Topic: Tangent lines

Question: Find the equation of the tangent line to the function at $(1,-2)$.

$$g(x)=3x^2-6x+1$$

Answer choices:

A $y=-2$

B $x+y=-2$

C $y=2$

D $x-y=2$

Solution: A

If we know that the tangent line intersects the curve at $(1, -2)$, we don't need to find $g(1)$, because it's the y-value where the tangent line intersects the curve, so $g(1) = -2$.

Take the derivative of the function.

$$g'(x) = 6x - 6$$

Find the slope of the tangent line at $(1, -2)$ by evaluating the derivative at that point.

$$g'(1) = 6(1) - 6$$

$$g'(1) = 0$$

The slope of the tangent line at $(1, -2)$ is $g'(1) = 0$.

Finally, substitute both $g(1)$ and $g'(1)$ into the tangent line formula, along with $a = 1$, since this is the value at which we're finding the equation of the tangent line.

$$y = g(a) + g'(a)(x - a)$$

$$y = -2 + 0(x - 1)$$

$$y = -2$$

Topic: Tangent lines

Question: Find the equation of the tangent line to the function at $(1,1/2)$.

$$f(x) = \frac{1}{x^2+1}$$

Answer choices:

A $\quad y = -\frac{1}{2}x + 1$

B $\quad y = x - 1$

C $\quad y = -2x + 2$

D $\quad y = \frac{1}{2}x - 1$

Solution: A

Use quotient rule to take the derivative of the function.

$$f'(x) = \frac{(0)(x^2+1) - (1)(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{0 - 2x}{(x^2+1)^2}$$

$$f'(x) = -\frac{2x}{(x^2+1)^2}$$

Find the slope of the tangent line at $(1,1/2)$ by evaluating the derivative at that point.

$$f'(1) = -\frac{2(1)}{(1^2+1)^2}$$

$$f'(1) = -\frac{1}{2}$$

Now we can find the equation of the tangent line by plugging the slope $f'(1) = 1/2$ and the point $(1,1/2)$ into the formula for the equation of the tangent line.

$$y = f(a) + f'(a)(x - a)$$

$$y = f(1) - \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2} - \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2} - \frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + 1$$

Topic: Tangent lines

Question: Where on the interval $-1 \leq x \leq 1$ does the function have horizontal tangent lines?

$$f(x) = x^3 - x - 3$$

Answer choices:

A At $x = 0$

B At $x = \pm \frac{\sqrt{3}}{3}$

C At $x = \pm \sqrt{3}$

D At $x = \pm 3$

Solution: B

Take the derivative of the function.

$$f'(x) = 3x^2 - 1$$

Horizontal tangent lines exist when $f'(x) = 0$, so we'll set the derivative equal to 0.

$$3x^2 - 1 = 0$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{1}}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \pm\frac{\sqrt{3}}{3}$$

On the interval $-1 \leq x \leq 1$, the function has two horizontal tangent lines, located at

$$x = \pm\frac{\sqrt{3}}{3}$$