

# Model Predictive Control for Autonomous Vehicles

TSFS12: Autonomous Vehicles – Planning, Control,  
and Learning Systems

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# Purpose of this Lecture

- Give a **background on receding-horizon control and model predictive control (MPC)**.
- Discuss different types of **MPC problems for autonomous vehicles** and how they can be **solved**.
- Illustrate use of **MPC for trajectory-tracking and path-following applications**.

## Expected Take-Aways from this Lecture

- Basic **knowledge** about **model predictive control**.
- Be familiar with **applications of MPC for autonomous vehicles**.
  - **Trajectory tracking.**
  - **Path following.**
- Familiarity with **solution of MPC problems**.

# Literature Reading

The following book and article sections are the main reading material for this lecture. References to further reading are provided throughout the slides and at the end of the lecture slides.

- Chapter 1 in Rawlings, J. B., D. Q. Mayne, & M. Diehl: *Model Predictive Control: Theory, Computation, and Design*. 2nd Edition. Nob Hill Publishing, 2017.
- Section V-C in Paden, B., Čáp, M., Yong, S. Z., Yershov, D., & Frazzoli, E: "A survey of motion planning and control techniques for self-driving urban vehicles". *IEEE Transactions on Intelligent Vehicles*, 1(1), 33-55, 2016.

## Outline of the Lecture

- **Application example:** Look-ahead control for autonomous trucks.
- **Fundamentals of model predictive control (MPC).**
- **Trajectory tracking** for autonomous vehicles using **MPC**.
- **Path following** for autonomous vehicles using **MPC**.
- **Tools and libraries for optimization and MPC.**

# Context in the Architecture for an Autonomous Vehicle

**Model predictive control is possible to apply at several different layers in the architecture for different tasks.**

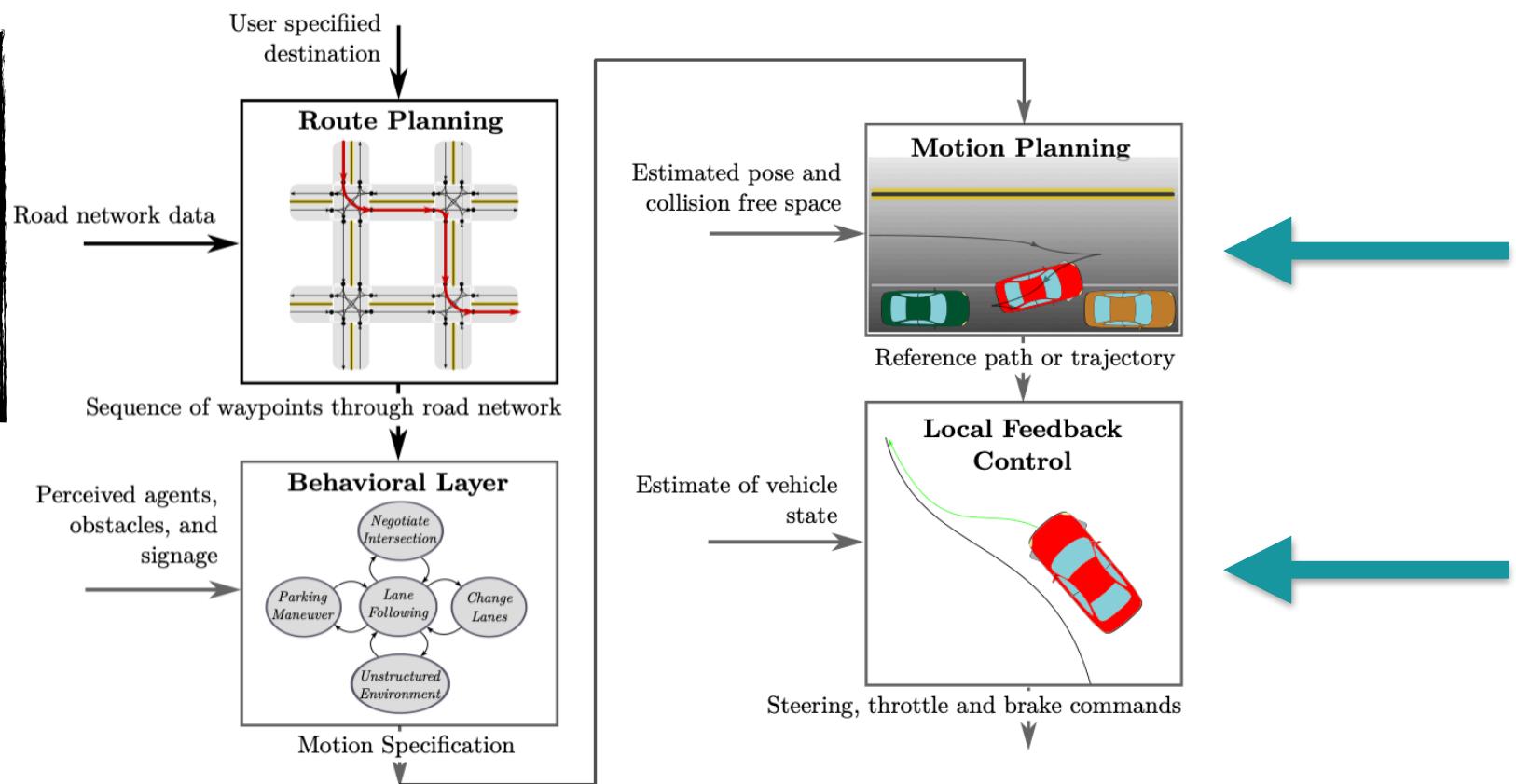


Figure from: Paden, B., Čáp, M., Yong, S. Z., Yershov, D., & Frazzoli, E: "A survey of motion planning and control techniques for self-driving urban vehicles". IEEE Transactions on Intelligent Vehicles, 1(1), 33-55, 2016.

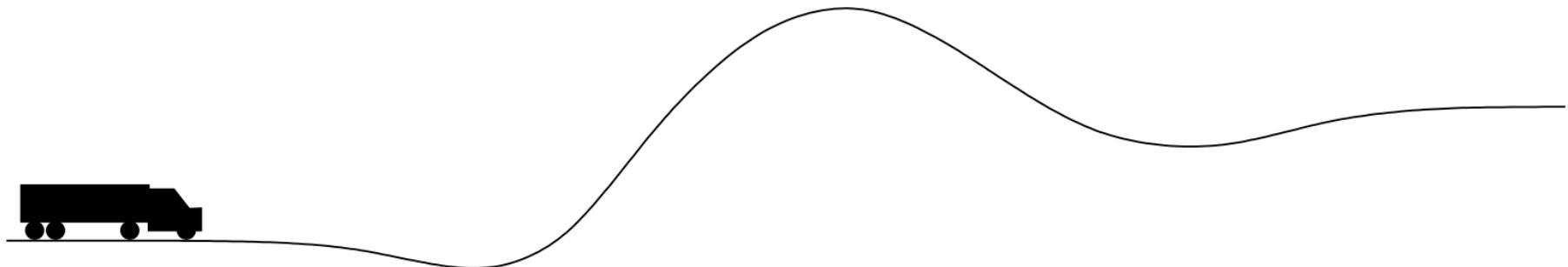
# Overview of Control Methods for Autonomous Vehicles

	Controller		Model	Stability	Time Complexity	Comments/Assumptions
Lecture 6	Pure Pursuit	(V-A1)	Kinematic	LES* to ref. path	$O(n)^*$	No path curvature
	Rear wheel based feedback	(V-A2)	Kinematic	LES* to ref. path	$O(n)^*$	$C^2(\mathbb{R}^n)$ ref. paths
	Front wheel based feedback	(V-A3)	Kinematic	LES* to ref. path	$O(n)^*$	$C^1(\mathbb{R}^n)$ ref. paths; Forward driving only
Lecture 6	Feedback linearization	(V-B2)	Steering rate controlled kinematic	LES* to ref. traj.	$O(1)$	$C^1(\mathbb{R}^n)$ ref. traj.; Forward driving only
	Control Lyapunov design	(V-B1)	Kinematic	LES* to ref. traj.	$O(1)$	Stable for constant path curvature and velocity
This lecture	Linear MPC	(V-C)	$C^1(\mathbb{R}^n \times \mathbb{R}^m)$ model <sup>#</sup>	LES* to ref. or path	$O\left(\sqrt{N} \ln\left(\frac{N}{\varepsilon}\right)\right)^\dagger$	Stability depends on horizon length
This lecture	Nonlinear MPC	(V-C)	$C^1(\mathbb{R}^n \times \mathbb{R}^m)$ model <sup>#</sup>	Not guaranteed	$O\left(\frac{1}{\varepsilon}\right)^\ddagger$	Works well in practice

# Application Example: Look-Ahead Control for Autonomous Trucks

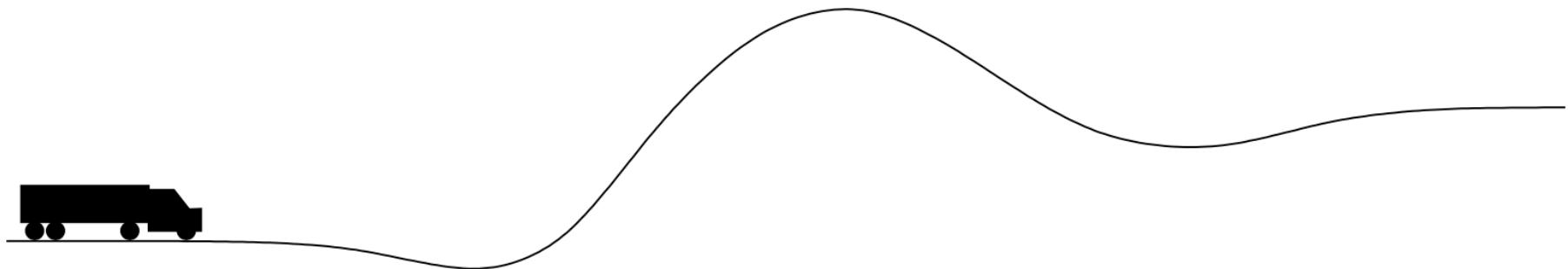
## Scenario for Look-Ahead Control (1/2)

- An autonomous **long-haulage heavy truck** on open road.
- An **on-board road topography map** and **on-board GPS**.
- **Control inputs:** engine torque, brake torque, and gear.



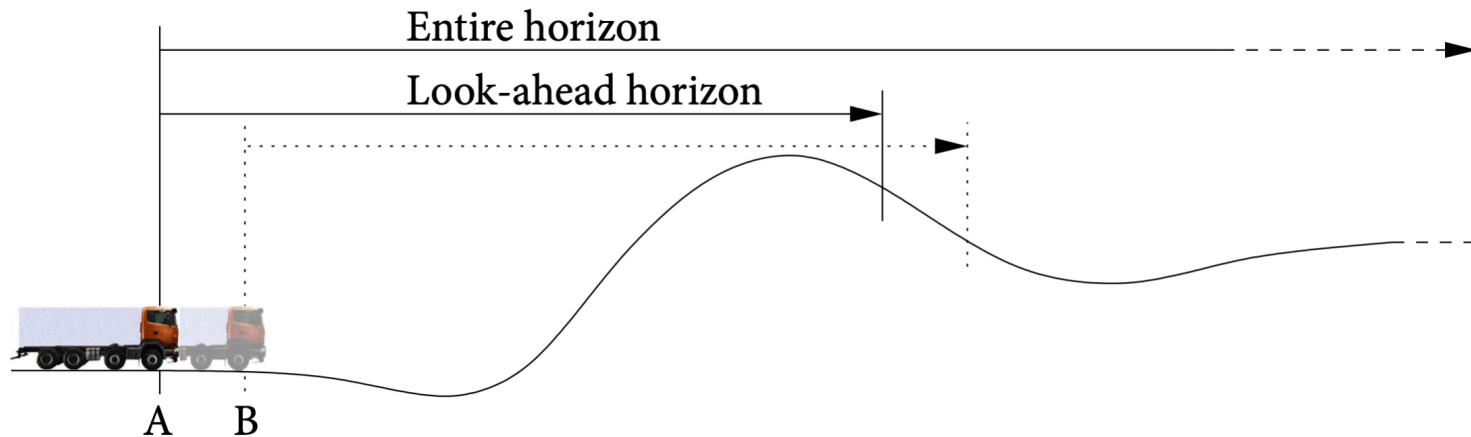
## Scenario for Look-Ahead Control (2/2)

- Objective: **Minimize consumption of fuel**, with a maximum trip time.
  - An **autonomous cruise controller** with **optimal control actions**, considering road topography.
- Can be formalized as a **mathematical optimization problem** (recall Lecture 5).



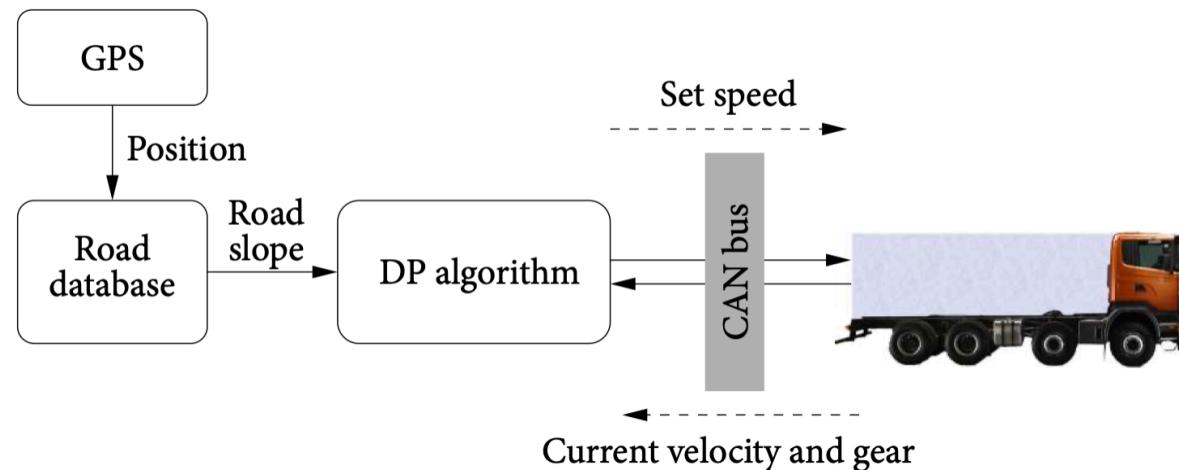
# Receding Horizon

- Objective is to **minimize a performance criterion** over the complete driving mission – however, **high computational cost** and **uncertainty**.
- Consider a **finite horizon** (look-ahead horizon) at point A, **solve problem** and **apply control inputs** in first part of the interval until point B, then **solve problem again** at point B over a shifted finite horizon – i.e., a **receding horizon**.



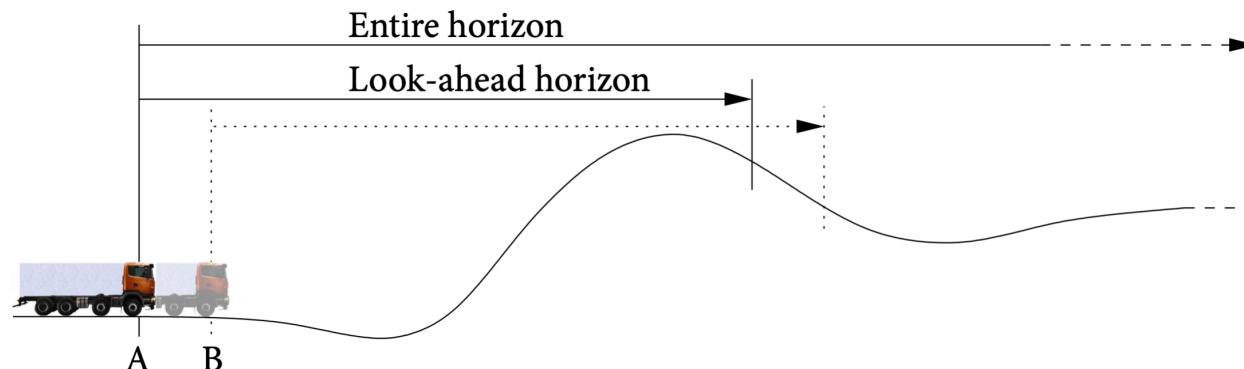
# Application of Look-Ahead Control

- A **GPS** gives the position of the autonomous truck.
- A **road data base** gives the road slope ahead of the vehicle from the current position over a **limited distance ahead of the vehicle** (typically about 2 km).
- A dynamic programming (DP) algorithm, together with a **model of the vehicle**, is used to **determine the control inputs** that minimize the fuel consumption over this intervall.



# Key Characteristics of Look-Ahead Control

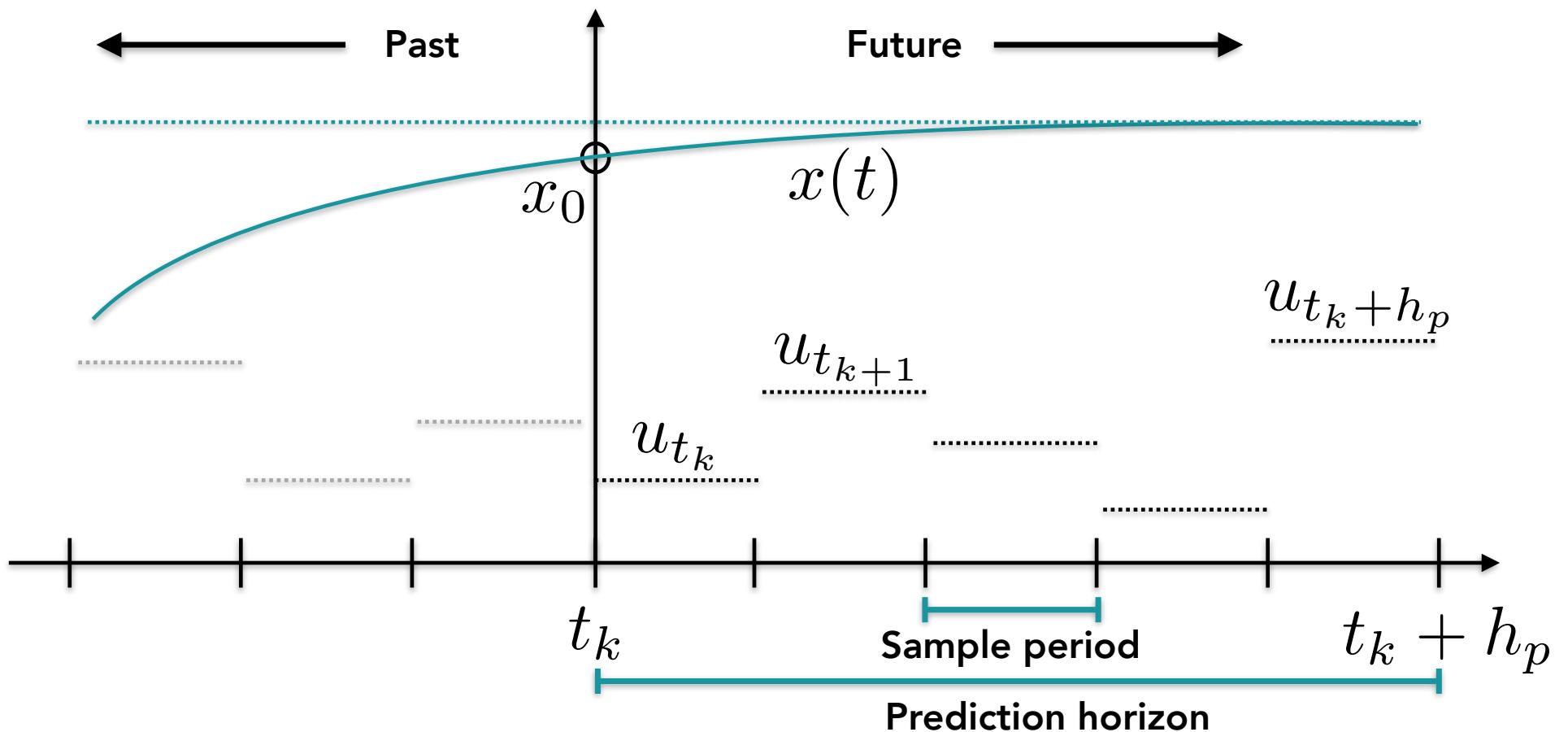
- The look-ahead approach can take into account **limitations on control signals** and **internal vehicle states**.
- In this driving segment, the truck will **accelerate before an uphill** if it is necessary to avoid that the **speed decreases below a lower limit** on speed because of limits on the propelling force.



- Can handle **changing driving conditions**, thanks to re-optimization.

# Fundamentals of Model Predictive Control

# Model Predictive Control (MPC)



# Key Concepts in MPC (1/2)

- A **model** of the system is used for **prediction**.
- The length of the finite horizon is called the **prediction horizon**.
- The **control horizon** is the **subset of the prediction horizon** over which the control inputs are allowed to vary.
  - Could also be the **full set**, i.e., the horizons coincide.

# Key Concepts in MPC (2/2)

- **Cost function** describing desired **objective** and **constraints** on control inputs and states.
- User-specified **weights** often used to trade **different components** to each other in the **cost function** of the MPC formulation.

# Recall Optimization Problem from Lecture 5

- **Optimization problem** over time horizon  $[0, T]$ , where  $T$  possibly is a free optimization variable:

$$\begin{aligned}
 & \text{minimize} && \int_0^T L(x(t), u(t)) dt + \Gamma(x(T)) \\
 & \text{subject to} && x(0) = x_0, \dot{x}(t) = f(t, x(t), u(t)), \\
 & && x(t) \in \mathbb{X}, u(t) \in \mathbb{U}, x(T) \in \mathbb{X}_T, t \in [0, T]
 \end{aligned}$$

Lagrange integrand
Mayer term

Initial conditions
System dynamics

State and control constraints
Terminal constraints

# A General MPC Formulation

- Solve **optimization problem** over the **time horizon**  $[t_k, t_k + h_p]$  at selected time instants  $t_k, k = 0, 1, 2, \dots$ , with  $h_p$  the prediction horizon:

$$\text{minimize} \quad \int_{t_k}^{t_k+h_p} L(x(t), u(t)) dt + \Gamma(x(t_k + h_p))$$

subject to  $x(t_k) = x_0, \dot{x}(t) = f(t, x(t), u(t)),$   
 $x(t) \in \mathbb{X}, u(t) \in \mathbb{U}, x(t_k + h_p) \in \mathbb{X}_{t_k + h_p}, t \in [t_k, t_k + h_p]$

- The **computed control inputs** are applied until next time step  $t_{k+1}$ .
- The **current states**  $x_0$  are updated at each new iteration.

# A QP MPC Formulation (1/2)

- Recall the **LQR control design** from Lecture 6. Assume a **linear model**:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

- Choose the **cost function** as the following:

$$L(x(t), u(t)) = x^T(t)Qx(t) + u(t)^T Ru(t)$$

$$\Gamma(x(t_k + h_p)) = x(t_k + h_p)^T P_f x(t_k + h_p)$$

## A QP MPC Formulation (2/2)

- The possibly time-varying **weight matrices**  $Q, R, P_f$  are positive (semi-)definite and are specified by the user.
  - Trade **different objectives** to each other.
  - With all remaining constraints linear or affine, the problem is a **quadratic program (QP)**, i.e., a convex optimization problem.

# Model Considerations in MPC

- The model equations of the system appear as a **constraint**, therefore imply properties of the resulting **optimization problem**.
- Recall the discussion on **convex vs. non-convex optimization** from Lecture 5.
- **Non-linear models** possible to include in the **MPC formulation**.
- **Linearizing a non-linear model** along a **nominal/reference trajectory** leads to a linear time-varying (LTV) system:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

# Hard vs. Soft Constraints

- Explicit constraint that needs to be fulfilled exactly – **hard constraint**.
- **Penalty in cost function** with certain weight – **soft constraint** (fulfil it as good as possible, given other objectives).
  - Could be beneficial in real-time applications.
- **Slack variables for constraints** can be introduced in the MPC formulation to avoid infeasible problems because of **constraints**.

# Hard vs. Soft Constraints – Example

- Assume that the speed  $v$  of an autonomous car should be kept close to a **desired reference velocity**  $v_{\text{ref}}$ .
- A **soft constraint** can be formulated as a component in the **cost function** as:

$$\int_{t_k}^{t_k + h_p} (v - v_{\text{ref}})^2 \, dt$$

- A **hard constraint** can be formulated as an **explicit constraint** as:

$$v_{\text{ref}} - \varepsilon \leq v \leq v_{\text{ref}} + \varepsilon$$

# Discretization of MPC Problem

- **Continuous-time MPC problems** need to be transformed to **finite-dimensional optimization problem** (a non-linear program, **NLP**) for solution in a computer.
- Recall the methods from Lecture 5:
  - Direct **multiple shooting** or **collocation**.
  - **Discretization of system dynamics** and **cost function**, with polynomial approximation of control inputs and possibly states.

# Computational Considerations

- A **longer prediction horizon**  $h_p$  implies a larger optimization problem, and thus higher computational cost.
- Also the **length of the sampling period**  $\Delta t = t_{k+1} - t_k$ , i.e., how often the MPC problem is solved, affects the computational cost.
- A **control horizon shorter than the prediction horizon** can be used to save computational resources.

# Stability of MPC

- **Length of prediction horizon** important in practice.
  - Sufficiently long horizon avoids short-term optimization.
- Choice of **cost** or **constraint** at **end of prediction horizon**.
  - Compare with having a sufficiently good **cost-to-go estimate**.
- **Extensive theory** exists for different formulations, see references for further details.

# Why MPC for Autonomous Vehicles?

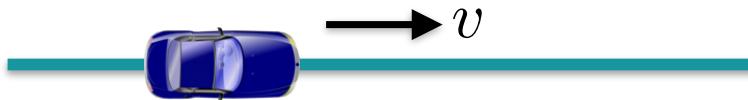
- Constantly **changing environment** with **inherent uncertainty** typical for autonomous vehicles.
- Different **sensor systems** provide updated information periodically (**situation awareness**).
- MPC offers **re-planning capabilities** combined with **optimal feedback control**.



# MPC for Autonomous Trajectory Tracking

# Trajectory Tracking vs. Path Following

- Recall the difference between a **path** (curve in space) and a **trajectory** (time-parameterized).
- **MPC** can be formulated both for **trajectory tracking** and **path-following** applications.
- Consider the **example of driving along a straight line**.
  - Driving exactly at the straight line with a lower speed than according to the reference trajectory, gives a large trajectory error but no path error.



# Problem Formulation for Trajectory Tracking

- Assume that we have **reference values**  $x_{\text{ref}}$  for the **states** that should be tracked.
- An **MPC cost function** for trajectory tracking could then be:

$$\int_{t_k}^{t_k + h_p} \left\{ (x(t) - x_{\text{ref}}(t))^T Q (x(t) - x_{\text{ref}}(t)) + u(t)^T R u(t) \right\} dt$$

# Example Constraints in Trajectory Following

- A key feature of MPC is that **constraints** can be included in the formulation and thus considered in the controller.
- **Linear constraints** on **inputs** and **states** formulated as:

$$u_{\min} \leq u(t) \leq u_{\max}$$

$$x_{\min} \leq x(t) \leq x_{\max}$$

- Input can be **acceleration** and states could be **position** and **velocity** for an autonomous vehicle, thus specifying, e.g., **maximum forward acceleration** or **not allowed areas in geometric space**.

# MPC for Kinematic Path Following

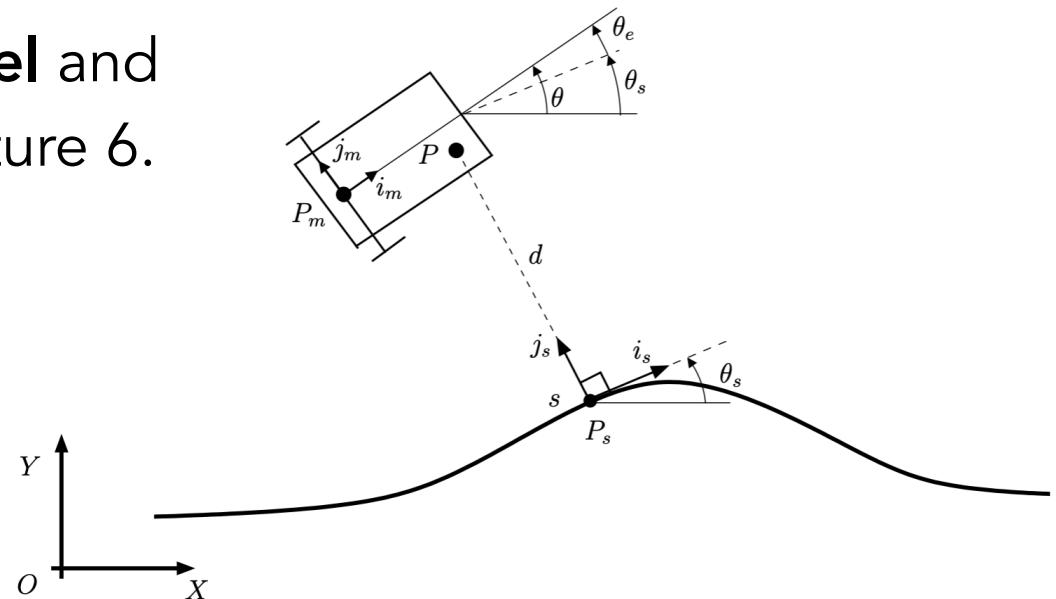
# Path Following for Kinematic Model

- Recall the **kinematic motion model** and **path-following** problem from Lecture 6.

$$\dot{s} = \frac{v}{1 - dc(s)} \cos(\theta_e),$$

$$\dot{d} = v \sin(\theta_e),$$

$$\dot{\theta}_e = vu - \dot{s}c(s)$$



- States  $x = (d \quad \theta_e)$  and system dynamics  $f(x, u) = \begin{pmatrix} v \sin(\theta_e) \\ vu - \dot{s}c(s) \end{pmatrix}$

# MPC Formulation for Kinematic Path Following (1/2)

- An **MPC problem** can be formulated for **path following**, where the objective is to achieve that  $x \rightarrow 0$ .
- With the **weights**  $Q = \text{diag}(q_1, q_2)$ ,  $R = r$ , the MPC problem for **path following** can be formulated as:

$$\text{minimize} \quad \int_{t_k}^{t_k + h_p} q_1 d^2 + q_2 \theta_e^2 + ru^2 dt$$

$$\begin{aligned} \text{subject to} \quad & x(t_k) = x_0, \quad \dot{x}(t) = f(x(t), u(t)), \\ & u(t) \in \mathbb{U}, \quad t \in [t_k, t_k + h_p] \end{aligned}$$

## MPC Formulation for Kinematic Path Following (2/2)

- The **constraint** on the control input can be determined based on **steering-angle limitations** as:

$$\mathbb{U} = \{u \mid 1/L \tan(\delta_{\min}) \leq u \leq 1/L \tan(\delta_{\max})\}$$

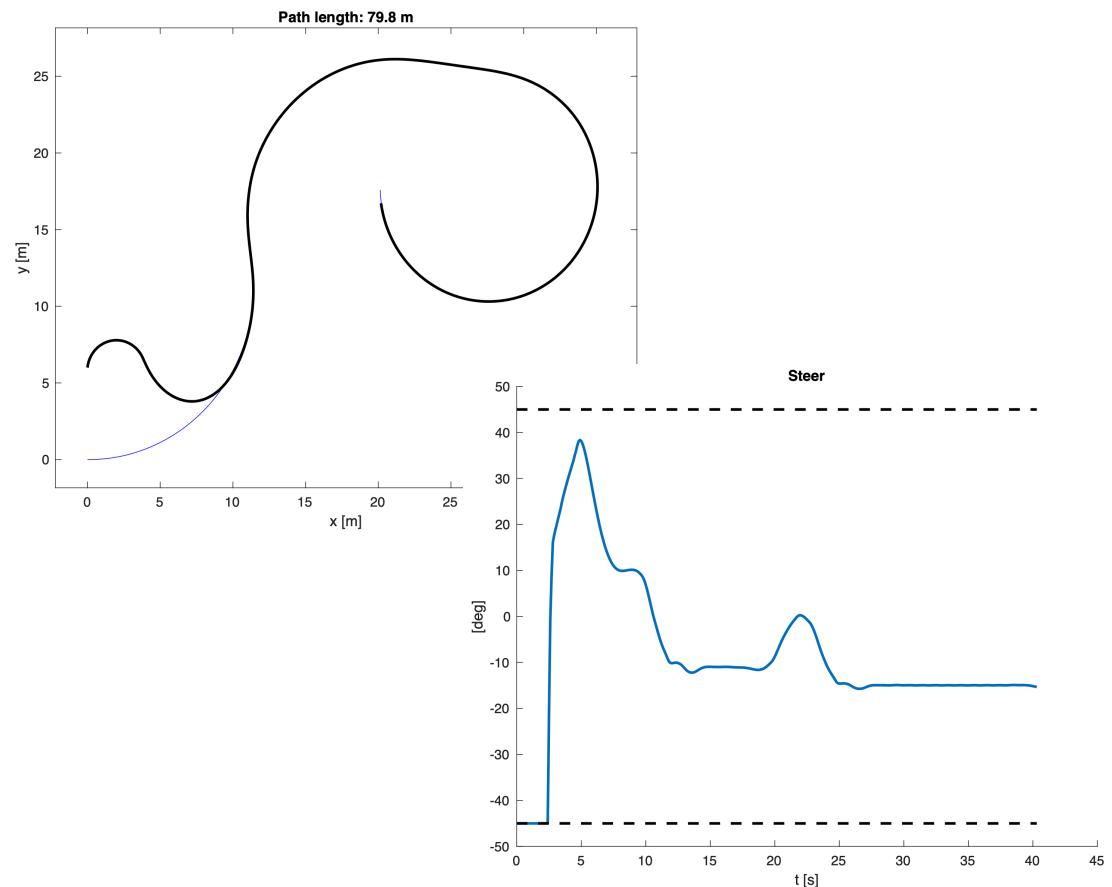
- Compared to the LQR approach, with **MPC** a **finite-horizon optimization problem** is instead solved at each sample instant, considering both the **non-linear vehicle model** and **constraints** on the input.
  - Results in a **computationally more expensive** controller.

# Example: Extra Assignment for Hand-in 3 (1/2)

- Part of extra assignment for **Hand-in Exercise 3** to implement this **path-following MPC**.
- **Linearization** of the non-linear kinematic single-track model at each MPC update step.
  - **Quadratic cost function** and **linear constraints, convex optimization problem** (efficient solvers exist).

# Example: Extra Assignment for Hand-in 3 (2/2)

- Application on an **example path** from Hand-in Exercise 3.
- **Constraints on control signal** (steering angle).
- Choices of **prediction horizon** and **sample period**.
- **Multiple shooting** with **explicit Runge-Kutta** method of fourth order for **discretization** of vehicle motion equations (see Lecture 5).



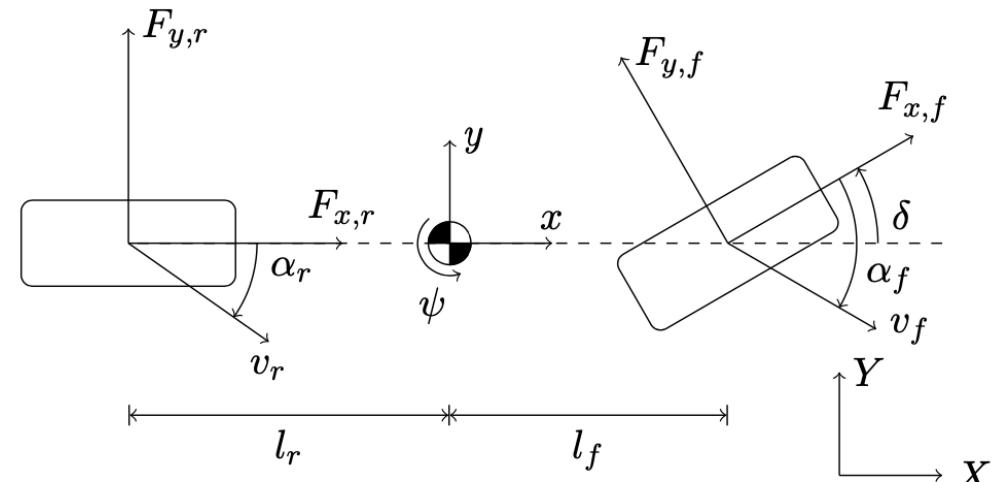
# MPC for Autonomous Path Following

# Problem Formulation

- The task is to **follow a desired path** (position and orientation) and a nominal **velocity profile**.
- To account for **dynamic effects** of a ground vehicle, a **single-track model** with a **linear tire–road interaction model** is used.
  - More **complex model** than the kinematic model in the previous example, beneficial for **higher velocities** and in **more advanced maneuvers**.
- Design an **MPC** (objective and constraints) for this task.

# Dynamic Vehicle Model (1/4)

- Consider the **single-track vehicle model** (lumped wheels on each axle).
- Front steering and front-wheel driven.**



$$m(\dot{v}_x - v_y \dot{\psi}) = F_{x,f} \cos(\delta) + F_{x,r} - F_{y,f} \sin(\delta),$$

$$m(\dot{v}_y + v_x \dot{\psi}) = F_{y,f} \cos(\delta) + F_{y,r} + F_{x,f} \sin(\delta),$$

$$I_Z \ddot{\psi} = l_f F_{y,f} \cos(\delta) - l_r F_{y,r} + l_f F_{x,f} \sin(\delta)$$

# Dynamic Vehicle Model (2/4)

- The **slip angles** (angle between the velocity vector and the direction of the wheel) are given by:

$$\alpha_f = -\arctan\left(\frac{v_{f,y}}{v_{f,x}}\right), \quad \alpha_r = -\arctan\left(\frac{v_{r,y}}{v_{r,x}}\right)$$

- A low-complexity **tire–road interaction model** for normal driving with linear tire stiffnesses is adopted:

$$F_{y,f} = C_{\alpha,f} \alpha_f, \quad F_{y,r} = C_{\alpha,r} \alpha_r$$

## Dynamic Vehicle Model (3/4)

- A **path parameter** is introduced to describe the traversal along the reference path (*traversal computed in the MPC*):

$$\dot{s} = \frac{ds}{dt}$$

- The **vehicle position** in the **global coordinate frame** is obtained by integration of the quantities:

$$\begin{pmatrix} \dot{p}_X \\ \dot{p}_Y \end{pmatrix} = \begin{pmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

## Dynamic Vehicle Model (4/4)

- Collect the **states** and the **control inputs** in the vectors:

$$x(t) = (p_X \quad p_Y \quad \psi \quad v_x \quad v_y \quad \dot{\psi} \quad s)^T$$

$$u(t) = (\delta \quad F_{x,f} \quad F_{x,r} \quad \dot{s})$$

- With these variables, the **vehicle dynamics** can be written as an **explicit ordinary differential equation system** as:

$$\dot{x}(t) = f_{\text{car}}(x(t), u(t))$$

# MPC Problem (1/4)

- **Reference values** for the vehicle consist of **position** in global coordinate frame, **longitudinal velocity**, and **yaw orientation**.
- Introduce the **vector of reference values** as:

$$(p_{X,\text{ref}} \quad p_{Y,\text{ref}} \quad v_{x,\text{ref}} \quad \psi_{\text{ref}})^T$$

## MPC Problem (2/4)

- Introduce a function that describes the **error quantities** that the controller should **drive to zero** (reference values as function of the path parameter  $s$ ):

$$\Phi(x(t), s) = \begin{pmatrix} p_{X,\text{ref}}(s) - p_X(t) \\ p_{Y,\text{ref}}(s) - p_Y(t) \\ e_\perp(t, s) \\ \psi_{\text{ref}}(s) - \psi(t) \\ v_{x,\text{ref}}(s) - v_x(t) \end{pmatrix}$$

- The **orthogonal projection** of the path error is:

$$e_\perp(t, s) = \sin(\psi_{\text{ref}})(p_{X,\text{ref}}(s) - p_X(t)) - \cos(\psi_{\text{ref}})(p_{Y,\text{ref}}(s) - p_Y(t))$$

## MPC Problem (3/4)

- The **finite-horizon MPC problem** to be solved at each sample instant can then be stated as:

$$\text{minimize} \quad \int_{t_k}^{t_k + h_p} \Phi^T Q \Phi + u^T R u \, dt$$

subject to  $x(t_k) = x_0, \dot{x}(t) = f_{\text{car}}(x(t), u(t)),$   
 $x(t) \in \mathbb{X}, u(t) \in \mathbb{U}, t \in [t_k, t_k + h_p]$

- The weights  $Q, R$  are used to **trade different objectives** in the cost function to each other.

# MPC Problem (4/4)

- There is **large flexibility** in terms of the choice of **constraints**.
- There are typically constraints on the **steering angle** and the **driving and braking forces**.
- Update of **path parameter** also constrained.
- **Geometric constraints** also possible.

$$\delta_{\min} \leq \delta \leq \delta_{\max},$$

$$F_{x,f,\min} \leq F_{x,f} \leq F_{x,f,\max},$$

$$F_{x,r,\min} \leq F_{x,r} \leq F_{x,r,\max},$$

$$1 - s_{\Delta} \leq \dot{s} \leq 1 + s_{\Delta}$$

# Solution of the MPC Problem

- At **each sample instant**, the finite-horizon **MPC problem is solved** with updated state measurements (or often, in practice state estimates).
- The model is non-linear, and thus the problem is a **non-linear MPC** (NMPC).
- **Numerical solution** of the **optimization problem** required.

# Solving MPC Optimization Problems

- A large number of optimization problems are solved online in MPC.
- Often there are real-time computational constraints for an MPC controller executing online.
- How can the associated optimization problem be solved efficiently and the special structure of it be utilized?

# Solving MPC Optimization Problems Using RTI

- One proposed method is the **Real-Time Iteration (RTI)** scheme:
  - Multiple shooting for discretization of continuous dynamics.
  - Sequential quadratic programming (**SQP**), using one single Newton step, and an effective warm starting by using the state and control input trajectories from the previous MPC iteration.
  - Separation of computations in ***preparation phase*** and ***feedback response phase*** (highly important in real-time implementations).

# Summary and Outlook of Model Predictive Control for Autonomous Vehicles

# Characteristics of Model Predictive Control

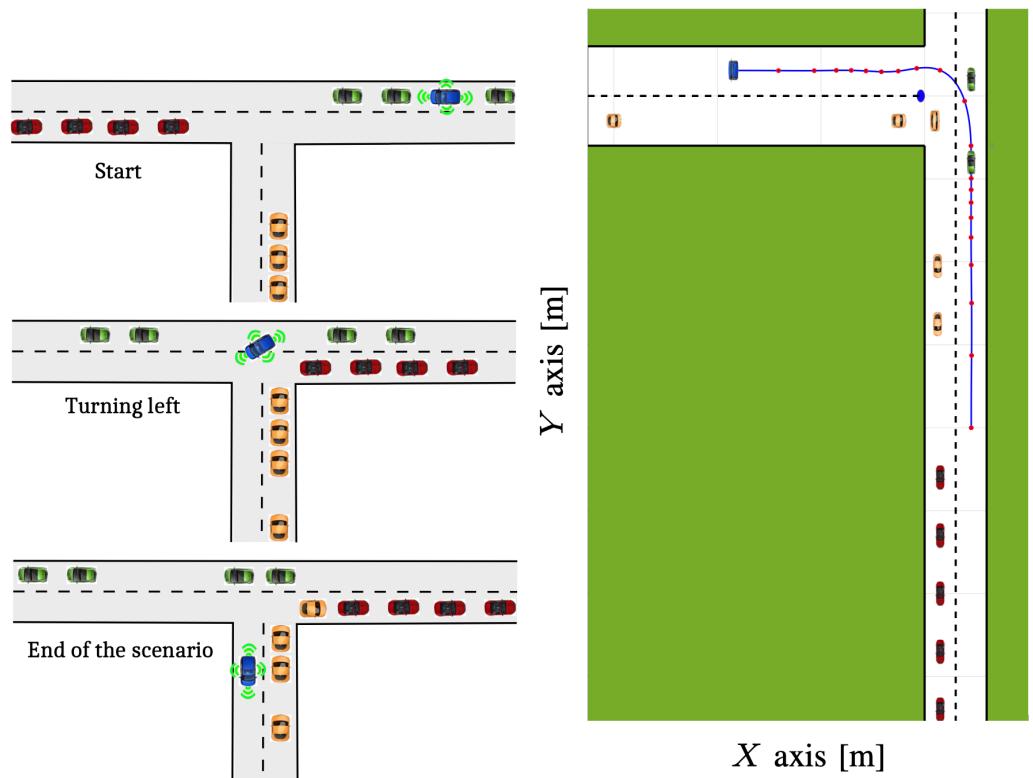
- **Integrated planning** and **control** possible.
- Possible to apply at **different layers** of the **decision-making architecture** of an autonomous vehicle, **MPC offers a framework**.
- **Automatic inherent re-planning** obtained at each sample instant, ***implicit feedback mechanism***.
- **High flexibility** in terms of **modeling** and **constraints**.
  - Also one of the main challenges in the MPC design.

## MPC Outlook (1/2)

- **Decentralized formulations and optimization for multiple vehicles** – reduce computational cost at the local node.
  - Trade-off with **communication load**.
- Dual to MPC for **state estimation** – **Moving horizon estimation (MHE)**.
  - Allows explicit constraints in the estimation problem.

## MPC Outlook (2/2)

- Integrated **collision avoidance** and **local planning** in **complex traffic scenarios** with MPC.
- **Obstacle representation, obstacle motion prediction, and collision-risk estimates.**



# Tools and Libraries for Numerical Optimal Control and Model Predictive Control

# CasADi



- Open-source tool for **numerical optimization and optimal control**.
- Includes efficient algorithms for **automatic differentiation** and computing Jacobians and Hessians in forward and backward mode.
- **Interfaces** to various NLP solvers and numerical integrators for ODEs/DAEs.
- Homepage: <https://web.casadi.org>

# IPOPT

Andreas Wächter · Lorenz T. Biegler

**On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming**

- Software package for **solving large-scale NLPs**.
- Implements a **primal-dual interior-point method** for finding local minima to the NLP.
- **Scaling** of model equations for numerical stability.
- Relies on external software packages for solving the **inherent linear equation systems** (e.g., MA27 and MA57 from HSL Mathematical Software Library).
- Homepage: <https://github.com/coin-or/Ipopt>

# YOP

- YOP – Yet Another Optimal Control Problem Parser.
- **Matlab toolbox** for **numerical optimal control**, with an interface to CasADi for access to **integrators** and **solvers of nonlinear programs**.
- Developed at Div. Vehicular Systems at Linköping University.
- Several **examples** available on toolbox homepage: <https://www.yoptimalcontrol.se/index.html>

# References and Further Reading

# References and Further Reading (1/2)

All the following books and articles are not part of the reading assignments for the course, but cover the topics studied during this lecture in more detail.

- Andersson, J., J. Gillis, G. Horn, and J. B. Rawlings, & M. Diehl: "CasADi–A software framework for nonlinear optimization and optimal control", *Mathematical Programming Computation*, 2018.
- Berntorp, K., Quirynen, R., Uno, T., & Di Cairano, S: "Trajectory tracking for autonomous vehicles on varying road surfaces by friction-adaptive nonlinear model predictive control". *Vehicle System Dynamics*, 58(5), 705-725, 2020.
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- Di Cairano, S., Kalabić, U. V., & Berntorp, K: "Vehicle tracking control on piecewise-clothoidal trajectories by MPC with guaranteed error bounds". In IEEE Conference on Decision and Control (CDC), 709-714, 2016.
- Diehl, M., Bock, H. G., & Schlöder, J. P: "A real-time iteration scheme for nonlinear optimization in optimal feedback control". *SIAM Journal on Control and Optimization*, 43(5), 1714-1736, 2005.

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