -1/2 into the second derivative.

$$f''\left(-\frac{1}{2}\right) = 2$$

Because the second derivative is positive at the critical point, it means there's a local minimum at x = -1/2.