UNIVERSIDAD DE GRANADA

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EXCELENCIA MARÍA DE MAEZTU

Orthogonal Polynomials: Applications and Bivariate Multiple Orthogonality

Orthogonal Polynomials in one variable

$$(\mathbb{R}[x], \langle \cdot, \cdot \rangle_\mu)$$

$$\text{OPS: } \{P_0, P_1, \dots, P_n, \dots\}$$

$$\langle P_n, P_k \rangle_\mu = 0 \quad (k = 0, \dots, n-1)$$

Increasing the number of variables

Orthogonal polynomials in two variables

$$(\mathbb{R}[x,y], \langle \cdot, \cdot \rangle_\mu)$$

$$\text{OPS: } \{\underbrace{P_{0,0}}_{\deg=0}, \underbrace{P_{1,0}, P_{1,1}}_{\deg=1}, \dots, \underbrace{P_{n,0}, \dots, P_{n,n}}_{\deg=n}, \dots\}$$

$$\mathbb{P}_n = (P_{n,0}, \dots, P_{n,n})^t \qquad \{\mathbb{P}_0, \mathbb{P}_1, \dots, \mathbb{P}_n, \dots\}$$

$$\langle \mathbb{P}_n, \mathbb{P}_k \rangle_\mu = 0_{(n+1)\times(k+1)} \quad (k = 0, \dots, n-1)$$

Increasing the number of measures

Multiple Orthogonality

$$\vec{n} = (n_1, \dots, n_r) \in \mathbb{N}^r, \quad (\mu_1, \dots, \mu_r) \text{ measures}$$

Type II Multiple Orthogonality

$$P_{\vec{n}} \text{ monic of degree } |\vec{n}|$$

$$\left\langle P_{\vec{n}}, x^k \right\rangle_{\mu_j} = 0, \quad (k = 0, \dots, n_j - 1)$$

Bivariate Multiple Orthogonality

$$\vec{n} = (n_1, \dots, n_r) \in \mathbb{N}^r,$$

$$(\mu_1, \dots, \mu_r) \text{ 2-dimensional measures}$$

First approach

$$\mathbb{P}_{\vec{n}} = \mathbb{X}_n + \sum_{k=0}^{n-1} G_{n,k}\mathbb{X}_k, \text{ satisfying}$$

$$\langle \mathbb{P}_{\vec{n}}, \mathbb{X}_k \rangle_{\mu_j} = 0_{(n+1)\times(k+1)},$$

$$(k = 0, \dots, n_j - 1)$$

Second approach

$$P_{\vec{n}} \text{ monic with } \pi(\exp(P_{\vec{n}})) = |\vec{n}|$$

$$\left\langle P_{\vec{n}}, x^t y^s \right\rangle_{\mu_j} = 0,$$

$$(\pi(t,s) = 0, \dots, n_j - 1)$$

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