



Calculus 1 Workbook

Definition of the limit

IDEA OF THE LIMIT

■ 1. The table below shows some values of a function $g(x)$. What does the table show for the value of $\lim_{x \rightarrow 4} g(x)$?

x	$g(x)$
3.9	1.9748
3.99	1.9975
3.999	1.9997
4.001	2.0002
4.01	2.0025
4.1	2.0248

■ 2. How would we express, mathematically, the limit of the function $f(x) = x^2 - x + 2$ as x approaches 3?

■ 3. How would you write the limit of $g(x)$ as x approaches ∞ , using correct mathematical notation?

$$g(x) = \frac{5x^2 - 7}{3x^2 + 8}$$



- 4. Explain what is meant by the equation.

$$\lim_{x \rightarrow -2} (x^3 + 2) = -6$$

- 5. Evaluate the limit.

$$\lim_{x \rightarrow -1} \frac{-x^2 + 3x - 1}{5}$$

- 6. Evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{x^2 - 5}{2}$$

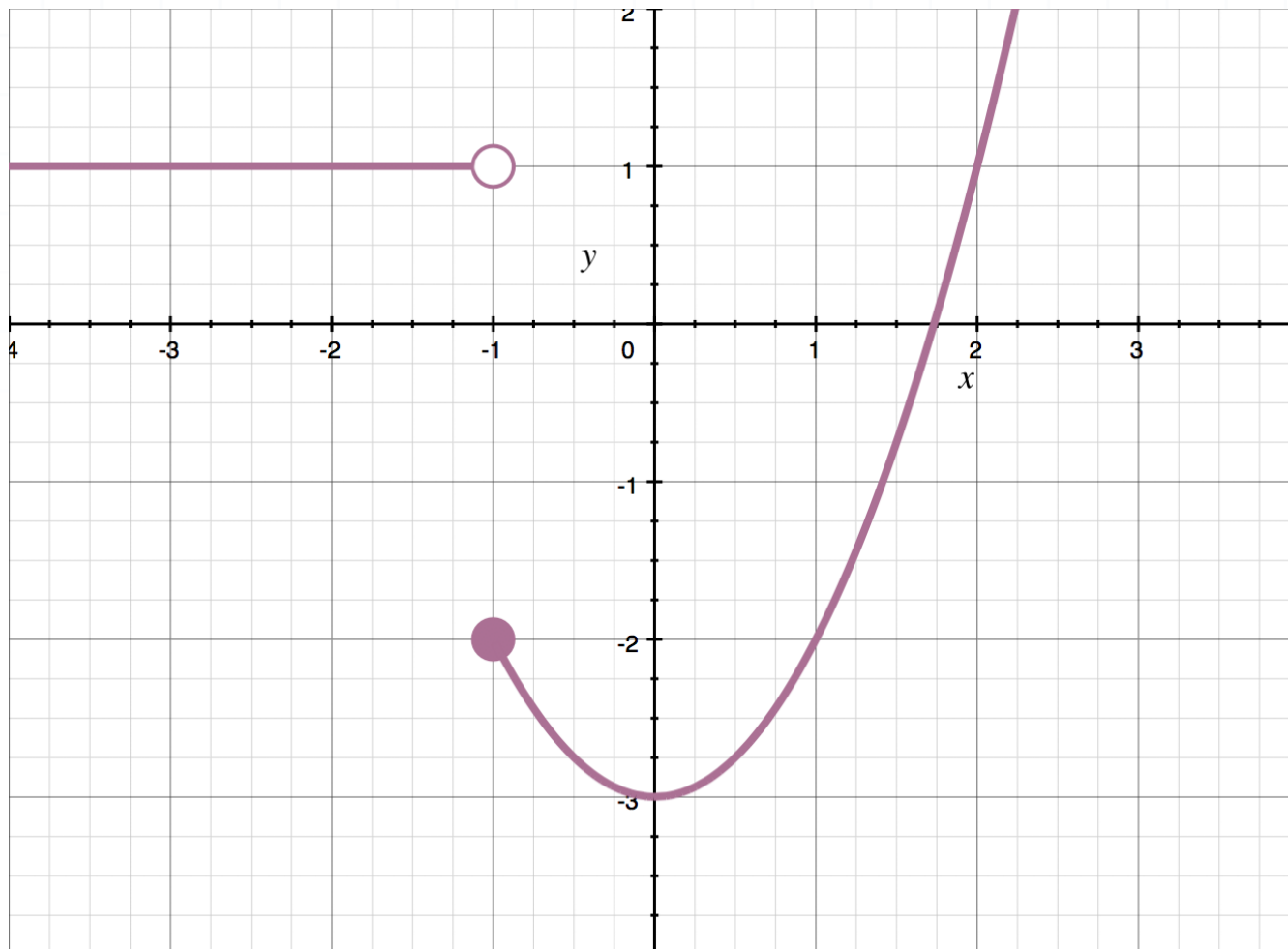


ONE-SIDED LIMITS

- 1. Find the limit.

$$\lim_{x \rightarrow -7^+} x^2 \sqrt{x+7}$$

- 2. What does the graph of $f(x)$ say about the value of $\lim_{x \rightarrow -1^+} f(x)$?



- 3. The table shows values of $k(x)$. What is $\lim_{x \rightarrow -5^-} k(x)$?



x	-5.1	-5.01	-5.0001	-5	-4.999	-4.99	-4.9
k(x)	-392.1	-3,812	-38,012	?	37,988	3,788	368.1

■ 4. What is $\lim_{x \rightarrow -2^-} h(x)$?

$$h(x) = \begin{cases} -2x - 1 & x < -2 \\ x & -2 \leq x < 2 \\ 2x - 3 & x \geq 2 \end{cases}$$

■ 5. What is $\lim_{x \rightarrow 6^+} g(x)$?

$$g(x) = \frac{x^2 + x - 42}{x - 6}$$

■ 6. Find the left- and right-hand limits of the function at $x = 3$.

$$f(x) = \frac{|x - 3|}{x - 3}$$



PROVING THAT THE LIMIT DOES NOT EXIST

- 1. Prove that the limit does not exist.

$$\lim_{x \rightarrow 0} \frac{-2|3x|}{3x}$$

- 2. Prove that the limit does not exist.

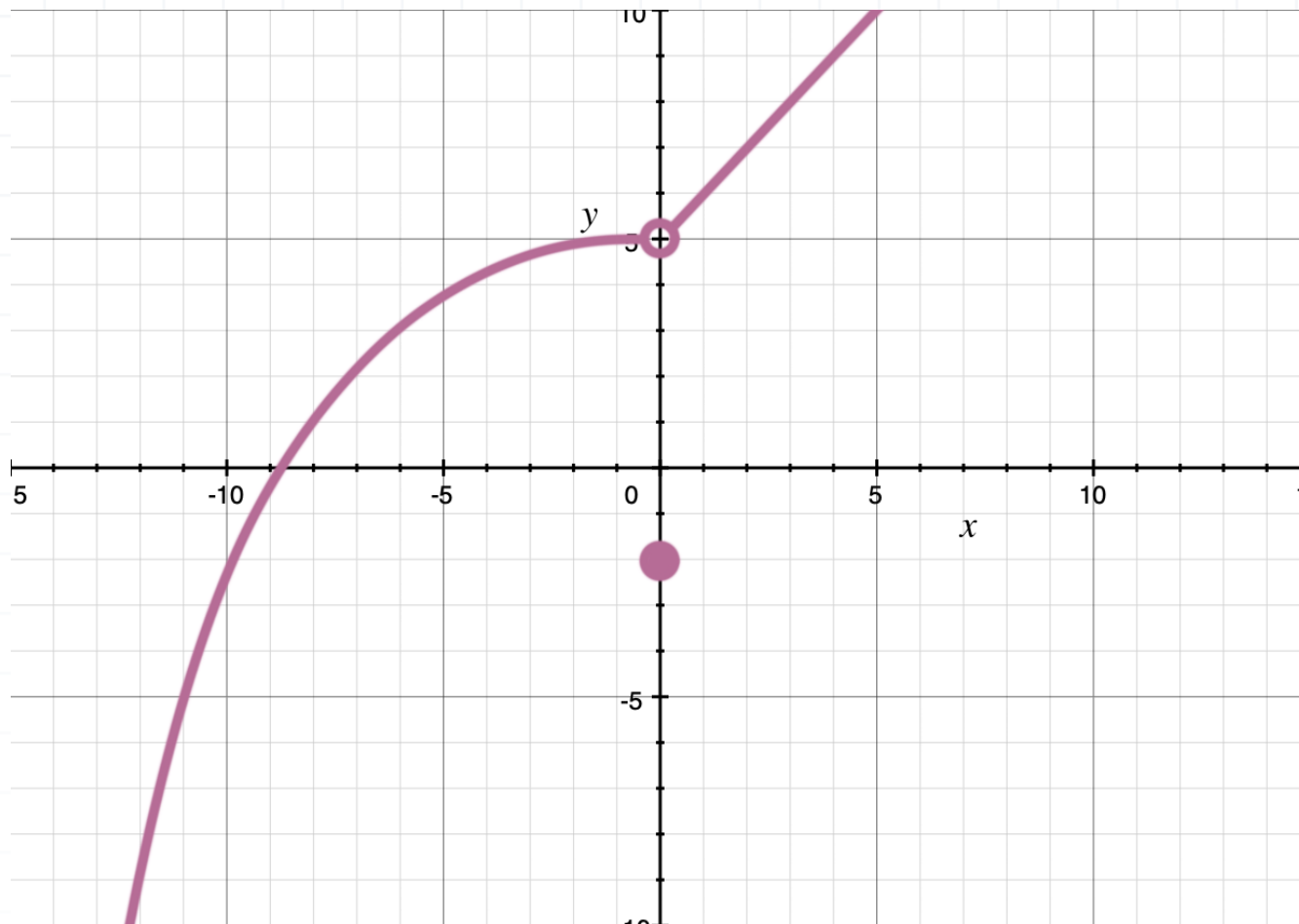
$$\lim_{x \rightarrow -5} \frac{x^2 + 7x + 9}{x^2 - 25}$$

- 3. Prove that $\lim_{x \rightarrow 1} f(x)$ does not exist.

$$f(x) = \begin{cases} -3x + 2 & x < 1 \\ 3x - 2 & x \geq 1 \end{cases}$$

- 4. Use the graph to determine whether or not the limit exists at $x = 0$.





- 5. Suppose we know that $\lim_{x \rightarrow 5} f(x) = 12$. If possible, determine the values of the one-sided limits.

$$\lim_{x \rightarrow 5^-} f(x)$$

$$\lim_{x \rightarrow 5^+} f(x)$$

- 6. Prove that the limit does not exist.

$$\lim_{x \rightarrow -2} \frac{x^2 - 4}{(x + 2)^2}$$



PRECISE DEFINITION OF THE LIMIT

- 1. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \rightarrow 4} (5x - 16) = 4$$

- 2. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \rightarrow -7} (-2x + 15) = 29$$

- 3. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \rightarrow 16} \left(\frac{2}{5}x - \frac{17}{5} \right) = 3$$

- 4. Use the precise definition of the limit to prove the value of the limit.

$$\lim_{x \rightarrow 7} \frac{x^2 - 15x + 56}{x - 7} = -1$$

- 5. Find δ when $f(x) = 2x - 5$, such that if $0 < |x - 1| < \delta$ then $|f(x) + 3| < 0.1$.



- 6. Find a value of δ given $\epsilon = 0.04$.

$$\lim_{x \rightarrow 2} (x - 2)^2 = 0$$



