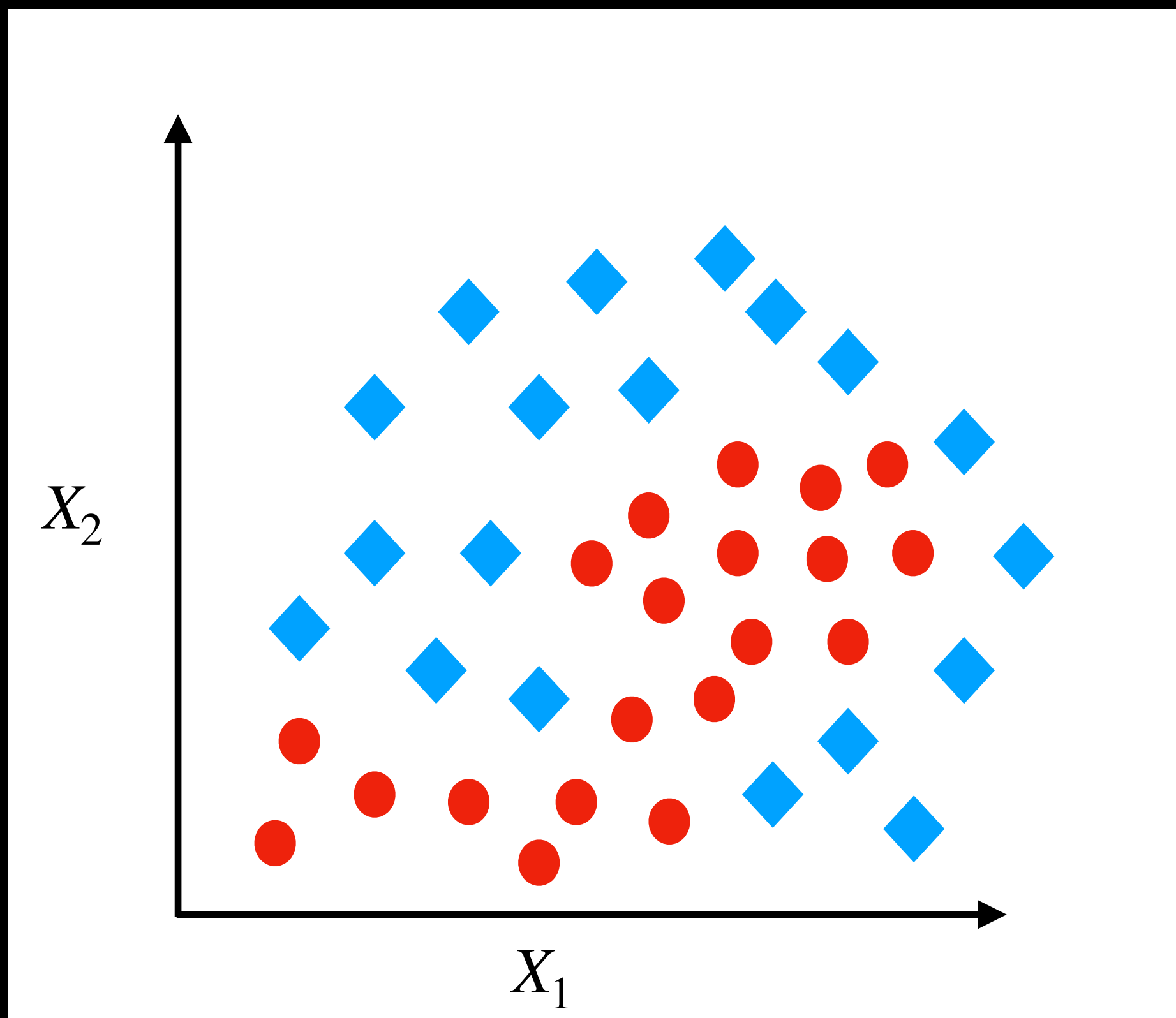
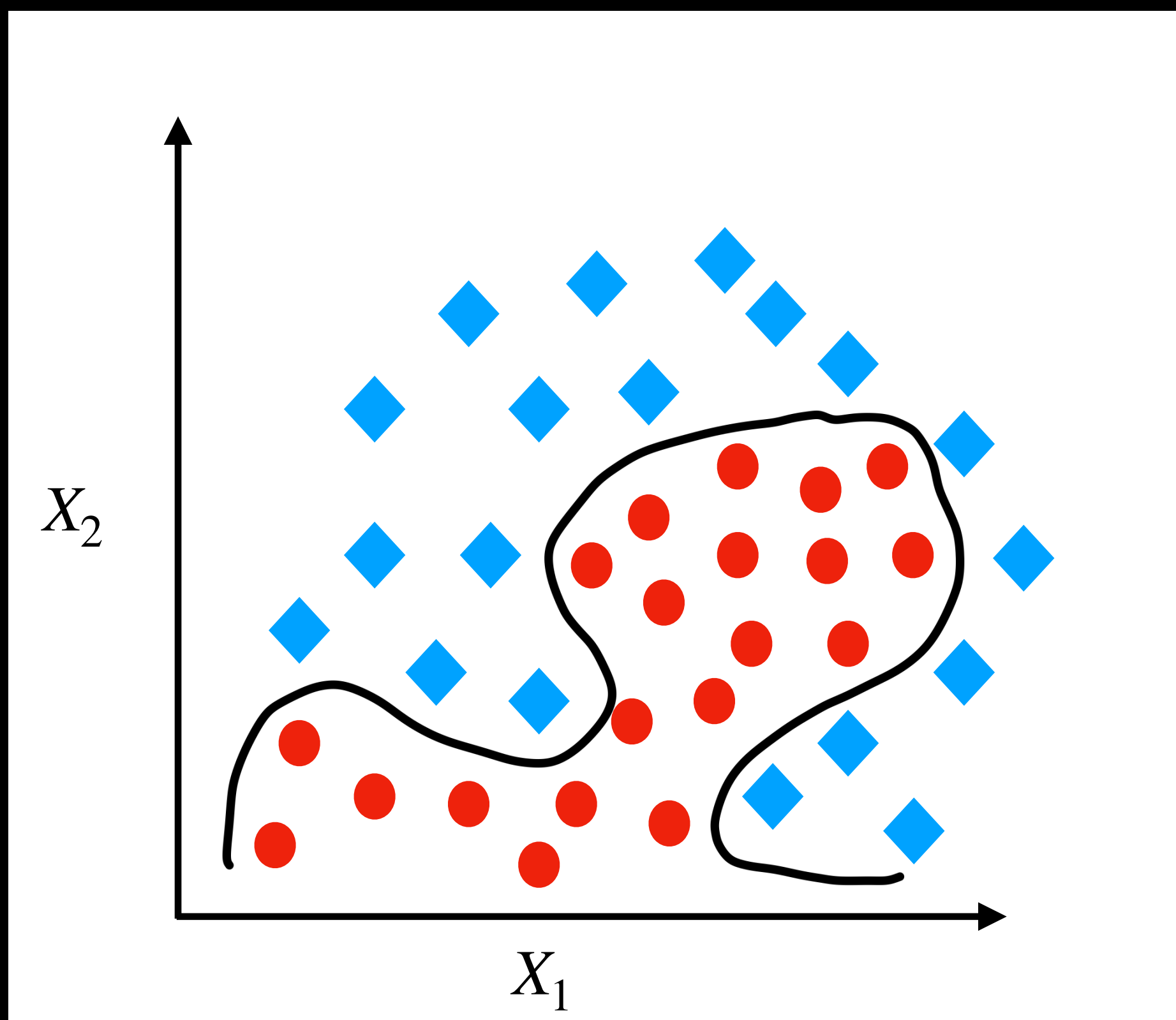
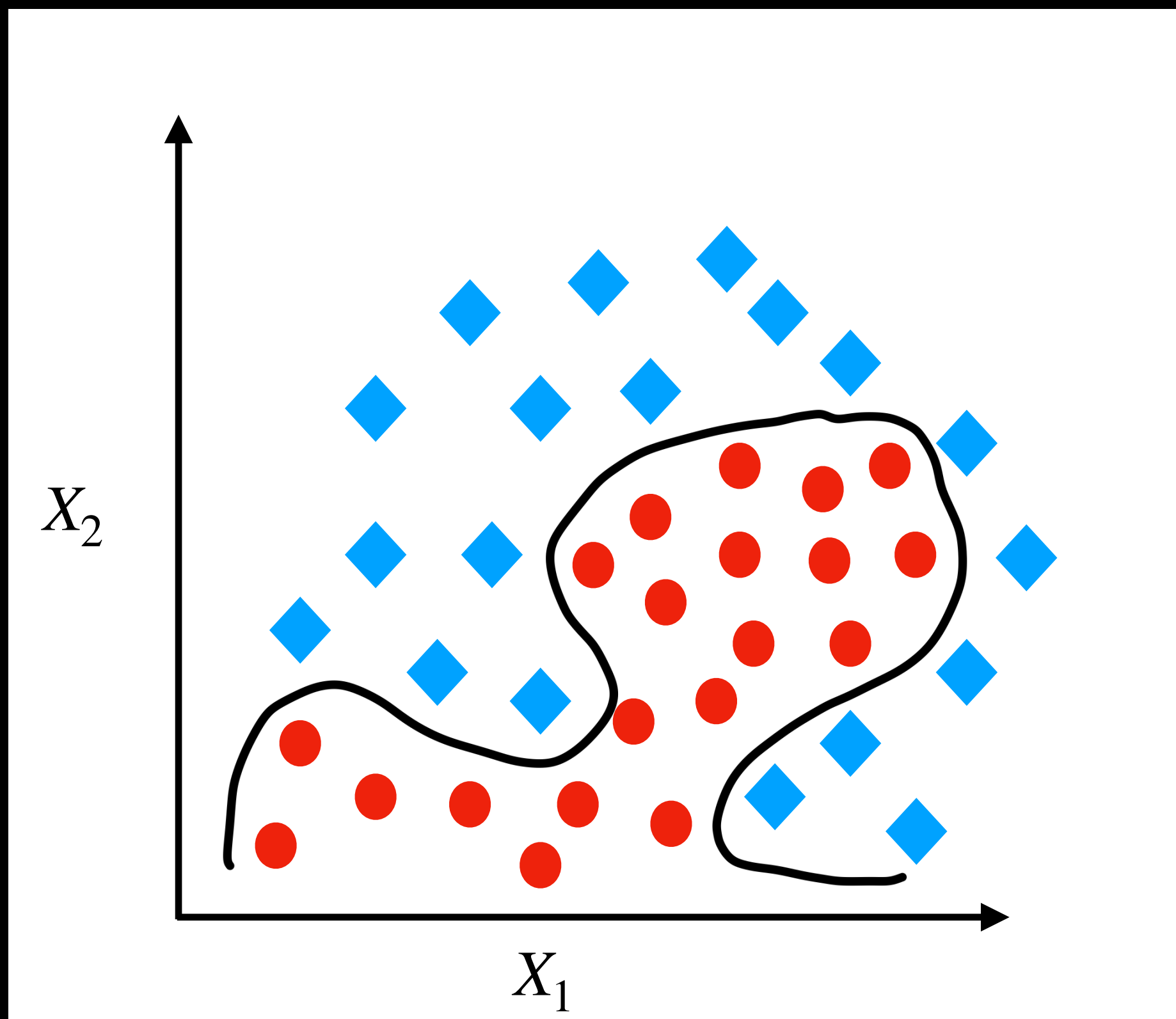


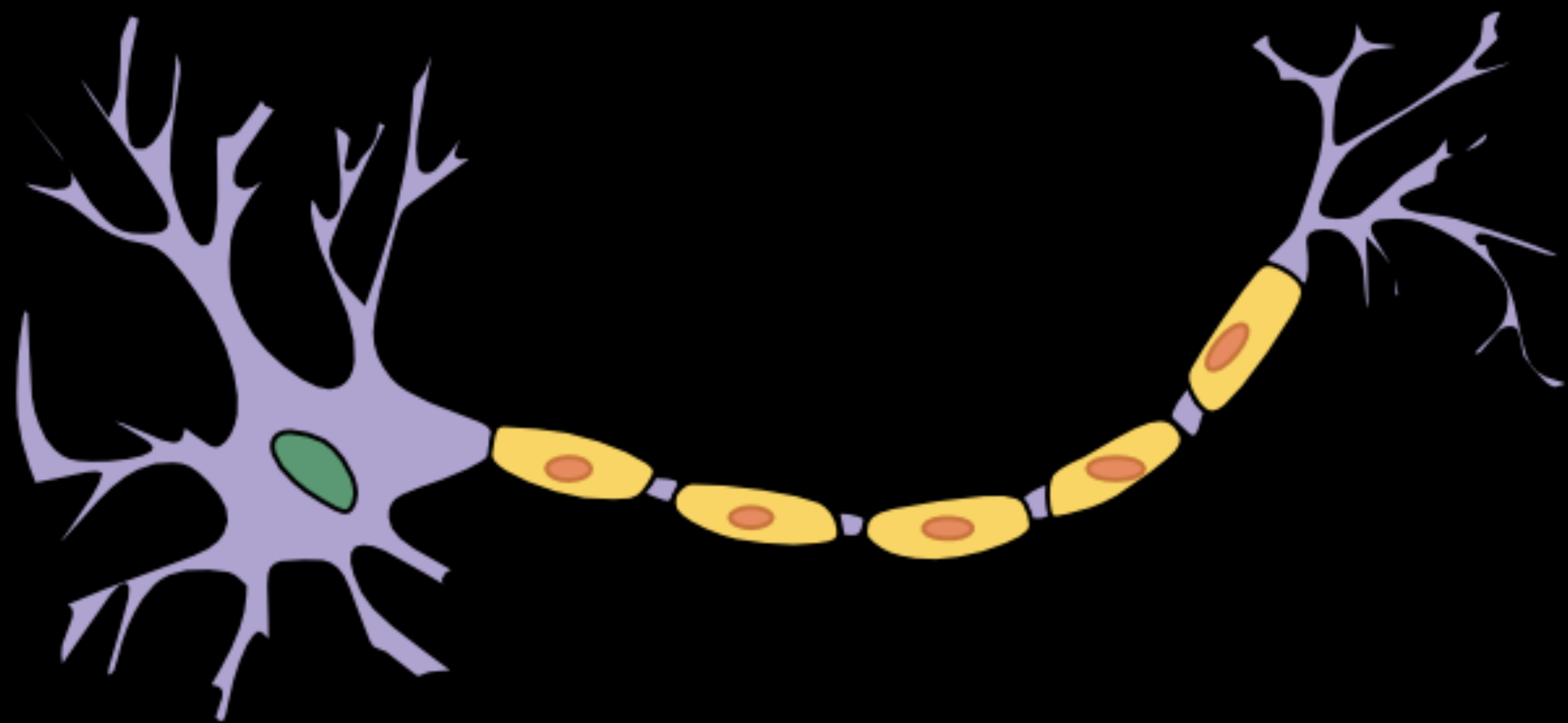
# Neural Networks

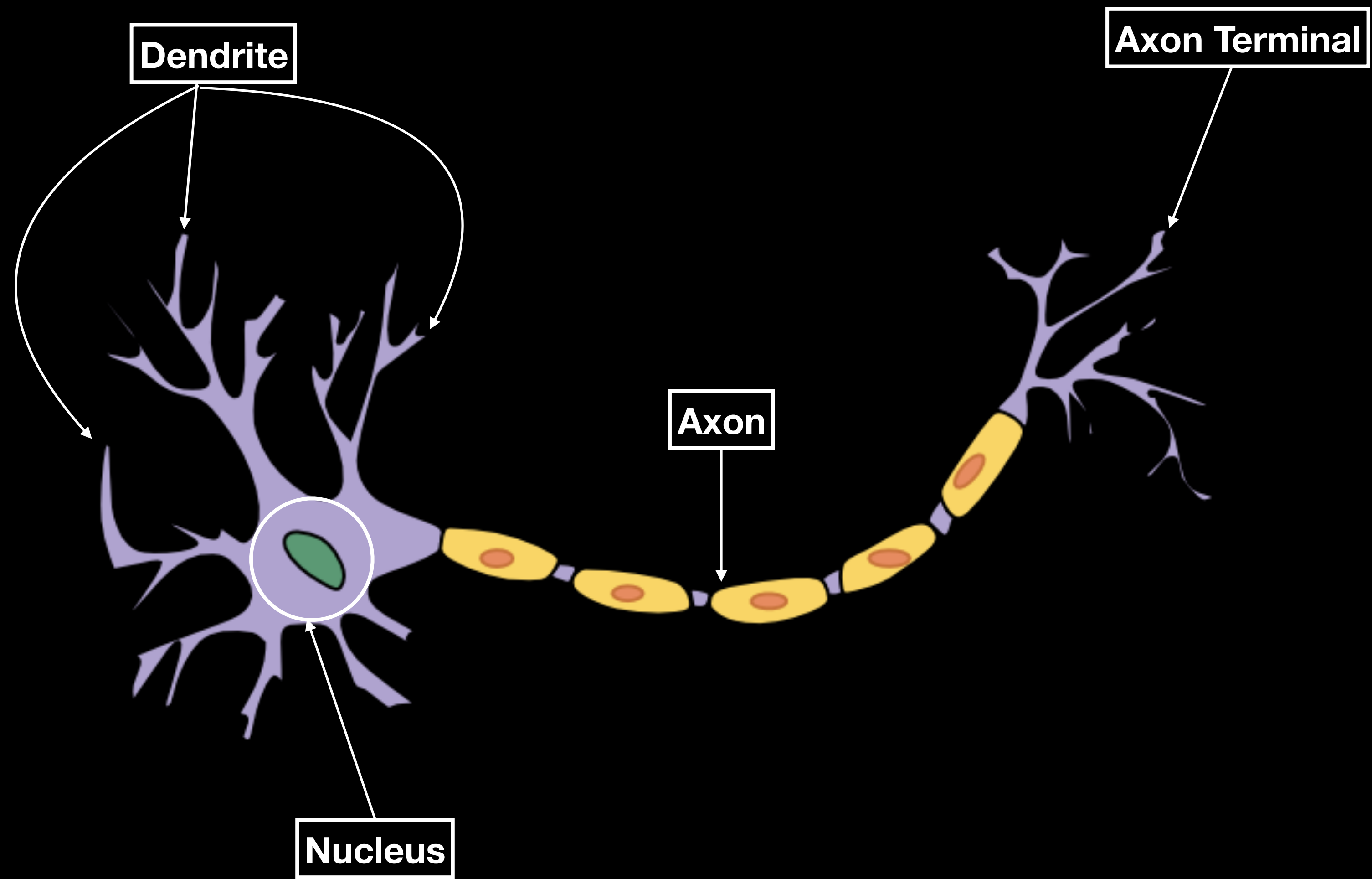


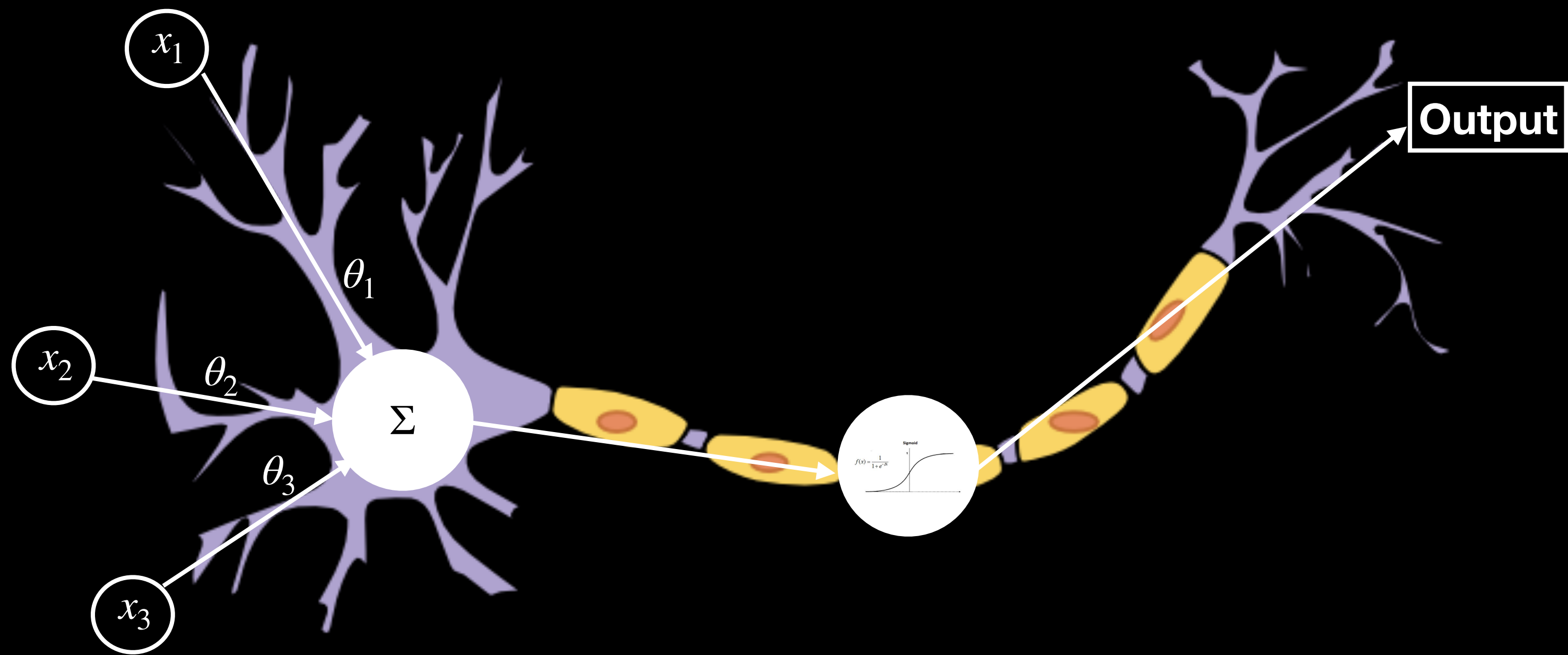




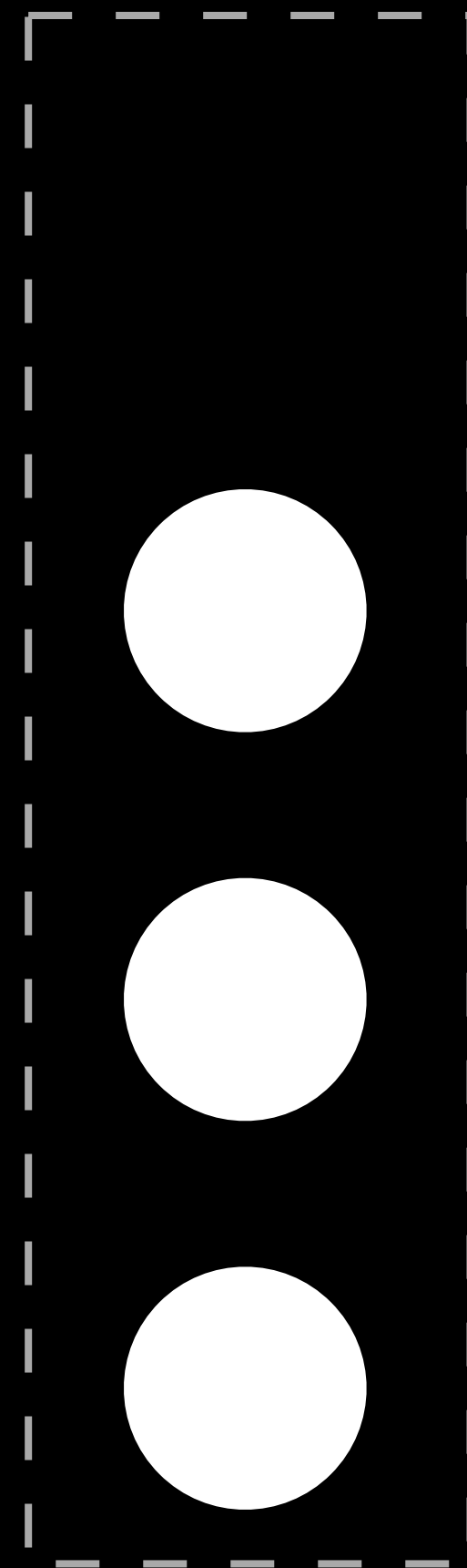
$$y = aX_1 + bX_2 + cX_1X_2 + fX_1^3X_2^4 + \dots$$



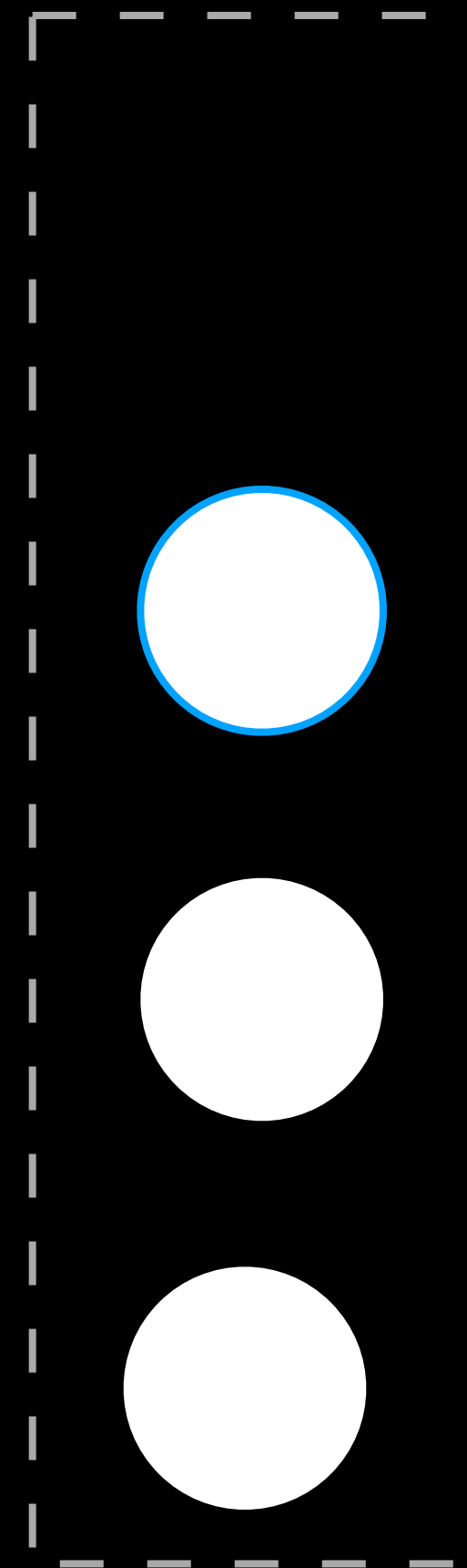




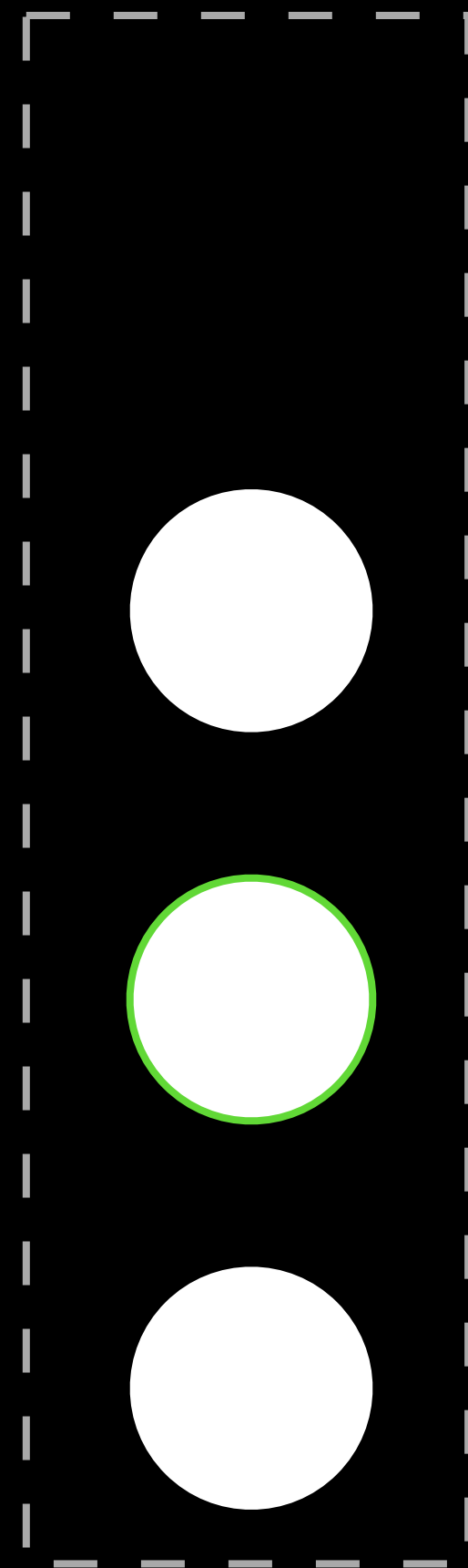
Input layer



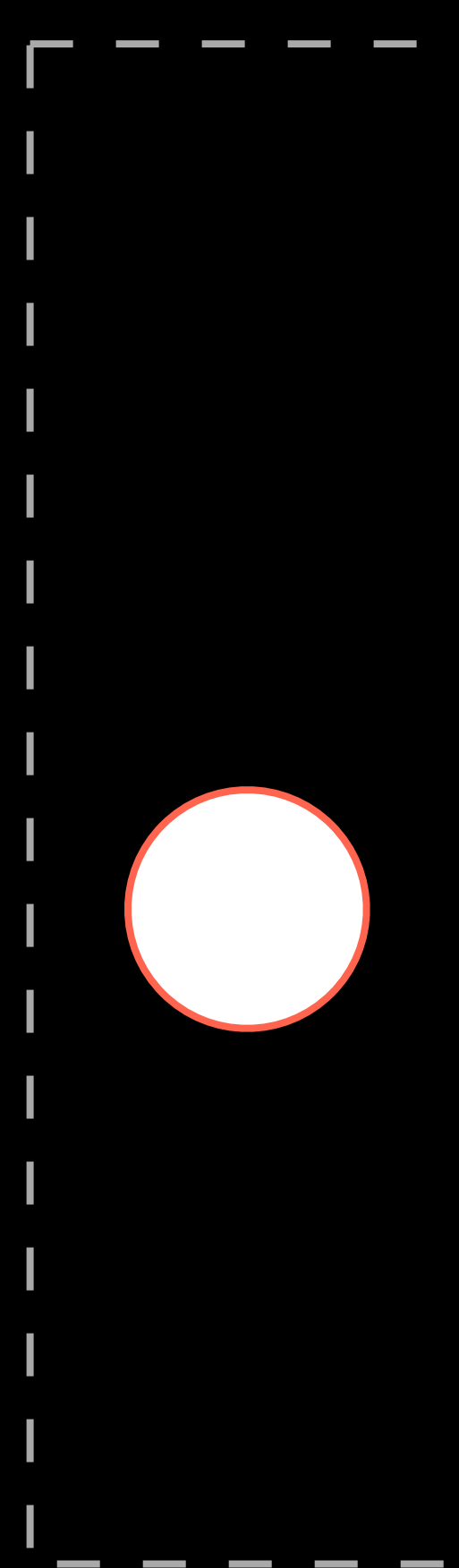
Hidden layer



Hidden layer



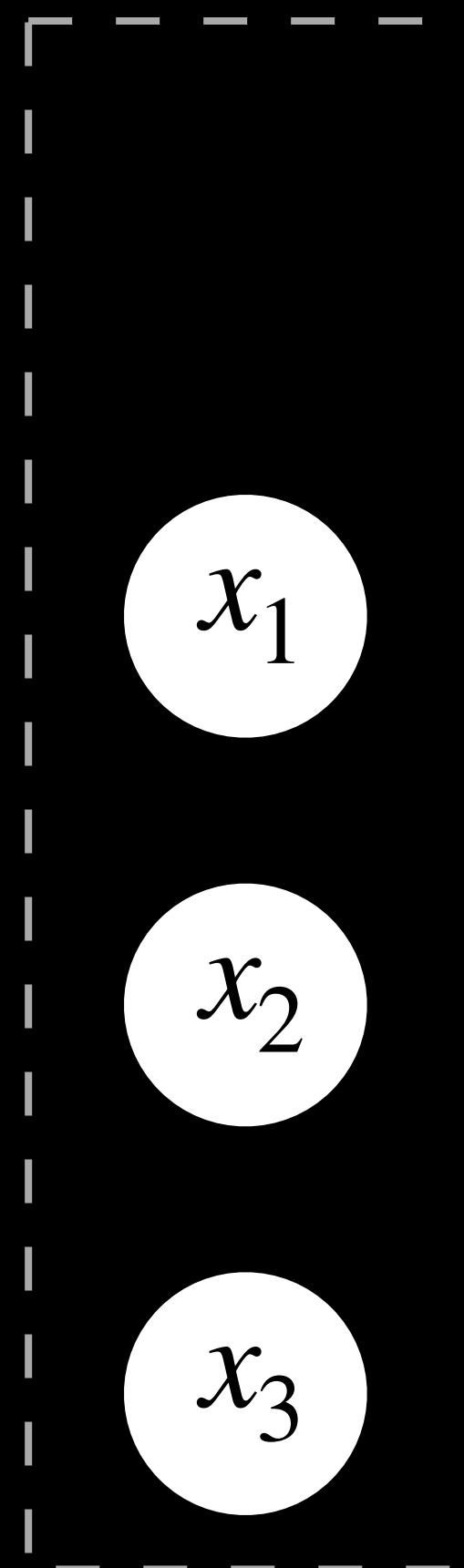
Output layer



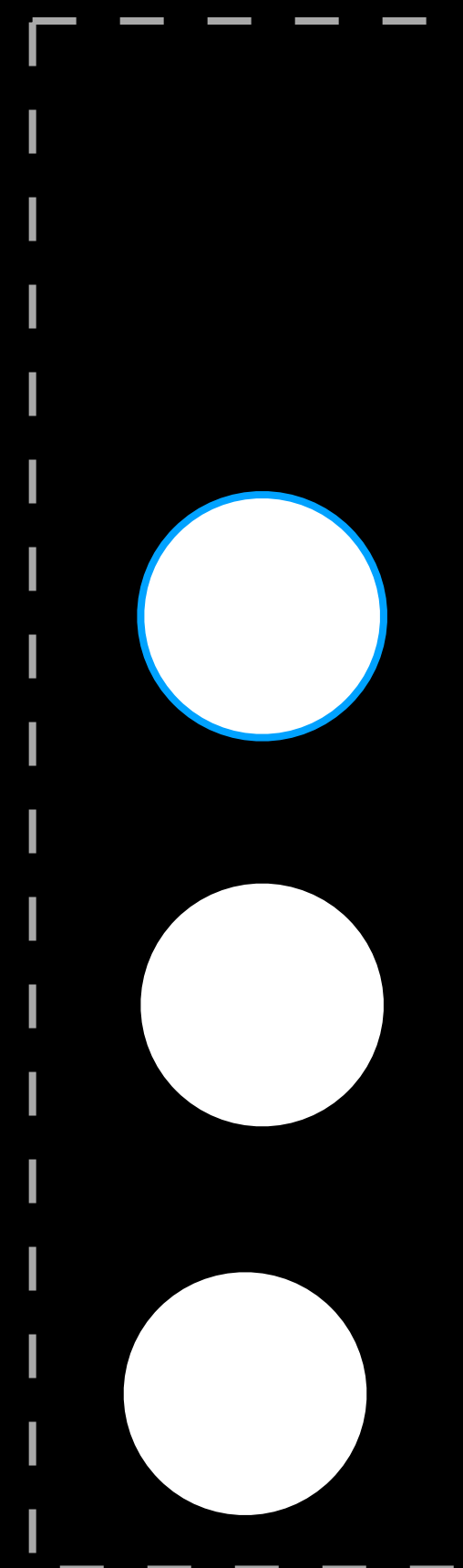
$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



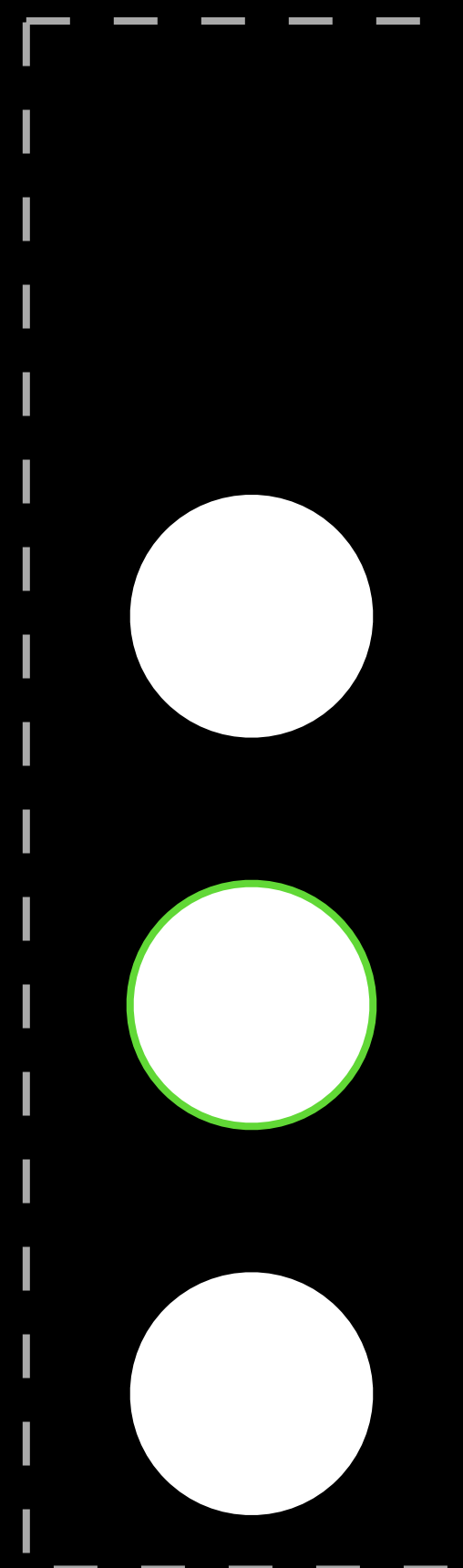
Input layer



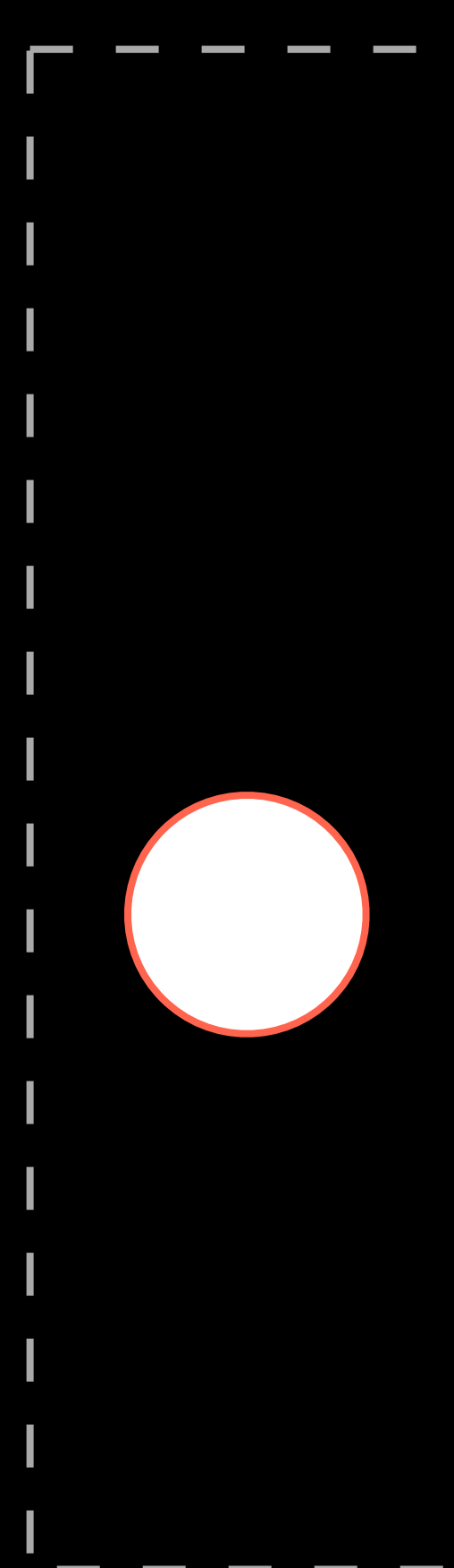
Hidden layer



Hidden layer

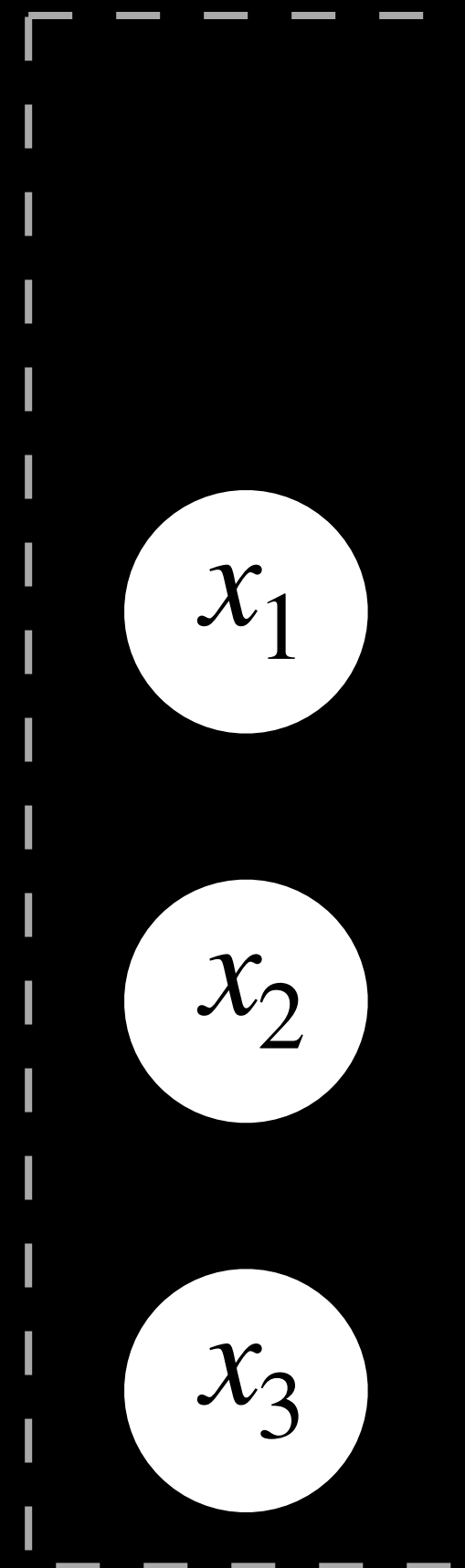


Output layer

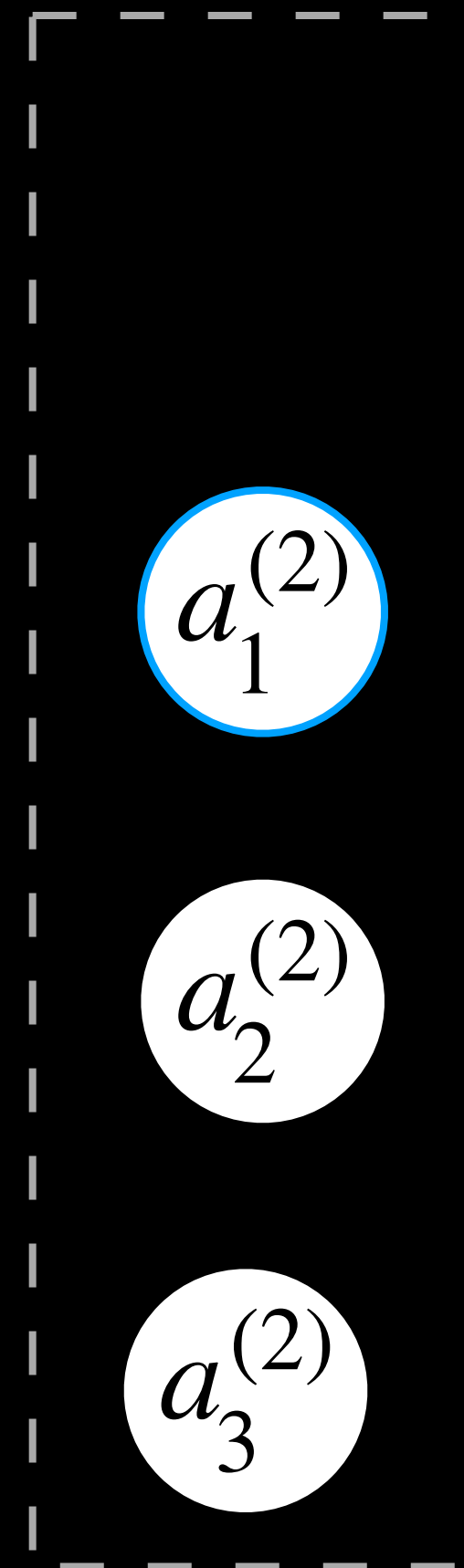


$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

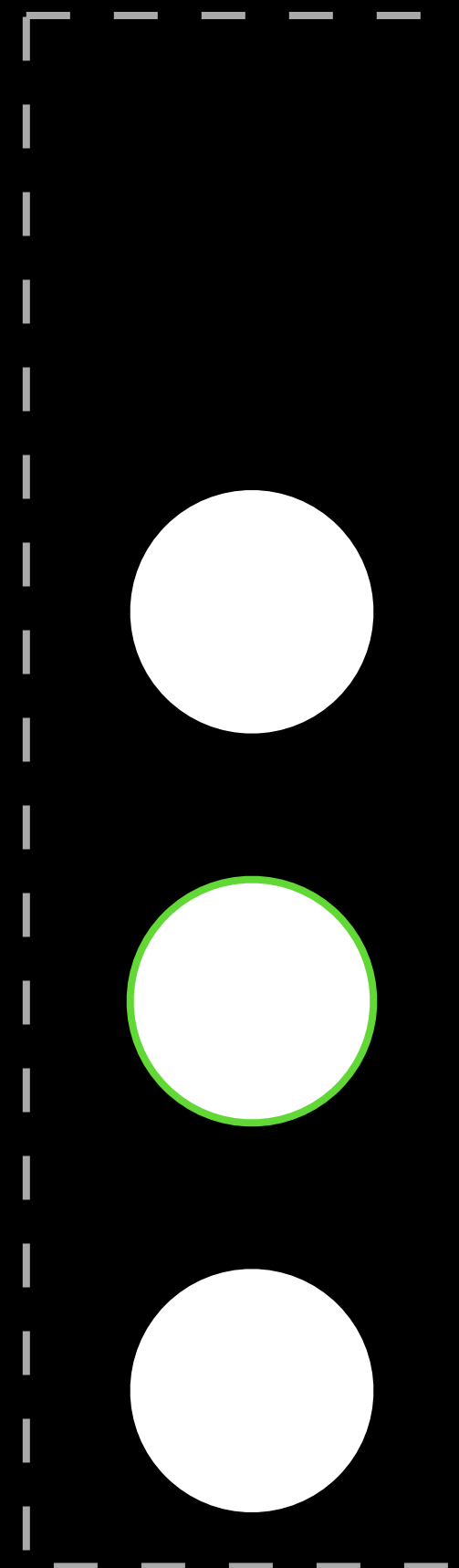
Input layer



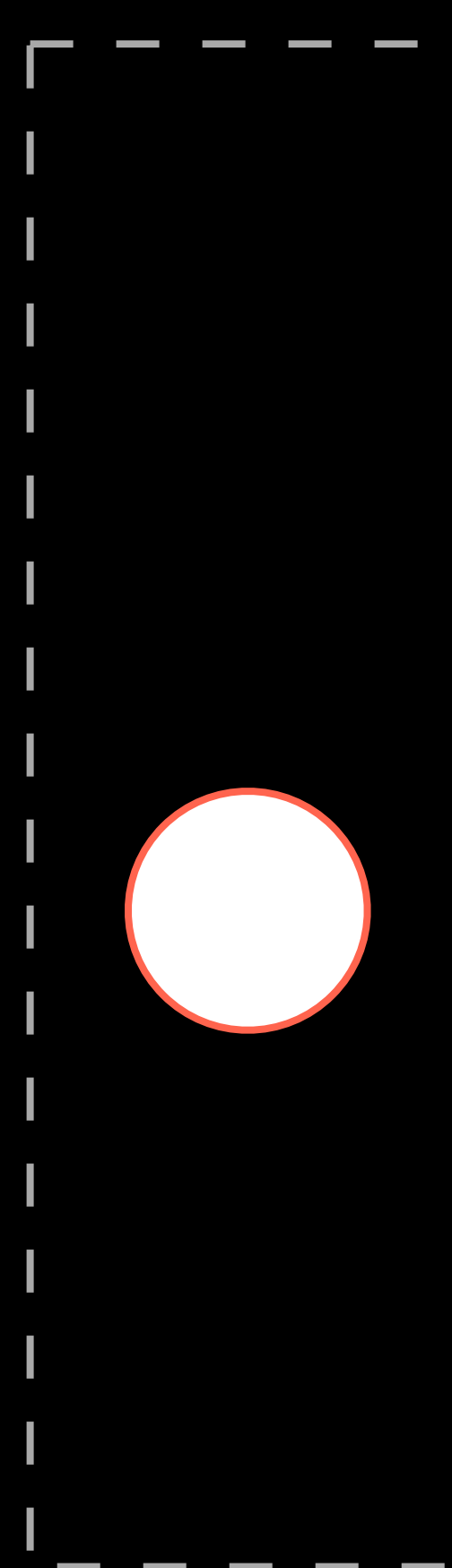
Hidden layer



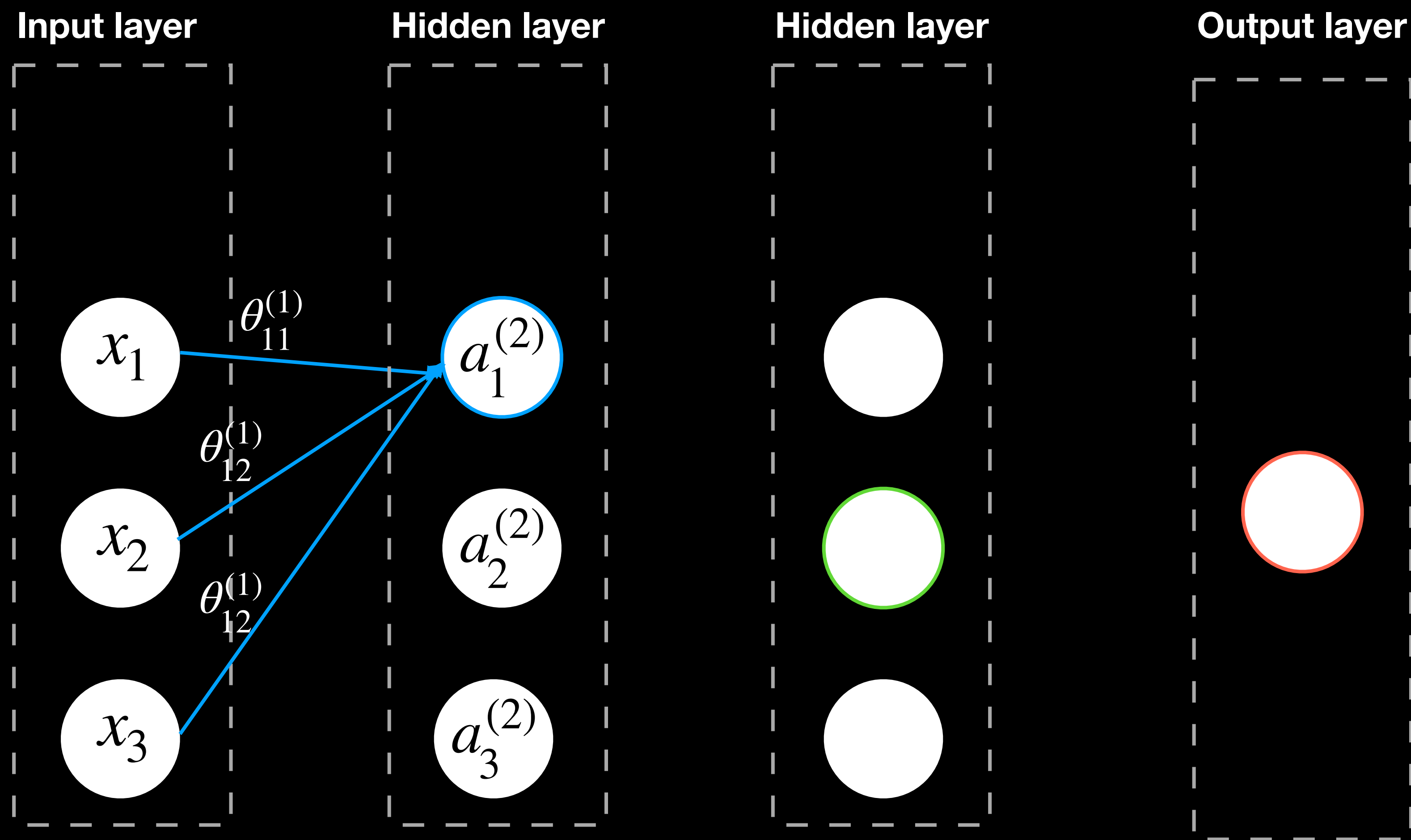
Hidden layer



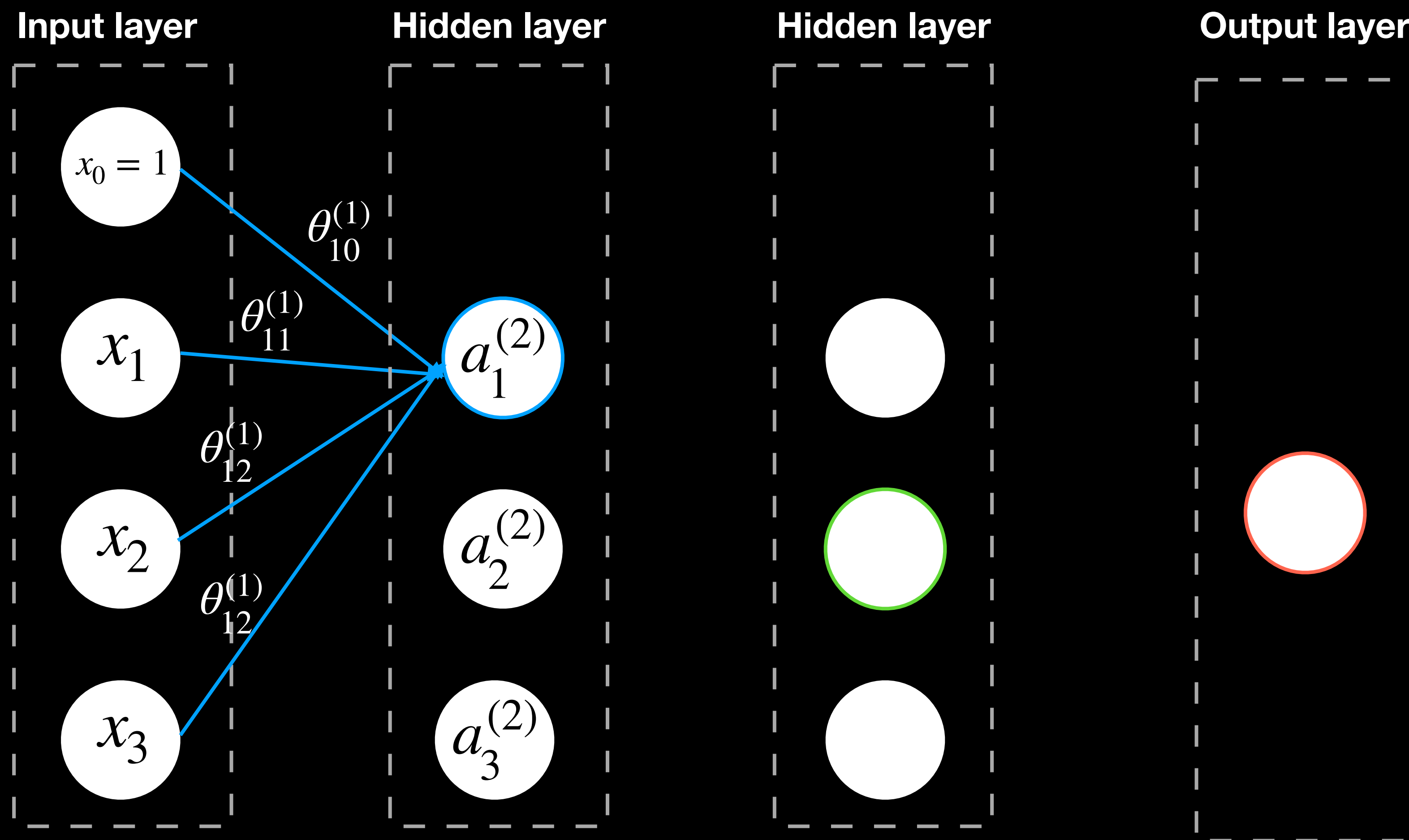
Output layer



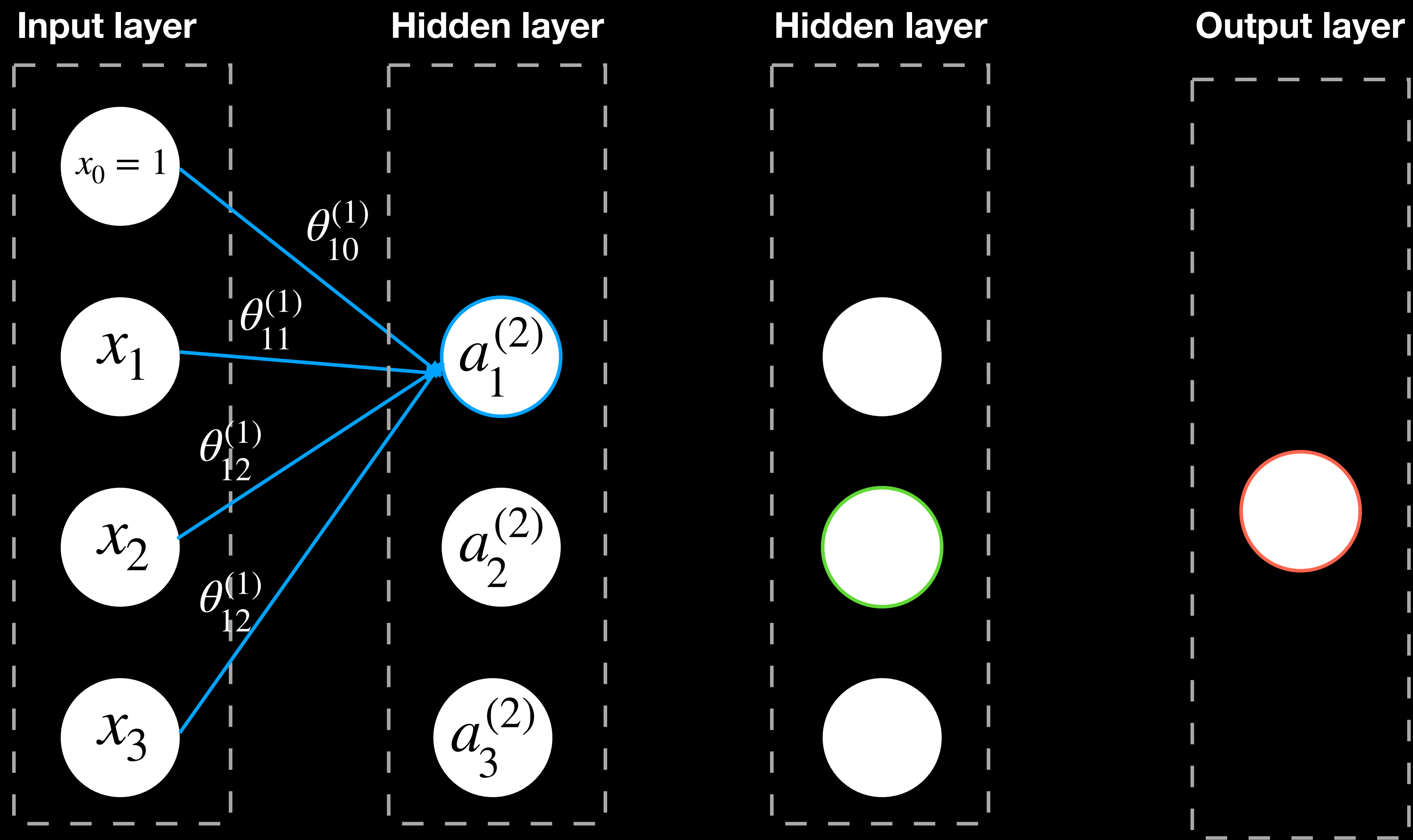
$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



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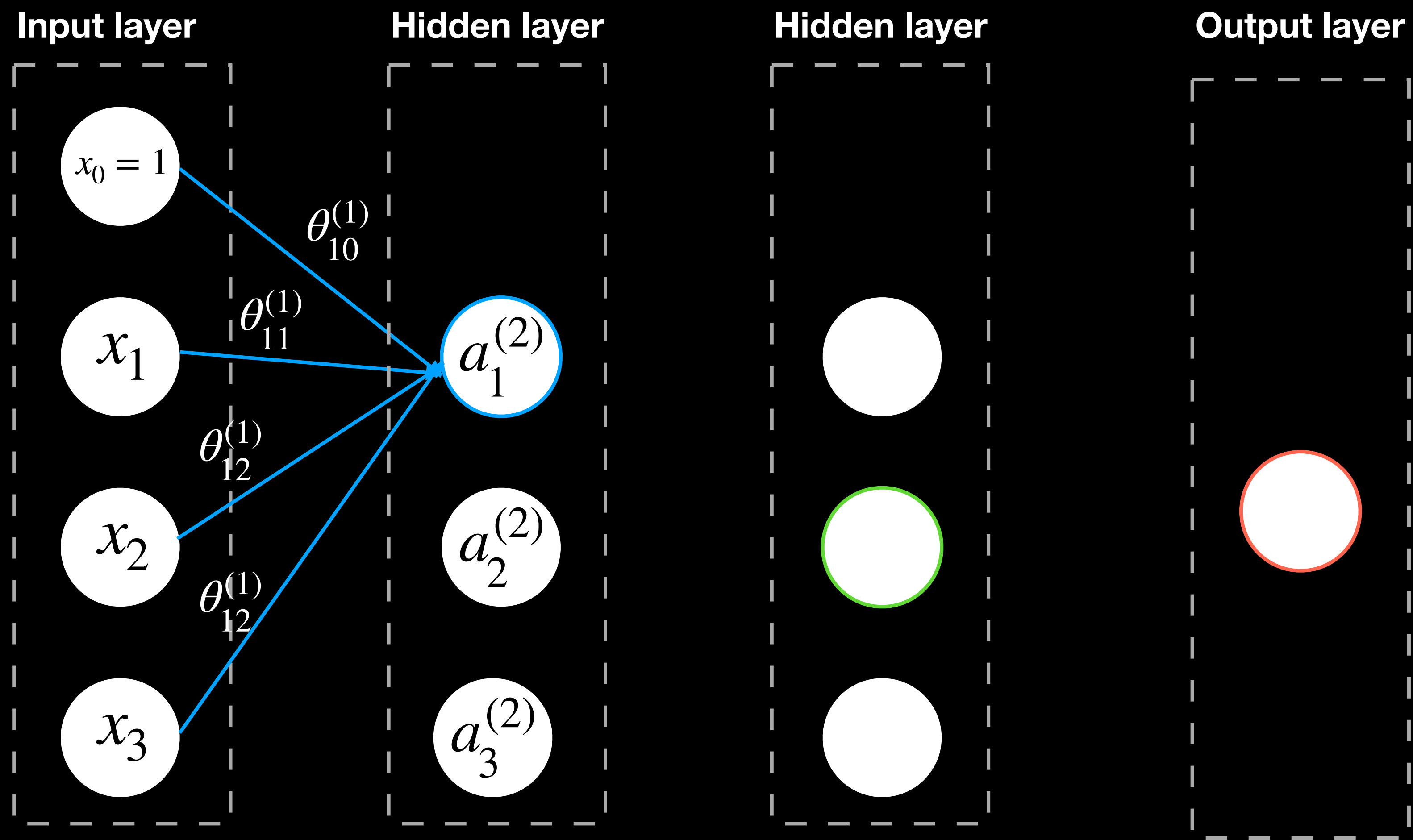


$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



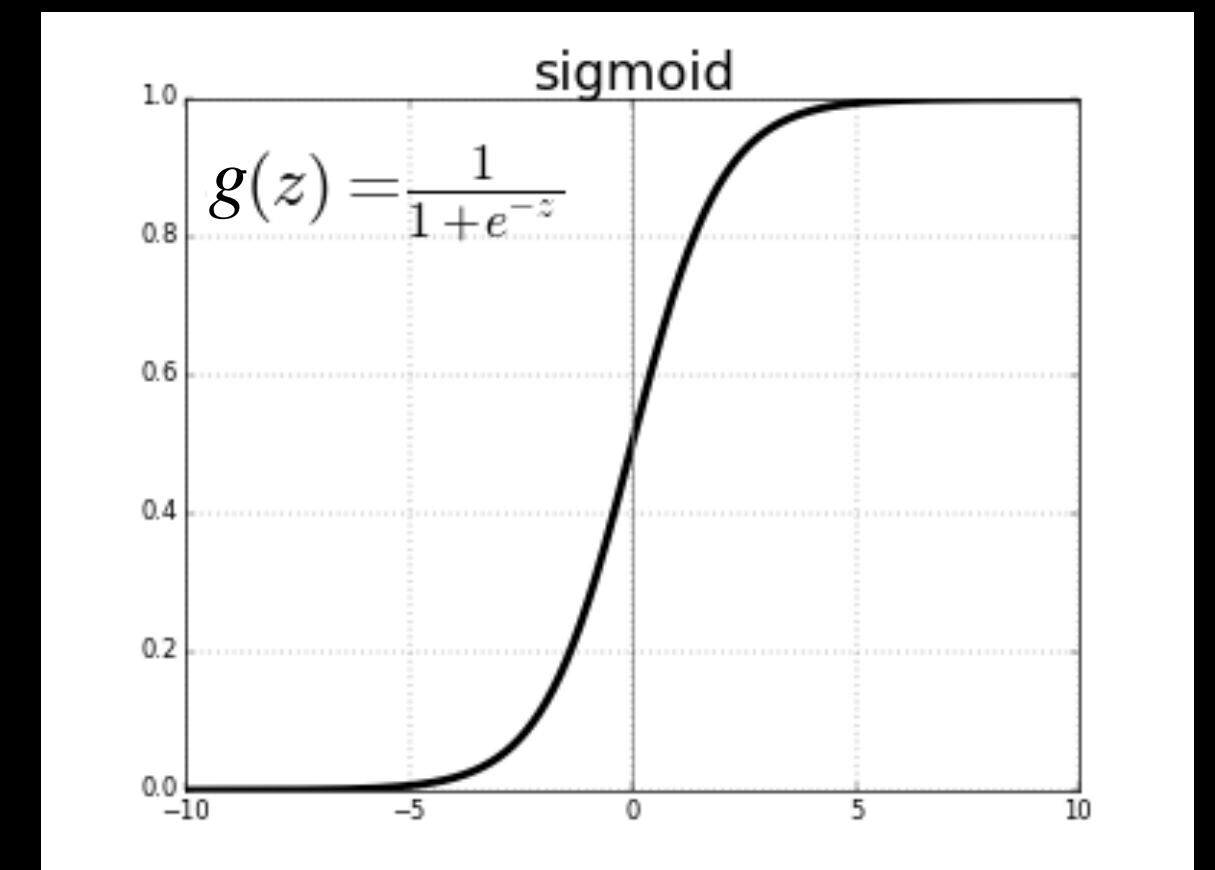
$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

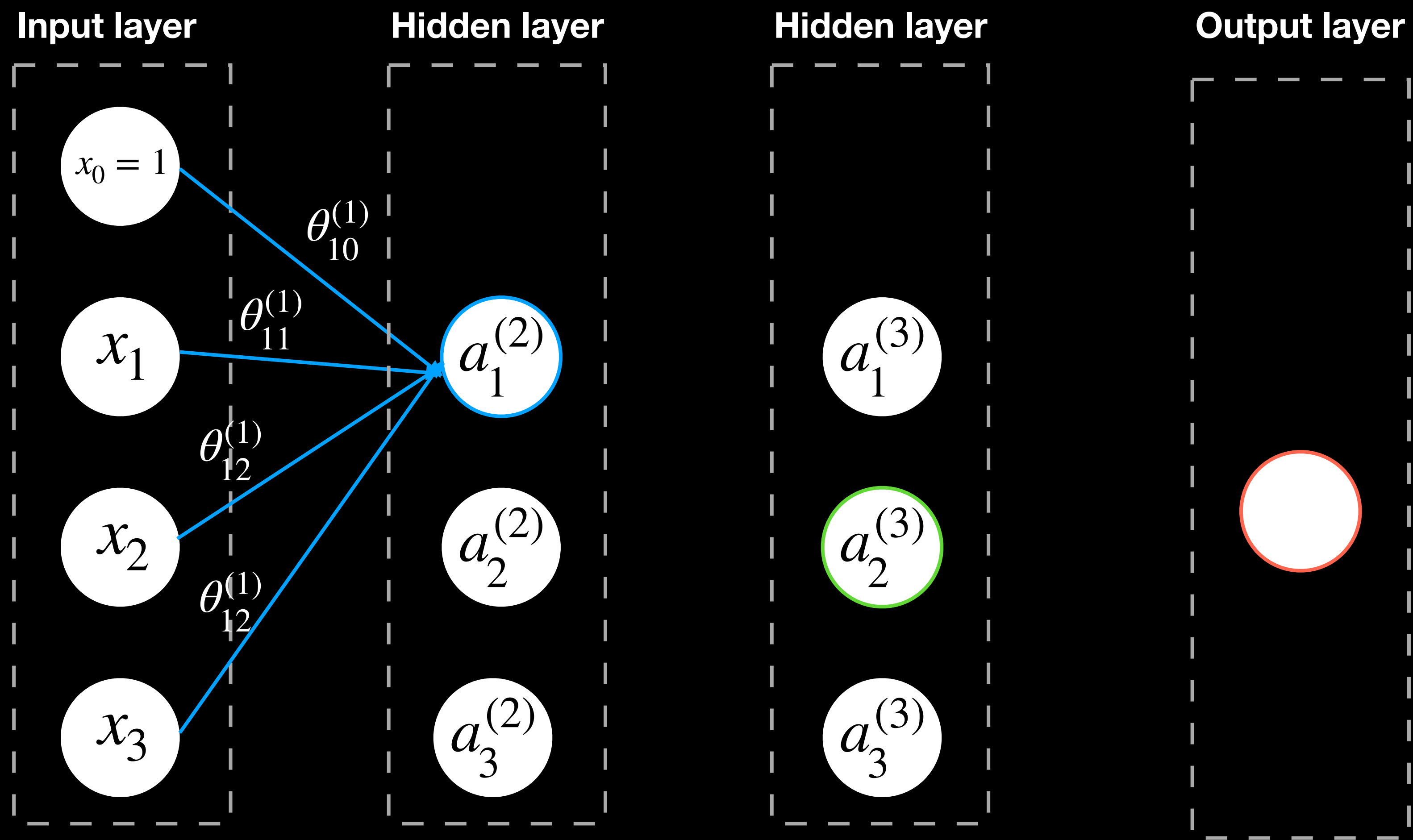
$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

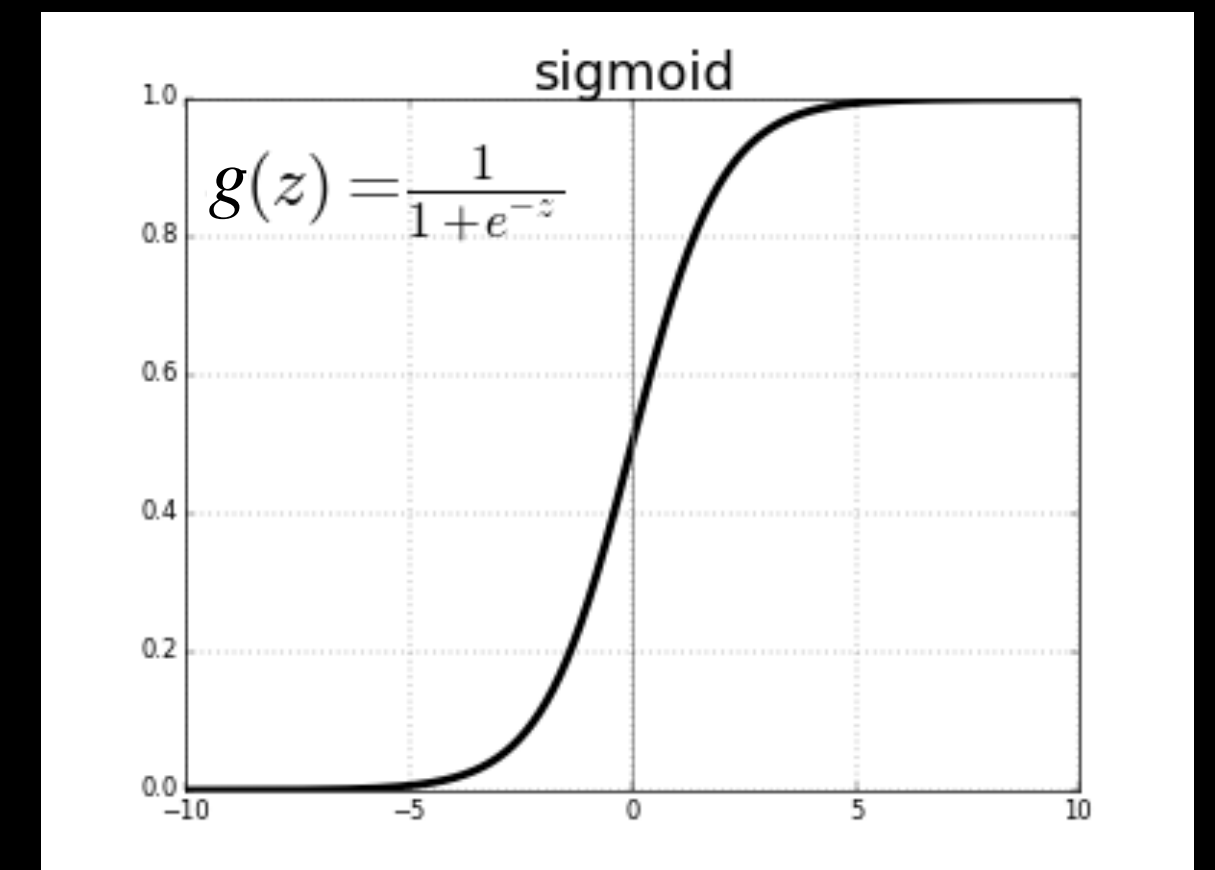
$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

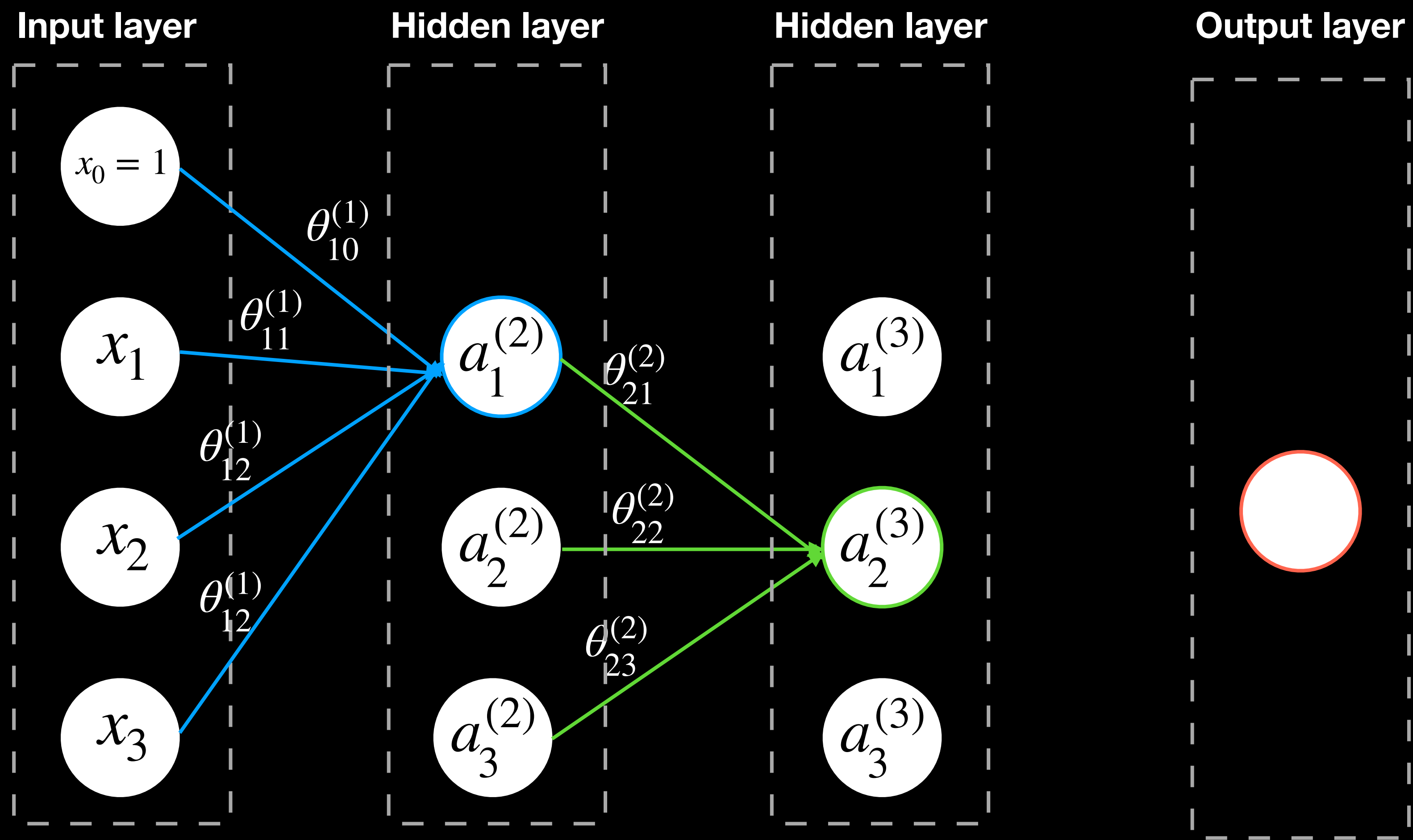




$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

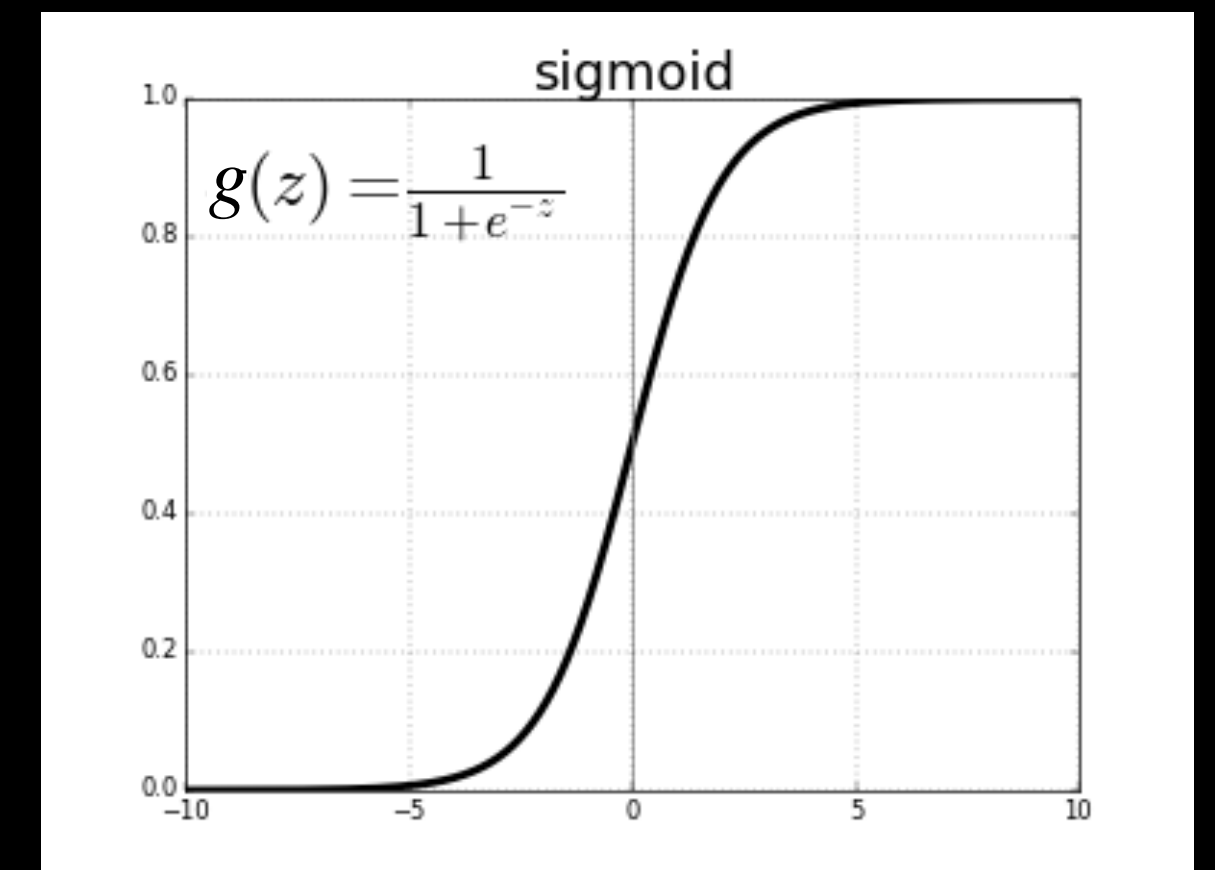
$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



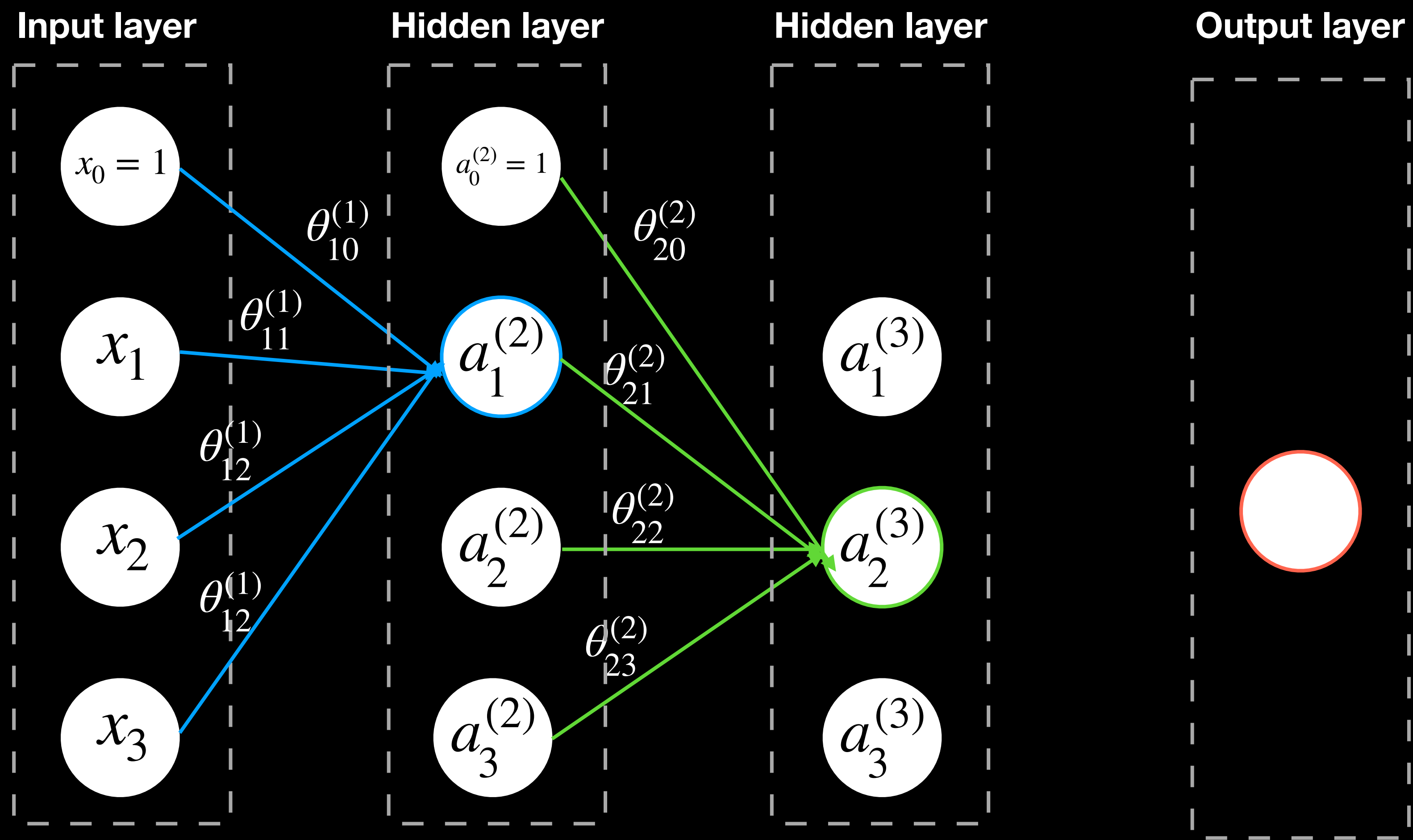


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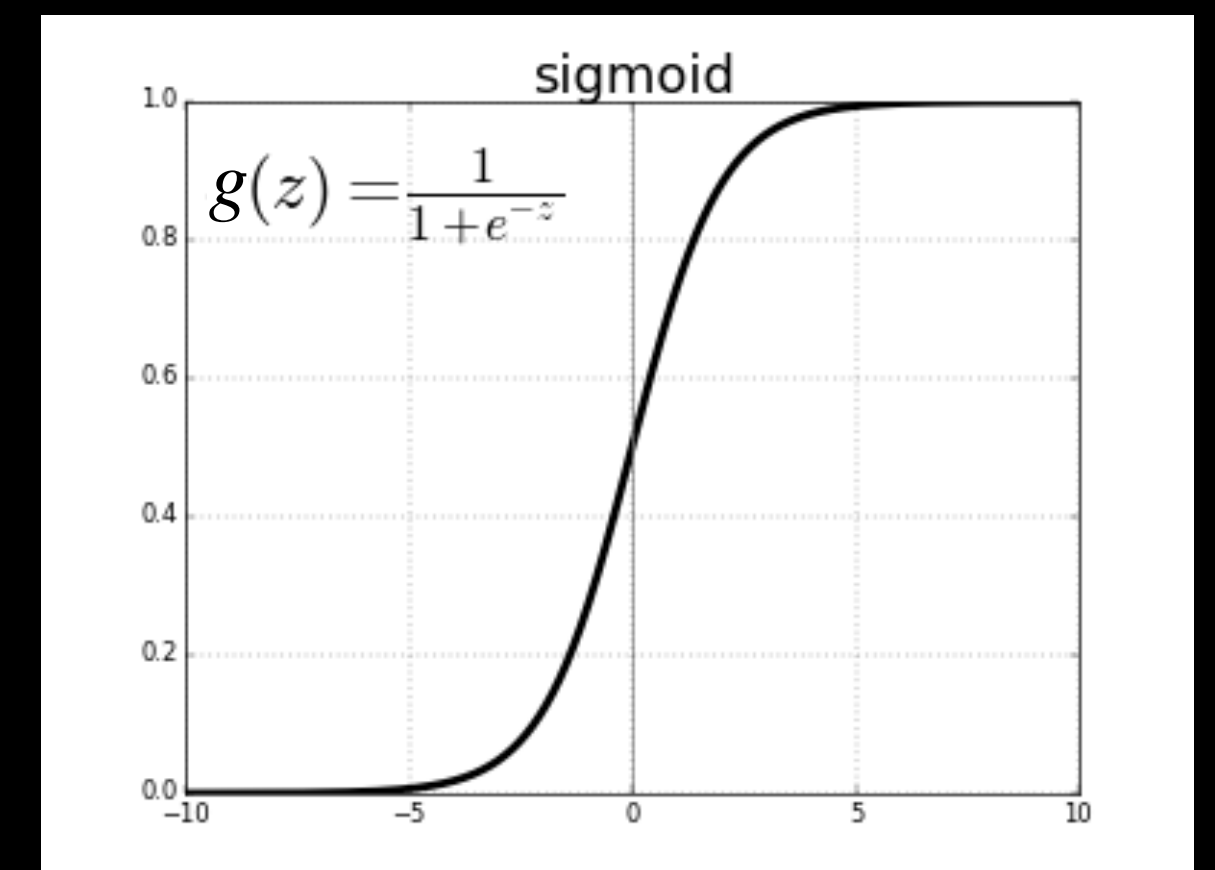


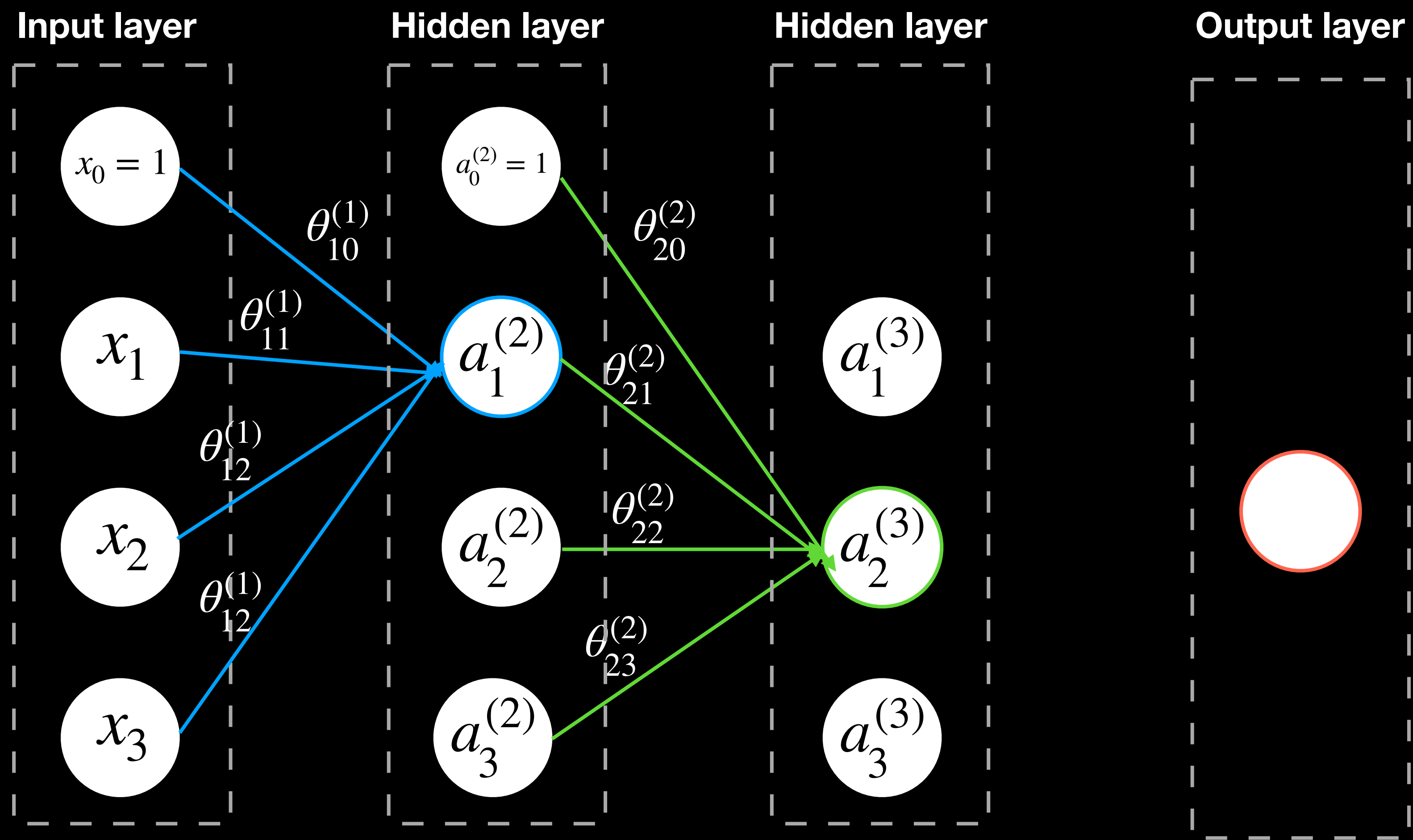




$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

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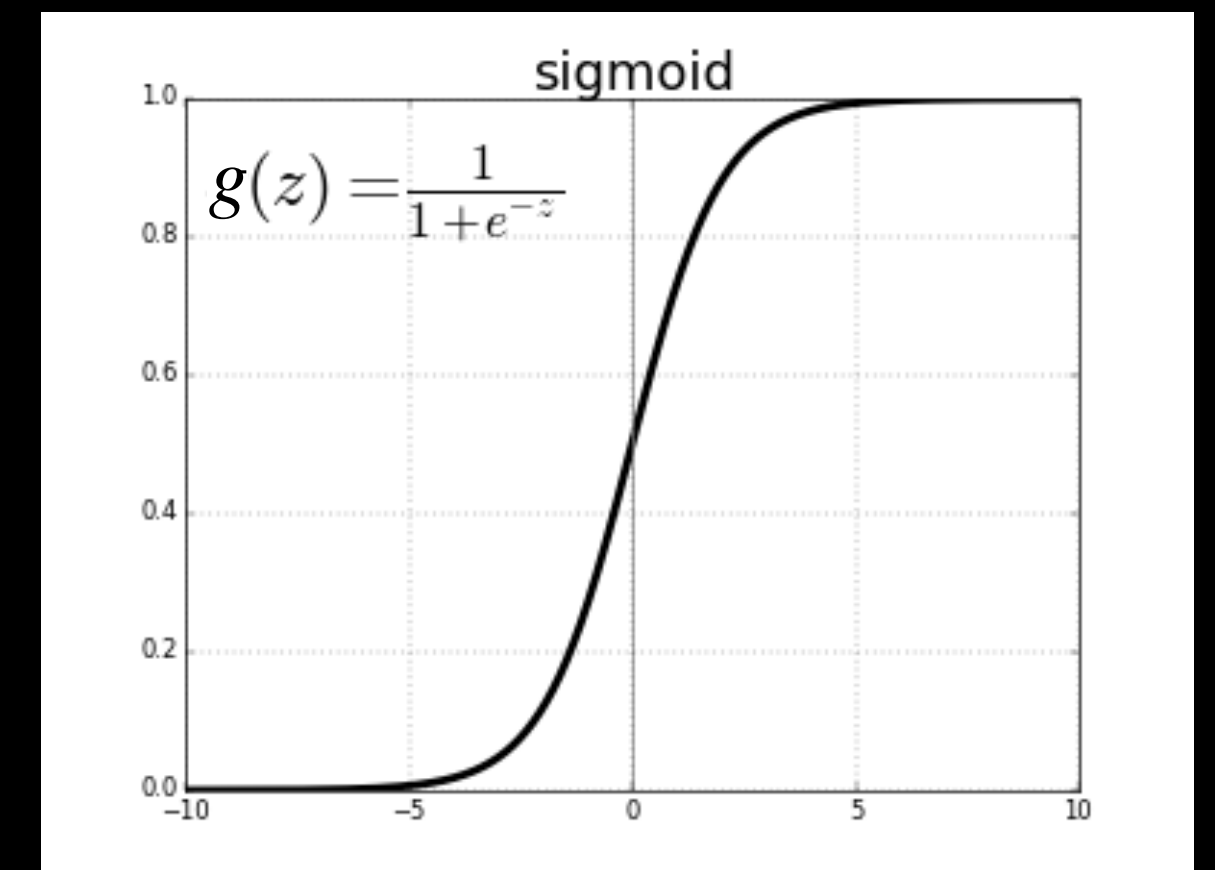


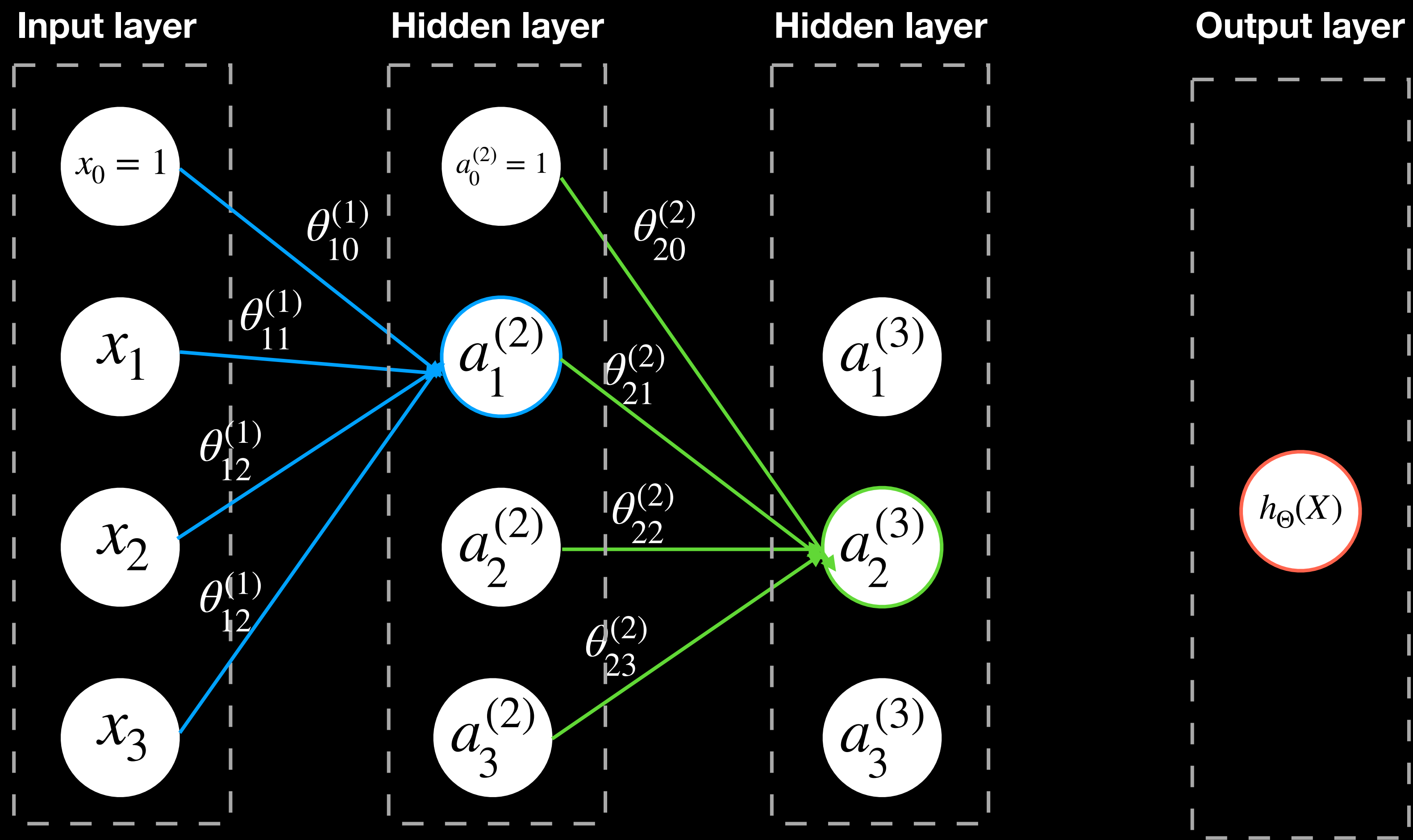


$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

$$a_2^{(3)} = g \left( \theta_{20}^{(2)} a_0^{(2)} + \theta_{21}^{(2)} a_1^{(2)} + \theta_{22}^{(2)} a_2^{(2)} + \theta_{23}^{(2)} a_3^{(2)} \right) = g \left( z_2^{(3)} \right)$$

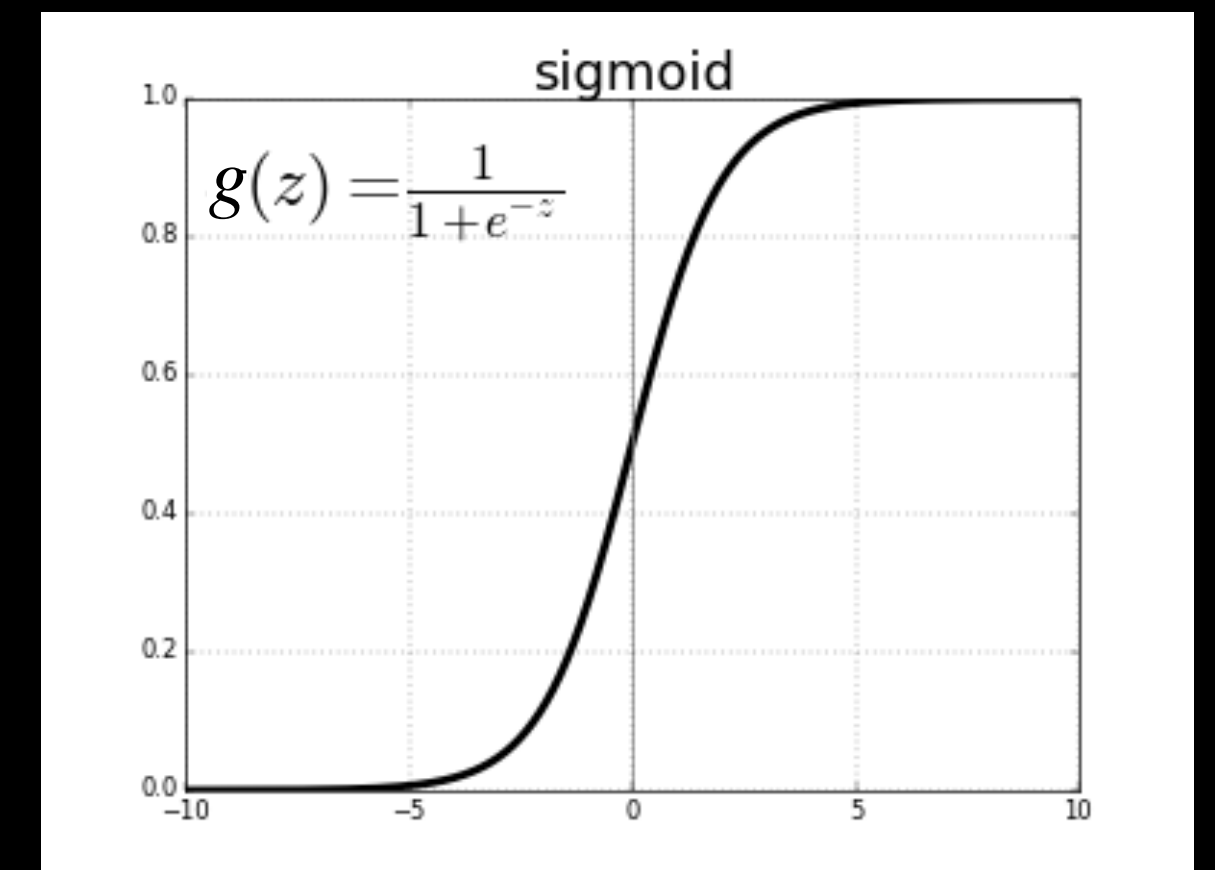


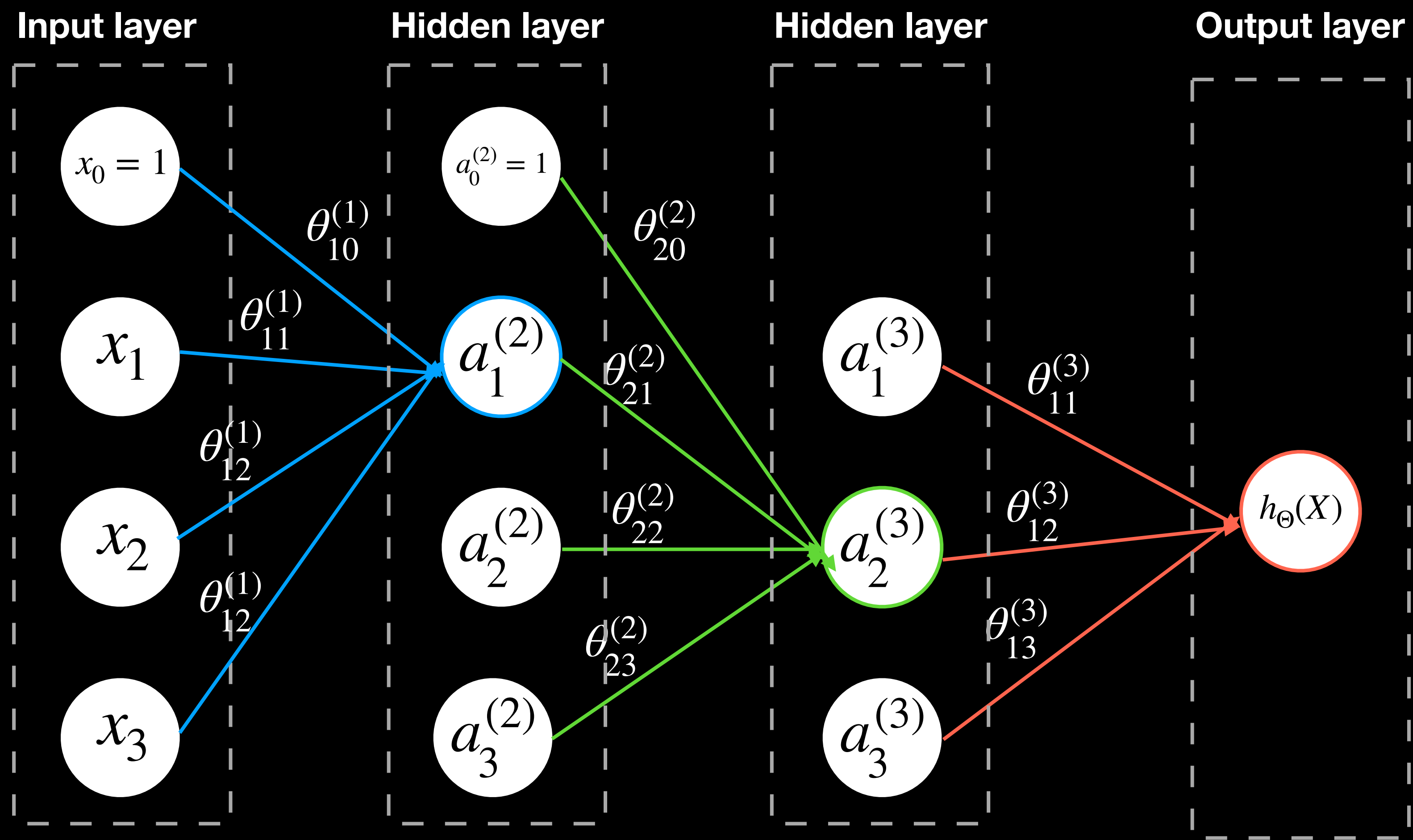


$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

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$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

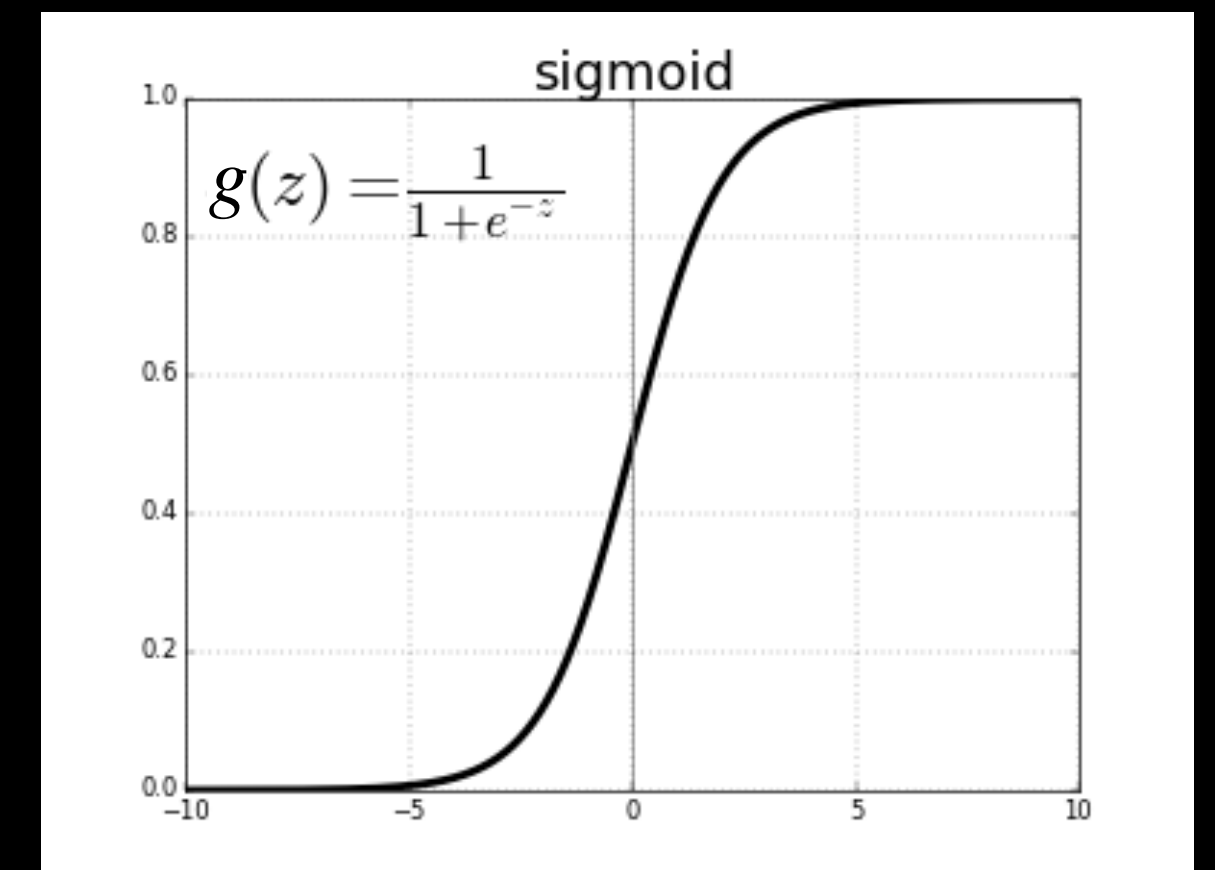


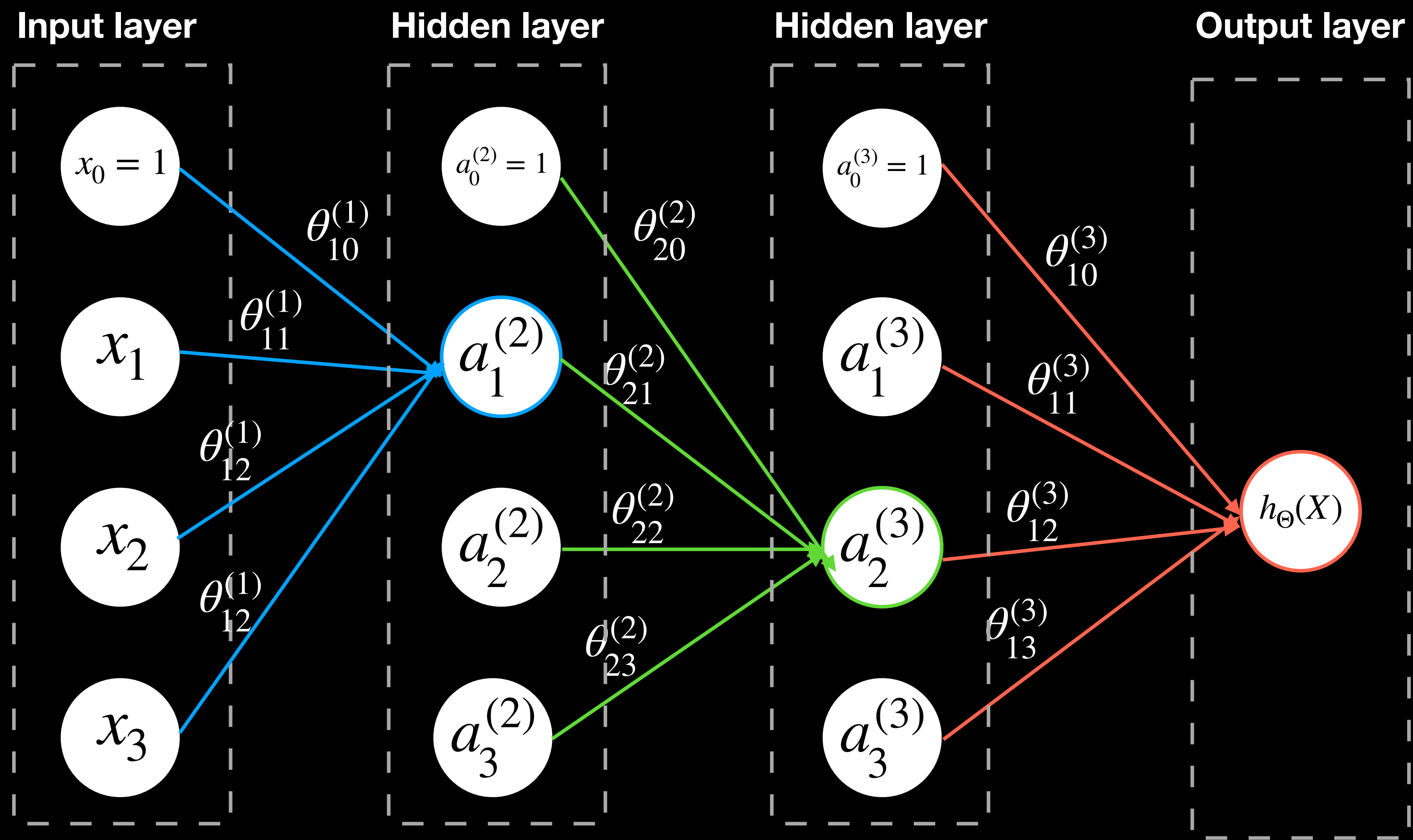


$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

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$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

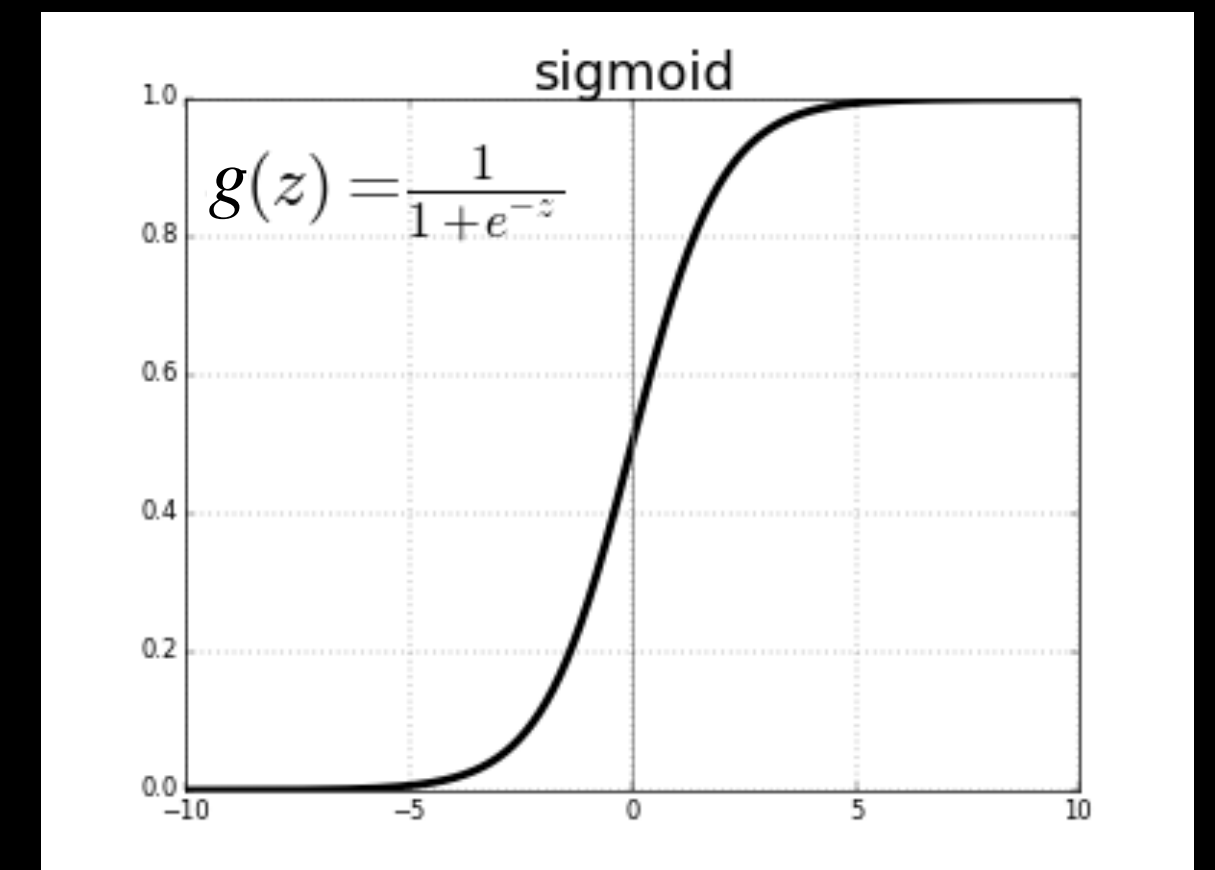




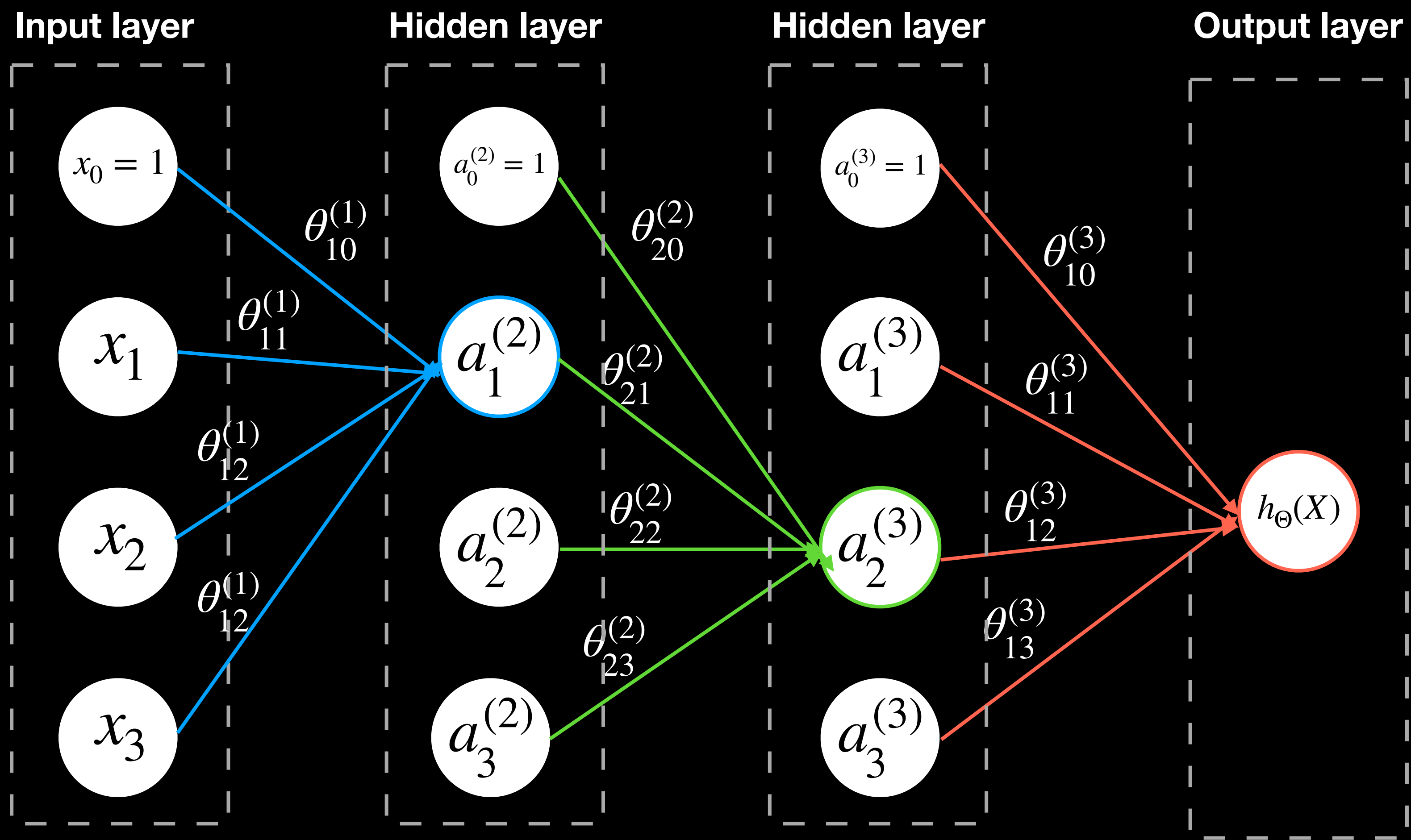
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$$a_2^{(3)} = g \left( \theta_{20}^{(2)} a_0^{(2)} + \theta_{21}^{(2)} a_1^{(2)} + \theta_{22}^{(2)} a_2^{(2)} + \theta_{23}^{(2)} a_3^{(2)} \right) = g \left( z_2^{(3)} \right)$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$





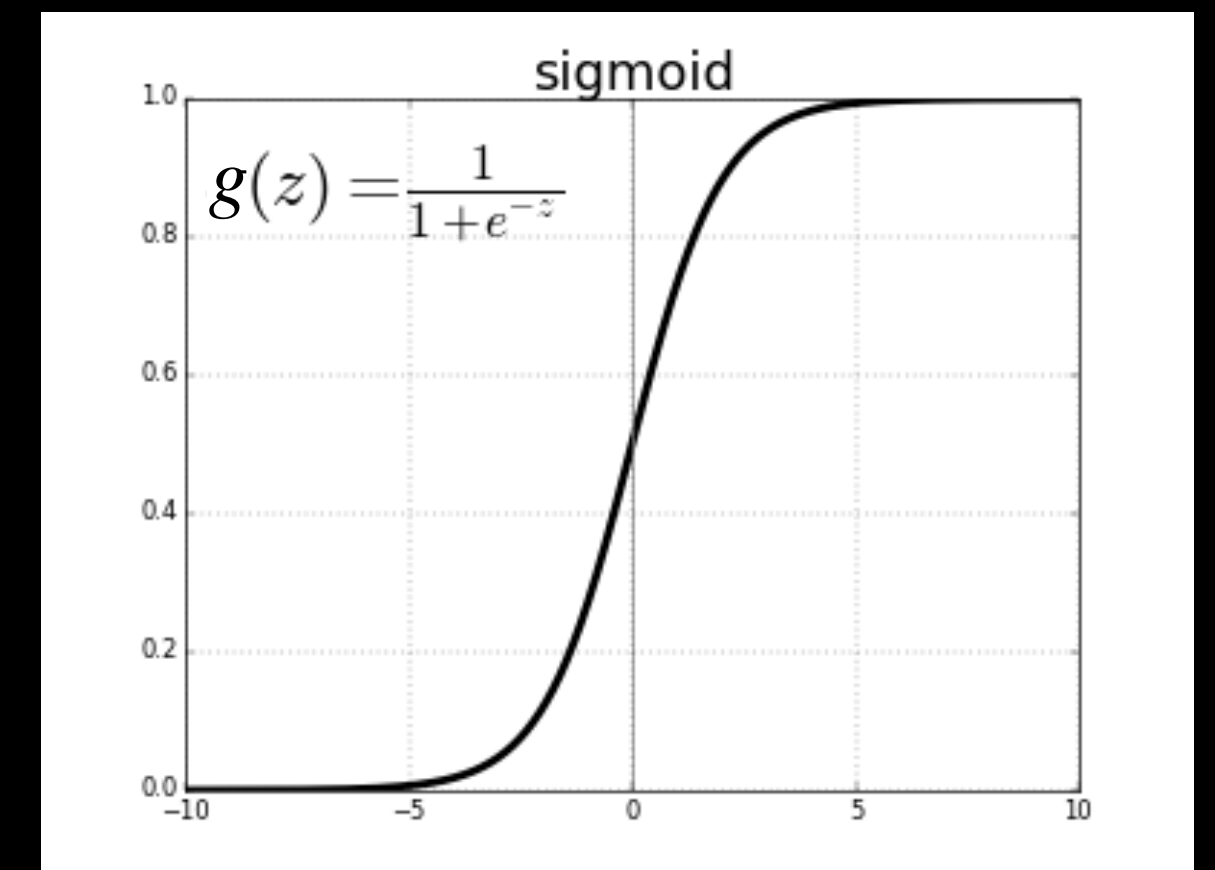


$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$a_1^{(2)} = g \left( \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 \right) = g \left( z_1^{(2)} \right)$$

$$a_2^{(3)} = g \left( \theta_{20}^{(2)} a_0^{(2)} + \theta_{21}^{(2)} a_1^{(2)} + \theta_{22}^{(2)} a_2^{(2)} + \theta_{23}^{(2)} a_3^{(2)} \right) = g \left( z_2^{(3)} \right)$$

$$h_{\Theta}(X) = g \left( \theta_{10}^{(3)} a_0^{(3)} + \theta_{11}^{(3)} a_1^{(3)} + \theta_{12}^{(3)} a_2^{(3)} + \theta_{13}^{(3)} a_3^{(3)} \right) = g \left( z^{(4)} \right)$$



# Cost Function

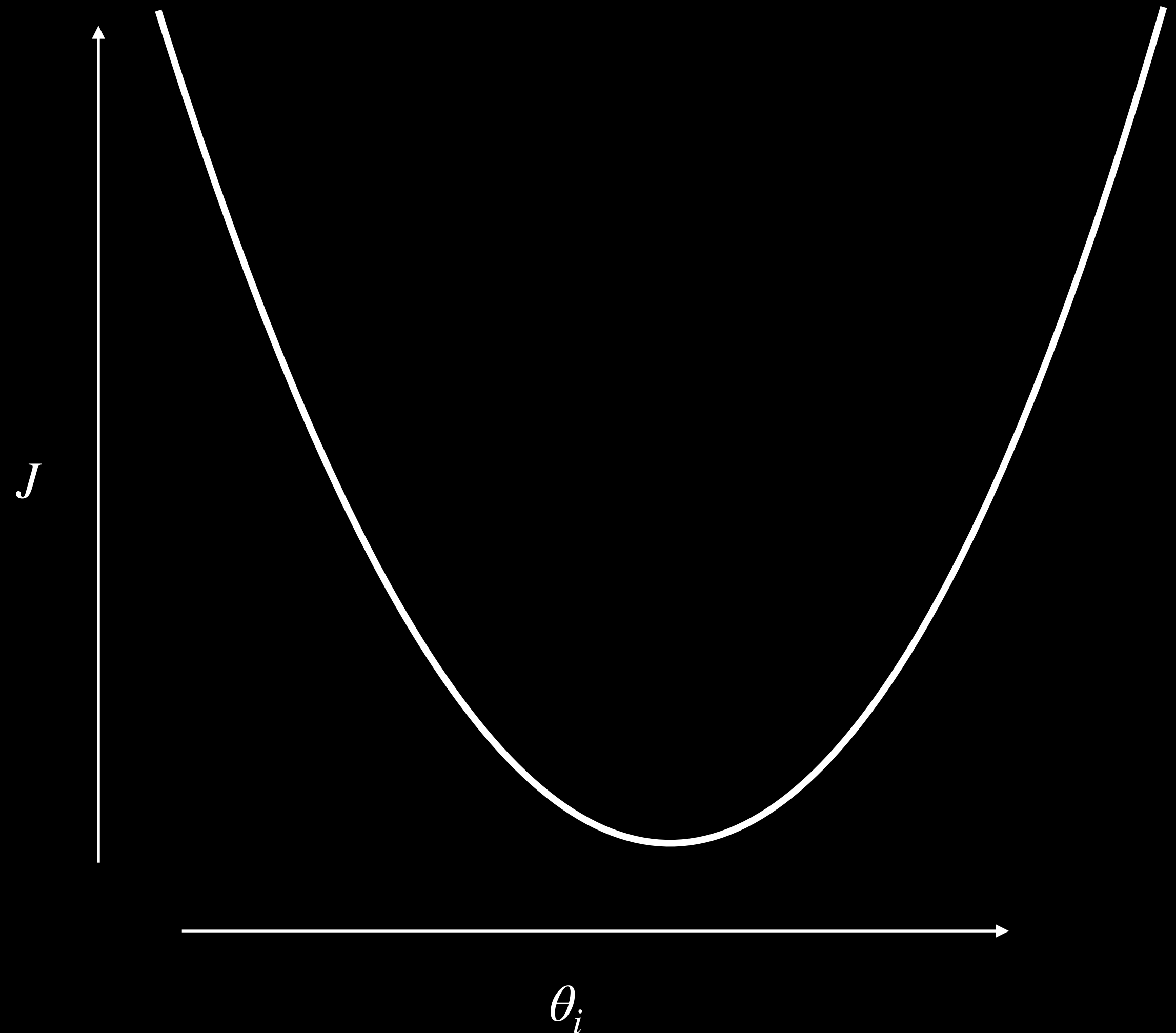
$$J(\Theta) = -\frac{1}{m} \left[ \sum_{i=1}^m \sum_{k=1}^K \boxed{y_k^{(i)} \log \left( h_{\Theta} \left( x^{(i)} \right) \right)_k} + \boxed{\left( 1 - y_k^{(i)} \right) \log \left( 1 - h_{\Theta} \left( x^{(i)} \right) \right)_k} \right] + \boxed{\frac{\lambda}{2m} \sum_{l=1}^{L-1} \sum_{i=1}^{s_l} \sum_{j=1}^{s_{l+1}} \left( \theta_{ji}^{(l)} \right)^2}$$

Diagram annotations: A blue box labeled **y=1** points to the first term of the log-likelihood. A green box labeled **y=0** points to the second term of the log-likelihood. A red box labeled **Regularisation Term** points to the third term.

- $m$  is the number of inputs
- $K$  is the number of classes
- $L$  is the number of layers in network
- $s_l$  is the number of units in the layer  $l$  (not including bias unit)
- $\lambda$  is the regularisation parameter

# Minimising J

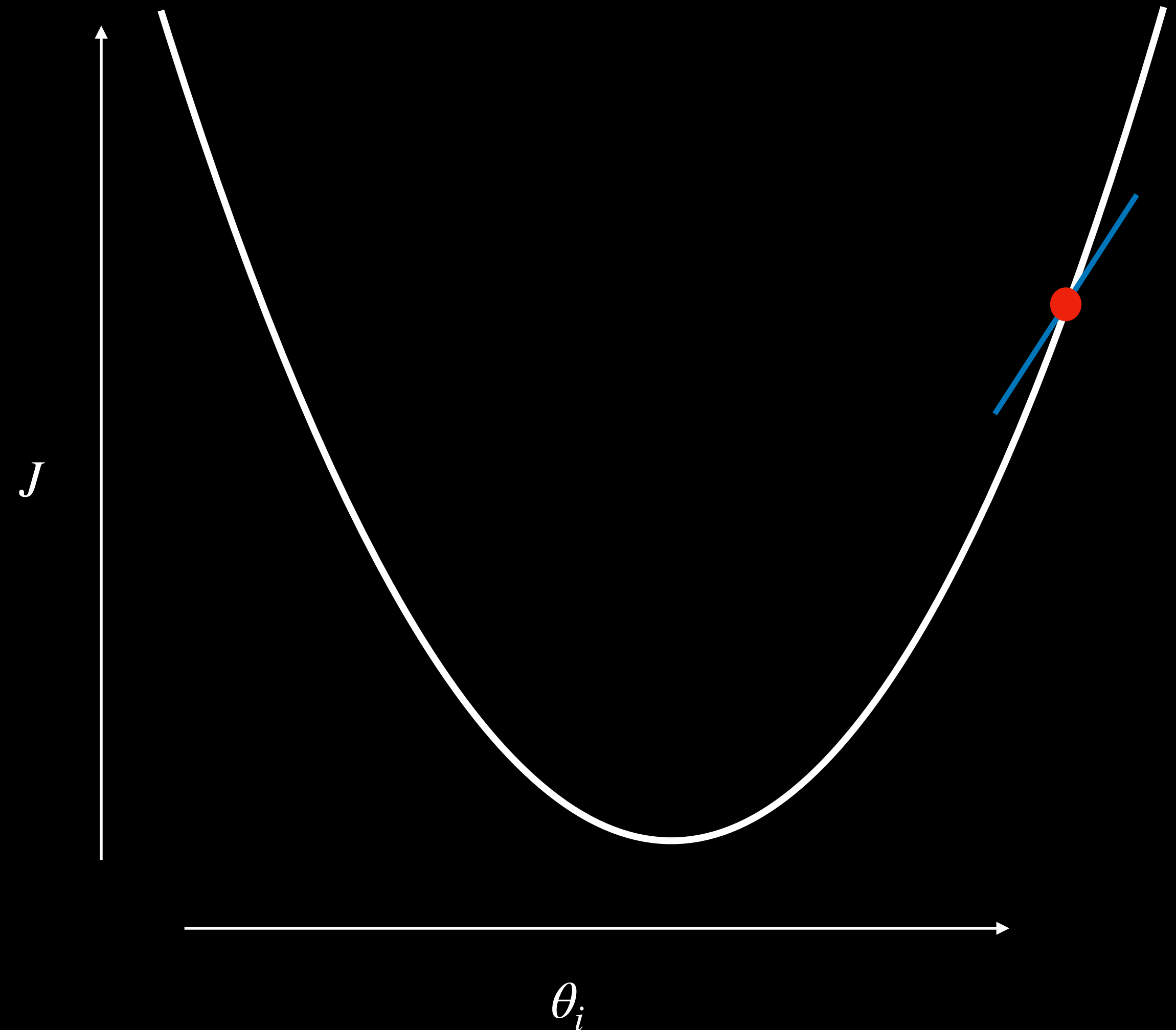
- Gradient Decent:
  - ✦ Calculate  $\frac{\partial}{\partial \theta_i} J$
  - ✦ Update  $\theta_i$  as:  $\theta_i := \theta_i - \alpha \frac{\partial}{\partial \theta_i} J$
  - ✦ Recalculate  $J$





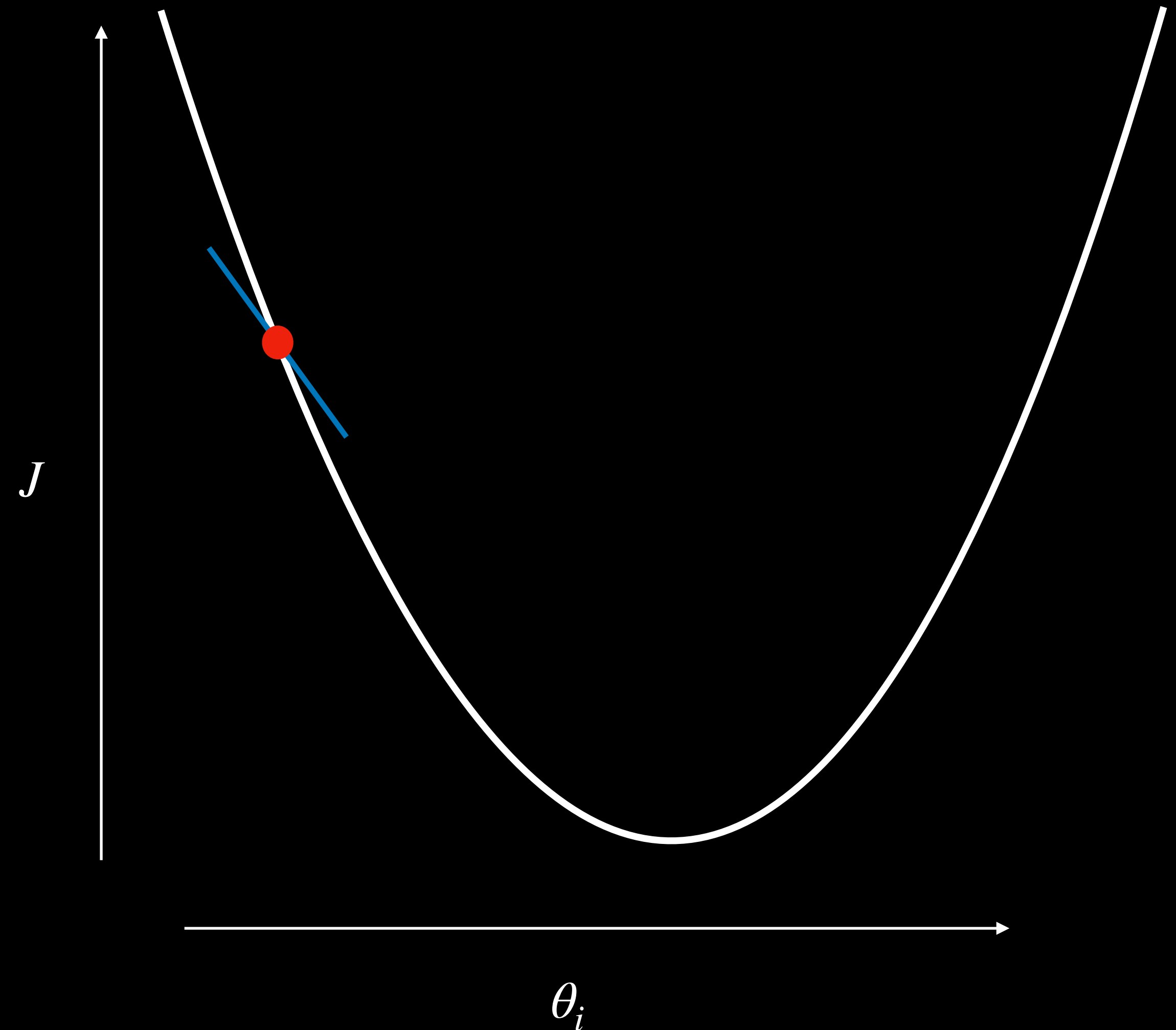
# Minimising J

- Gradient Decent:
  - ✦ Calculate  $\frac{\delta}{\delta\theta_i}J$
  - ✦ Update  $\theta_i$  as:  $\theta_i := \theta_i - \alpha \frac{\delta}{\delta\theta_i}J$
  - ✦ Recalculate  $J$



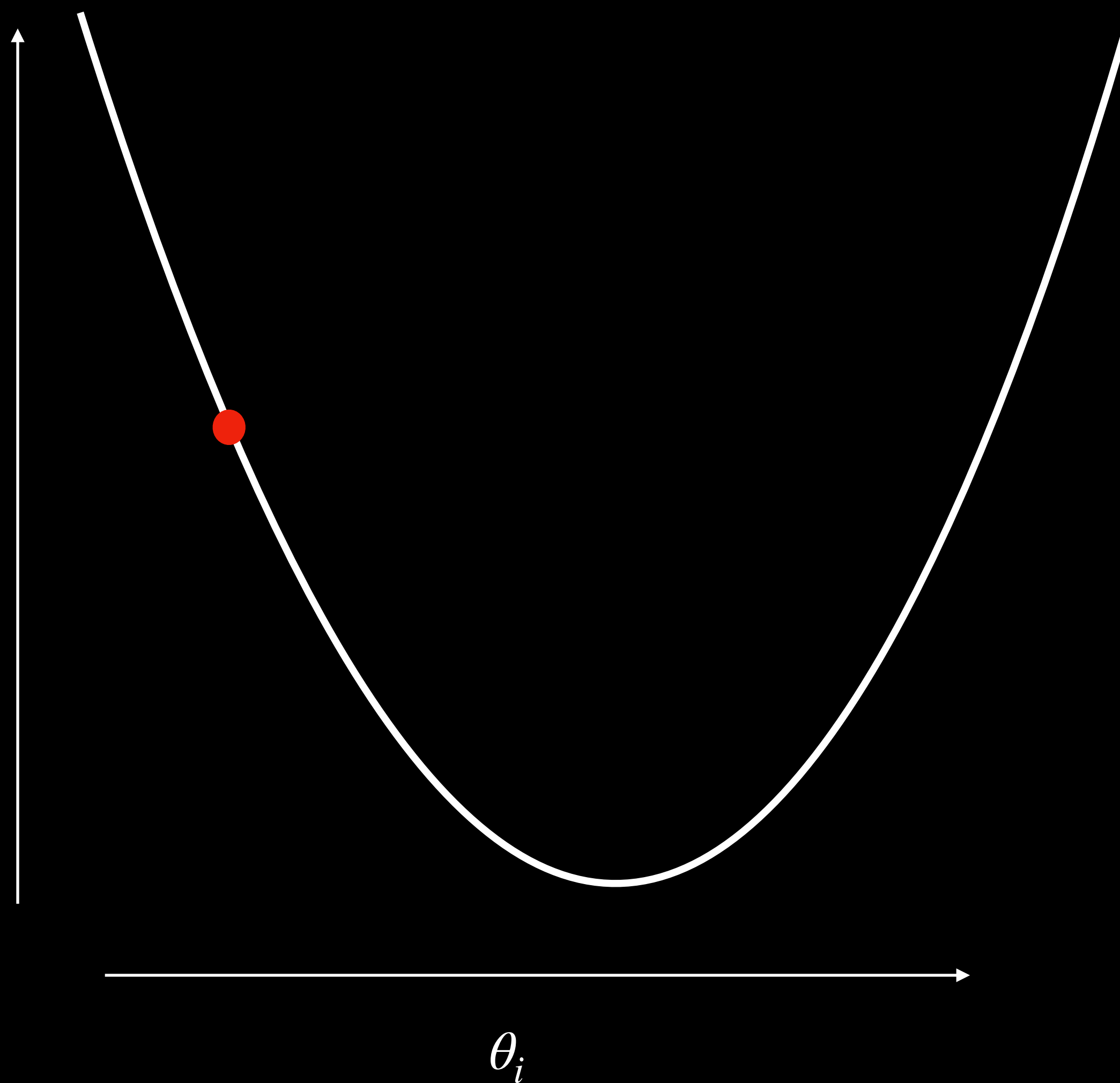
# Minimising J

- Gradient Decent:
  - ✦ Calculate  $\frac{\delta}{\delta\theta_i}J$
  - ✦ Update  $\theta_i$  as:  $\theta_i := \theta_i - \alpha \frac{\delta}{\delta\theta_i}J$
  - ✦ Recalculate  $J$



# Minimising J

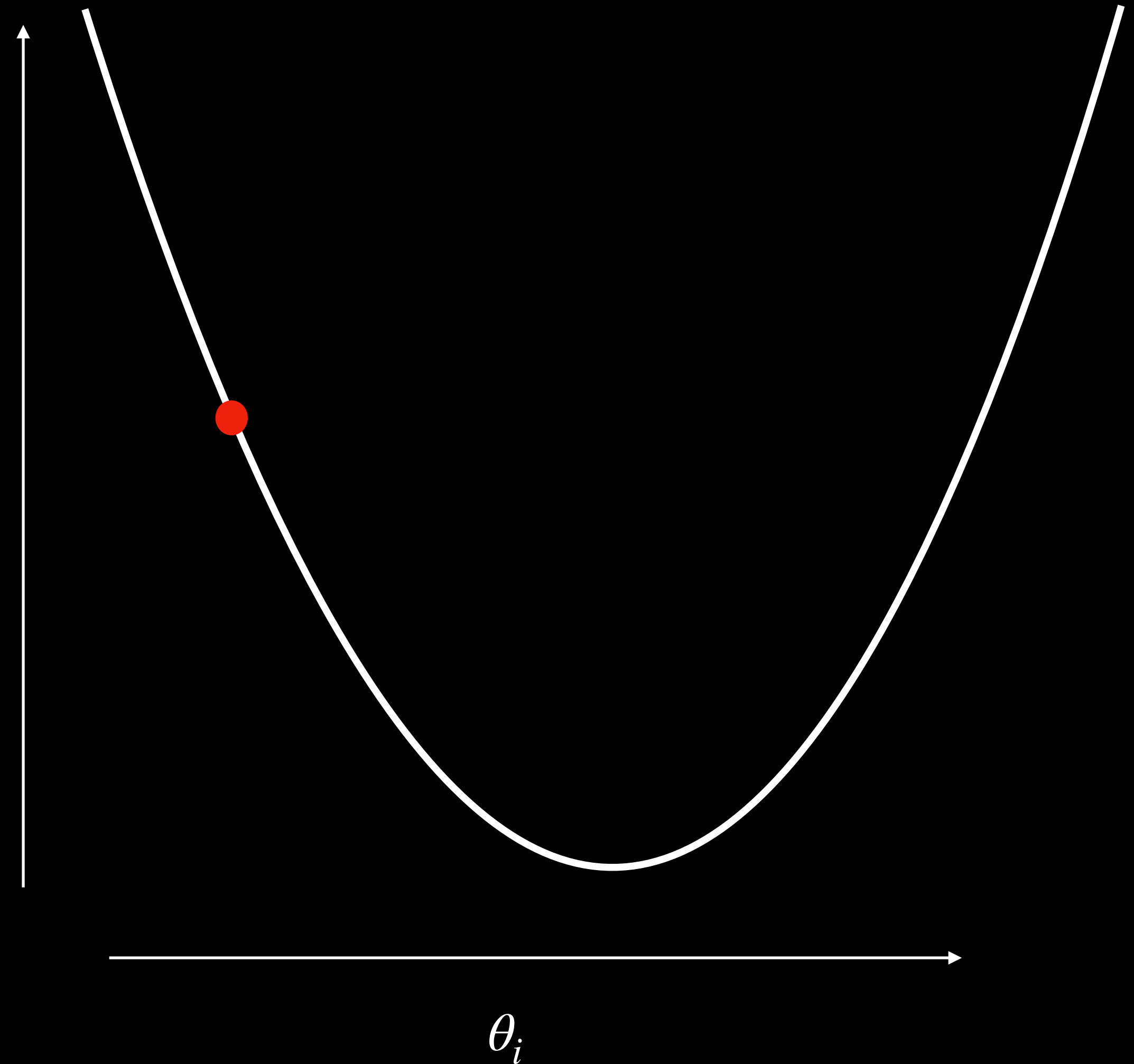
$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$



# Minimising J

$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

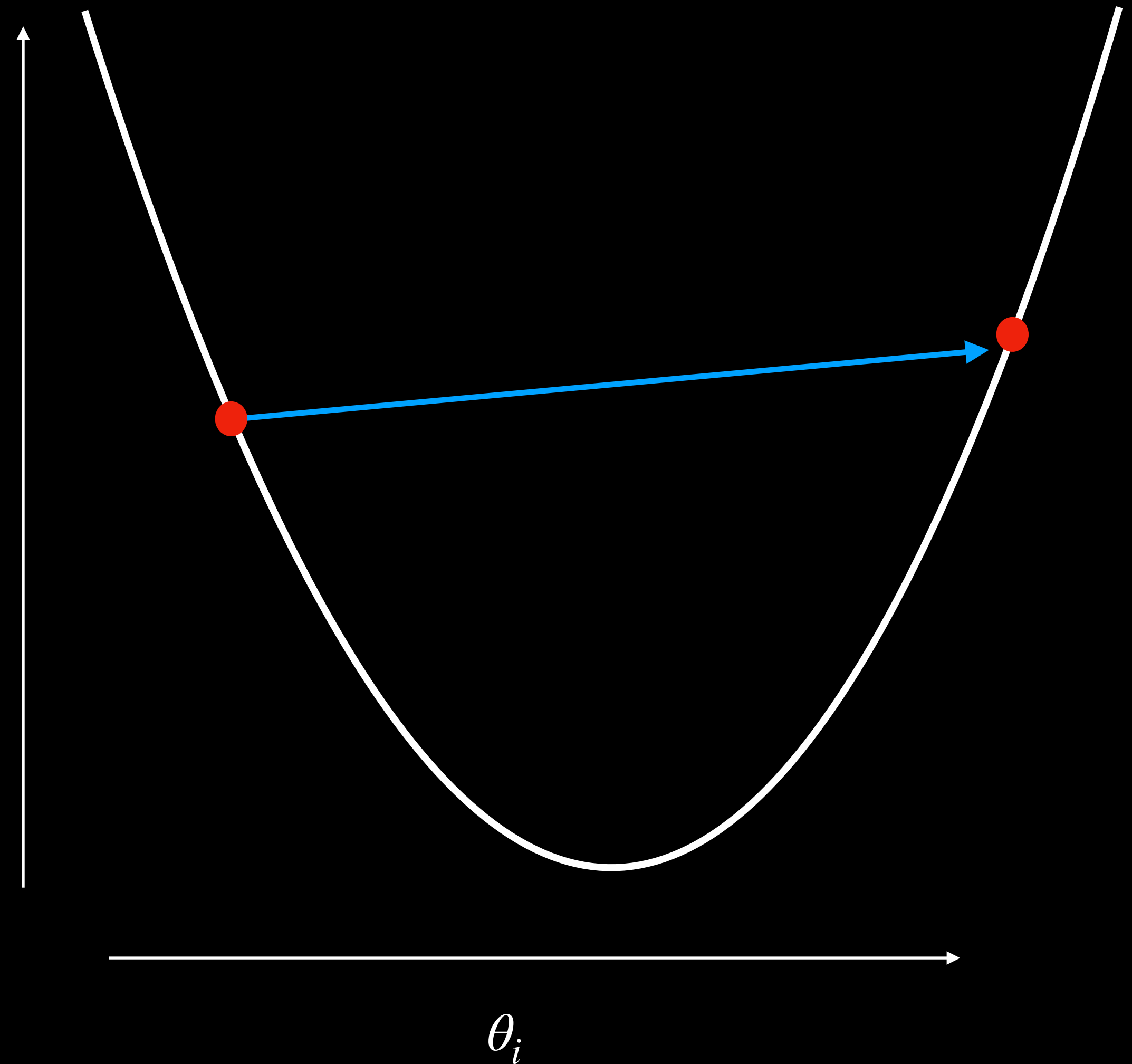
- If  $\alpha$  too large end up diverging



# Minimising J

$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

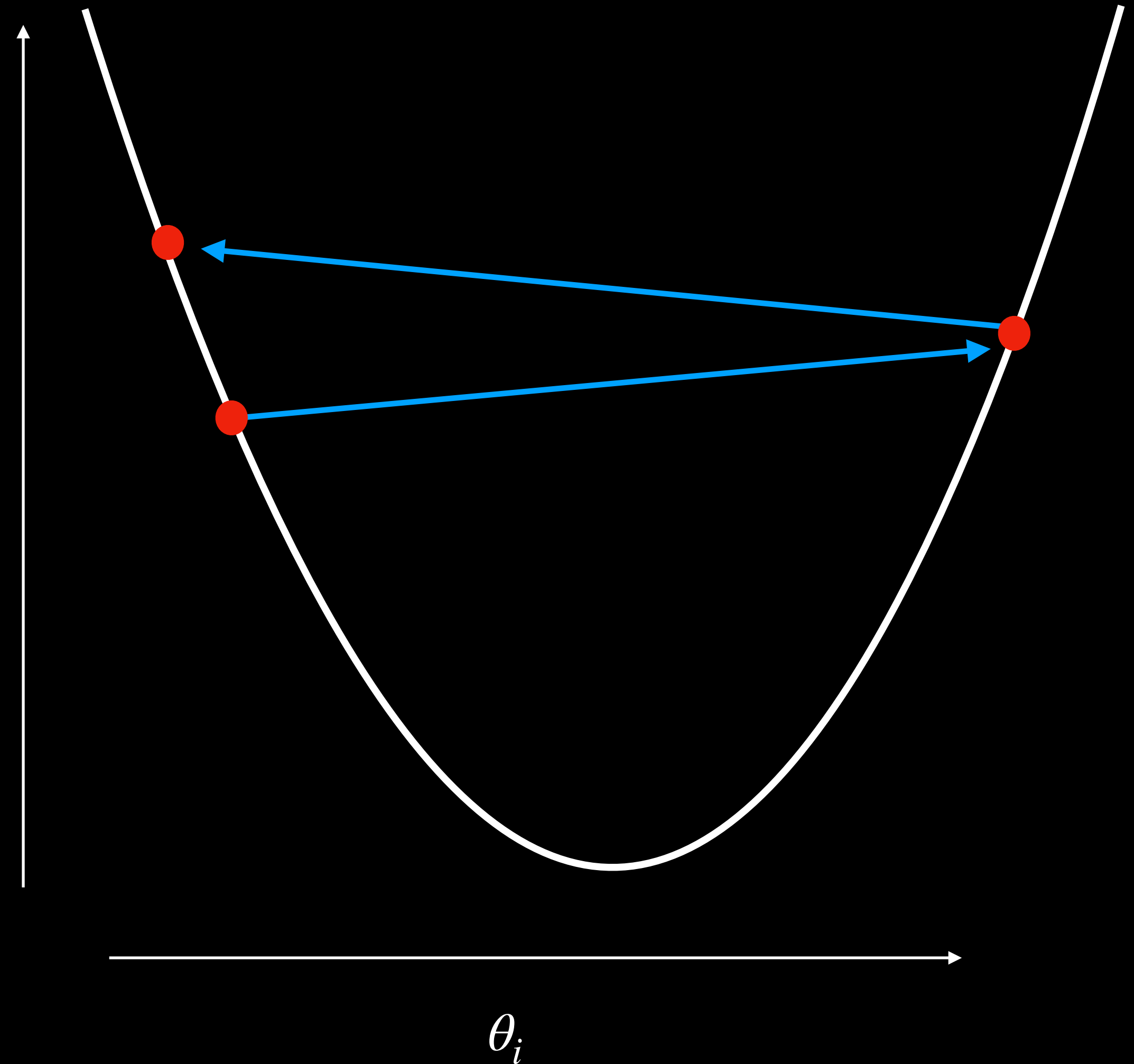
- If  $\alpha$  too large end up diverging



# Minimising J

$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

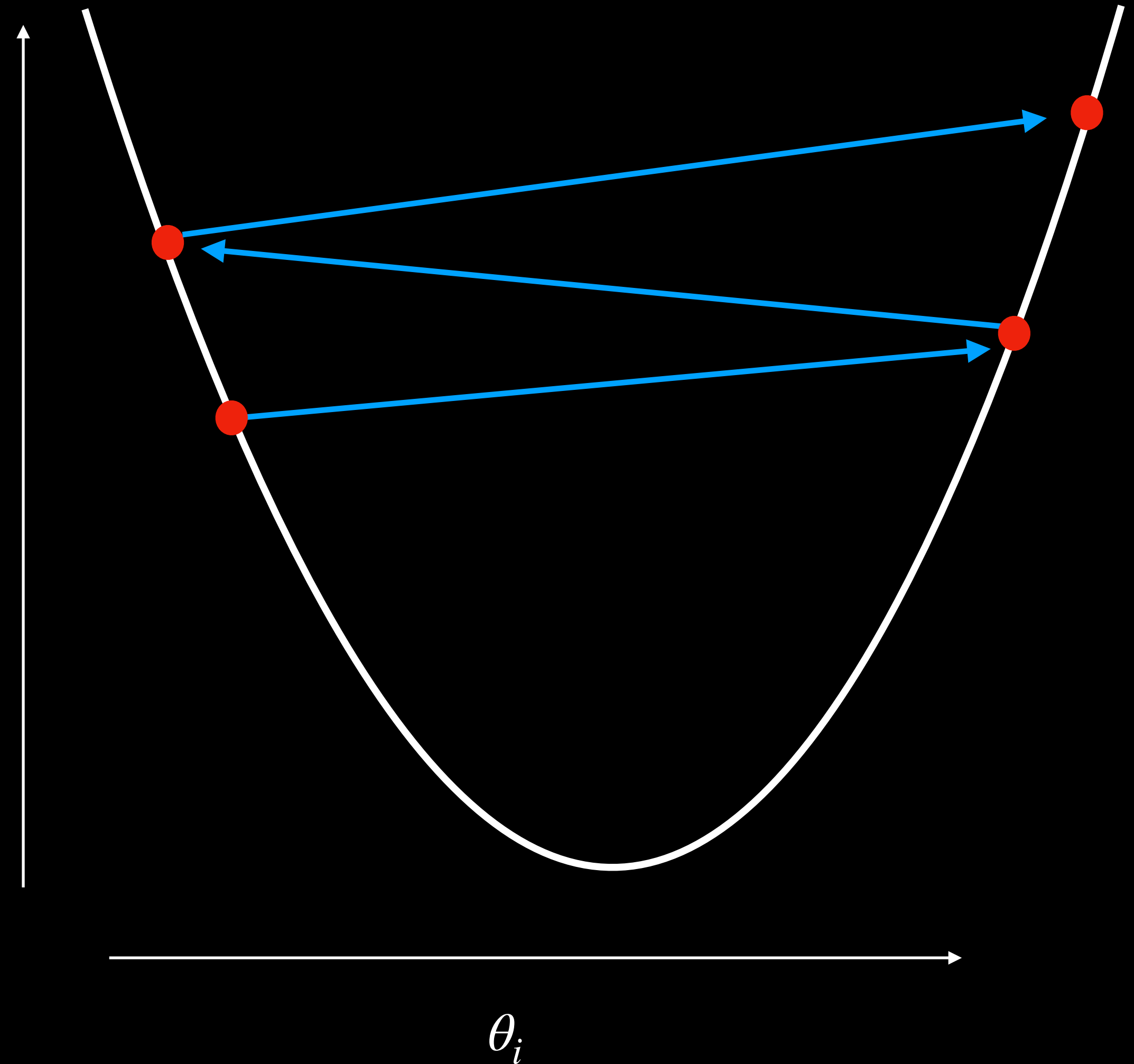
- If  $\alpha$  too large end up diverging



# Minimising J

$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

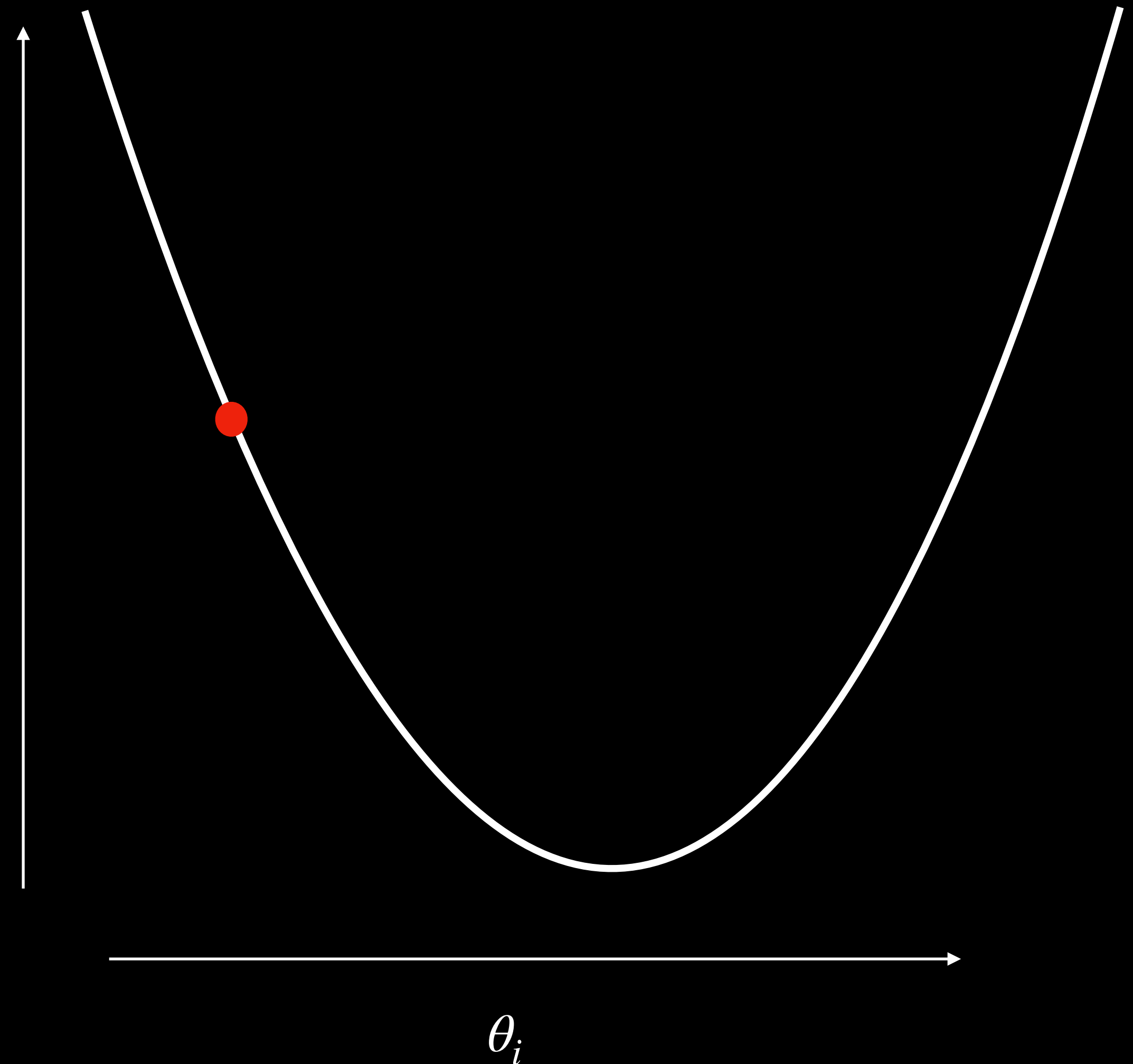
- If  $\alpha$  too large end up diverging



# Minimising J

$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

- If  $\alpha$  too large end up diverging
- If  $\alpha$  too small can take too long to converge

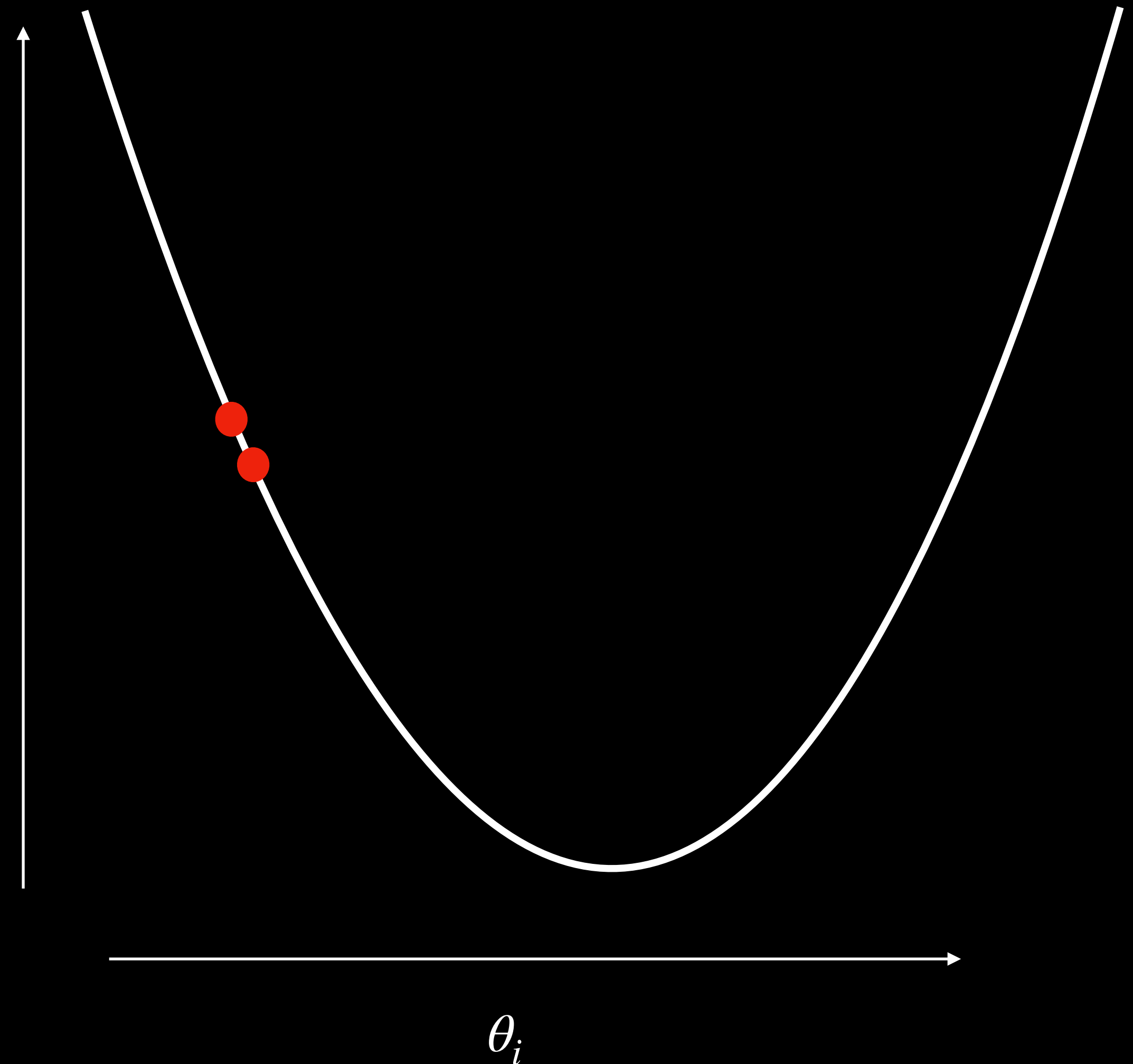




# Minimising J

$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

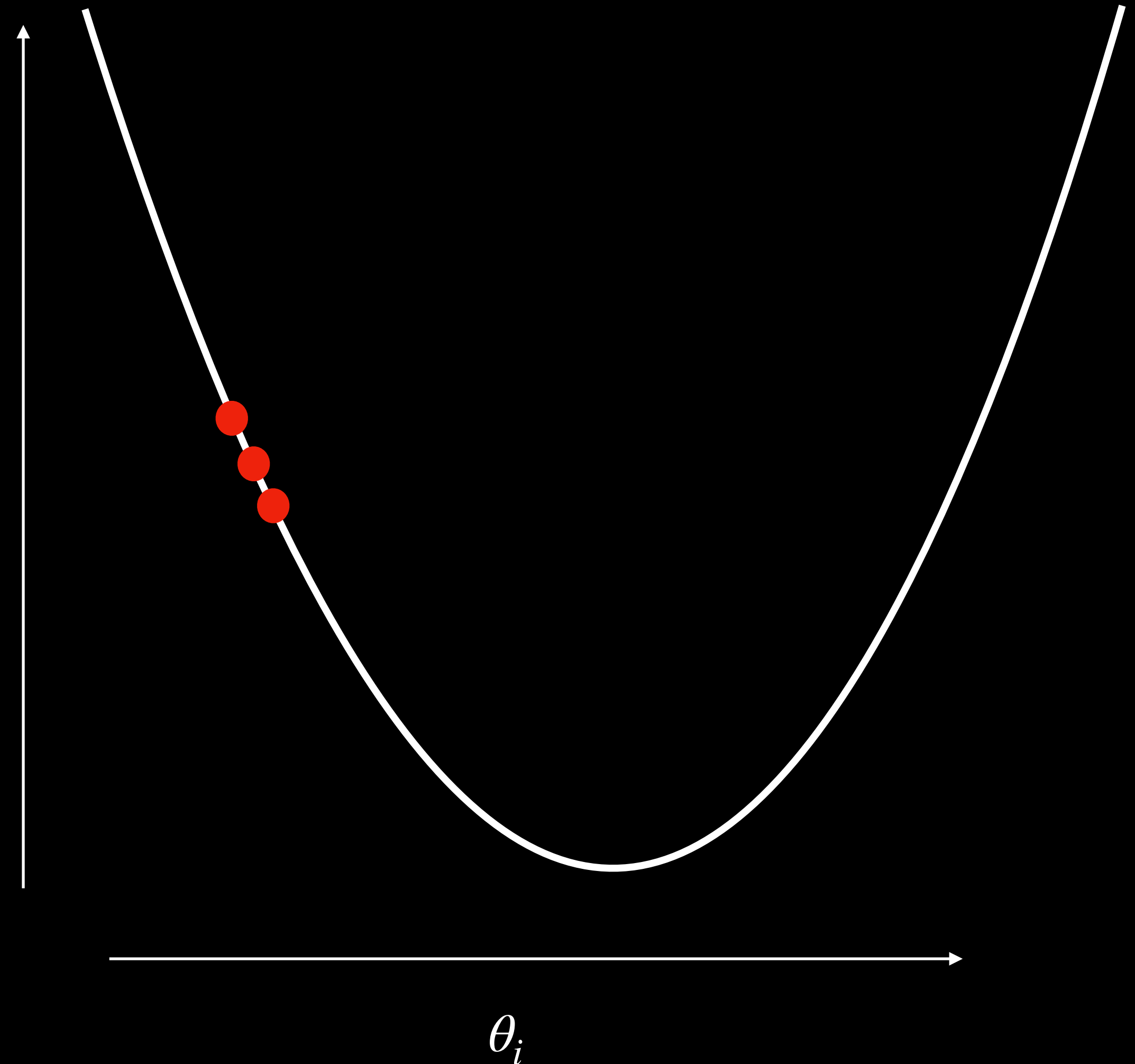
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# Minimising J

$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

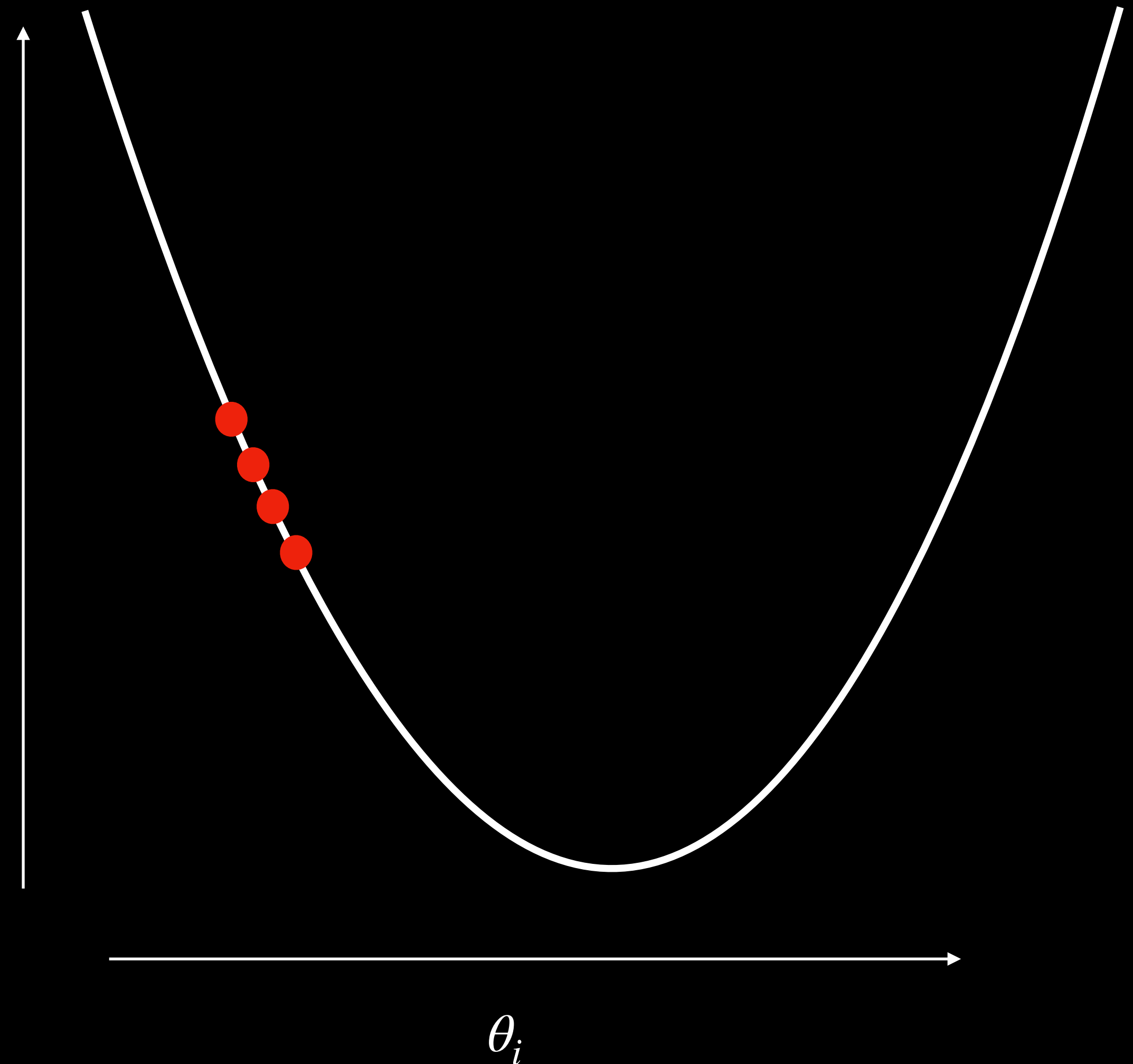
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# Minimising J

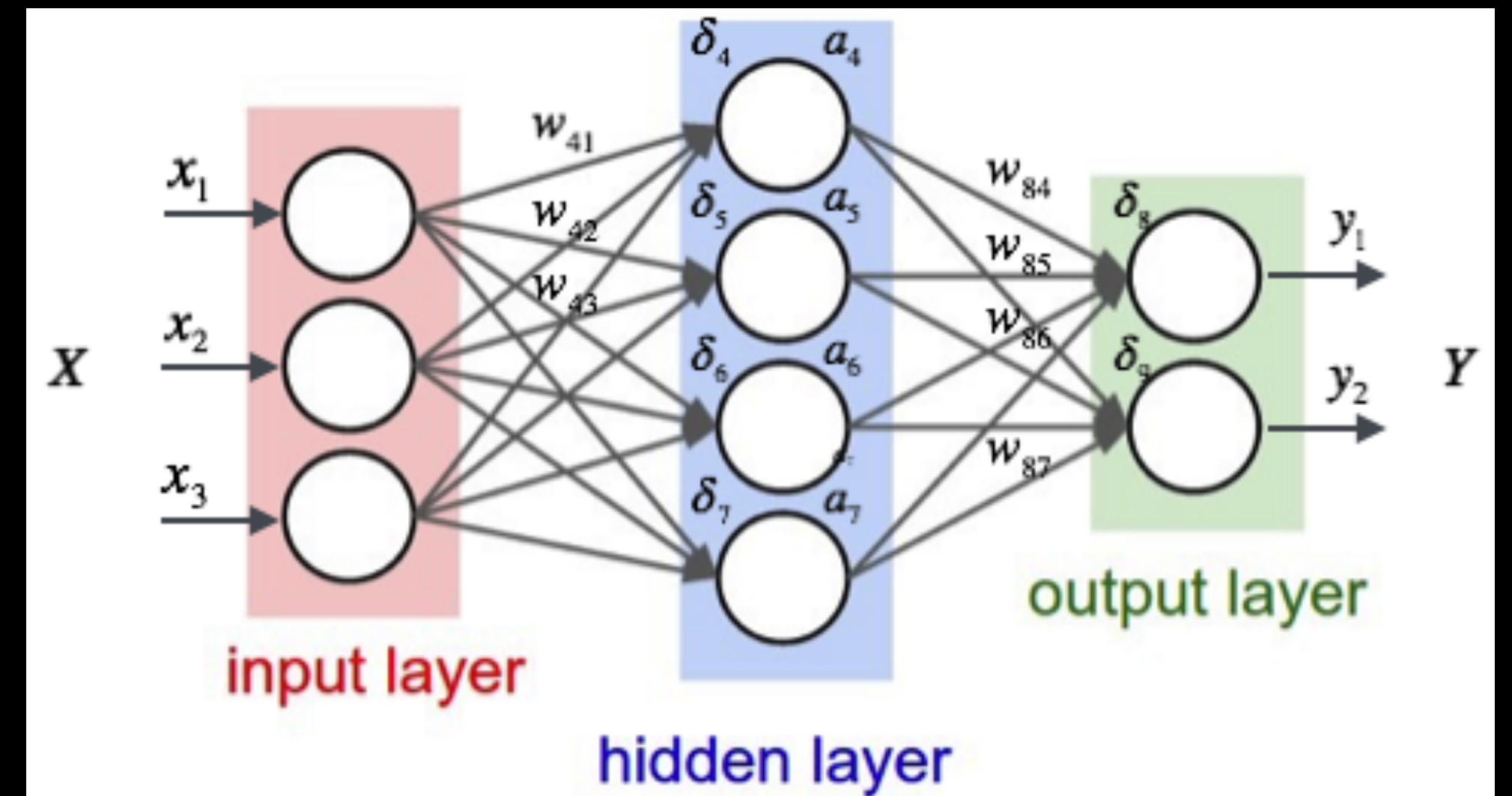
$$\theta_i := \theta_i - \alpha \frac{\delta}{\delta \theta_i} J$$

- If  $\alpha$  too large end up diverging
- If  $\alpha$  too small can take too long to converge



# Gradient descent with NN

- Forward Propagation to calculate J
- Backward propagation:
  - Working backwards from output calculate the 'error' on each node.
  - Use this to calculate  $\frac{\delta}{\delta \theta_i} J$  and update  $\theta_i$
- Repeat



```
In [4]: import keras
from keras.datasets import mnist
from keras.models import Sequential
from keras.layers import Dense, Dropout
from keras.optimizers import RMSprop

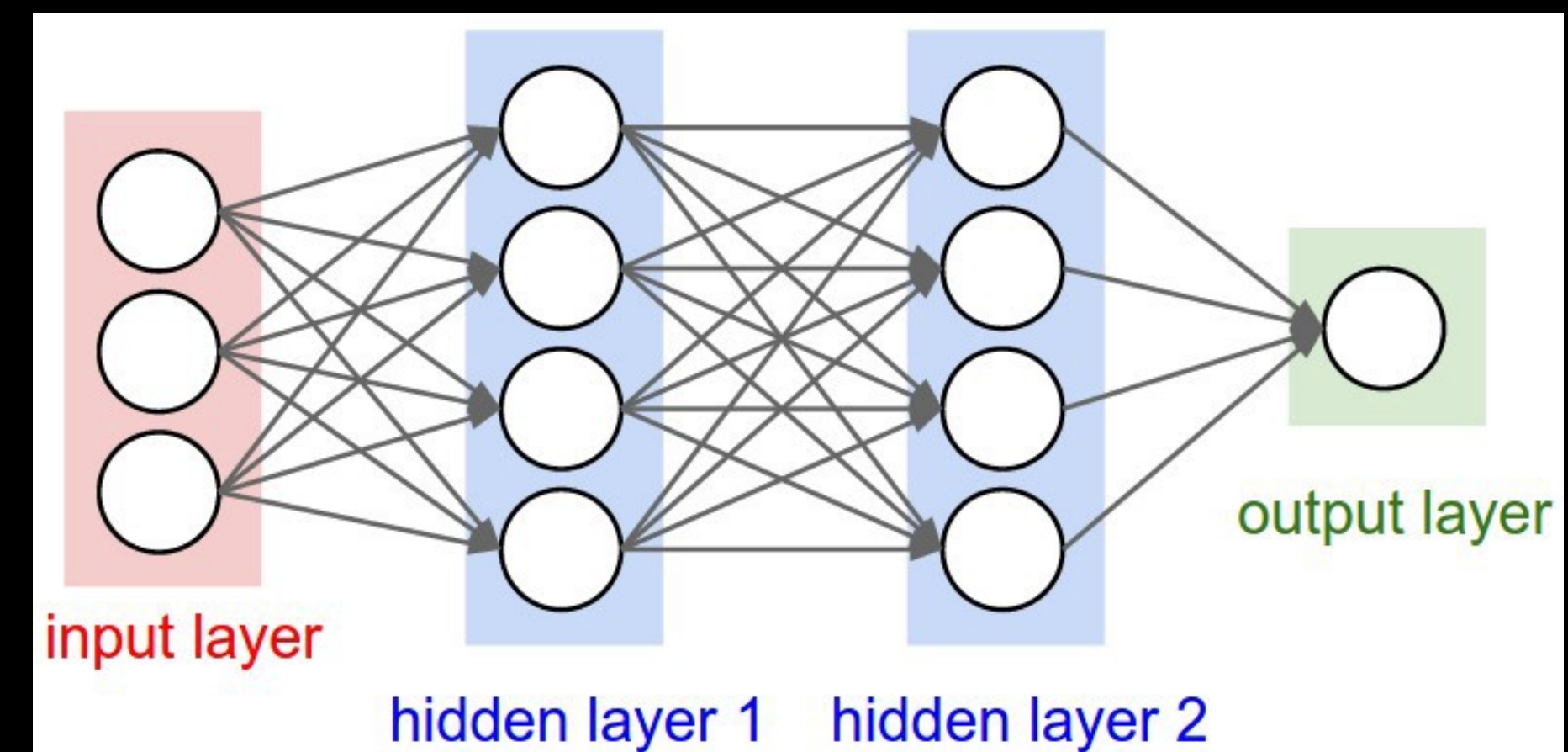
model = Sequential()
model.add(Dense(512, activation='relu', input_shape=(784,)))
model.add(Dropout(0.2))
model.add(Dense(512, activation='relu'))
model.add(Dropout(0.2))
model.add(Dense(num_classes, activation='softmax'))

model.summary()

model.compile(loss='categorical_crossentropy',
              optimizer=RMSprop(),
              metrics=['accuracy'])
```

<https://github.com/keras-team/keras/tree/master/examples>

<https://keras.io/>





```

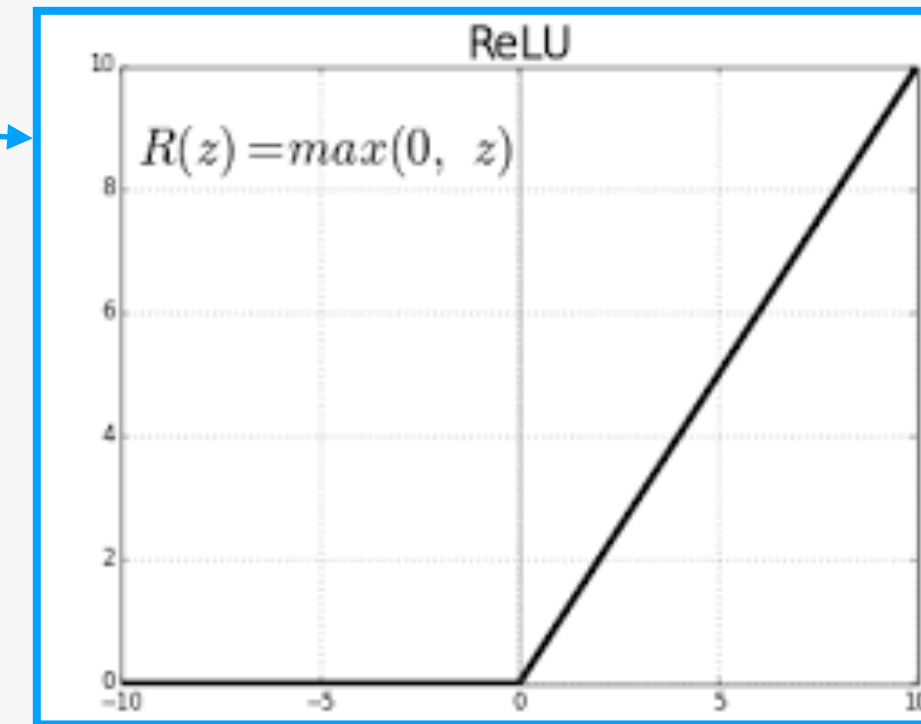
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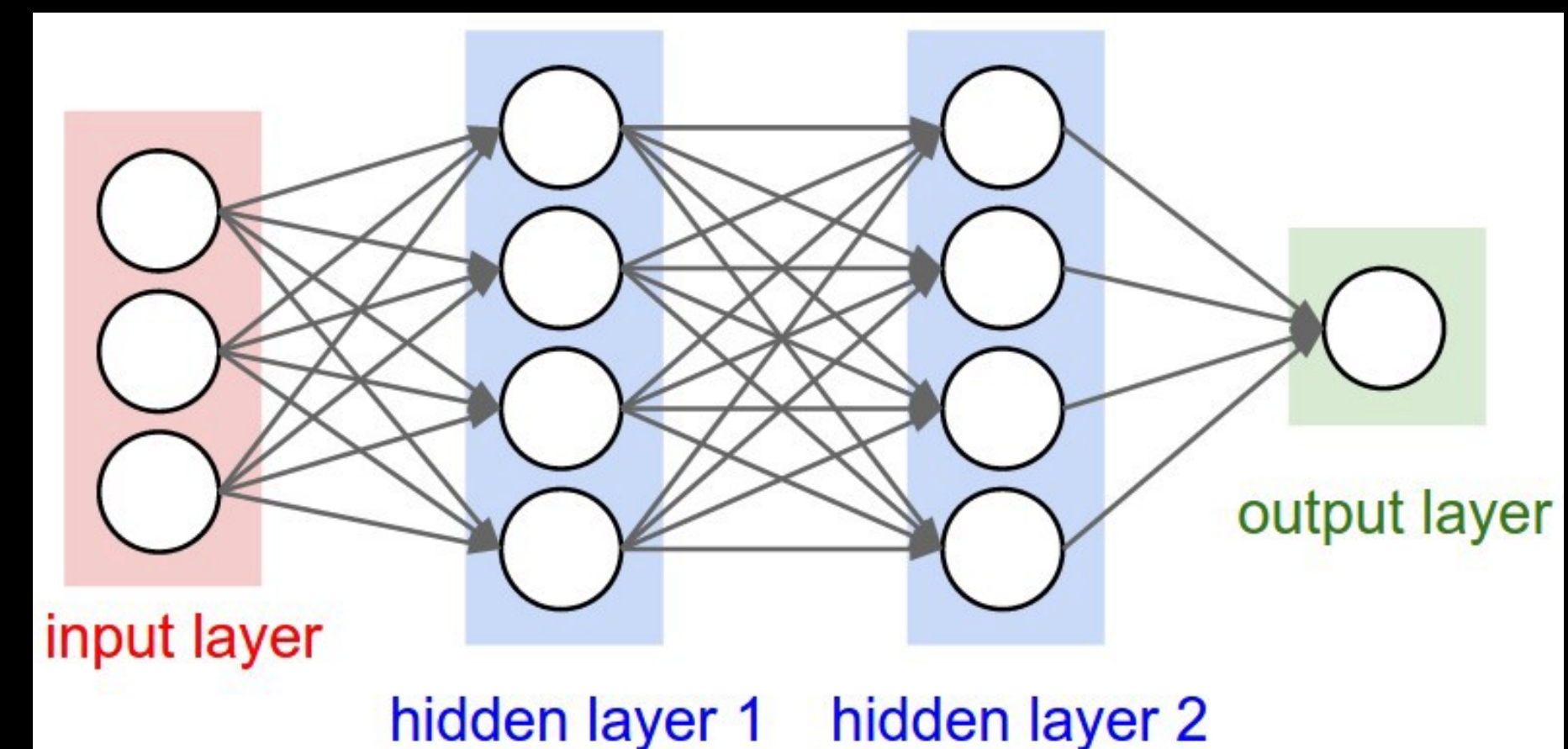
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                      metrics=['accuracy'])

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<https://github.com/keras-team/keras/tree/master/examples>

<https://keras.io/>



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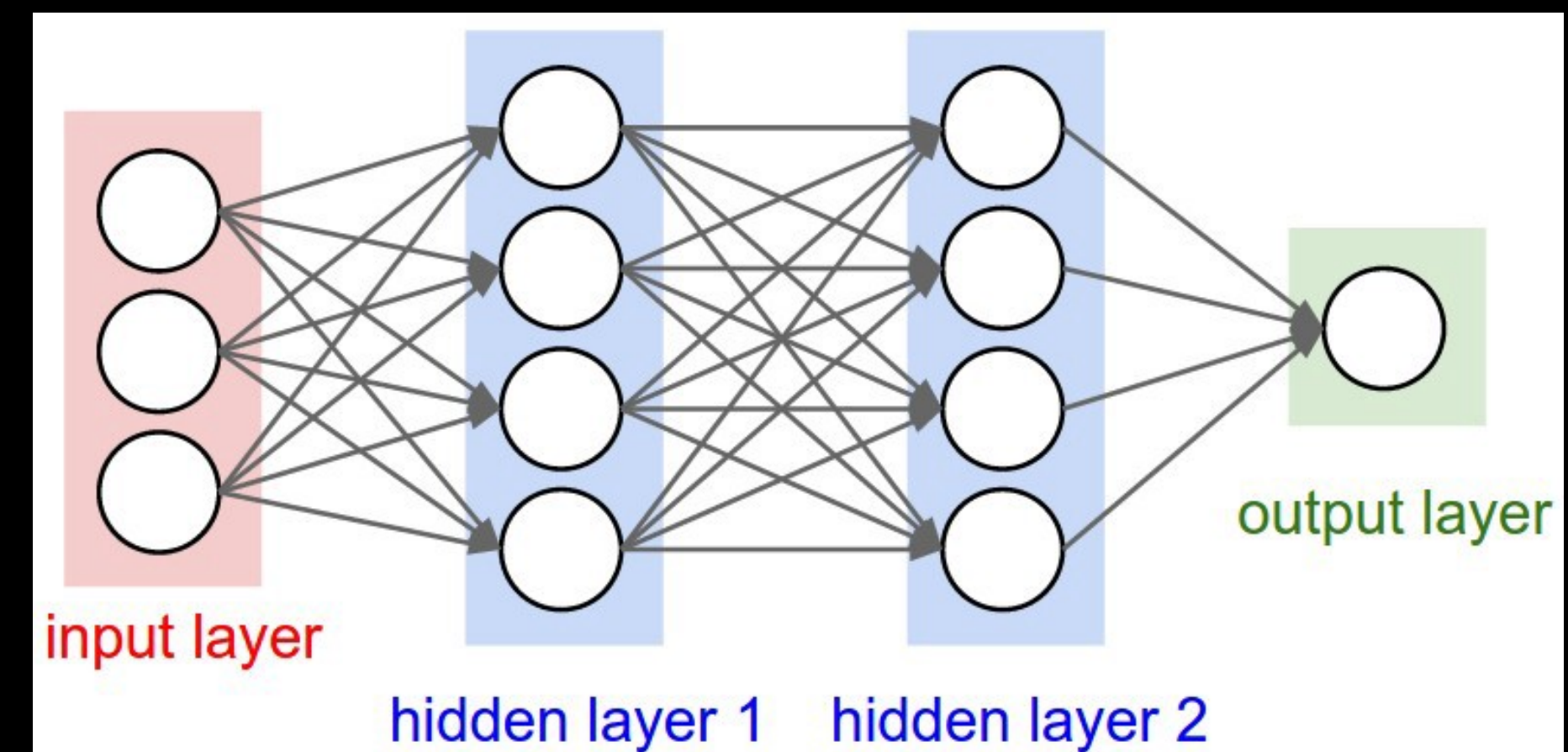
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model.add(Dense(512, activation='relu'))
model.add(Dropout(0.2))
model.add(Dense(num_classes, activation='softmax'))

model.summary()

model.compile(loss='categorical_crossentropy',
              optimizer=RMSprop(),
              metrics=['accuracy'])
```

<https://github.com/keras-team/keras/tree/master/examples>

<https://keras.io/>





In [5]:

```
history = model.fit(x_train, y_train,  
                    batch_size=batch_size,  
                    epochs=epochs,  
                    verbose=1,  
                    validation_data=(x_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] - 6s 92us/step - loss: 0.2488 - acc: 0.9238 - val\_loss: 0.1489 - val\_acc  
: 0.9509

Epoch 2/20

60000/60000 [=====] - 5s 90us/step - loss: 0.1031 - acc: 0.9690 - val\_loss: 0.0789 - val\_acc  
: 0.9743

Epoch 3/20

60000/60000 [=====] - 5s 89us/step - loss: 0.0743 - acc: 0.9772 - val\_loss: 0.0800 - val\_acc  
: 0.9768

Epoch 4/20

60000/60000 [=====] - 5s 90us/step - loss: 0.0590 - acc: 0.9818 - val\_loss: 0.0769 - val\_acc  
: 0.9793

Epoch 5/20

60000/60000 [=====] - 6s 95us/step - loss: 0.0494 - acc: 0.9848 - val\_loss: 0.0887 - val\_acc  
: 0.9778

Epoch 6/20

60000/60000 [=====] - 6s 99us/step - loss: 0.0440 - acc: 0.9866 - val\_loss: 0.0832 - val\_acc  
: 0.9784

Epoch 7/20

60000/60000 [=====] - 6s 101us/step - loss: 0.0383 - acc: 0.9884 - val\_loss: 0.0834 - val\_ac  
c: 0.9812

Epoch 8/20

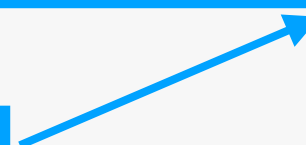
60000/60000 [=====] - 6s 102us/step - loss: 0.0352 - acc: 0.9900 - val\_loss: 0.0793 - val\_ac  
c: 0.9825



In [5]:

```
history = model.fit(x_train, y_train,  
                    batch_size=batch_size,  
                    epochs=epochs,  
                    verbose=1,  
                    validation_data=(x_test, y_test))
```

```
history = model.fit(X_train,y_train,epochs = 50,validation_split=0.10,class_weight=class_weight)
```



Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] - 6s 92us/step - loss: 0.2488 - acc: 0.9238 - val\_loss: 0.1489 - val\_acc: 0.9509

Epoch 2/20

60000/60000 [=====] - 5s 90us/step - loss: 0.1031 - acc: 0.9690 - val\_loss: 0.0789 - val\_acc: 0.9743

Epoch 3/20

60000/60000 [=====] - 5s 89us/step - loss: 0.0743 - acc: 0.9772 - val\_loss: 0.0800 - val\_acc: 0.9768

Epoch 4/20

60000/60000 [=====] - 5s 90us/step - loss: 0.0590 - acc: 0.9818 - val\_loss: 0.0769 - val\_acc: 0.9793

Epoch 5/20

60000/60000 [=====] - 6s 95us/step - loss: 0.0494 - acc: 0.9848 - val\_loss: 0.0887 - val\_acc: 0.9778

Epoch 6/20

60000/60000 [=====] - 6s 99us/step - loss: 0.0440 - acc: 0.9866 - val\_loss: 0.0832 - val\_acc: 0.9784

Epoch 7/20

60000/60000 [=====] - 6s 101us/step - loss: 0.0383 - acc: 0.9884 - val\_loss: 0.0834 - val\_acc: 0.9812

Epoch 8/20

60000/60000 [=====] - 6s 102us/step - loss: 0.0352 - acc: 0.9900 - val\_loss: 0.0793 - val\_acc: 0.9825

In [5]:

```
history = model.fit(x_train, y_train,  
                    batch_size=batch_size,  
                    epochs=epochs,  
                    verbose=1,  
                    validation_data=(x_test, y_test))
```

`history = model.fit(X_train,y_train,epochs = 50,validation_split=0.10,class_weight=class_weight)`

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] - 6s 92us/step - loss: 0.2488 - acc: 0.9238 - val\_loss: 0.1489 - val\_acc: 0.9509

Epoch 2/20

60000/60000 [=====] - 5s 90us/step - loss: 0.1031 - acc: 0.9690 - val\_loss: 0.0789 - val\_acc: 0.9743

Epoch 3/20

60000/60000 [=====] - 5s 89us/step - loss: 0.0743 - acc: 0.9772 - val\_loss: 0.0800 - val\_acc: 0.9768

Epoch 4/20

60000/60000 [=====] - 5s 90us/step - loss: 0.0590 - acc: 0.9818 - val\_loss: 0.0769 - val\_acc: 0.9793

Epoch 5/20

60000/60000 [=====] - 6s 95us/step - loss: 0.0494 - acc: 0.9848 - val\_loss: 0.0887 - val\_acc: 0.9778

Epoch 6/20

60000/60000 [=====] - 6s 99us/step - loss: 0.0440 - acc: 0.9866 - val\_loss: 0.0832 - val\_acc: 0.9784

Epoch 7/20

60000/60000 [=====] - 6s 101us/step - loss: 0.0383 - acc: 0.9884 - val\_loss: 0.0834 - val\_acc: 0.9812

Epoch 8/20

60000/60000 [=====] - 6s 102us/step - loss: 0.0352 - acc: 0.9900 - val\_loss: 0.0793 - val\_acc: 0.9825

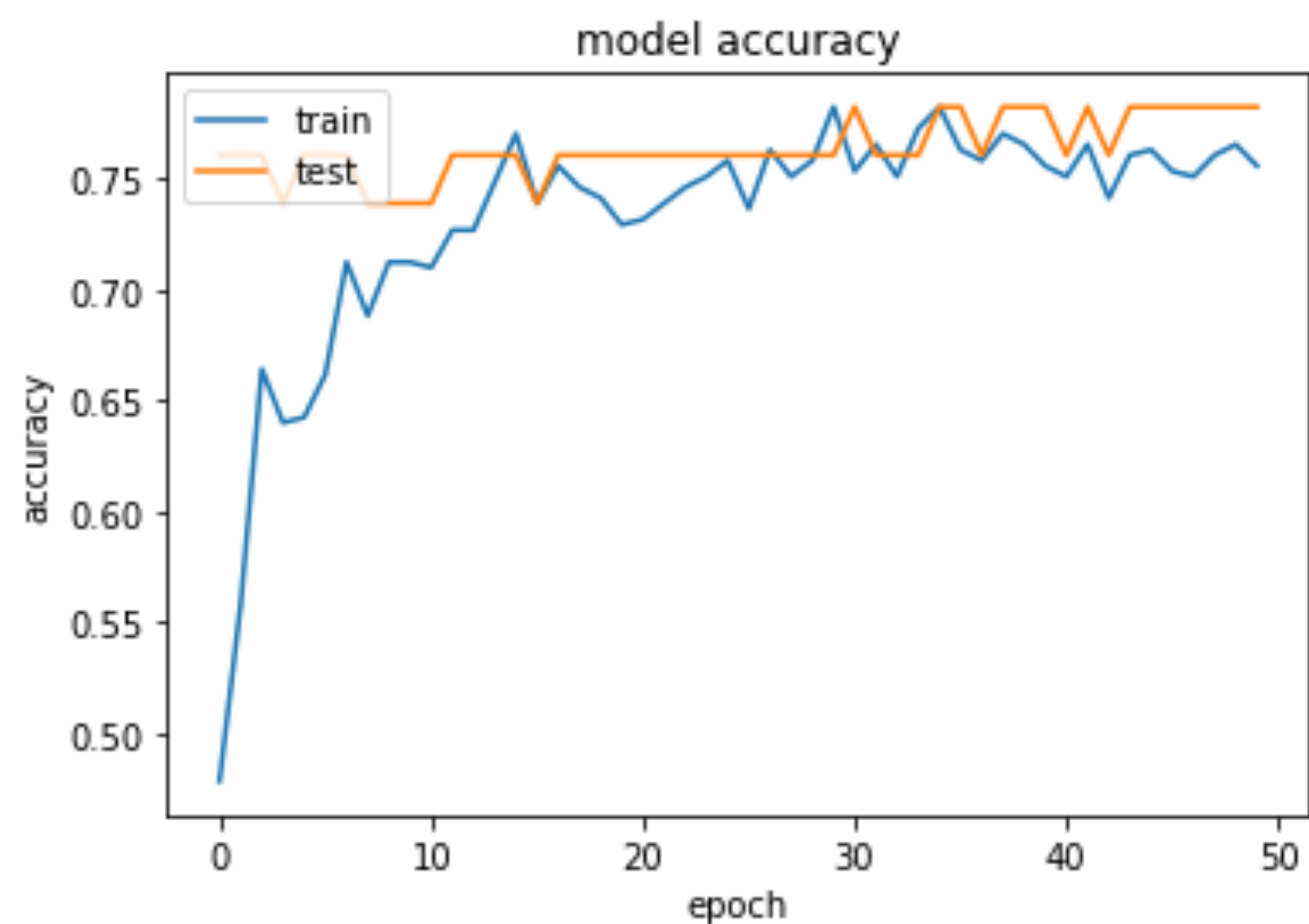
In [6]:

```
score = model.evaluate(x_test, y_test, verbose=0)  
print('Test loss:', score[0])  
print('Test accuracy:', score[1])
```

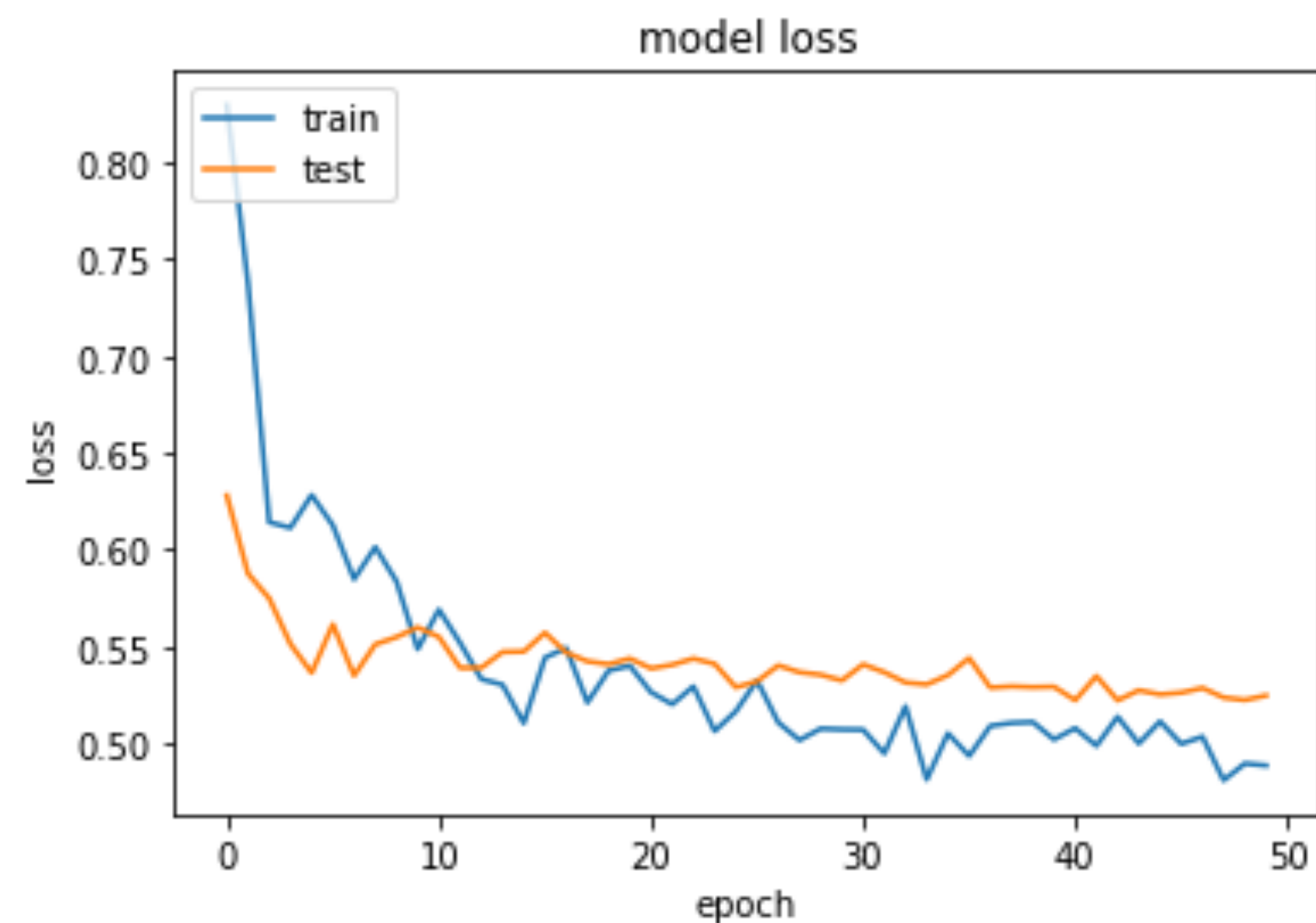
Test loss: 0.11100612514  
Test accuracy: 0.9832



```
In [18]: plt.plot(history.history['acc'])
plt.plot(history.history['val_acc'])
plt.title('model accuracy')
plt.ylabel('accuracy')
plt.xlabel('epoch')
plt.legend(['train', 'test'], loc='upper left')
plt.show()
```



```
# summarize history for loss
plt.plot(history.history['loss'])
plt.plot(history.history['val_loss'])
plt.title('model loss')
plt.ylabel('loss')
plt.xlabel('epoch')
plt.legend(['train', 'test'], loc='upper left')
plt.show()
```



# Useful references

- Free coursera course: <https://www.coursera.org/learn/machine-learning>
- Keras documentation: <https://keras.io/>
- Previous NN hack night: <https://github.com/JBCA-MachineLearning/Typhoons-and-Hurricanes-Hacknight>
- More info on activation functions: <https://medium.com/the-theory-of-everything/understanding-activation-functions-in-neural-networks-9491262884e0>