# The Ammunition Market, 2006-2015:
# A Time Series Analysis

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# 1 Abstract

While the phenomena of large increases in gun sales following changes in aggregate demand factors are well documented, similar conclusions about the behavior of the ammunition market in response to such alterations remain undetermined. The domestic ammunition market has received comparatively little analysis relative to other types of comparable commodities in terms of economic and public policy research. This study seeks to contribute to this literature by analyzing the responsiveness of ammunition prices to changes in the producer price index of its constitutive elements, and to establish a general overview of the domestic ammunition market as a whole. This analysis presents 3 key findings. First, 7 out of 119 firearm calibers sold account for over half of all ammunition sales. Second, there are unexpected differences in price elasticity of demand between these 7 common calibers. Third, changes in the real price per cartridge of commonly available brass cased, full metal jacket calibers do not increase or decrease commensurately to changes in the producer price indexes of input elements brass, copper, and lead. These results suggest that the primary determinant of changes in the real price per round of ammunition originate in aggregate demand, rather than from producers passing along increases in production costs.

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## 2 Introduction

On December 14th, 2012 a 20 year old Connecticut man clad entirely in black and armed with his mother's semiautomatic rifle, two semiautomatic handguns, and multiple loaded magazines approached the Sandy Hook elementary school in Newtown, Connecticut. Five minutes later this active shooting would leave 27 victims dead and 2 injured, with 20 of those victims children under the age of 8 years old.[1] This single depraved act occurred barely a month after the second election of Democratic President Barack Obama, defining a new era of mass shootings targeting school aged children and renewed calls for federal gun control legislation. Policy suggestions within public debate ranged from reintroduction of the previously passed Assault Weapons Ban, a subsection of the Violent Crime Control and Law Enforcement Act of 1994 allowed to sunset in 2004, to increased funding and visibility for mental health services. With full public support from then President Barack Obama, Democratic Senator Dianne Feinstein of California would introduce the Assault Weapons Ban of 2013 almost a month after the shooting, which sought to ban the possession of semiautomatic firearms and detachable magazines capable of accepting more than 10 rounds of ammunition.[2]

The proposed legislation managed to leave committee but ultimately was defeated after a vote in the Senate. However, the aftermath of this shooting with its heightened calls for gun control and newly proposed legislation resulted in an observable overnight increase in the demand for the kinds of firearms facing a ban. Despite a policy agenda seeking to minimize access to the kinds of firearms that render one person capable of committing a mass atrocity that takes the lives of dozens in a matter of minutes, the mere signalling of future firearms legislation caused the exact opposite effect of its intended purpose. From both an economics and policy perspective, these casual observations raised the following questions. Why did a commodity whose mere possession carried with it the possibility of imminent future legal consequence increase in quantity sold? What

1. Federal Bureau of Investigation U.S. Department of Justice, *Active Shooter Incidents in the United States from 2000-2018*, https://www.fbi.gov/file-repository/active-shooterincidents-2000-2018.pdf/view, Accessed: 2023-10-01, 2018, 16.

2. *Assault Weapons Ban of 2013, S. 150, 113th Congress*, Accessed: 2023-10-01, 2013.

motivations drove this change in behavior? Additionally, assuming that the aggregate demand for firearms primarily drives their consumption, does the ammunition market behave in a comparable manner?

This research project sought to investigate these questions by collecting ammunition sales data from 2006 to 2015, generating a descriptive overview, calculating the average monthly real price per round, and converting these values to a time series in the given period. Once determined, this time series was plotted against the price movements of ammunitions' major inputs (copper, brass, and lead) in the form of their Producer Price Indexes from 2006 to 2015. The dynamics of the relationship between the monthly price per round and its underlying inputs were analyzed via a vector autoregression model to determine the extent to which, if any, alterations in input prices explains changes in the monthly price per round. Ultimaely, this methodology seeks to investigate whether short run ammunition prices are primarily determined by movements in aggregate supply or demand.

## 2.1 Literature Review

Consistent with previous findings, the academic literature has documented the increased demand for firearms after high profile shootings.[3] A study on the effect of mass shootings between 1999 and 2016 sought to compare the effect of each state's dominant political party on firearms demand against the political party composition of the national legislature. In comparison to previous qualitative research that utilized personal interviews at gun shops and shows to determine increases in gun sales, this research sought to conduct quantitative analysis by employing data acquired from the Federal Bureau of Investigation's National Instant Check System (FBI NICs) on monthly background check totals from each state as a proxy for firearm sales.[4] Under federal law any gun sale occurring through a licensed federal firearms dealer requires the conduct of a background check, whereby certain criminal convictions bar a person from purchasing a firearm

3. Tae-Young Pak, "The effects of mass shootings on gun sales: Motivations, mechanisms, policies and regulations," *Journal of Policy Modeling* 44, no. 6 (September 2022): 1148–1164, https://doi.org/\url{https://doi.org/10.1016/j.jpolmod.2022.10.005}; DM Studdert et al., "Handgun Acquisitions in California After Two Mass Shootings," *Annals of Internal Medicine* 166, no. 10 (May 2016): 698–706, https://doi.org/\url{https://doi.org/10.7326/M16-1574}.

4. Pak, "The effects of mass shootings on gun sales: Motivations, mechanisms, policies and regulations," 1150.

[18 U.S.C. 922(t) and 925(a)(1)]. Though this method may undercount the number of firearms sold due to one background check possibly covering an individual purchasing multiple firearms at one time, or the sale taking place outside of a licensed dealership, the non-existence of any federal tracking of firearm ownership makes it a feasible alternative.

These numbers on gun sales were measured against an analysis of both the FBI's and magazine *Mother Jones'* database on mass public shootings and mass murders taking place in private places, with a lower threshold of 3 fatalities set for each MCE to account for differences in data collection. Regional effects were accounted for through variables assigning the occurrence of each MCE to its respective state and nationwide variables linking fatality counts to all states. Demographic information was obtained from the 2010 federal Census, along with state GDP & unemployment data from the Bureau of Economic Analysis and violent crime statistics from the FBI's uniform crime report. Finally, binary variables were defined for the majority political party in each state house, governorship, and presidencies of Bill Clinton, George W. Bush, Barack Obama, and Donald Trump to account for the effect of dominant political affiliation. This data was used to conduct a dynamic Ordinary Least Squares regression to estimate the values of the predictor variable parameters for firearm sales.

The research found a statistically significant, temporary increases in gun sales 3-6 months following the occurrence of a MCE meeting the previous threshold, but only under Democratic presidents- the majority political affiliation of state legislatures did not cause similar increases. Additionally, the study found little evidence suggestive that theses increases in gun purchases following an MCE were commensurate with an increase in gun owners; the majority of these purchases were initiated by fear driven consumers seeking to enlarge their stockpile of weapons in response to the possibility of a ban. The work concludes by suggesting that legislative efforts at curbing gun violence through controlling access would more effectively come through passage from state legislatures, due to the lack of consumer responsiveness to political signals following an MCE. As mentioned previously, the author claims these findings consistent with the counter-intuitive position that proposals for gun control under Democratic presidents results in demand spikes for firearms.

Similar conclusions were determined in other research on gun sales in California following mass shootings in Newtown, Connecticut in 2012 and San Bernardino, California in 2015 between 2007 and 2016.[5] This particular study utilized time series analysis with a seasonal autoregressive integrated moving-average model to track general gun sales over the given time period and changes in trends in response to particular events. Rather than focus on all gun sales this study focused on handguns specifically.[6] This methodological difference in comparison to the aforementioned research was noteworthy for its applicability to changes in consumer trends over a selected period. This relatively recent study provides the first detailed picture of firearm purchasing behaviour in the immediate aftermath of mass shootings, made possible by the California specific laws around firearms that mandate a detailed level of statewide data collection.

This particular study stands out due to the subject of its analysis occurring within California. In contrast to the rest of the nation, California law requires *all* firearm purchases to take place through a licensed federal firearms retailers, which additionally requires an application for approval through the California Department of Justice, who then retains a permanent record of each firearm acquisition. Data on these sales was then aggregated across the period in question and codified based on demographic, transaction by firearm type, and the geographic location of each sale. The time series analysis was conducted at a weekly frequency and the number of excess handgun purchases was calculated as the difference occurring between the expected quantity from a baseline and those taking place immediately post-shooting.

The research compared its findings with federal NICs data during the same period for the state of California and found similar trends of handgun sales spiking sharply immediately following the mass shooting in Newtown and San Bernardino, followed by a reversion to expected levels approximately 2 months after the MCE. This work additionally found a pronounced relative acquisition response amongst women, white persons, and first time handgun purchasers, though men still acquired handguns at comparatively higher levels in absolute terms. These findings differ from

5. Studdert et al., "Handgun Acquisitions in California After Two Mass Shootings."
6. Studdert et al.

the national results of the previous study, suggesting potential variation by regional difference in acquisition patterns. However, both works establish statistically significant, conclusive findings about the pattern of increases in firearm sales after mass casualty events.

## 2.2 Motivations

The previous research on this topic specifically focuses on firearm sales. There exists a general dearth of academic research on ammunition sales, let alone in response to political signals. Unlike firearm sales, there are limited federal regulations around the prohibition and sale of ammunition on the consumer market. Under current statute 18 is the minimum age to purchase rifle or shotgun ammunition and 21 to purchase handgun ammunition [18 U.S.C. 922(b)(1) and (b)(2); 27 CFR 478.99(b)]. Persons prohibited from possessing or purchasing firearms share the same restrictions on ammunition, however unlike firearms no federal statute stipulates the regulation of background checks for the purchase of ammunition [18 U.S.C. 922(g)]. As a necessary component for the operation of a firearm for the purpose of accurately dispensing projectiles, this absence of study on the behavior and trends within the domestic ammunition market highlights a possible avenue for policy to reduce gun violence in the United States.

This research paper seeks to establish an initial investigation into the ammunition market through econometric analysis. In addition to establishing some general facts about the ammunition market it specifically aims to analyze the dynamic relationship between ammunition prices and their constitutive producer price indexes for copper, brass, and lead. Though but one aspect of study on the behavior of the ammunition market, this research paper hopes to provide descriptive insights to generate increased regulation to control access to ammunition. The author of this paper finally hopes that his background as a subject matter expert within the gun community, with 8 years spent on active duty in the United States Marine Corps as a professional gunsmith, competitive shooter, marksmanship coach, and foreign weapons instructor, combined with his graduate level education as an economist, will lend itself to a uniquely insightful approach to this topic. Rather than serving as the final word on this subject, this work aspires to the initial point of departure for further study.

# 3 Methods

## 3.1 Data Description

As previously mentioned, no centralized consumer database or publicly available federal tracking of ammunition sales currently exist for analysis. To substitute for this absence, this study utilized ammunition sales data graciously provided by Dr. Matthew Chambers of Clemson University, who collected sales data from online firearms and ammunition retailer Guns.com.[7] While constituting only a fraction of the total quantity of ammunition sold within the United States, this data set represents an adequate sample size for establishing an initial methodology for larger future analysis.

The provided ammunition data set initially included 43,137 rows of sales transaction information by caliber, bullet type, case type, primer type, corrosive status, price, and number of rounds purchased. The transaction period occurred from August 2006 to July 2015. Pertinent to analysis, this study focused on caliber, bullet type, case type, price, and number of rounds sold. Caliber refers to the diameter of the projectile encased in each cartridge, with the cartridges cases themselves possessing different physical form specifications referred to as ”chamberings”. A chamber is cut into the barrel of a rifle or handgun to accommodate specific cartridges, themselves selected and designed for particular applications of the firearm, such as target shooting or hunting. The case material of a cartridge typically consists of an alloyed brass or softer steel material, the selection of which engenders particular capabilities for the manufacturing process or shooter. The bullet type (the projectile which leaves the case of a cartridge), typically consists of some combination of steel, lead, or copper, the selection of which similarly depends on the bullets external or terminal applications.

The analysis primarily examined the monthly price per round. All ammunition sales transactions were converted to price per cartridge by dividing the given price by the number of cartridges sold. To extricate changes in the price level due to inflation all prices were converted from nom-

7. Matthew Chambers, “Gun-Deals.com Sales Data,” Accessed: 2023-09-30, February 2023.

inal to real 2022 dollars before division by the number of rounds sold. Consumer Price Index (CPI) data was obtained from the Bureau of Labor Statistics, and all years utilized for conversion to 2022 dollars were based on the provided average CPI values for that year, opposed to by the specific month.[8]

$$\text{Converting ammunition prices to 2022 real dollars:}$$
$$\text{2022 price} = \text{ammo price} * \frac{\text{2022 yearly average CPI (292.7)}}{\text{ammo price yearly average CPI}} \tag{1}$$
$$\text{Calculating real price per round:}$$
$$\text{real price per round} = \frac{\text{2022 price}}{\text{number of rounds sold}} \tag{2}$$

This conversion to real dollars works by CPI's accounting for alterations in the general price level from a given base year, making price comparisons possible across different periods of time. This method was selected to provide consistent points of comparison across the entire time series.

The time series component of this study seeks to determine the relationship between changes in ammunition prices as explainable through changes in the cost of production by the manufacturer. Concurrent with the research on increases in firearm sales in response to MCE's,[9] establishing the extent to which ammunition prices change in response to these supply side factors offers a necessary alternative to changes in aggregate demand. To explore this possibility Producer Price Index (PPI) data on the major constituent elements of a loaded cartridge (brass, copper, and lead) was obtained from the Federal Reserve's Economic Database consistent with the sales data from August, 2006 to July, 2015.[10] In contrast to CPI data, the PPI reflects alterations in the average price received by domestic producers of certain goods and services over a selected time period. It similarly operates as an index of inflation, except that occurring at the wholesale level opposed to in consumption.

Previous research explored the hypothesis that gun purchases after mass casualty events derive

8. Minneapolis Federal Reserve Bank, *Consumer Price Index*, https://www.minneapolisfed.org/about-us/monetary-policy/inflation-calculator/consumerprice-index-1913-, Accessed: 2023-09-30, September 2023.

9. Pak, "The effects of mass shootings on gun sales: Motivations, mechanisms, policies and regulations"; Studdert et al., "Handgun Acquisitions in California After Two Mass Shootings."

10. St. Louis Federal Reserve Bank, *Producer Price Index by Commodity; Metals and Metal Products: Copper and Brass Mill Shapes*, https://fred.stlouisfed.org/series/WPU102502, Accessed: 2023-09-30, September 2023; St. Louis Federal Reserve Bank, *Producer Price Index by Industry; Copper, Nickel, Lead and Zinc Mining*, https://fred.stlouisfed.org/series/PCU212230212230, Accessed: 2023-09-30, September 2023.

their motivation from fear of future terrorist attacks, criminal violence, or the possibility of a future firearm's ban that would prevent citizens from acquiring lethal means to defend themselves.[11] In exploration of such a hypothesis this analysis sought to investigate this explanation by determining whether the predominant ammunition sales types reflected calibers commonly used in firearms designed for combat applications. The top 10 most commonly sold calibers were calculated from the data set and then sorted by further criteria as the variables of interest for the analysis.

To model the dynamics of the ammunition market in response to PPI alterations, once the top 10 most common calibers in the data set were determined further reduction was necessary to select for cartridges composed of the elements included in the Producer Price Indexes and those commonly used for combative application. This included aggregating data for calibers 9mm Luger, .380 ACP, .45 ACP, .40 S&W, 5.56x45mm, and 7.62x51mm with brass cases & full metal jacket bullets. Additionally, all .22 LR was included irrespective of case and bullet type as a reference point against the behavior of other calibers- as a popular small caliber cartridge not generally useful for self defense or criminal ends it was not included in the federal assault weapons ban.[12] The first 4 calibers reflect the top 4 most common amongst centerfire handguns, the latter 2 the most common amongst centerfire rifles, and the .22 LR as the only rimfire. This methodology necessitated removing the 3 other top 10 most popular calibers, 7.62x39mm, 5.45x39mm, and 7.62x54R due to their overwhelming representation with steel case types in the data set .

With the top 7 calibers specified by application, case material, and bullet type we additionally calculated their price elasticities of demand . After conducting an OLS regression of the number of rounds sold on the price for each caliber to determine the slope coefficient, we employed the following formula:

$$\text{price elasticity of demand} = \text{ slope coefficient} * \frac{\text{mean price}}{\text{mean quantity}} \tag{3}$$

Once finalized all included data was aggregated by month. Before employment into any regression model, the real average price per round and selected Producer Price Index data were tested for

11. Pak, "The effects of mass shootings on gun sales: Motivations, mechanisms, policies and regulations," 1149.
12. *Assault Weapons Ban of 2013, S. 150, 113th Congress.*

normality with the Shapiro-Wilk test at the p=.05 significance level:

$$W = \frac{\sum_{i=1}^{n} \alpha_i x_i^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{4}$$

where,

$W$ = Shapiro-Wilk test statistic,

$\alpha_i$ = the calculation parameter,

$H_0$ : The random sample was drawn from a normal distribution

$H_a$ : The random sample does not follow a normal distribution

Initial normality testing determined the necessity of further preprocessing, justifying the log transformation of the real price per round and application of the the Box-Cox formula to both PPI columns. Following these transformations the Shapiro-Wilk test was run again to conclude the finalized variables were normally distributed in preparation for regression analysis.

## 3.2 Statistical Analysis

### 3.2.1 Vector Autoregressive Model

The time series analysis was conducted at a monthly frequency. Due to the multivariate nature of the analysis, a vector autoregressive model (VAR) was selected to determine dynamics between the real monthly cost per round of the selected calibers and the two selected Producer Price Indexes.[13] The real monthly cost per round served as the dependent variable of primary interest, while the combined PPI of copper & brass and the PPI of lead were treated as independent variables. The primary objective of the analysis was to determine the statistical significance of changes in the Producer Price Indexes on fluctuations in the real monthly price per cartridge.

Python was used in conjunction with Jupyter Notebook as the statistical analysis software for initial data preparation and analysis, while RStudio was utilized for model fitting in the VAR

13. Robert H. Shumway and David S. Stoffer, *Time Series Analysis and Its Applications* (Springer Cham, April 2017), ISBN: 978-3-319-52452-8, https://doi.org/https://doi.org/10.1007/978-3-319-52452-8; D.A. Dickey, "Time Series: Nonstationary Distributions and Unit Roots," in *International Encyclopedia of the Social & Behavioral Sciences*, ed. Neil J. Smelser and Paul B. Baltes (Oxford: Pergamon, 2001), 15731–15735, ISBN: 978-0-08-043076-8, https://doi.org/https://doi.org/10.1016/B0-08-043076-7/00524-6, https://www.sciencedirect.com/science/article/pii/B0080430767005246.

analysis portion. This software was selected due to its modularity, popularity, and widely available support. Multiple dependencies and additional libraries were employed in support of this main objective.

VAR models take the form of treating each variable as a linear function of its previous lags, along with the lags of the other included variables.[14] The ”autoregressive” portion of a VAR model refers to regressing values from the previous periods on the current period, while the ”vector” component refers to operating on the variable values within the model in vectorized form. Due to the model specification of this analysis requiring interaction between 3 variables (real monthly cost per round, copper & brass PPI, and the lead PPI), the VAR model takes the following general equation:

$$x_t = c_1 + \sum_{i=1}^{3} \alpha_{1,i} y_{t-i} + \sum_{i=1}^{3} \beta_{1,i} x_{t-i} + \sum_{i=1}^{3} \gamma_{1,i} z_{t-i} + \varepsilon_{x,t} \tag{5}$$

$$y_t = c_2 + \sum_{i=1}^{3} \alpha_{2,i} y_{t-i} + \sum_{i=1}^{3} \beta_{2,i} x_{t-i} + \sum_{i=1}^{3} \gamma_{2,i} z_{t-i} + \varepsilon_{y,t} \tag{6}$$

$$z_{t,} = c_3 + \sum_{i=1}^{3} \alpha_{3,i} y_{t-i} + \sum_{i=1}^{3} \beta_{3,i} x_{t-i} + \sum_{i=1}^{3} \gamma_{3,i} z_{t-i} + \varepsilon_{z,t} \tag{7}$$

where,

$x_t$ =real monthly cost per round,

$y_t$ =copper and brass PPI,

$z_t$ =lead PPI,

$\varepsilon_{t,n}$= an unobservable zero mean white noise vector process,

$c_1, c_2, c_3$ = constants,

$\alpha, \beta, \gamma$ = variable coefficient matrices.

Testing for stationarity was initially conducted with Augmented Dickey Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Sin (KPSS) tests to determine the necessity of differencing,[15] then following differencing to ensure the establishment of stationarity for all included variables. Critical values were utilized at the 5% significance level.

14. Walter Enders, *Applied Econometric Time Series* (Wiley, 2015), 289, ISBN: 978-1-118-80856-6.
15. Dickey, “Time Series: Nonstationary Distributions and Unit Roots.”

Augmented Dickey-Fuller testing for a unit root, illustrated via the simple model $y_t = \alpha_1 y_{t-1} + \varepsilon_t$:

$$1.\triangle y_t = \gamma y_{y-1} + \varepsilon_t \tag{8}$$

$$2.\triangle y_t = \alpha_0 + \gamma y_{y-1} + \varepsilon_t \tag{9}$$

$$3.\triangle y_t = \alpha_0 + \gamma y_{y-1} + \alpha_2 t + \varepsilon_t \tag{10}$$

$$DF_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \tag{11}$$

where,

1 = pure random walk model,

2 = random walk with a drift,

3 = random walk with a drift and a linear time trend

$\gamma$ = the coefficient of the first lag in the differenced time series,

$DF_\tau$ = unit root test statistic,

If $DF_\tau$ = 0, the sequence contains a unit root.

KPSS unit root testing of trend stationarity employs a three component representation of the observed time series $y_1, y_2, ...y_n$ as:

$$y_t = \beta t + (r_t + \alpha) + \varepsilon_t \tag{12}$$

where,

$r_t = r_{t-1} + \mu_t$ is a random walk and $r_0 = \alpha$ is the intercept,

$t =$ the time index,

$\mu_t$ are i.i.d,

$H_0$ : $y_t$ is trend stationary.

$H_a$ : $y_t$ is a unit root process.

The number of lags included in the model specifies the model's order, with a VAR of order 1 referring to a model that includes lagged values from one period prior. Selecting the optimal lag length entails the employment of selection criteria,based in part on the likelihood function, towards minimizing forecast errors.

Lag length was determined by comparing the calculated Bayes Information criterion (BIC) and Akaike Information criterion (AIC):

$$BIC(p) = \ln(\frac{SSR(p)}{T}) + (p+1)\frac{ln(T)}{T} \tag{13}$$

$$AIC(p) = \ln(\frac{SSR(p)}{T}) + (p+1)\frac{2}{T} \tag{14}$$

where,

$p =$ the optimal lag order,

SSR= the sum of squared residuals,

T= the total length of the series.

The lag length was selected based on the lowest selection criteria value when compared amongst both methods.[16] After the selection of optimal lag order the VAR model was fitted via the VAR package in RStudio and followed up by serial correlation testing. For this purpose both the Breusch-Godfrey and Portmanteau tests were employed at the p=.05 significance level, whereby each test measures the amount of similarity between the fitted model's residuals with its own lagged versions in a statistical phenomenon known as 'autocorrelation'.

Breusch-Godfrey test for serial correlation:

$$\hat{\mu}_t = \rho_1 \hat{\mu}_{t-1} + \rho_2 \hat{\mu}_{t-2} + \rho_3 \hat{\mu}_{t-1} + \varepsilon_t \tag{15}$$

where,

$\hat{\mu}_t =$ regression of the residuals of the estimated VAR model,

$\varepsilon_t =$ an error term,

$H_0 : \rho_1 = \rho_2 = \rho_3 = 0,$

$H_a : \rho_1 \neq \rho_2 \neq \rho_3 \neq 0$

16. Enders, *Applied Econometric Time Series*.

Portmanteau test statistic for serial correlation:

$$Q = n \sum_{k=1}^{k} r_k^2(\hat{\alpha}) \tag{16}$$

where,

Q= Portmanteau test statistic,

$r_k^2(\hat{\alpha})$ = Square of k autocorrelations of residual time series,

n= N - d,

K= number of autocorrelations,

Degrees of Freedom (v) = K-*p*-*q*

Unlike the widely used Durbin-Watson tests both the Breusch-Godfrey and Portmanteau tests are applicable to multivariate analysis. The null hypothesis of each test assumes no difference in error variance between the different lags of the time series, initially claiming for rejection that the series has no autocorrelation. While the observation of statistically significant autocorrelation may either be desirable or undesirable depending on the study, the absence of autocorrelation between the residuals of the monthly price per round and the Producer Price Indexes of either constitutive elements would give evidence to suggest that PPI alterations do not influence the real price per round.

Normality testing was conducted on the fitted VAR model's residuals to determine the model's goodness of fit. The Jarque-Bera test was selected for this purpose, with the null hypothesis of this test assuming a normal distribution amongst the residuals. Critical values were utilized at the 5% significant level:

$$\text{Jarque-Bera} = \frac{n}{6} * [S^2 + \frac{(K-3)^2}{4}] \tag{17}$$

where,

S= skewness,

K=kurtosis,

and n=number of observations

With normality established or rejected the determined VAR model was examined for forecasting volatility via Autoregressive Conditional Heteroskedacity (ARCH) testing.[17] This method investigates the variance of the error term in a time series as a function of the previous term's error term. When time series exhibit a large amount of volatity, displaying periods of relative calm interspersed with periods of large swings, it undermines the ability to make accurate forecasts about the behavior of the series. The null hypothesis of this test assumes no ARCH effects, established for rejection or acceptance at the 5% significance level.

ARCH(3) model estimation, illustrated with one variable for simplicity:

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \alpha_3 y_{t-3}^2 \tag{18}$$

$$y_t = \sigma_t \varepsilon_t \tag{19}$$

where,

$\sigma_t = \sqrt{\{\alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + \alpha_3 y_{t-3}^2}$, mean of zero,

$\varepsilon_t$ is i.i.d

The means of the variables within the determined VAR model were checked for constancy using the Ordinary Least Squares Cumulative Sum (OLS-CUSUM) test. Assuming a null hypothesis of constant mean parameters, this method generates cumulative sum of squares parameters with a 95% confidence interval, whereby their persistence within the confidence interval reflects their general stability across time. In a similar fashion to other time series statistical tests, OLS-CUSUM results reflecting parameter stability permits clearer understanding of variable dynamics and allows for accurate forecasting.

#### 3.2.2 Structural Analysis

Following the establishment and initial statistical testing of the VAR model, innovation accounting was utilized to analyze the dynamics of the model. First, impulse response functions were employed to determine how each of the different variables respond to changes in another. This methodology allows us to qualitatively represent the positive, negative, or neutral alteration of one variable for determining the extent towards which the monthly real price per round responds to changes in the producer prices of copper, brass, and lead. In order to control for the possible

17. Shumway and Stoffer, *Time Series Analysis and Its Applications*.

influence of non-normal data and heteroskedasticity effects we used Monte Carlo simulation methods with 2000 replications and bootstrapped confidence intervals.

We considered two different ordering of the variables (Cholesky ordering) in order to accurately identify the system and derive meaningful impulse response functions. We initially imposed the ordering of PPI lead $\rightarrow$ PPI copper & brass $\rightarrow$ real monthly price per round. This ordering permits the assumption that the PPI of lead contemporaneously affects the PPI of copper & brass and the real monthly price per round, that the PPI of copper & brass affects the real monthly price per round, and that the real monthly price per round does not contemporaneously affect any other variable. Alternatively, we also considered the ordering of PPI copper & brass $\rightarrow$ PPI lead $\rightarrow$ real monthly price per round. This ordering theoretically assumes the contemporaneous affect of the PPI of copper & brass on the PPI of lead and the real monthly price per round, that the PPI of lead contemporaneously affects the real monthly price per round, and that the real monthly price per round does not contemporaneously affect any other variable. Hence, each Cholesky ordering assumes that one of the Producer Price Indexes direct prices in the domestic ammunition market, reflected in the monthly real price per round.

After Impulse Response Functions we analyzed Forecast Error Variance Decomposition (FEVDs) to quantify the extent to which an exogenous shock to one variable explains forecast variation in the other variables. We considered FEVD across 11 periods to ultimately examine variance decomposition across 10 periods, with period 0 only impacting the shocked variable. This component of structural analysis offers another important dimension to determining which constitutive Producer Price Index majorly impacts the real monthly price per round, if either.

Finally, we considered the statistical concept of Granger causality testing to check the helpfulness of each variable in forecasting the others.

Granger causality testing, shown in the bivariate case for simplicity:

$$\text{Restricted model}: y_t = c_2 + \sum_{i=1}^{3} \alpha_{2,i} y_{t-i} + \varepsilon_{x,t} \tag{20}$$

$$\text{Unrestricted model}: y_t = c_2 + \sum_{i=1}^{3} \alpha_{2,i} y_{t-i} + \sum_{i=1}^{3} \beta_{2,i} x_{t-i} + \varepsilon_{x,t} \tag{21}$$

where,

$H_0 : \beta_{2,1} = \beta_{2,2} = \beta_{2,3} = 0,$

$H_0 : \beta_{2,1} \neq \beta_{2,2} \neq \beta_{2,3} \neq 0$

The simplicity of this test rests on its insensitivity to Cholesky ordering, with a Granger causing variable reducing forecast error when added to the forecast model. The null hypothesis of no Granger causality was tested at the 5% significance level. While the importance of Granger causality for forecasting should not go understated, within the context of this study it was not interpreted as providing causal insight into the relationship between the variables.

# 4 Results

## 4.1 Descriptives

43,137 rows of provided ammunition sales data from August 2008 to July 2015 were analyzed for this study. A general descriptive overview was conducted to determine the 10 most popular calibers within the data set, followed by narrowing down this sample by the necessary condition of full metal jacket bullets and brass cases. Price elasticities of demand were calculated for all calibers. As mentioned previously all .22 LR transactions were included in the final sample. The sorted data frame was plotted as a time series for initial observation, reflecting the monthly real price per round for each caliber. The following figures present these findings:

| | Caliber | NumberOfRounds | PercentageSold |
|---|---|---|---|
| 0 | .223/5.56 | 4294800.0 | 22.66 |
| 1 | 9mm Luger | 2936307.0 | 15.50 |
| 2 | .22 LR | 2431218.0 | 12.83 |
| 3 | 7.62x39 | 1907509.0 | 10.07 |
| 4 | .45 ACP | 1015912.0 | 5.36 |
| 5 | .40 S&W | 902495.0 | 4.76 |
| 6 | 5.45x39 | 789615.0 | 4.17 |
| 7 | .308/7.62x51 | 671311.0 | 3.54 |
| 8 | .380 ACP | 603055.0 | 3.18 |
| 9 | 7.62x54R | 532801.0 | 2.81 |

Figure 1: Top 10 Calibers

| | Caliber | real_price_per_round | NumberOfRounds | PercentageSold |
|---|---|---|---|---|
| 0 | .308/7.62x51 | 0.87 | 444995.0 | 2.35 |
| 1 | .223/5.56 | 0.58 | 3120950.0 | 16.47 |
| 2 | .45 ACP | 0.52 | 665150.0 | 3.51 |
| 3 | .380 ACP | 0.45 | 471300.0 | 2.49 |
| 4 | .40 S&W | 0.42 | 539940.0 | 2.85 |
| 5 | 9mm Luger | 0.35 | 1994395.0 | 10.52 |
| 6 | .22 LR | 0.12 | 2431218.0 | 12.83 |
| Column_Total | | | 9667948.0 | 51.02 |

Figure 2: Top 10 Calibers, Brass Case and Full Metal Jacket

| | Caliber | Value | Price_Elasticity |
|---|---|---|---|
| 0 | .223/5.56 | 1.13 | elastic |
| 1 | .308/7.62 | 0.96 | inelastic |
| 2 | 9mm Luger | 1.23 | elastic |
| 3 | .45 ACP | 0.94 | inelastic |
| 4 | .40 S&W | 0.89 | inelastic |
| 5 | .380 ACP | 1.11 | elastic |
| 6 | .22 LR | 0.86 | inelastic |

Figure 3: Price Elasticity of Demand

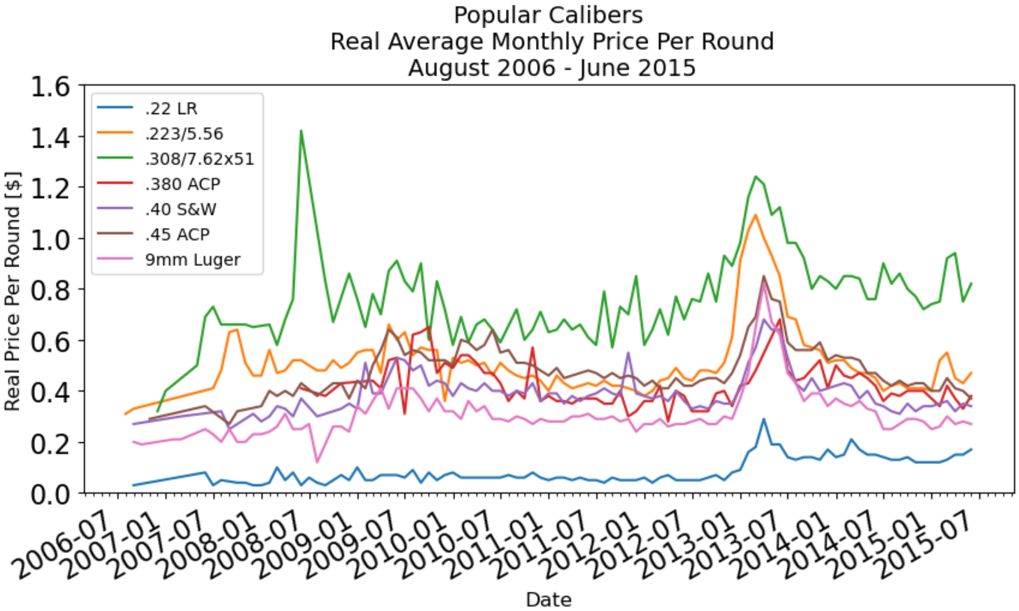



Figure 4: Real Monthly Cost Per Round, by Caliber

The monthly price per round for the most popular calibers reflects a few noteworthy trends. As expected, the rifle calibers (7.62x51mm and 5.56x45mm) were consistently more expensive than the handgun calibers and .22 LR proved the cheapest. All calibers, with the exception of 7.62x51mm, display their largest peak during the second quarter of 2013.

The following charts include the time series variables analyzed in the vectorized autoregressive model:

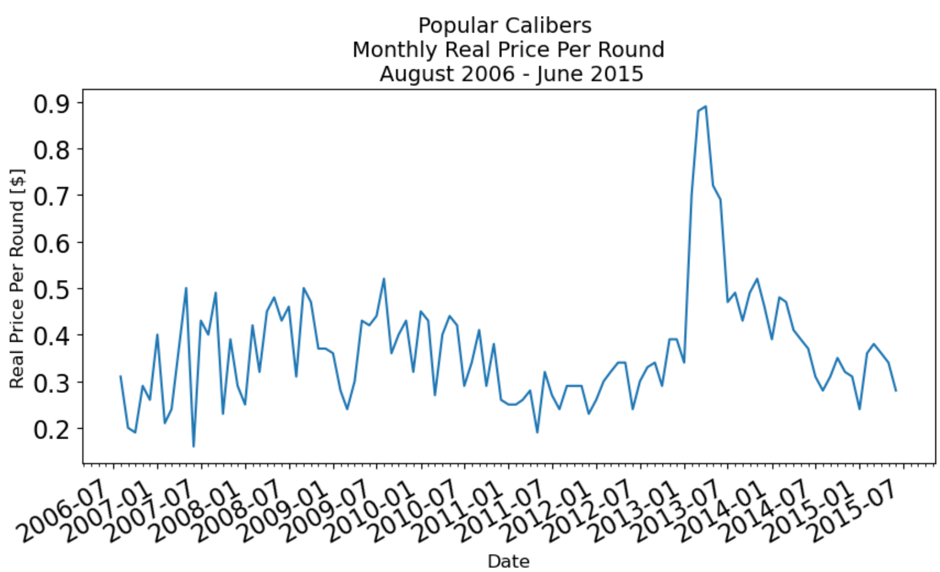



Figure 5: Real Monthly Cost Per Round

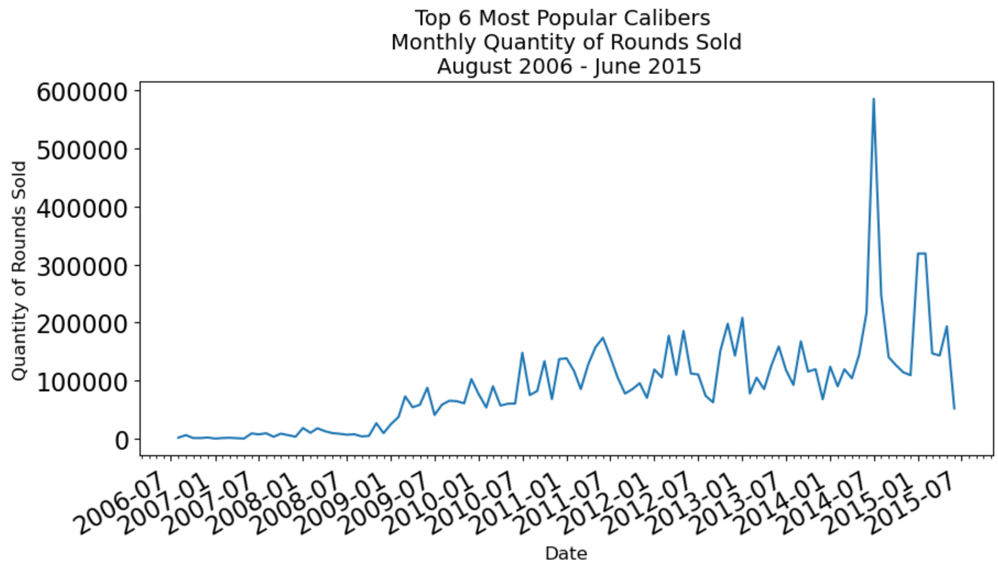



Figure 6: Monthly Number of Rounds

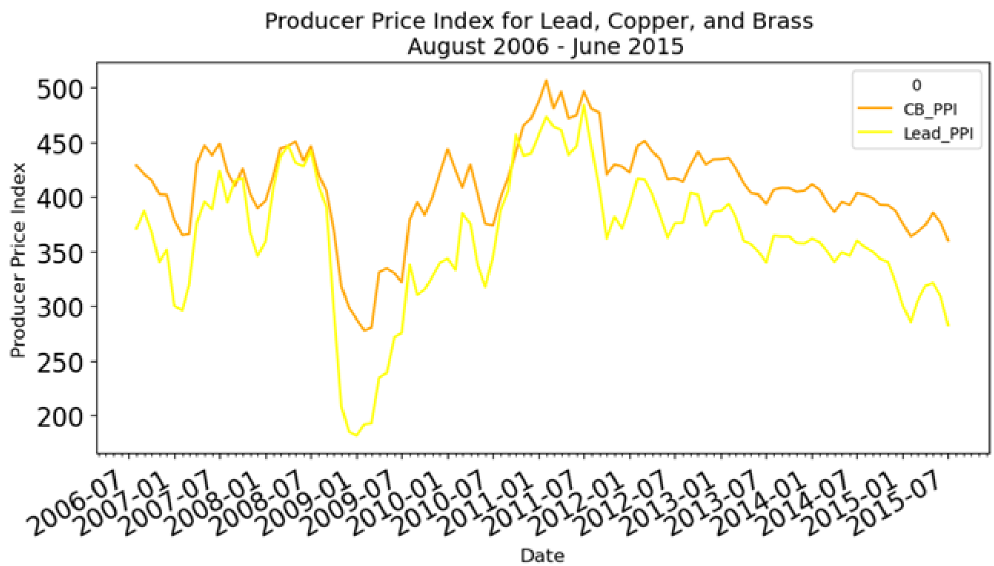



Figure 7: Producer Price Index for Copper, Brass, and Lead

Consistent with the by caliber observations, the aggregated monthly price per round time series displays its highest peak in the second quarter of 2013. Both the Producer Price Indexes for copper & brass and lead track closely in terms of their performance, with lead appearing consistently cheaper. Neither the monthly number of rounds sold or the Producer Price Indexes for any of the constitutive elements nominally exhibit performance closely consistent with the real monthly real cost per round.

## 4.2 VAR Model Dynamics

The multivariate VAR model was constructed with the VARS time series package in RStudio and initially examined over a prior period of 12 months. Before VAR model construction and testing all included time series variables were checked for normality and transformed as described previously to a normalized form. This study chose to test for Granger causality due to an interest in the forecasting ability of each independent variable towards predicting the monthly price per round, which necessitated inducing stationarity through differencing. Following normalization each series was first differenced and checked for stationarity (constant mean, variance, etc.) to ensure the absence of any unit roots. The results from this portion are reflected in the following table and graphs:

| **Table 1.** Stationarity | | |
|---|---|---|
| Test Name | Post Differencing | |
| **ADF** | Critical Value, P-value | Stationarity |
| Monthly Price Per Round | -4.247, .000 | stationary |
| Copper and Brass PPI | -5.254, .000 | stationary |
| Lead PPI | -6.439, .000 | stationary |
| **KPSS** | Critical Value, P-value | Stationarity |
| Monthly Price Per Round | .463, .084 | stationary |
| Copper and Brass PPI | .463, .077 | stationary |
| Lead PPI | .463, .072 | stationary |

**All critical values were tested at the 5% significance level.*

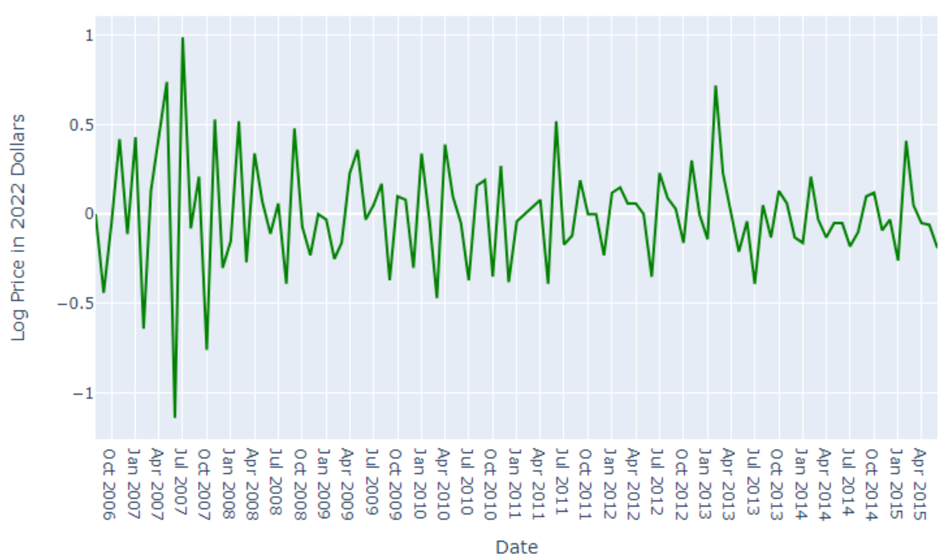



Figure 8: Stationary Log Price

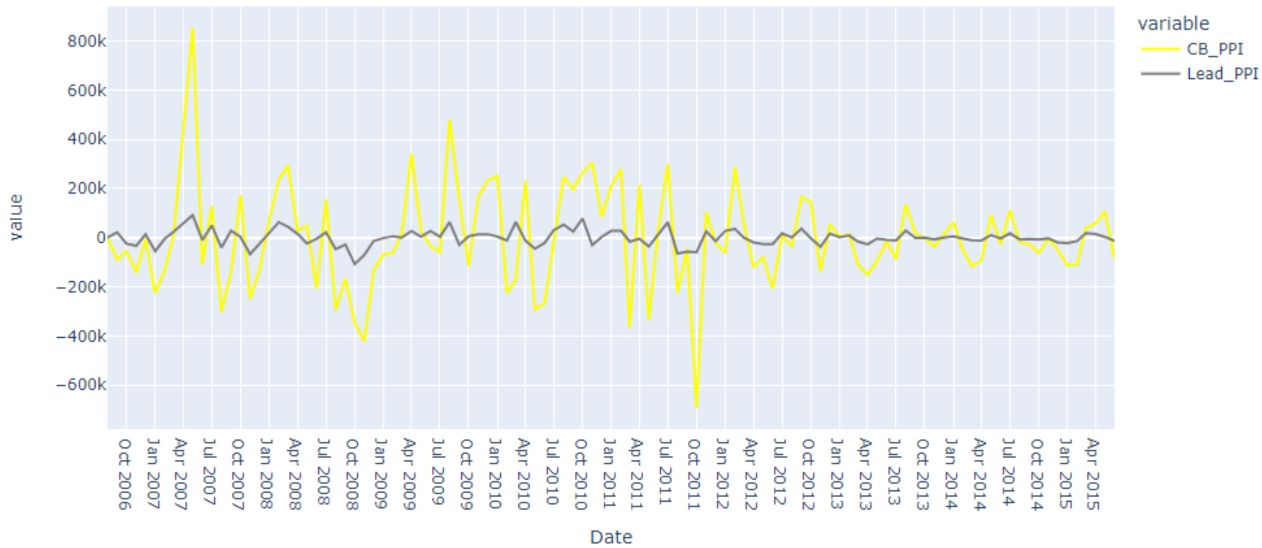



Figure 9: Stationary Producer Price Indexes

The stationarity of all included time series after taking the first differences means all variables are integrated of order one. The log transformed real price per round displays this stationarity around a mean of zero, generally alternating from period to period between a maximal height of 1 and minimum around approximately -1. Similarly, each transformed Producer Price Index reflects this stationarity around a mean of zero, with the lead PPI displaying less volatility than the PPI for copper and brass.

The VAR model's optimal lag length was determined based on minimizing the Akaike selection criteria. Initial length testing was conducted up to 12 lags, with optimal lag selection occurring at 3 lags. After this determination the VAR model was specified based on an optimal lag length of 3 periods (months). The below chart illustrates the model summary:

| Table 2. VAR Model Estimation Results | | | |
|---|---|---|---|
| Model: VAR | Method: OLS | No. of Equations: 3 | Log Likelihood: -2540.851 |
| **Eq: Monthly Real Price per Round** | Coefficient | Std. Error | T-stat, Probability |
| constant | 0.009 | 0.024 | 0.372 , 0.711 |
| Lead PPI.l1 | 0.000 | 0.000 | 1.830 , 0.070 |
| CB PPI.l1 | -0.000 | 0.000 | -2.560 , 0.012 |
| monthly real price per round.l1 | -0.572 | 0.102 | -5.588 , 0.000 |
| Lead PPI.l2 | 0.000 | 0.000 | 1.437 , 0.154 |
| CB PPI.l2 | 0.000 | 0.000 | 1.012 , 0.31399 |
| monthly real price per round.l2 | -0.361 | 0.111 | -3.256 , 0.001 |
| Lead PPI.l3 | -0.000 | 0.000 | -0.394 , 0.69482 |
| CB PPI.l3 | -0.000 | 0.000 | -0.073 , 0.942 |
| monthly real price per round.l3 | -0.084 | 0.101 | -0.831, 0.408 |
| **Eq: CB PPI** | Coefficient | Std. Error | T-stat, Probability |
| constant | -906.11 | 16782.764 | -0.054 , 0.957 |
| Lead PPI.l1 | 4.866 | 0.813 | 5.980 , 0.000 |
| CB PPI.l1 | -0.493 | 0.145 | -3.393 , 0.00102 |
| monthly price per round.l1 | -82227.560 | 71006.100 | -1.158 , 0.24981 |
| Lead PPI.l2 | 0.004 | 0.919 | 0.005, 0.995 |
| CB PPI.l2 | 0.261 | 0.150 | 1.742, 0.084 |
| monthly price per round.l2 | -139745.223 | 76808.475 | -1.819, 0.07207 |
| Lead PPI.l3 | -1.159 | 0.898 | -1.290 , 0.200 |
| CB PPI.l3 | -0.027 | 0.133 | -0.203 , 0.839 |
| monthly price per round.l3 | 46178.878 | 70175.996 | 0.658 , 0.512 |
| **Eq: Lead PPI** | Coefficient | Std. Error | T-stat, Probability |
| constant | -315.640 | 2907.4713 | -0.109, 0.913 |
| Lead PPI.l1 | 0.527 | 0.141 | 3.744 , 0.000 |
| CB PPI.l1 | -0.065 | 0.025 | -2.585 , 0.011 |
| monthly price per round.l1 | 1403.456 | 12301.203 | 0.114 , 0.909 |
| Lead PPI.l2 | -0.175 | 0.159 | -1.101 , 0.273 |
| CB PPI.l2 | 0.079 | 0.026 | 3.039 , 0.003 |
| monthly price per round.l2 | -9478.714 | 13306.415 | -0.712 , 0.478 |
| Lead PPI.l3 | -0.399 | 0.155 | -2.565 , 0.011 |
| CB PPI.l3 | 0.016 | 0.023 | 0.729 , 0.467 |
| monthly price per round.l3 | -9089.376 | 12157.394 | -0.748 , 0.456 |

**All critical values were tested at the 5% significance level.*

Focusing on the monthly price per round, the VAR(3) model's specification suggests that only the copper & brass PPI, first lag of the monthly real price per round, and second lag of the monthly real price per round are statistically significant (p-value less than .05) in determining the current monthly price per round. For the log-log difference relationship in the model between the monthly price per round and its lags, the calculated coefficient for the first lag of the monthly price per round means that a 1% increase results in a 57% decrease in the current real price per round. Given the size of each lags coefficients, it appears that the lagged values of the monthly price per round exhibit the most influence on ammunition prices in the current period.

The below table reflects the calculated model's residual correlation matrix, normality, and serial correlation testing results:

**Table 3.** VAR Model Residuals Correlation, Normality, and Autocorrelation Results

| **Correlation Matrix of Residuals** | Monthly Price per Round | CB PPI | Lead PPI |
|---|---|---|---|
| Monthly Price per Round | 1 | .009 | -.068 |
| CB PPI | .009 | 1 | .715 |
| Lead PPI | -.068 | .715 | 1 |
| **Jarque-Bera Normality Test** | Chi-Squared | P-value | Fail to Reject \Reject |
| Residuals | 17.362 | .008 | Reject |
| Skewness | .798 | .849 | Fail to Reject |
| Kurtosis | 16.564 | .000 | Reject |
| **Breutsch-Godfrey Serial Correlation Test** | Chi-Squared | P-Value | Fail to Reject \Reject |
| L1. Residuals | 13.472 | .142 | Fail to Reject |
| L2. Residuals | 25.253 | .118 | Fail to Reject |
| L3. Residuals | 33.009 | .196 | Fail to Reject |
| L4. Residuals | 56.844 | .014 | Reject |
| **Portmanteau Serial Correlation Test** | Chi-Squared | P-Value | Fail to Reject \Reject |
| Residuals | 135.39 | .117 | Fail to Reject |
| **ARCH Test** | Chi-Squared | P-Value | Fail to Reject \Reject |
| Residuals | 257.4 | .000 | Reject |

**All critical values were tested at the 5% significance level. For the correlation matrix the column variable is the response variable.*

Of note, the residuals of the two Producer Price Indexes display a large amount of correlation. The very small residual p-values calculated in normality testing allow us to reject the null hypothesis of normality. In both sets of serial correlation testing the results provided p-values in excess of 5% critical value for all included lags, which means that the VAR(3) model does not exhibit serial correlation issues. However, the autoregressive conditional heteroskedasticity (ARCH) testing produced a p-value of .000, which permits rejection of the no heteroskedasticity null hypothesis. This indicates that the determined model suffers from issues with heteroskedasticity and non-normality.

The VAR parameters were checked for constancy using the OLS-CUSUM test.

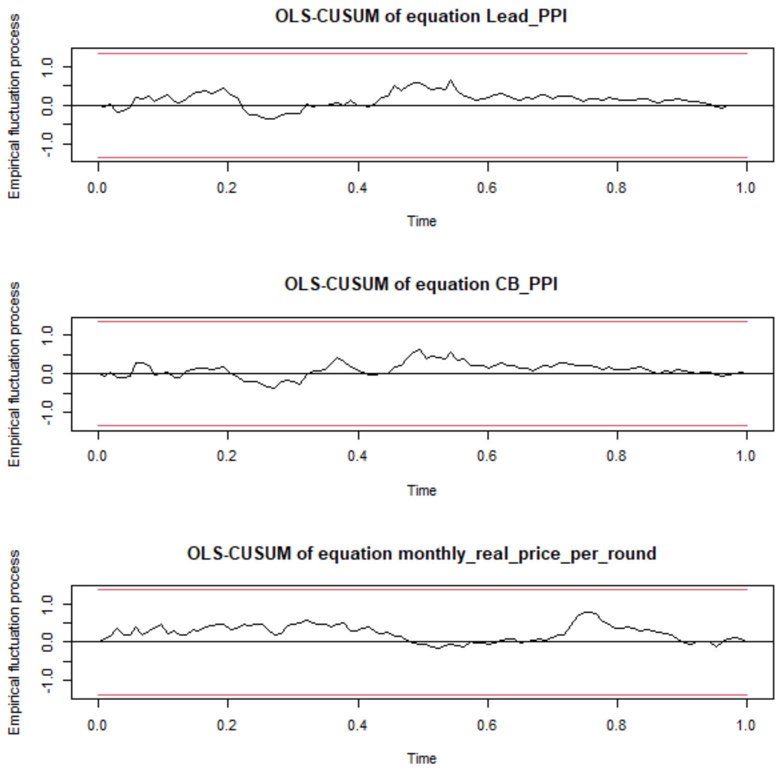
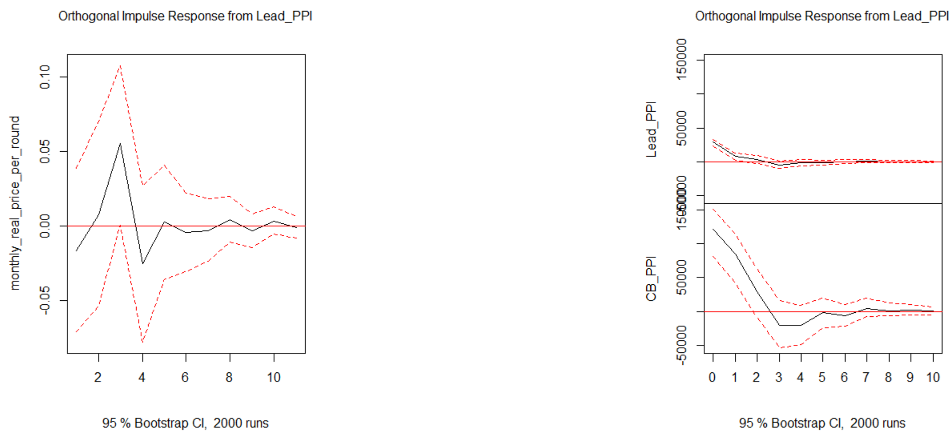



Figure 10: OLS-CUSUM Test Results

The results in Figure 10 illustrate the failure to reject the null hypothesis of constant mean parameters due to the generated sum of squares falling within the 95% confidence intervals for all 3 of the variables of interest. This outcome demonstrates the stability of the means of each series over time. The constancy reflected in Figure 10 reflects the utility of the calculated model for structural analysis.

Impulse response functions (IRF) were generated for the initial Cholesky ordering of PPI Lead $\rightarrow$ PPI Copper & Brass $\rightarrow$ real monthly price per round.

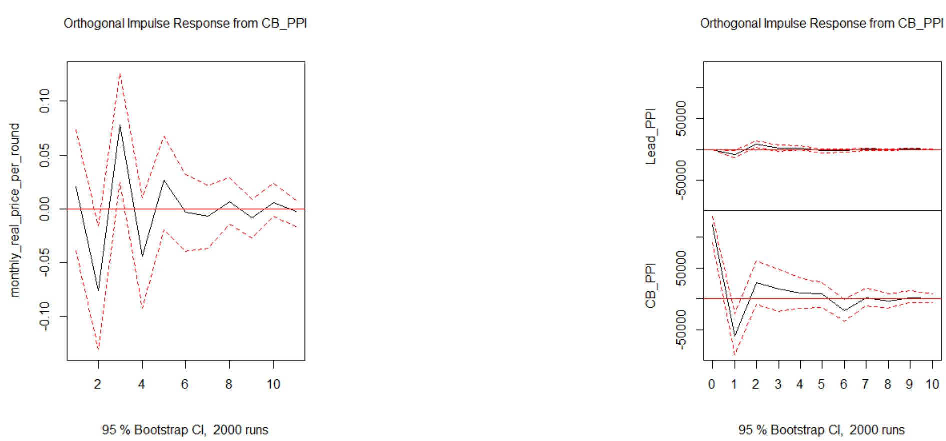



(a) IRF obtained from a lead shock on the monthly real price per round

(b) IRF obtained from a lead shock on the Producer Price Indexes

Figure 11: Lead Impulse Response Functions

The IRF results in Figure 11 demonstrate that a shock to the lead PPI generates a statistically insignificant response in the monthly real price per round, a positive response in itself for 3 months, and a large approximately 3 period positive effect in the PPI for copper & brass. These graphs illustrate that a one period shock to the lead PPI has a positive impact on the copper & brass PPI.



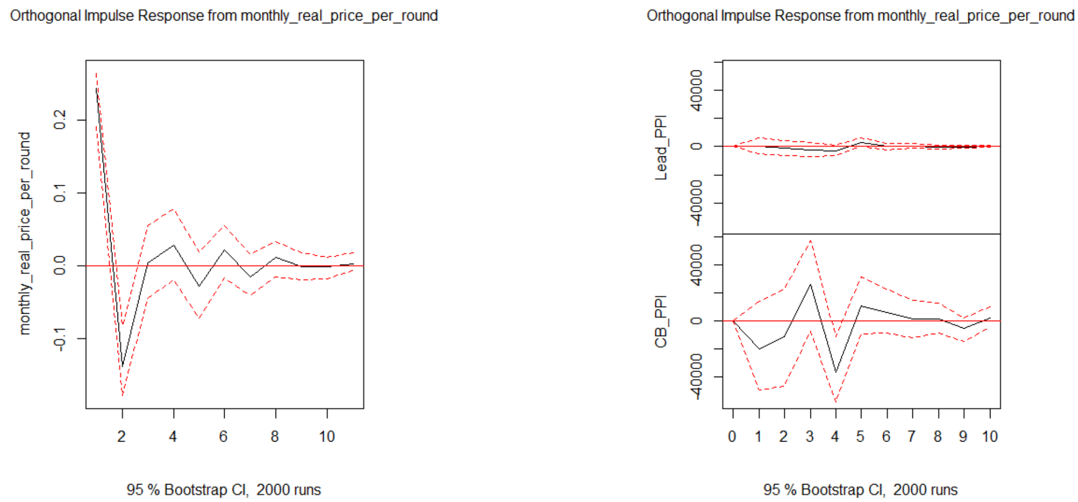

(a) IRF obtained from a copper & brass shock on the monthly real price per round

(b) IRF obtained from a copper & brass shock on the Producer Price Indexes

Figure 12: Copper & Brass Impulse Response Functions

The IRF results in Figure 12 demonstrate that a shock to the copper & brass PPI generates a statistically insignificant response in the monthly real price per round, a minimally negative impact on the PPI for lead, and a large positive effect on itself. From these results it appears that the copper

& brass PPI responds strongly to its own shocks.




(a) IRF obtained from a price per round shock on the monthly price per round

(b) IRF obtained from a price per round shock on the Producer Price Indexes

Figure 13: Monthly Price Per Round Impulse Response Functions

The IRF results in Figure 13 demonstrate that a shock to the price per round generates a strongly positive response to itself that dissipates in approximately 2 periods, a minimal impact on the PPI for lead, and a negative response in the PPI for copper & brass. Under these ordering conditions, the largest impact effect in the price per round appears to originate from its own shocks.

We next considered the FEVD results for the Cholesky ordering of PPI lead $\rightarrow$ PPI copper & brass $\rightarrow$ price per round. The results are displayed in the table below:

**Table 4.** Forecast Error Variance Decomposition

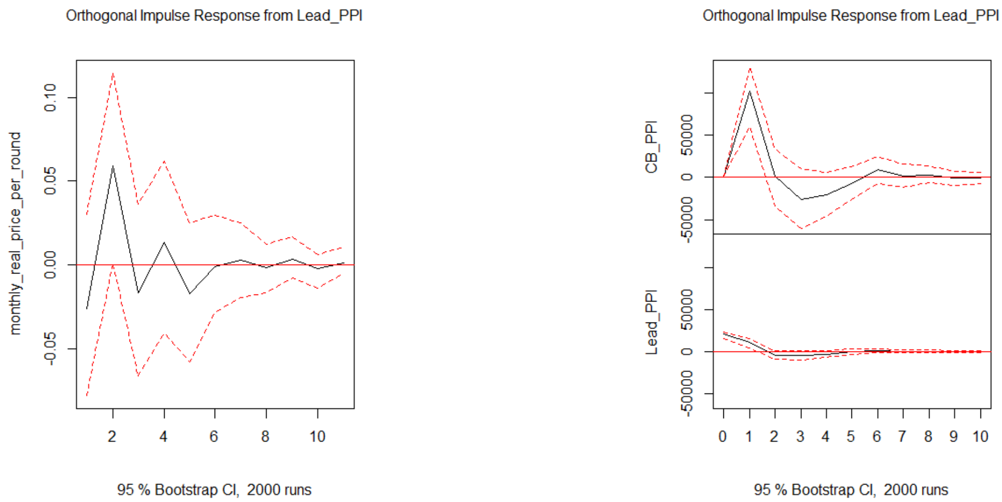
| Period | lead PPI shock | copper & brass PPI shock | monthly price per round shock |
|---|---|---|---|
| Response | lead , copper & brass , price | lead , copper & brass , price | lead , copper & brass , price |
| 0 | 1.000 0.000 0.000 | 0.511 0.489 0.000 | 0.005 0.007 0.988 |
| 1 | 0.939 0.060 0.000 | 0.547 0.443 0.010 | 0.004 0.072 0.924 |
| 2 | 0.869 0.130 0.001 | 0.545 0.442 0.013 | 0.036 0.129 0.835 |
| 3 | 0.863 0.131 0.006 | 0.538 0.435 0.027 | 0.041 0.144 0.815 |
| 4 | 0.852 0.135 0.013 | 0.526 0.419 0.055 | 0.041 0.149 0.810 |
| 5 | 0.843 0.136 0.022 | 0.523 0.419 0.058 | 0.041 0.148 0.811 |
| 6 | 0.841 0.138 0.022 | 0.519 0.423 0.058 | 0.041 0.149 0.811 |
| 7 | 0.840 0.138 0.022 | 0.519 0.423 0.058 | 0.041 0.149 0.810 |
| 8 | 0.840 0.138 0.022 | 0.519 0.423 0.058 | 0.041 0.149 0.810 |
| 9 | 0.840 0.138 0.022 | 0.519 0.422 0.059 | 0.041 0.150 0.809 |
| 10 | 0.840 0.138 0.022 | 0.519 0.422 0.059 | 0.041 0.150 0.809 |

Examining the 11 period variance decomposition originating in a lead PPI shock, it reflects a response of 1 at period 0 and .840 at period 10 to its own shock; for a shock to copper & brass it reflects a response of 0 at period 0 and .138 at period 10; and finally for a price per round shock it reflects a response of 0 at period 0 and .022 at period 10. These results show that 84% of the variance in the lead PPI originate in shocks from itself, while copper & brass alteration explains approximately 14% .

Examining the 11 period variance decomposition originating in a copper & brass shock, it reflects a response of .48 at period 0 and .422 at period 10; for a shock to the lead PPI it shows a response of .511 at period 0 and .519 at period 10; and finally for a monthly price per round shock it reflects a response of 0 at period 0 and .06 at period 10. These results show that 52% of the variance in the copper & brass PPI originate from alterations to the PPI of lead, with 42% explainable by shocks to its own PPI. Interestingly, the copper & brass PPI appears sensitive to changes in the lead PPI.

Examining the 11 period variance decomposition originating in a shock to the monthly price per round, it reflects a response of .988 at period 0 and .809 at period 10; for a shock to the lead PPI the monthly price per round shows a response of .005 at period 0 and .04 at period 10; and finally for a copper & brass shock it reflects a .006 at period 0 and .149 at period 10. These results show that approximately 81% of the variance in the monthly price per round originate in responses to itself, while the PPI for copper & brass explains around 15%. These findings are consistent with the statistically significant findings in the VAR(3) model, where the current real price per round was most predicted with its own lags.

Impulse response functions were generated for the second Cholesky ordering PPI copper & brass $\rightarrow$ PPI lead $\rightarrow$ real monthly price per round. These results are displayed below:

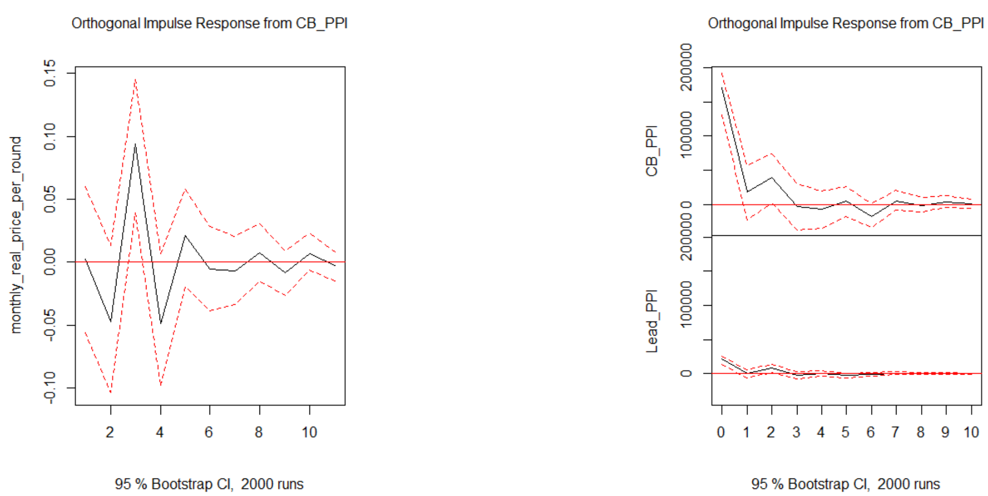



(a) IRF obtained from a lead PPI shock on the monthly price per round

(b) IRF obtained from a lead PPI shock on the Producer Price Indexes

Figure 14: Lead Impulse Response Functions, second ordering

The IRF results in Figure 14 demonstrate that a shock to the lead Producer Price Index generates a statistically insignificant response in the monthly price per round, while corresponding with a positive alteration in itself and the copper & brass PPI.



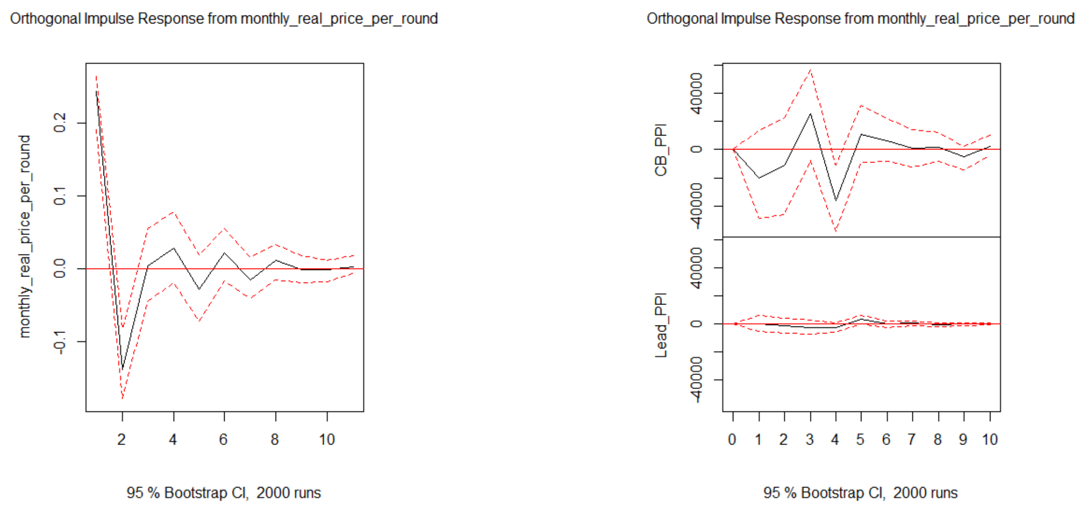

(a) IRF obtained from a copper & brass PPI shock on the monthly price per round

(b) Impulse response functions obtained from a copper & brass PPI shock on the Producer Price Indexes

Figure 15: Copper & Brass Impulse Response Functions, second ordering

The IRF results in Figure 15 demonstrates that a shock to the copper & brass Producer Price

Index also generates a statistically insignificant response in the monthly price per round, while producing a large positive effect in itself and a smaller positive change in the lead PPI. These outcomes thus behave in consonance with the alternative ordering shock to the copper & brass PPI.




(a) IRF obtained from a monthly price per round shock on the monthly price per round

(b) IRF obtained from a monthly price per round shock on the Producer Price Indexes

Figure 16: Monthly Price Per Round Impulse Response Functions, second ordering

The IRF results in Figure 16 illustrate that a shock to the monthly price per round corresponds with a large positive change in itself, no change in the lead PPI, and a small decrease in the copper & brass PPI. Similar to the previous ordering, the largest response in the monthly price per rounds appears to originate in its own shocks.

We similarly analyzed the FEVDs for this alternative Cholesky ordering. The results are displayed in the following table:

| **Table 5.** Forecast Error Variance Decomposition | | | |
|---|---|---|---|
| Period | lead PPI shock | copper & brass PPI shock | monthly price per round shock |
| Response | copper & brass , lead , price | copper & brass , lead , price | copper & brass , lead , price |
| 0 | 0.511 0.489 0.000 | 1.000 0.000 0.000 | 0.000 0.012 0.988 |
| 1 | 0.450 0.549 0.000 | 0.729 0.261 0.010 | 0.026 0.049 0.924 |
| 2 | 0.479 0.520 0.001 | 0.737 0.251 0.013 | 0.117 0.048 0.835 |
| 3 | 0.469 0.525 0.006 | 0.714 0.259 0.027 | 0.138 0.048 0.815 |
| 4 | 0.462 0.525 0.013 | 0.686 0.258 0.055 | 0.140 0.050 0.810 |
| 5 | 0.460 0.518 0.022 | 0.684 0.258 0.058 | 0.139 0.050 0.811 |
| 6 | 0.461 0.518 0.022 | 0.685 0.257 0.058 | 0.140 0.050 0.811 |
| 7 | 0.461 0.517 0.022 | 0.685 0.257 0.058 | 0.140 0.050 0.810 |
| 8 | 0.461 0.517 0.022 | 0.685 0.257 0.058 | 0.140 0.050 0.810 |
| 9 | 0.461 0.517 0.022 | 0.684 0.257 0.059 | 0.141 0.050 0.809 |
| 10 | 0.461 0.517 0.022 | 0.684 0.257 0.059 | 0.141 0.050 0.809 |

Considering an 11 period variance decomposition for a lead PPI shock, it reflects a response of .489 at period 0 and .517 at period 10 for itself; for a copper & brass PPI shock it shows a response of .511 at period 0 and .461 at period 10; and finally for a monthly price per round shock it shows a response of 0 at period 0 and .02 at period 10. These results show that 52% of the variance in the lead PPI originate from changes in the copper & brass PPI, with 46% explainable from its own variations.

Examining the 11 period variance decomposition for the copper & brass PPI, for its own shock it reflects a response of 1 at period 0 and .684 at period 10; for a lead PPI shock it shows a response of 0 at period 0 and .257 at period 10; finally for a monthly price per round shock the copper & brass PPI shows a response of 0 at period 0 and .059 at period 10. This decomposition illustrates that 68% of its variance comes from its own alteration, while approximately 26% derives from variation in the PPI for lead.

In the last variance decomposition for the monthly real price per round, its own shock at period 0 generates a response of .988 at period 0 and .809 at period 10; for a shock to the copper & brass PPI the monthly price per round shows a response of .000 at period 0 and .141 at period 10; and finally for a lead PPI shock it reflects a response of .012 at period 0 and .050 at period 10. This decomposition demonstrates that 81% of the variation in the monthly price per round comes from itself, while shocks in the copper & brass PPI explain approximately 14%. Similar to the findings of the VAR(3) coefficients and the earlier Cholesky ordering, between the two Producer Price Indexes the copper & brass PPI exhibits the most influence.

Finally, we conducted Granger causality testing to determine the extent to which each time series assists as a factor in forecasting the others. Granger causality testing does not suffer from sensitivity to Cholesky orderings, permitting a singular summation of these results unlike the impulse response functions and forecast error variance decomposition. In the relationship between the lead PPI as a source of Granger causality for the copper & brass PPI and monthly price per round, the p-value test result of .000 provides the basis to strongly reject the null hypothesis of no Granger causality, which means the lead PPI assists in forecasting the other two variables. In the relationship between the copper & brass PPI as a source of Granger causality, the calculated p-value of .000 strongly indicates that the copper & brass PPI assists in forecasting the other variables. Finally, in the case of the monthly price per round as an origin of Granger causality for the other variables, the calculated p-value of .014 indicates we can reject the null hypothesis and infer that the monthly price per round assists in forecasting the other 2 series. From these results we can observe that all of the included variables Granger cause the others.

# 5 Conclusion

This study examined the dynamic relationship between the monthly price per round and changes in the ammunition production input factors of copper, brass, and lead using a vector autoregression model. This approach tested the hypothesis that supply side factors, captured in the selected Producer Price Index time series, primarily influence the short run price per round of ammunition. Due to the limit of this focus we cannot entirely discount the counterinfluence of aggregate demand on ammunition prices. This methodology was chosen to investigate an alternative explanation to the consumption trends for firearms illustrated in previous research.[18] Additionally, it sought to offer a missing general descriptive overview of the domestic ammunition market and its trends as

18. Studdert et al., "Handgun Acquisitions in California After Two Mass Shootings"; Pak, "The effects of mass shootings on gun sales: Motivations, mechanisms, policies and regulations."

a justification for further study.

The finding that 7 of the 10 most popular calibers in the data set accounted for 51% of all recorded sales, with 6 of those 7 calibers popularly used in military or law enforcement applications, supports the earlier finding that consumers prefer to purchase calibers with specific utility in combat. This result was genuinely surprising due to the initial hypothesis of a more even consumption distribution among the 119 calibers contained within the data set. In terms of price elasticity of demand, the two most popular calibers in the data set (9mm and 5.56x45mm) displayed the highest price elasticity, while the most expensive popular caliber on average (7.62x51mm) was inelastic. Of note, .40 S&W, a caliber extremely popular among American law enforcement, displayed the highest inelasticity for the popular calibers.

The study determined that neither the Producer Price Index for copper, brass, or lead between August 2006 and July 2015 majorly explained deviations in the monthly price per round. Between the two indexes, irrespective of Cholesky ordering, both the calculated VAR(3) model results and IRF models showed that neither PPI exerted a statistically significant effect on the monthly price per round. This conclusion was further supported by the evidence contained within the forecast error variance decomposition; the calculated effect of a shock to either the lead or copper & brass PPI explained no more than 6% of the change in the monthly price per round, irrespective of ordering.

While this research cannot definitively deny the existence of a stronger alternative explanation, the calculated results from structural analysis show that previous lags in the monthly price per round offer the most plausible explanation for variation in the current price of ammunition. From the examined VAR(3) model the coefficient of the first two lags of monthly ammunition prices offer the strongest statistical support for predicting ammunition prices in the current period. Under both examined Cholesky orderings, the FEVD results showed that monthly price per round shocks accounted for approximately 81% of variation in ammunition prices. Commensurate with the Granger causality p-values, the referenced time series offer forecasting utility for the monthly price per round. Given the above, these findings suggest that short run changes in the real price per round likely originate from changes in demand side factors.

The VAR models residual correlation and normality results illustrate possible shortcomings in the model's reliability and validity. The residuals of the lead, copper, and brass PPIs were highly correlated, indicating potential additional relationships unaccounted for within the model. Given that all of the elements included in the PPIs are common industrial manufacturing inputs, one possible explanation suggests a time varying relationship among the variables that was inaccurately accounted for. This shortcoming could also indicate model misspecification in the lag order or general functional form.

Conclusively, this study was conducted to provide an initial overview and general insight into the dynamics of the domestic ammunition market. Rather than offering an authoritative final declaration on the topic, it was executed to provide a preliminary investigation on a topic in much need of further econometric analysis. These early findings suggest that a larger data set on ammunition sales, added explanatory variables, and a more experimental approach could render significant results on how the ammunition market behaves in response to political signals.

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