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The Mean-Variance-Skewness-Kurtosis Problem (Draft)

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Note type project
Tags #NAG #finance #optimization

Abstract

Introduction

Problem

Mean-Varaince-Skewness-Kurtosis (MVSK) optimization problem comes from portfolio optimization in finance. It is an extension of the better known Markowitz or Mean-Variance optimization problem. The task is to diversify a selection of stocks, thereby protecting against the possibility that a significant portion of the portfolio suffers losses at the same time. This extension promises to also consider rare events, i.e. fat tails.

Other peoples work

Our contribution

Paper structure

Preliminaries

In this section we revisit well know results of pop with the added modificaiton of cardinality constraints. The results we consider here will be used in Section ?? to bound the optimization problems we model in the following Section ??.

Polynomial optimization

Utility - References > ^LV2018
We begin with the tools of polynomial optimization and show how the cardinality constraints reduce complexity in contradistinction to the case of mixed i-integer programming [CITATIONS].

For polynomials $f, g_1, g_2, \dots, g_m, h_1, \dots, h_k \in \mathbb{R}[\mathbf{x}]$ in n variables with the cardinality constraint that at most $\ell < n$ variables can be nonzero we define the following polynomial optimization problem:

$$\begin{aligned} f_* := \min \;& f(\mathbf{x}) \\ s.t. \;& \mathbf{x} \in \mathbb{R}^n \\ & g_i(\mathbf{x}) \geq 0 \;\forall\, i \in [m] \\ & h_j(\mathbf{x}) = 0 \;\forall\, j \in [k] \\ & \mathbf{x}^\alpha = 0 \;\forall\, \alpha \in \mathbb{N}^n \; s.t. \; |\mathrm{supp}(\alpha)| > \ell \end{aligned} \tag{POP}$$

One can consider the cardinality constraints, $\mathbf{x}^\alpha = 0 \, \forall \, \alpha \in \mathbb{N}^n \; s.t. \; |\mathrm{supp}(\alpha)| > \ell$, as ideal constraints. Doing this we can reformulate (POP) into an optimization problems over measures. Once this is done we can isolate the cardinality constraints again:

$$\begin{aligned} \inf \;& \int f(x) d\mu \\ s.t. \;& \mu \in \mathcal{M}_+(K), \\ & \int d\mu = 1, \\ & \int \mathbf{x}^\alpha d\mu = 0 \, \forall \, \alpha \in \mathbb{N}^n \; s.t. \; |\mathrm{supp}(\alpha)| > \ell \end{aligned} \tag{MEA}$$

Where, $K := \{x \in \mathbb{R}^n \mid g_i(x) \geq 0 \, , \; h_j(x) = 0\}$ and $\mathcal{M}_+(K)$ is the set of all positive measures supported on K.
Proof: Result Polynomial Optimization is Equivalent to Optimization over Probability Measures

We can further reformulate (MEA) into an optimization problem over moments:

$$\begin{aligned} \inf \;& L(f) \\ s.t. \;& L : \mathbb{R}[\mathbf{x}] \to \mathbb{R} \text{ linear} \\ & L(1) = 1 \\ & M(g_0 L) := L([\mathbf{x}][\mathbf{x}]^T) \succeq 0 \\ & M(g_j L) := L(g_j(\mathbf{x})[\mathbf{x}][\mathbf{x}]^T) \succeq 0 \, \forall \, j \in [m] \\ & M(h_k L) := L(h_k[\mathbf{x}][\mathbf{x}]^T) = 0 \, \forall \, k \in [d] \\ & L(\mathbf{x}^\alpha) = 0 \, \forall \, \alpha \in \mathbb{N}^n \; s.t. \; |\mathrm{supp}(\alpha)| > \ell \end{aligned} \tag{MOM}$$

We use the standard proof Result Optimization over Probability Measures is equivalent to Optimization over Moments and then single out the cardinality constraints afterwards.

Form this formulation we can use the cardinality constraints to reduce the space of possible moments thereby reducing the problems size. The optimization problem (MOM) is equivalent to (MOMcr)

$$\begin{aligned} \inf \;& L(f) \\ s.t. \;& L : \mathbb{R}\widetilde{[\mathbf{x}]} \to \mathbb{R} \text{ linear} \\ & L(1) = 1 \\ & \widetilde{M}(g_0 L) := L(\widetilde{[\mathbf{x}]}\widetilde{[\mathbf{x}]}^T) \succeq 0 \\ & \widetilde{M}(g_i L) := L(g_i(\mathbf{x})\widetilde{[\mathbf{x}]}\widetilde{[\mathbf{x}]}^T) \succeq 0 \, \forall \, j \in [m] \\ & \widetilde{M}(h_j L) := L(h_k\widetilde{[\mathbf{x}]}\widetilde{[\mathbf{x}]}^T) = 0 \, \forall \, j \in [k] \\ & \color{red}{L(\mathbf{x}^\alpha \mathbf{x}^\beta) = 0 \, \forall \, \alpha, \beta \in \mathbb{N}^n_{t-1} \; s.t. \; |\mathrm{supp}(\alpha + \beta)| > \ell} \end{aligned} \tag{MOMcr}$$

Where $\widetilde{[\mathbf{x}]}$ is the vector of monomials excluding all monomials with support larger than ℓ

Proof
Let L be a feasible solution to (MOM) then define a feasible solution L' to (MOMcr) as follows:

$$L'(x^\alpha) = \begin{cases} L'(x^\alpha) & x^\alpha \in \widetilde{[\mathbf{x}]} \\ 0 & \text{else} \end{cases}$$

From this definition is clearly follows that $L : \mathbb{R}\widetilde{[\mathbf{x}]} \to \mathbb{R}$ linear, $L(f) = L'(f)$, $L'(1) = 1$ and $L(\mathbf{x}^\alpha \mathbf{x}^\beta) = 0 \, \forall \, \alpha, \beta \in \mathbb{N}^n_{t-1} \; s.t. \; |\mathrm{supp}(\alpha + \beta)| > \ell$. Let $\widehat{[\mathbf{x}]}$ denote all the monomials that are in $[\mathbf{x}]$ but not in $\widetilde{[\mathbf{x}]}$. For any polynomial $p(x)$ it follows that

$$L(p(x)[\mathbf{x}][\mathbf{x}]^T) = L\Big(p(x) \begin{pmatrix} \widehat{[\mathbf{x}]}\widehat{[\mathbf{x}]}^T & \widetilde{[\mathbf{x}]}\widehat{[\mathbf{x}]}^T \\ \widehat{[\mathbf{x}]}\widetilde{[\mathbf{x}]} & \widetilde{[\mathbf{x}]}\widetilde{[\mathbf{x}]}^T \end{pmatrix}\Big) \text{ (upto permutation)}$$

Since L' is not supported on $\widehat{[\mathbf{x}]}$ it follows that:

$$L(\widetilde{[\mathbf{x}]}\widetilde{[\mathbf{x}]}^T) \succeq 0$$
$$L(g_i(\mathbf{x})\widetilde{[\mathbf{x}]}\widetilde{[\mathbf{x}]}^T) \succeq 0 \,\forall\, j \in [m]$$
$$L(h_k\widetilde{[\mathbf{x}]}\widetilde{[\mathbf{x}]}^T) = 0 \,\forall\, j \in [k]$$

Conversely, start with a L' that satisfies (MOMcr) and define a linear $L : \mathbb{R}[\mathbf{x}] \to \mathbb{R}$ follows:

$$L(x^\alpha) = \begin{cases} L'(x^\alpha) & x^\alpha \in \widetilde{[\mathbf{x}]} \\ 0 & \text{else} \end{cases}$$

Clearly we have that $L : \mathbb{R}[\mathbf{x}] \to \mathbb{R}$ linear, $L(f) = L'(f)$, $L(1) = 1$ and $L(\mathbf{x}^\alpha) = 0 \,\forall\, \alpha \in \mathbb{N}^n \; s.t. \; |\text{supp}(\alpha)| > \ell$. Let $\widehat{[\mathbf{x}]}$

□

Claim

For $x \geq 0$ and L defined via $\mu \in \mathcal{M}_+(K)$ we have

$$L(\mathbf{x}^\alpha \mathbf{x}^\beta) = 0 \,\forall\, \alpha, \beta \in \mathbb{N}^n_{t-1} \; s.t. \; |\text{supp}(\alpha + \beta)| > \ell$$
$$\iff$$
$$L(\mathbf{x}^\alpha)L(\mathbf{x}^\beta) = 0 \,\forall\, \alpha, \beta \in \mathbb{N}^n_{t-1} \; s.t. \; |\text{supp}(\alpha + \beta)| > \ell$$

Proof (Incomplete)
Since $x \geq 0$ we have that

$$L(\mathbf{x}^\alpha \mathbf{x}^\beta) = \int \mathbf{x}^\alpha \mathbf{x}^\beta d\mu \leq \int \mathbf{x}^\alpha d\mu \int \mathbf{x}^\beta d\mu = L(\mathbf{x}^\alpha)L(\mathbf{x}^\beta)$$

For the other direction consider again

$$L(\mathbf{x}^\alpha \mathbf{x}^\beta) = \int \mathbf{x}^{\alpha+\beta} d\mu = 0.$$

Since μ is a positive measure and $x \geq 0$ it must follow that $\mathbf{x}^\alpha \mathbf{x}^\beta$ must be zero on K. This in turn means that either $\mathbf{x}^\alpha$ or $\mathbf{x}^\beta$ is zero on K. This then in turn implies that $L(\mathbf{x}^\alpha) = \int \mathbf{x}^\alpha d\mu = 0$ or $L(\mathbf{x}^\alpha) = \mathbf{x}^\beta d\mu = 0$

□

Lattice based grid search

Signomial optimization

Modeling

The Data

We are given the returns $R \in \mathbb{R}^{N \times M}$ of N risky assets where M is the number of data points. The data is daily measurements of a stock price over the span of several years. We will be using ≈ 100 days of data and ≈ 20.

The Variables

Portfolio weights can fall into any of the following domains:
- **Standard Simplex** in this setting there is no shorting nor leveraged positions: $w \in \Delta^N$, where $\Delta^N := \{x \in [0,1]^N \;:\; \sum_{i=1}^N x_i = 1\}$.
- **The bounded box** In this setting the investor has shorted and leveraged positions in the market. Obviously

there is a bound on how leveraged a position can be, we denote this bound by b: $w \in \mathbb{B}^N$, where $\mathbb{B}^N := \{x \in [-b, b]^N\}$.
- **"Quadratic domain"** $\backslash$todo : look into what Sh. sent me.

(Centralized) Moments:

We define the k^{th}-moment with weights w to be:

$$\phi_k(r, w) := \mathbb{E}[(w^T r)^k] = w^T \Phi^{(k)} (\underbrace{w \otimes w \otimes \cdots \otimes w}_{(k-1)-many})$$

where $\Phi^{(k)} := \mathbb{E}[r(\underbrace{r \otimes r \otimes \cdots \otimes r}_{(k-1)-many})^T)$. In order to centralize the moments around the mean we use $\bar{r} := r - \Phi^{(1)}$.
This then give the first 4 moments:

1. **Mean:** $\mathbb{E}[w^T r] = w^T \Phi^{(1)}$, $\Phi^{(1)}$ expected returns
2. **Variance:** $\mathbb{E}[w^T \bar{r}\bar{r}^T w] = w^T \Phi^{(2)} w$, where $\Phi^{(2)}$ is the covariance matrix
3. **Skewness:** $\mathbb{E}[w^T \bar{r}\bar{r}^T w r^T w] = \mathbb{E}[w^T \bar{r}(\bar{r} \otimes \bar{r})^T (w \otimes w)] = w^T \Phi^{(3)} (w \otimes w)$, where $\Phi^{(3)}$ is the "Skewness matrix".
4. **Kurtossis:** $\mathbb{E}[w^T \bar{r}(\bar{r} \otimes \bar{r} \otimes \bar{r})^T (w \otimes w \otimes w)] = w^T \Phi^{(4)} (w \otimes w \otimes w)$, where $\Phi^{(3)}$ is the "Kurtosis matrix".

Cardinality Constraints

Practically speaking it would be better have fewer rather more stocks in ones portfolio. The reasons being that it reduces complexity and transaction costs. One way to do this is to say that we are only interested in portfolios that consist of $k < N$ stocks where N, recall is the number of candidate stocks to choose from. We hence, impose the constraint that at most $k < N$ for the weights we assign may be nonzero. This can be phrased as a polynomial constraint:

$$\mathbf{x}^\alpha = 0 \;\; \forall \;\; \alpha \in \mathbb{N}^n \; s.t. \; |\text{supp}(\alpha)| = k + 1$$

This constraint is saying is that among $k + 1$ distinct variables at least one variable is zero. This will have consequences in the relaxations [Support Constraint on Variables in Polynomial Optimization]

The Multi Objective Optimization Problem

The goal is to "solve" the following **multi-objective polynomial optimization** problem:

$$\begin{aligned} \max \;\; & f_1(w) := w^T \mu \\ \min \;\; & f_2(w) := w^T \Phi^{(2)} w \\ \max \;\; & f_3(w) := w^T \Phi^{(3)} (w \otimes w) \\ \min \;\; & f_4(w) := w^T \Phi^{(4)} (w \otimes w \otimes w) \\ s.t. \;\; & w \in \Delta^N \end{aligned} \tag{0}$$

The idea is that we take a 100 days worth of data for 50 stocks we "solve" the above problem and in doing so we find a "optimal" portfolio w^* in terms of the stock weights.

The Objective Function

Formulation **(0)** however, does not give a clear notion of what a solution would be. As such we must construct a suitable objective function that we can optimize over whilst modeling the problem accurately. We consider three choices of objective function.

Linear Objective Function

One way to balance the different objectives would be to extend the idea of the mean-variance version and simply consider a linear combination of the four objectives:

$$\begin{aligned} &\max\ \lambda_1 f_1(w) - \lambda_2 f_2(w) + \lambda_3 f_3(w) - \lambda_4 f_4(w) \\ &s.t.\ \ w \in \Delta^N \end{aligned} \tag{?}$$

Where $\lambda \in \mathbb{R}^4_+$ or (Δ^4) depending on modeling choices. The problem with this objective function is the difference in scale between the different moments. $TODO$ show. The advantage of this formulation is its conceptual simplicity.

Polynomial Objective Function

We can extend the previous idea to arbitrary polynomial functions in $f_1(w), \ldots, f_4(w)$ as follows:

$$\begin{aligned} &\max\ P(f_1(w), f_2(w), f_3(w), f_4(w)) \\ &s.t.\ \ w \in \Delta^N \end{aligned} \tag{?}$$

Where $P \in \mathbb{R}[x_1, x_2, x_3, x_4]$ is a polynomial. For example:

$$P(f_1(w), f_2(w), f_3(w), f_4(w)) := f_1(w) + -10 f_2(w) + 40 f_4(w) - 39 f_3(w) f_4(w)^2.$$

However, great care must be taken to ensure that the chosen polynomial will have a meaning relating to the underlying problem. The Advantage of this formulation is that we can lower bound it using tools from POP that we will discuss later.

Minkowski Distance Objective Function

An idea that is prevalent in portfolio optimization is to choose as objective the distance from an idealized solution \citations. To begin consider the individual optimization problems whose solutions are sometimes called the *aspirational* solution \cite{}.

$$\begin{aligned} \psi_{\pm}^{(1)} = \max \setminus \min\ f_1(w) &:= w^T \Phi^{(1)} \\ s.t.\ \ w &\in \Delta^N \end{aligned} \tag{1}$$

$$\begin{aligned} \psi_{\pm}^{(2)} = \max \setminus \min\ f_2(w) &:= w^T \Phi^{(2)} w \\ s.t.\ \ w &\in \Delta^N \end{aligned} \tag{2}$$

$$\begin{aligned} \psi_{\pm}^{(3)} = \max \setminus \min\ f_3(w) &:= w^T \Phi^{(3)} (w \otimes w) \\ s.t.\ \ w &\in \Delta^N \end{aligned} \tag{3}$$

$$\begin{aligned} \psi_{\pm}^{(4)} = \max \setminus \min\ f_4(w) &:= w^T \Phi^{(4)} (w \otimes w \otimes w) \\ s.t.\ \ w &\in \Delta^N \end{aligned} \tag{4}$$

For a fixed choice (made by the investor) of $\lambda \in \Delta^4$ define the following optimization problem:

$$\begin{aligned} \min\ \not{f}(w) := &\left(\frac{\psi_+^{(1)} - w^T \Phi^{(1)}}{\psi_+^{(1)}}\right)^{\lambda_1} + \left(\frac{w^T \Phi^{(2)} w - \psi_-^{(2)}}{\psi_-^{(2)}}\right)^{\lambda_2} \\ &+ \left(\frac{\psi_+^{(3)} - w^T \Phi^{(3)} (w \otimes w)}{\psi_+^{(3)}}\right)^{\lambda_3} + \left(\frac{w^T \Phi^{(4)} (w \otimes w) - \psi_-^{(4)}}{\psi_-^{(4)}}\right)^{\lambda_4} \\ s.t.\ \ w \in \Delta^N & \end{aligned} \tag{5}$$

Observations:

- (1,2,3,4) is an instance of polynomial optimization over the [simplex].
- (2,3,4) Contain STABLE SET $TODO$ show
- (2,3) add non-convexity $TODO$ show.
- (1) is just an LP more over the solution is the stock with highest expected returns

- (2) is a convex optimization problem, hence we can apply gradient decent with a proof of convergence to the unique global optima (that we know exists) *TODO* show
- The objective function $\not f(d)$ is not polynomial nor rational *TODO* show
- We can attack (3,4) with Polynomial optimization techniques
- Problem (5) can in fact be reformulated as a signomial optimization problem.

Recasting the objective as a signomial

This may even be a posynomial!!!!

The problems in **(5)** can be recast into the form of a *signomial* which is the following form:

$$f(\mathbf{x}) = \sum_{j=1}^{\ell} c_j e^{\alpha^{(j)T} x}$$

we do this as follows: Define

- $z_1 := \log(1 - \frac{1}{\psi_+^{(1)}} w^T \Phi^{(1)})$
- $z_2 := \log(\frac{1}{\psi_-^{(2)}} w^T \Phi^{(2)} w - 1)$
- $z_3 := \log(1 - \frac{1}{\psi_+^{(3)}} w^T \Phi^{(3)} (w \otimes w))$
- $z_4 := \log(\frac{1}{\psi_-^{(4)}} w^T \Phi^{(4)} (w \otimes w \otimes w) - 1)$

Since $w \in \Delta^N$ we have that

$$\begin{aligned} z_3 \in &\ [\log(1 - \max_{w \in \Delta^N} \frac{1}{\psi_+^{(3)}} w^T \Phi^{(3)} (w \otimes w)), \log(1 - \min_{w \in \Delta^N} \frac{1}{\psi_+^{(3)}} w^T \Phi^{(3)} (w \otimes w))] \\ = &\ [\log(0), \log(1 - \frac{\psi_-^{(3)}}{\psi_+^{(3)}}] \end{aligned}$$

Then equation **(5)** becomes

$$\begin{aligned} \min \ \ & \not f(z) := \sum_{j=1}^{4} e^{\lambda_i z_i} \\ s.t. \ \ & z_1 \in (-\infty, \log(1 - \frac{\psi_-^{(1)}}{\psi_+^{(1)}})] \\ & z_2 \in (-\infty, \log(\frac{\psi_-^{(2)}}{\psi_+^{(2)}} - 1)] \\ & z_3 \in (-\infty, \log(1 - \frac{\psi_-^{(3)}}{\psi_+^{(3)}})] \\ & z_4 \in (-\infty, \log(\frac{\psi_-^{(4)}}{\psi_+^{(4)}} - 1)] \end{aligned} \tag{6}$$

Optimization

In this section we apply the general results of the Preliminaries section to problems (2,3,4,6). In particular we apply the Lasserre hierarchy to to lower bound (2,3,4) and the grid search to upper bound them. We use signomial optimization to approximately solve (6)

Polynomial optimization

Lower bounds for skewness and kurtosis

Upper bounds for skewness and kurtosis

Signomial optimization

[Signomial Optimization]

Numerical results

Without cardinality constraints

Skewness

level	#stocks	primal_status	dual_status	obj_val	comp_time(s)
2	5	FEASIBLE_POINT	FEASIBLE_POINT	-1.020754455691659e-6	0.031
2	6	FEASIBLE_POINT	FEASIBLE_POINT	-2.920655586956662e-6	0.047
2	7	FEASIBLE_POINT	FEASIBLE_POINT	-2.915511951729846e-6	0.125
2	8	FEASIBLE_POINT	FEASIBLE_POINT	-2.907559228471547 3e-6	0.219
2	9	FEASIBLE_POINT	FEASIBLE_POINT	-2.928634529578934 7e-6	0.515
2	10	FEASIBLE_POINT	FEASIBLE_POINT	-2.925969315287471 8e-6	1.079
2	11	FEASIBLE_POINT	FEASIBLE_POINT	-2.922883206666286 6e-6	2.578
2	12	FEASIBLE_POINT	FEASIBLE_POINT	-5.343330270946647 6e-5	4.609
2	13	FEASIBLE_POINT	FEASIBLE_POINT	-5.34352809246628e-5	9.235
2	14	FEASIBLE_POINT	FEASIBLE_POINT	-5.343514597411791e-5	17.531
2	15	FEASIBLE_POINT	FEASIBLE_POINT	-5.343466851953268e-5	33.234
mem.	---	---	---	---	---
3	5	FEASIBLE_POINT	FEASIBLE_POINT	-1.002897385505069e-6	0.782
3	6	FEASIBLE_POINT	FEASIBLE_POINT	-2.928965166961877 4e-6	4.593
3	7	FEASIBLE_POINT	FEASIBLE_POINT	-2.849054542182358e-6	24.485
3	8	FEASIBLE_POINT	FEASIBLE_POINT	-2.929254429232411e-6	142.0
mem.	---	---	---	---	---

Kurtosis (currently under revision)

The "UNKNOWN_RESULT_STATUS" seems to stem from the objective value. That is to say, using 1 instead of $f_4(w)$ makes the program feasible again which is weird.

level	#stocks	primal_status	dual_status	obj_val
2	5	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-5.462410379852476e6
2	6	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-6.003107515609454e6
2	7	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-1.160944950805242 2e7
2	8	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-883351.2287677532
2	9	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-210134.43819556796

level	#stocks	primal_status	dual_status	obj_val
2	10	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-7.149278898484302e6
2	11	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-21849.10743635129
2	12	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-30742.43665861959
2	13	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-1072.1779065118176
2	14	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-728.9859449572551
mem.	---	---	---	---
2	15	UNKNOWN_RESULT_STATUS	UNKNOWN_RESULT_STATUS	-413.9730106071591
3	5	FEASIBLE_POINT	FEASIBLE_POINT	-9.814954929710079e-7
3	6	FEASIBLE_POINT	FEASIBLE_POINT	-1.0110424132951022e-6
3	7	FEASIBLE_POINT	FEASIBLE_POINT	-0.0001658148457561855
3	8	FEASIBLE_POINT	FEASIBLE_POINT	-0.0001658149202996121
mem.	---	---	---	---

With cardinality constraints

Skewness

Kurtosis

Grid search upper bounds

Signomial optimization.

Extensions and related topics

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