

ASSIGNMENT 2 : ICSE 12,2019

AI21BTECH11016

PROBLEM 3-A :

Solve for x :

$$\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$$

Solution: Applying tan on both sides,

$$\tan \left(\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) \right) = \tan \left(\frac{\pi}{4} \right) \quad (1)$$

$$\frac{\tan \left(\tan^{-1} \left(\frac{x-1}{x-2} \right) \right) + \tan \left(\tan^{-1} \left(\frac{x+1}{x+2} \right) \right)}{1 - \tan \left(\tan^{-1} \left(\frac{x-1}{x-2} \right) \right) \tan \left(\tan^{-1} \left(\frac{x+1}{x+2} \right) \right)} = 1 \quad (2)$$

$$\boxed{\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

$$\Rightarrow \frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2} \frac{x+1}{x+2}} = 1 \quad (3)$$

$$\boxed{\therefore \tan(\tan^{-1} x) = x, \forall x \in R}$$

$$\left(\frac{x^2 + x - 2 + x^2 - x - 2}{x^2 - 4 - x^2 + 1} \right) = 1 \quad (4)$$

$$\left(\frac{2x^2 - 4}{-3} \right) = 1 \quad (5)$$

$$2x^2 - 4 = -3 \quad (6)$$

$$2x^2 = 4 - 3 = 1 \quad (7)$$

$$\Rightarrow \boxed{x = \pm \frac{1}{\sqrt{2}}} \quad (8)$$

\therefore The solution to the given equation is $x = \pm \frac{1}{\sqrt{2}}$