ASSIGNMENT-1

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1 PROBLEM 3.6-B :

1.1 Determine the inverse z-transform of

$$X(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} \qquad |z| < \frac{1}{2} \tag{1.1}$$

and determine whether the Fourier transform exist.

Solution: The Z-transfrom of $x(n)$ is defines as

$$X(z) = \mathcal{Z}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{1.2}$$

Now, inverse Z-transform is defined by

$$x(n) = \mathcal{Z}^{-1}\left[\sum_{n=-\infty}^{\infty} x(n)z^{-n}\right] \tag{1.3}$$

$$X(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} \tag{1.4}$$

Because of the region of convergence,the sequence is a left-sided one and since $X(z)$ at $z = 0$ is finite,the sequence is zero for $n > 0$.Thus,we divide,so as to obtain a series in powers of z as follows:

$$\begin{array}{r|llll} & z & -2z^2 & +4z^3 & .. \\ \hline 1+2z & z & & & \\ & z & +2z^2 & & \\ \hline & & -2z^2 & & \\ & & -2z^2 & -4z^3 & \\ \hline & & & 4z^3 & \\ & & & .. & \end{array}$$

Therefore x(n) will be

$$x(n) = -2^n u[-n-1] \tag{1.5}$$

For fourier transform to exist the following condition should be satisfied,

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty \tag{1.6}$$

$$\Rightarrow \sum_{n=-\infty}^{0} |x(n)| = \sum_{n=-\infty}^{0} |(-2)^n| \tag{1.7}$$

$$= \sum_{n=0}^{\infty} \left|(-2)^{-n}\right| \tag{1.8}$$

$$= \sum_{n=0}^{\infty} \left|\left(-\frac{1}{2}^{n}\right)\right| \tag{1.9}$$

$$= \frac{1}{1 - \left(-\frac{1}{2}\right)} \tag{1.10}$$

$$= \frac{2}{3} < \infty \tag{1.11}$$

Therefore, Fourier Transform exists.