Assignment-2 : Oppenheim 2.10 - c

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1) Determine the output of a linear time-invariant system if the impulsive response $h[n]$ and the input $x[n]$ are as follows:

$$x[n] = u[n] \qquad \text{and} \qquad h[n] = (0.5)\, 2^n u[-n].$$

Solution: The output of linear time-invariant system is given by

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{0.1}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{0.2}$$

$$= \sum_{k=-\infty}^{\infty} u[k]2^{n-k-1}u[-n+k] \tag{0.3}$$

$$= \sum_{k=-\infty}^{\infty} 2^{n-k-1}u[k]u[k-n] \tag{0.4}$$

Now we can define eqn(0.1) as

$$y(n) = \begin{cases} \sum\limits_{k=n}^{\infty} 2^{n-k-1} & n \geq 0 \\ \\ \sum\limits_{k=0}^{\infty} 2^{n-k-1} & n \leq 0 \end{cases}$$

case 1: $n \geq 0$

$$\begin{aligned} y(n) &= 2^{n-1} \sum_{k=n}^{\infty} 2^{-k} \\ &= 2^{n-1} \times \frac{2^{-(n)}}{1-2^{-1}} \\ &= 2^{n-1} \times \left(2 \times 2^{-(n)}\right) \\ y(n) &= 1 \end{aligned} \tag{0.5}$$

case 2: $n \leq 0$

$$\begin{aligned} y(n) &= 2^{n-1} \sum_{k=0}^{\infty} 2^{-k} \\ &= 2^{n-1} \times \frac{2^{0}}{1-2^{-1}} \\ &= 2^{n-1} \times (2) \\ y(n) &= 2^{n} \end{aligned} \tag{0.6}$$

$$\therefore y(n) = \begin{cases} 1 & n \geq 0 \\ 2^n & n \leq 0 \end{cases} \tag{0.7}$$