On polynomials satisfying a special R_{II} type recurrence formula

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We consider the sequence of polynomials $\{P_n\}_{n\geq 0}$ satisfying the recurrence formula

$$P_{n+1}(x) = (x - c_{n+1})P_n(x) - d_{n+1}(x^2 + 1)P_{n-1}(x), \quad n \geq 1,$$

with $P_0(x) = 1$, $P_1(x) = x - c_1$, where $\{c_n\}_{n\geq 1}$ is a real sequence and $\{d_{n+1}\}_{n\geq 1}$ is a positive chain sequence. The above recurrence formula can be classified as belongs to the class of recurrence formulas known as R_{II} type recurrence formulas. It turns out that the polynomials P_n are characteristic polynomials associated with certain generalized eigenvalue problems involving two tri-diagonal matrices. Even though the zeros of P_n are simple and lie on the real line, with our R_{II} type recurrence formula one can always associate a positive measure on the unit circle. The orthogonality properties satisfied by the polynomials P_n with respect to this measure is also studied. Examples are given to justify the results.