Decision Trees

CS229: Machine Learning

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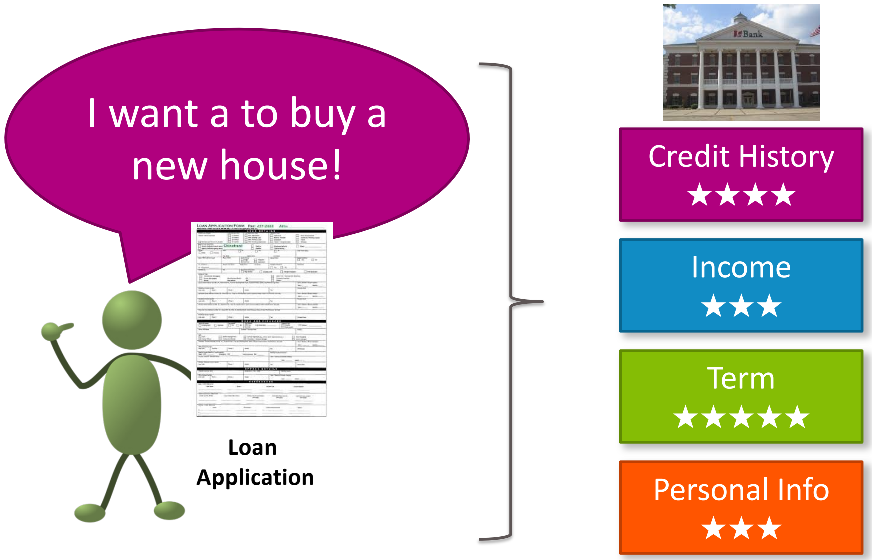
Slides include content developed by and co-developed with Emily Fox

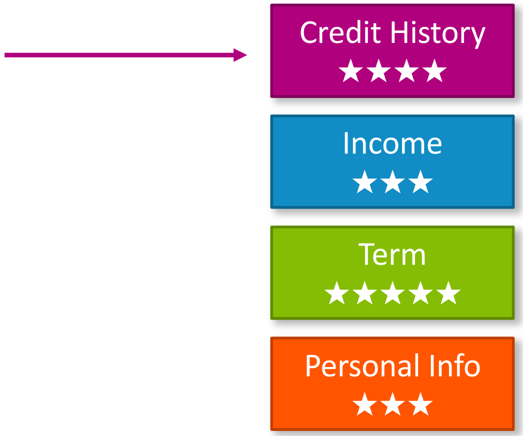
Predicting potential loan defaults

What makes a loan risky?



Credit history explained

Did I pay previous loans on time?

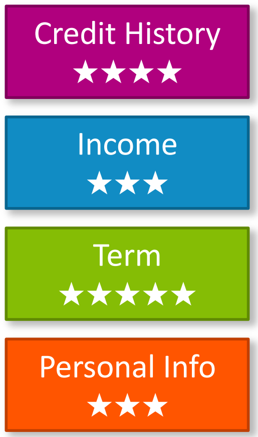


Example: excellent, good, or fair

Income

What’s my income?

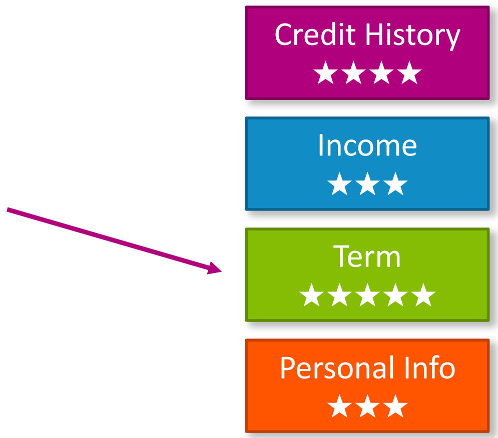
Example:
$80K per year



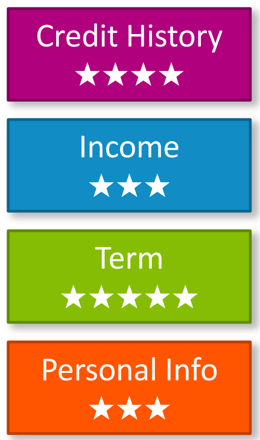
Loan terms

How soon do I need to pay the loan?

Example: 3 years, 5 years,…



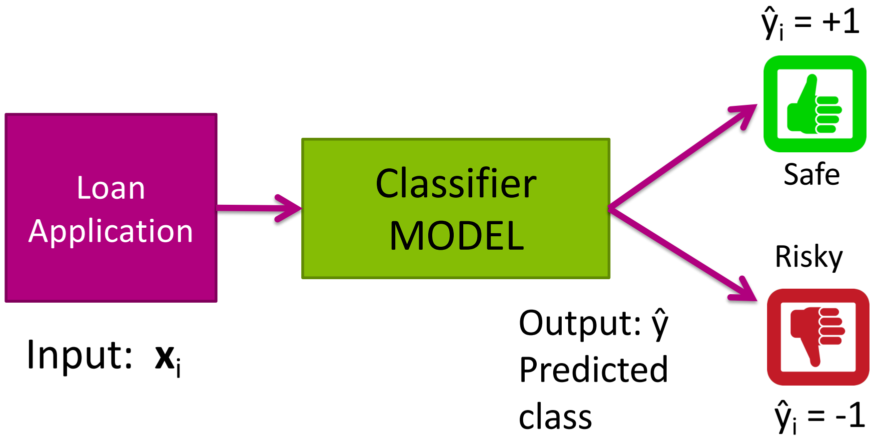
Personal information



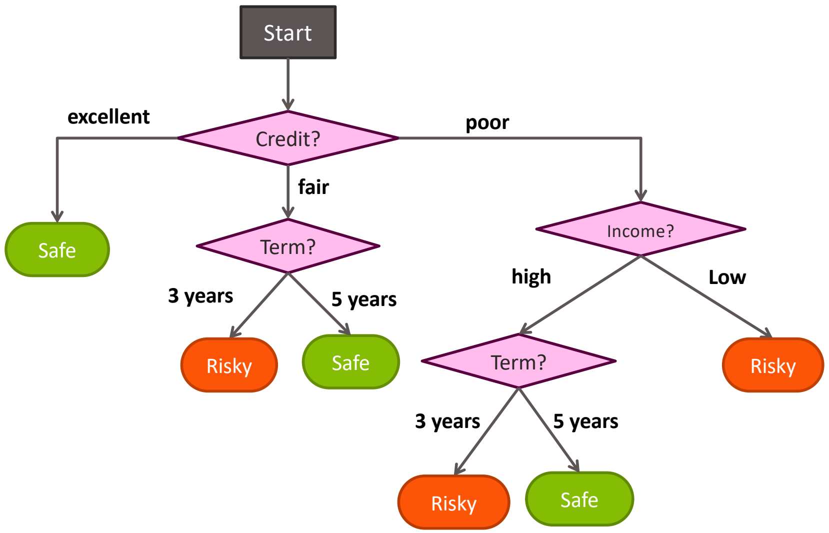
Age, reason for the loan, marital status,…

Example: Home loan for a married couple

Classifier review

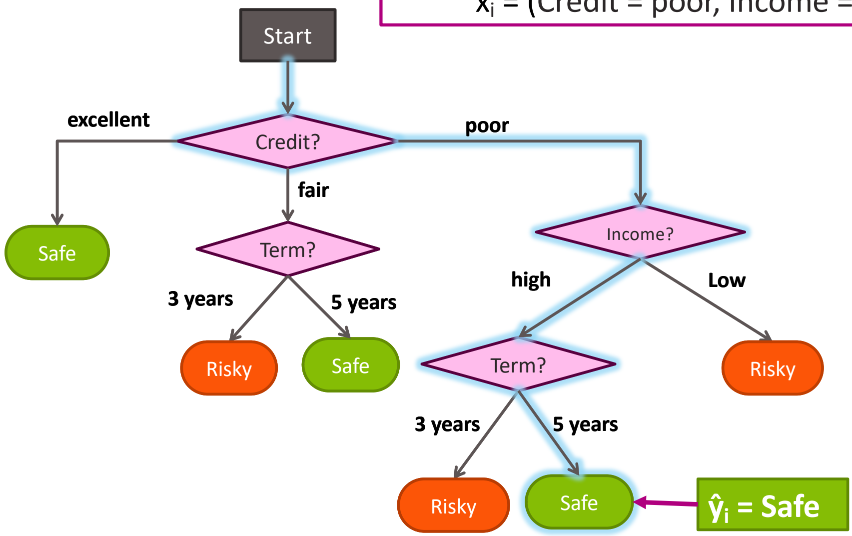


This module ... decision trees



Scoring a loan application

x_i = (Credit = poor, Income = high, Term = 5 years)

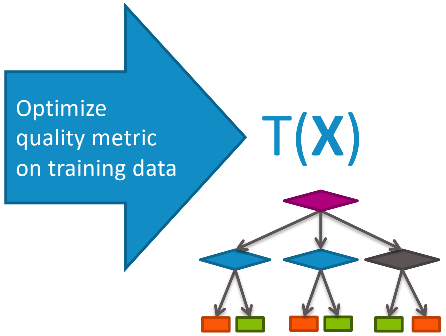


Decision tree learning task

Decision tree learning problem

Training data: N observations (x_i, y_i)

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe



Quality metric: Classification error

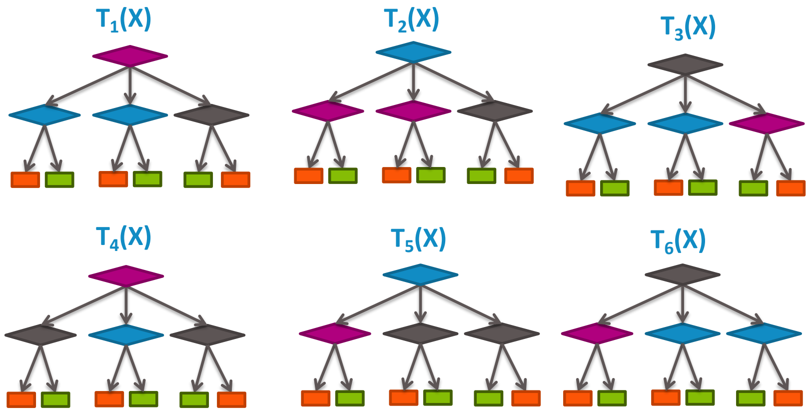
- Error measures fraction of mistakes

$$\text{Error} = \frac{\#\ \text{incorrect predictions}}{\#\ \text{examples}}$$

  – Best possible value : 0.0
  – Worst possible value: 1.0

How do we find the best tree?

Exponentially large number of possible trees makes decision tree learning hard!



Learning the smallest decision tree is an *NP-hard problem* [Hyafil & Rivest '76]

Greedy decision tree learning

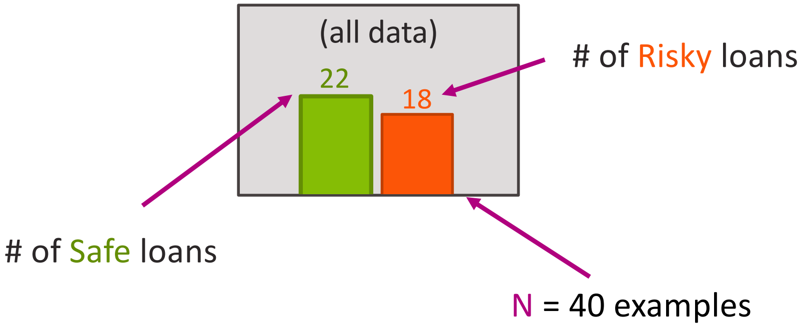
Our training data table

Assume N = 40, 3 features

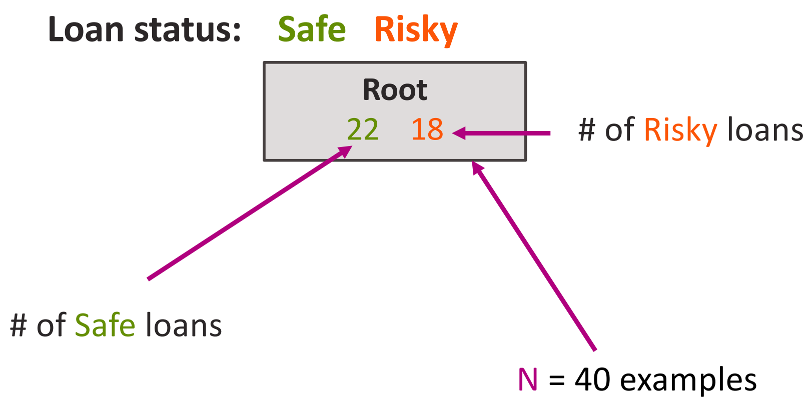
Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe

Start with all the data

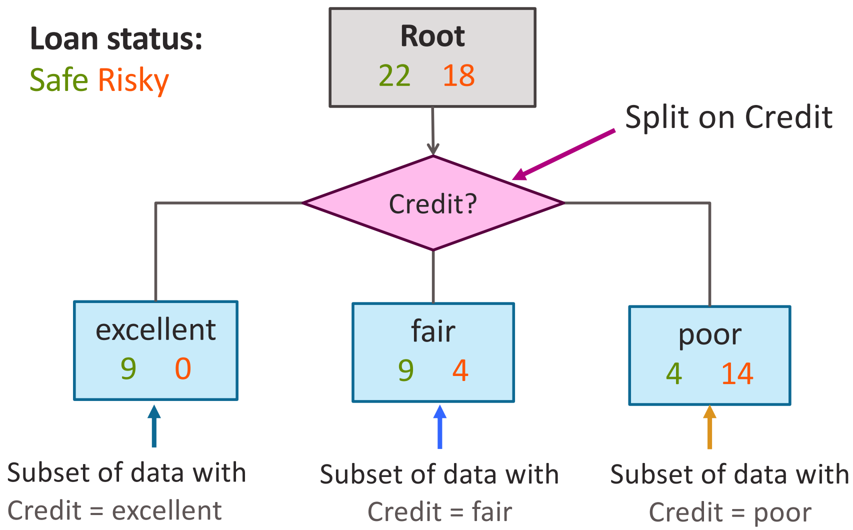
Loan status: **Safe** **Risky**



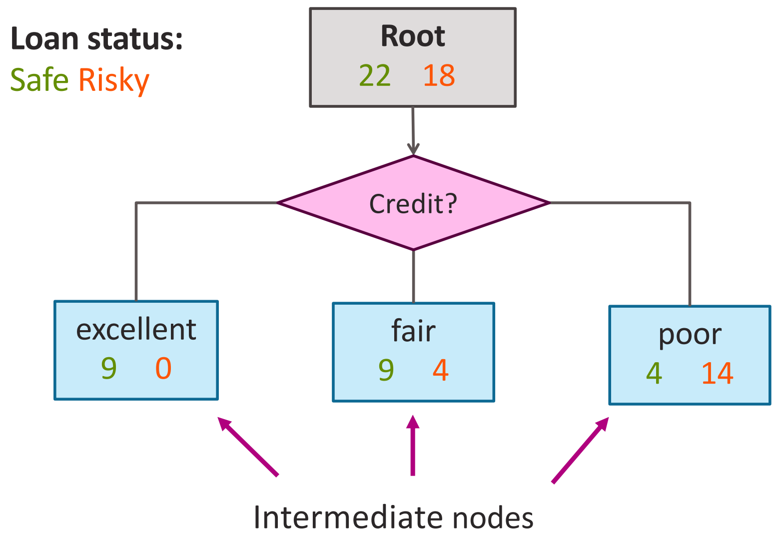
Compact visual notation: Root node



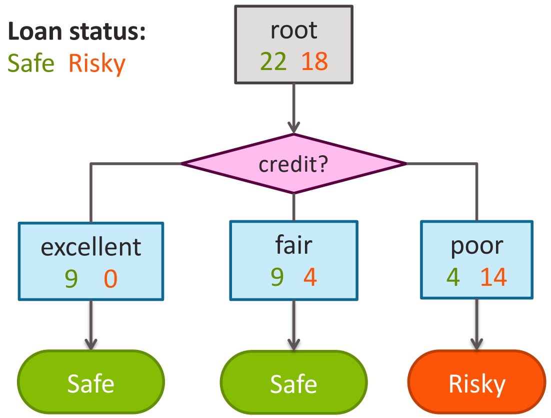
Decision stump: Single level tree



Visual notation: Intermediate nodes



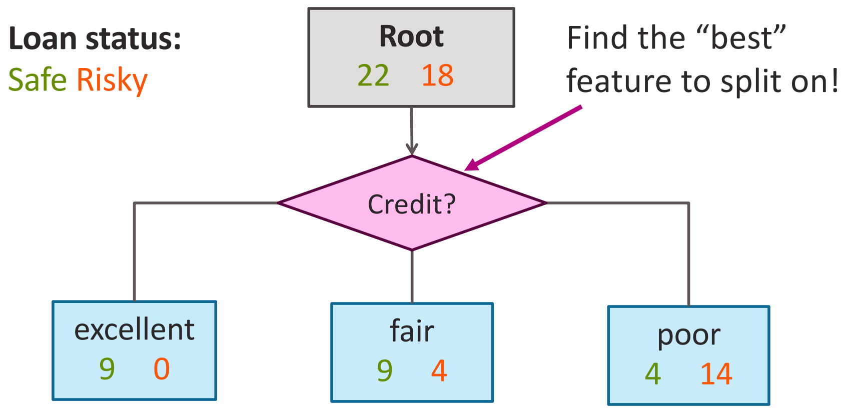
Making predictions with a decision stump



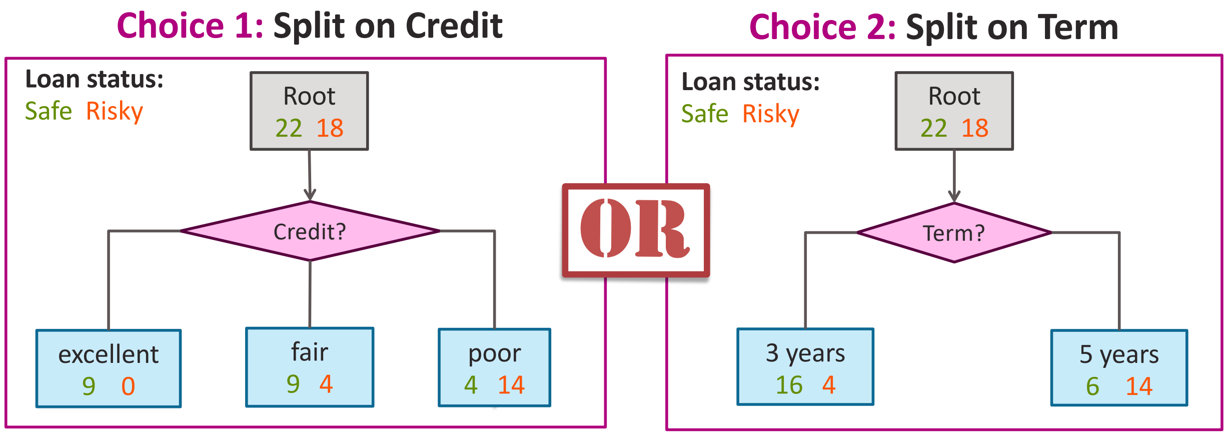
For each intermediate node, set $\hat{y}$ = majority value

Selecting best feature to split on

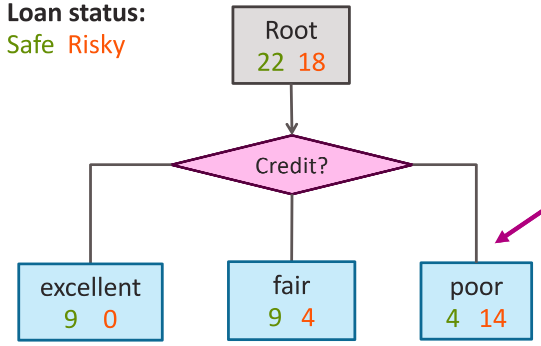
How do we learn a decision stump?



How do we select the best feature?



How do we measure effectiveness of a split?

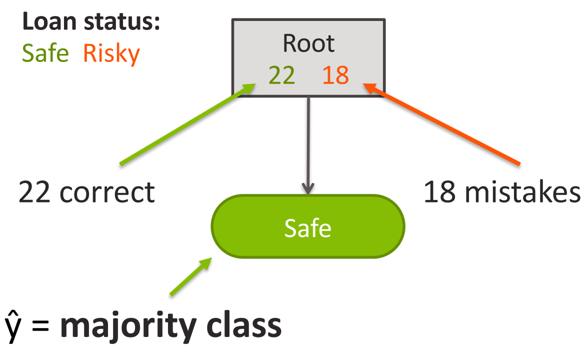


Idea: Calculate classification error of this decision stump

$$\text{Error} = \frac{\text{\# mistakes}}{\text{\# data points}}$$

Calculating classification error

- Step 1: ŷ = class of majority of data in node
- Step 2: Calculate classification error of predicting ŷ for this data

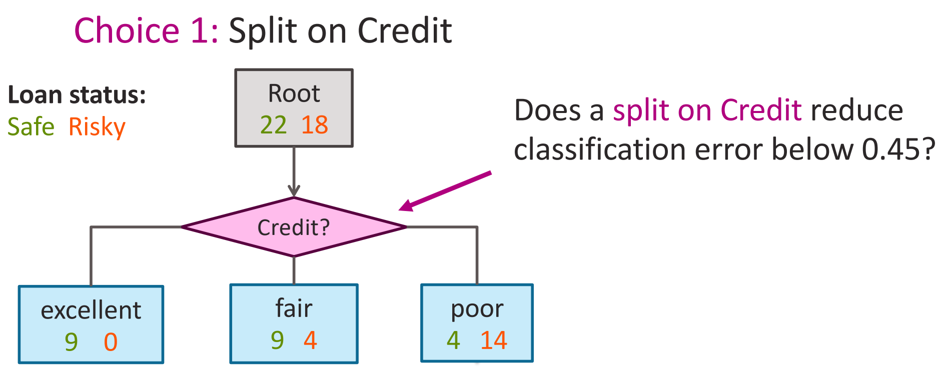


Error = ______

=

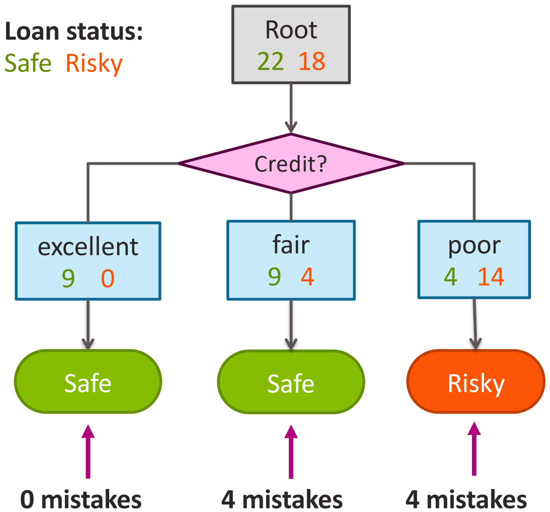
Tree	Classification error
(root)	0.45

Choice 1: Split on Credit history?



Split on Credit: Classification error

Choice 1: Split on Credit

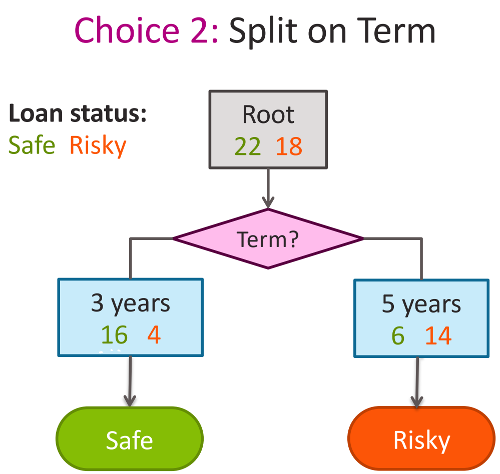


Error = ______

=

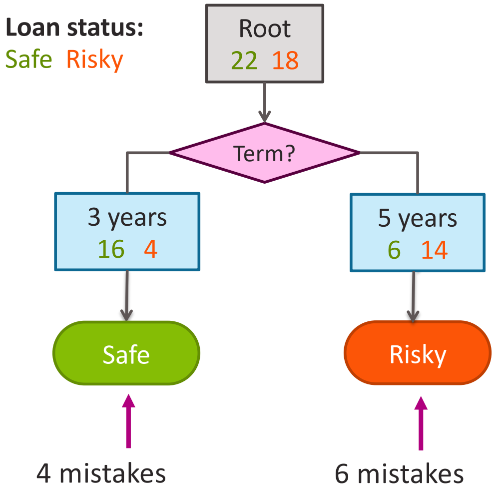
Tree	Classification error
(root)	0.45
Split on credit	0.2

Choice 2: Split on Term?



Evaluating the split on Term

Choice 2: Split on Term



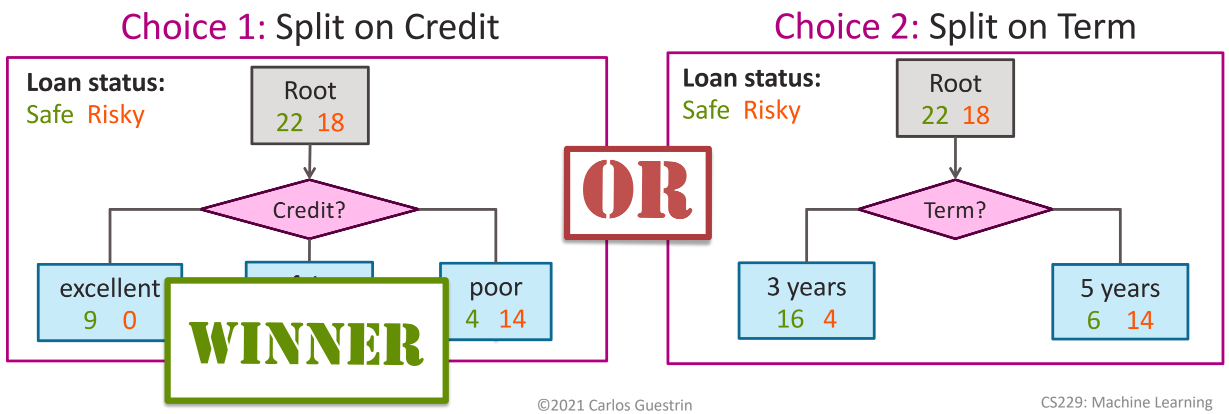
Error = ______

=

Tree	Classification error
(root)	0.45
Split on credit	0.2
Split on term	0.25

Choice 1 vs Choice 2: Comparing split on Credit vs Term

Tree	Classification error
(root)	0.45
split on credit	0.2
split on loan term	0.25

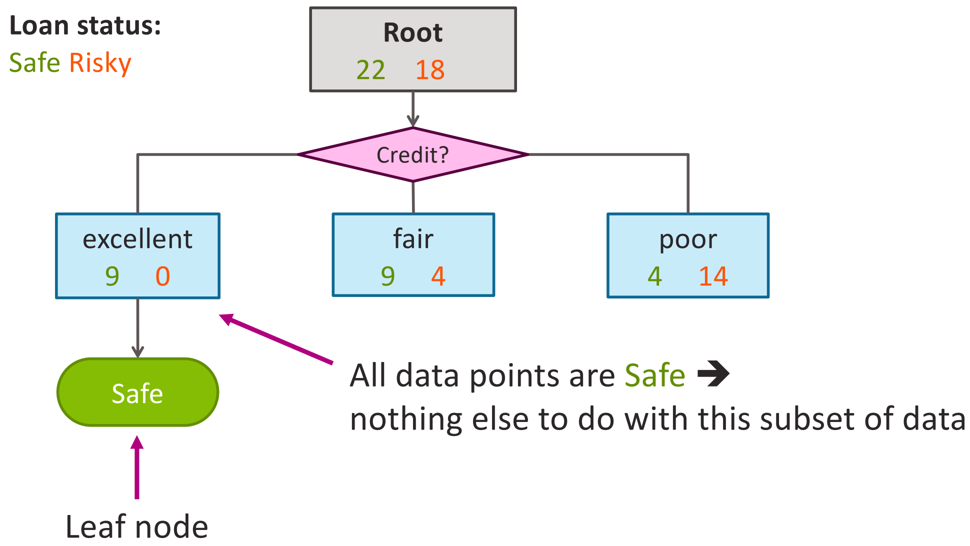


Feature split selection algorithm

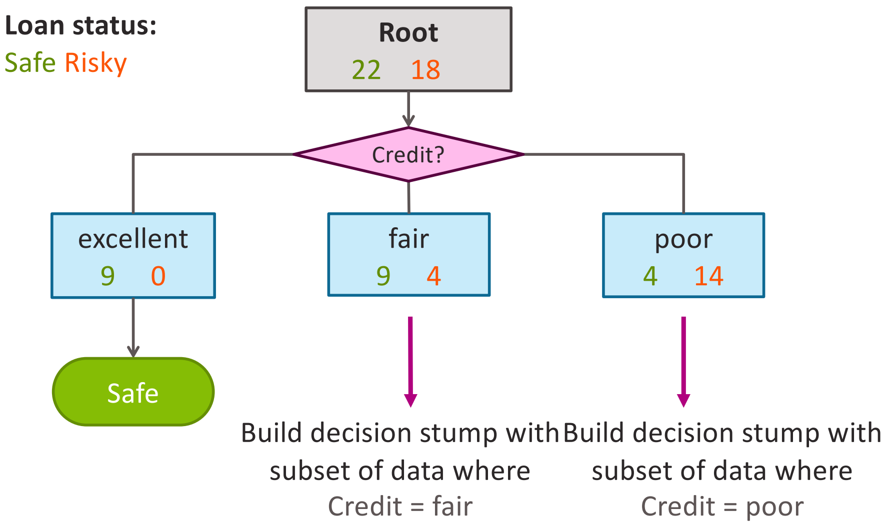
- Given a subset of data M (a node in a tree)
- For each feature $h_i(x)$:
  1. Split data of M according to feature $h_i(x)$
  2. Compute classification error of split
- Chose feature $h^*(x)$ with lowest classification error

Recursion & Stopping conditions

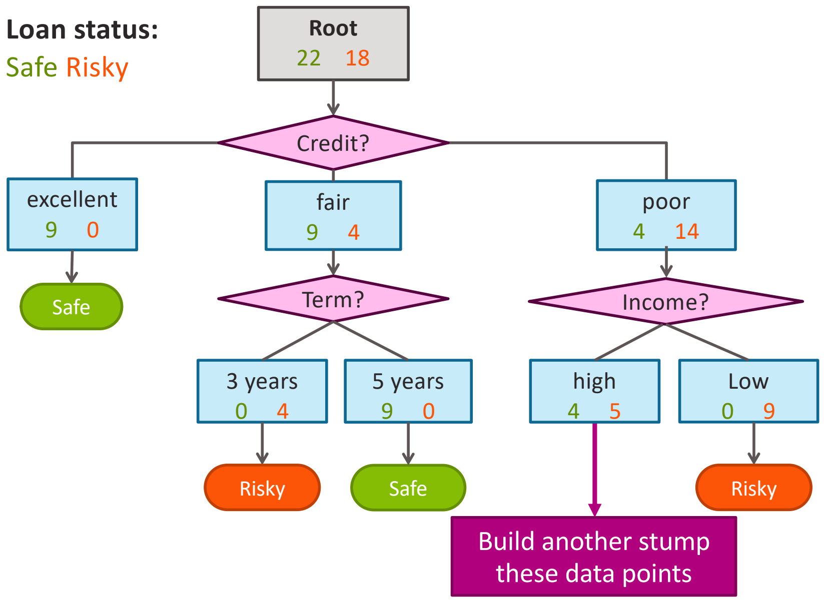
We've learned a decision stump, what next?



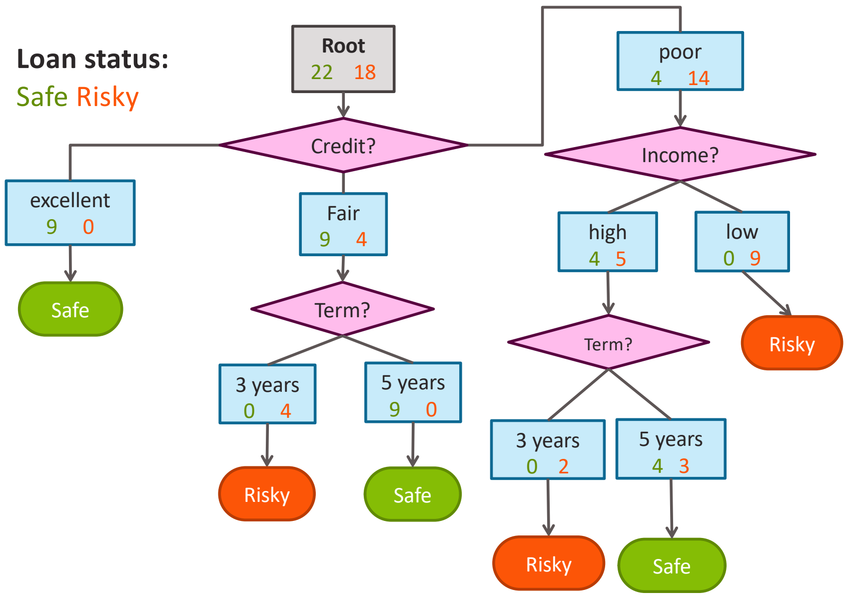
Tree learning = Recursive stump learning



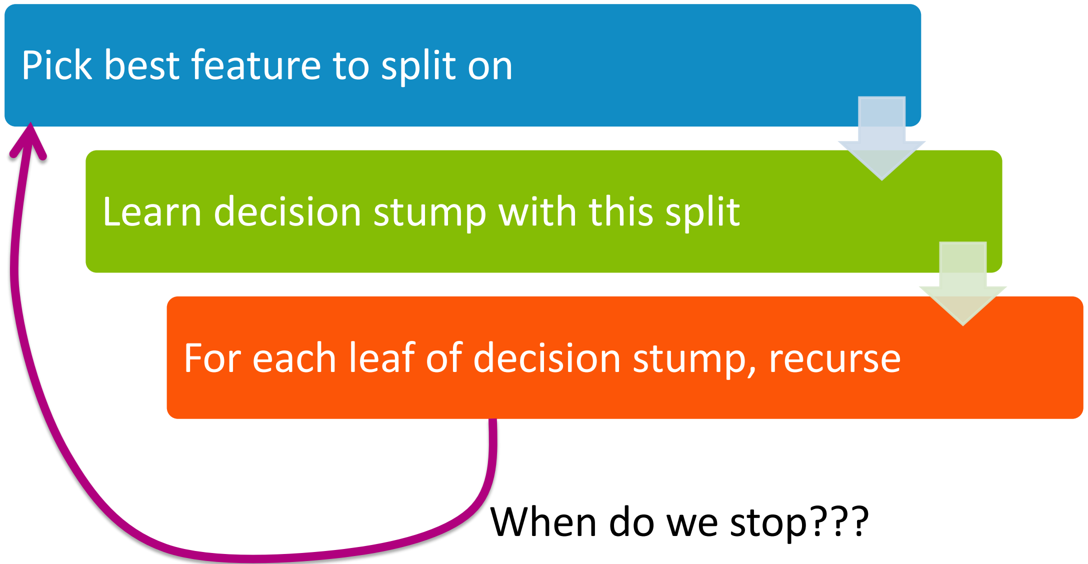
Second level



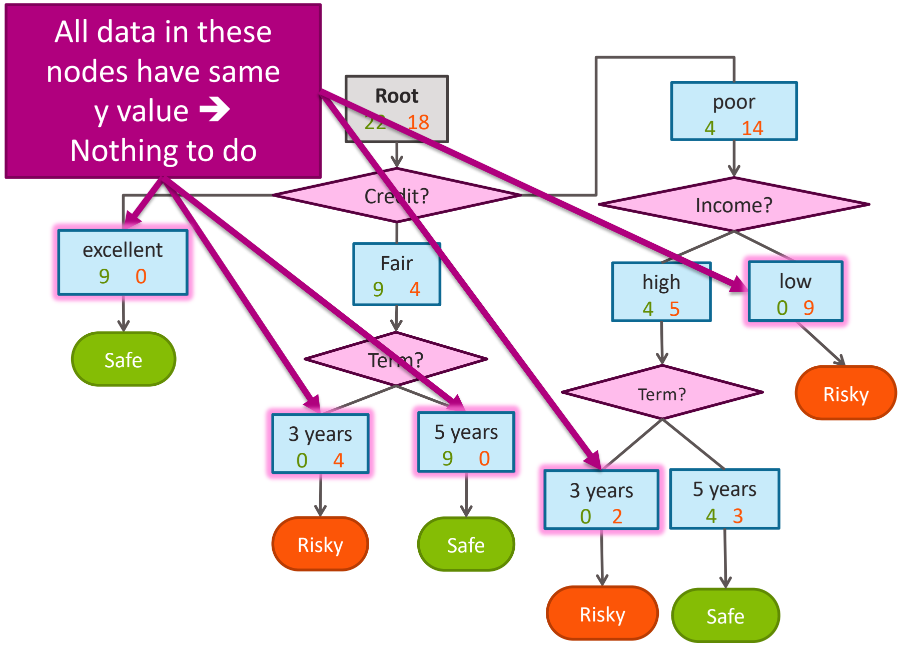
Final decision tree



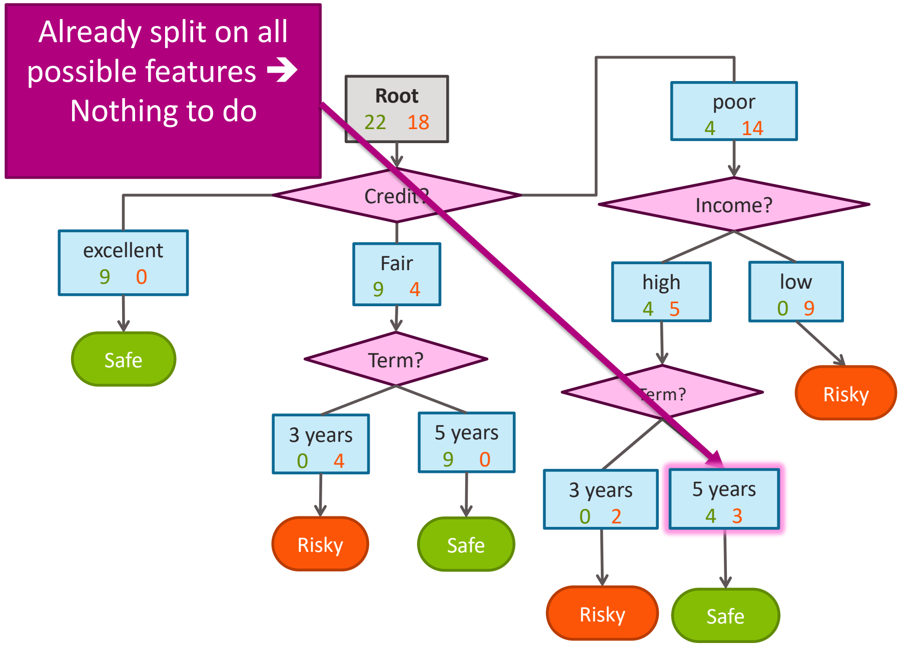
Simple greedy decision tree learning



Stopping condition 1: All data agrees on y



Stopping condition 2: Already split on all features



Greedy decision tree learning

- Step 1: Start with an empty tree
- Step 2: Select a feature to split data
- For each split of the tree:
  - Step 3: If nothing more to do, make predictions
  - Step 4: Otherwise, go to Step 2 & continue (recurse) on this split

Pick feature split leading to lowest classification error

Stopping conditions 1 & 2

Recursion

Is this a good idea?

Proposed stopping condition 3:
Stop if no split reduces the classification error

Stopping condition 3:
Don't stop if error doesn't decrease???

y = **x**[1] xor **x**[2]

x[1]	**x**[2]	**y**
False	False	False
False	True	True
True	False	True
True	True	False

y values
True False

Root	
2	2

Error = ______

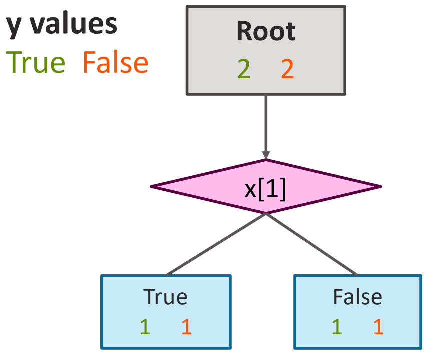
=

Tree	Classification error
(root)	0.5

Consider split on x[1]

y = **x**[1] **xor** **x**[2]

x[1]	**x**[2]	**y**
False	False	False
False	True	True
True	False	True
True	True	False



Error = ______

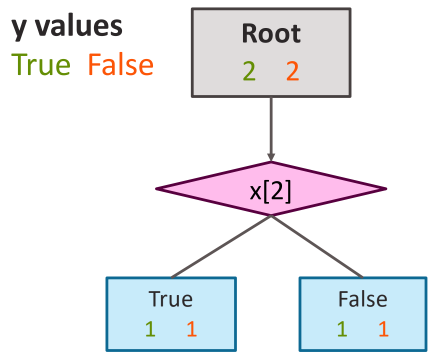
=

Tree	Classification error
(root)	0.5
Split on x[1]	0.5

Consider split on x[2]

y = **x**[1] **xor** **x**[2]

x[1]	**x**[2]	**y**
False	False	False
False	True	True
True	False	True
True	True	False



$$\text{Error} = \frac{1+1}{2+2} = 0.5$$

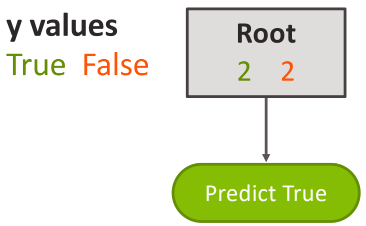
Neither features improve training error… Stop now???

Tree	Classification error
(root)	0.5
Split on x[1]	0.5
Split on x[2]	0.5

Final tree with stopping condition 3

y = **x**[1] xor **x**[2]

x[1]	**x**[2]	**y**
False	False	False
False	True	True
True	False	True
True	True	False



Tree	Classification error
with stopping condition 3	0.5

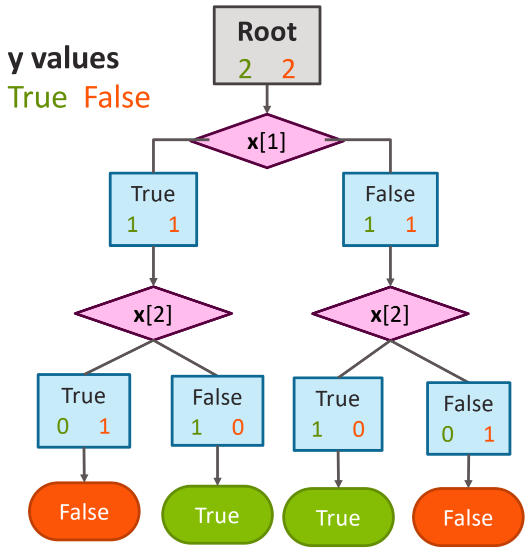
Without stopping condition 3

Condition 3 (stopping when training error doesn't't' improve) is not recommended!

$y = x[1]$ **xor** $x[2]$

x[1]	x[2]	y
False	False	False
False	True	True
True	False	True
True	True	False

Tree	Classification error
with stopping condition 3	0.5
without stopping condition 3	

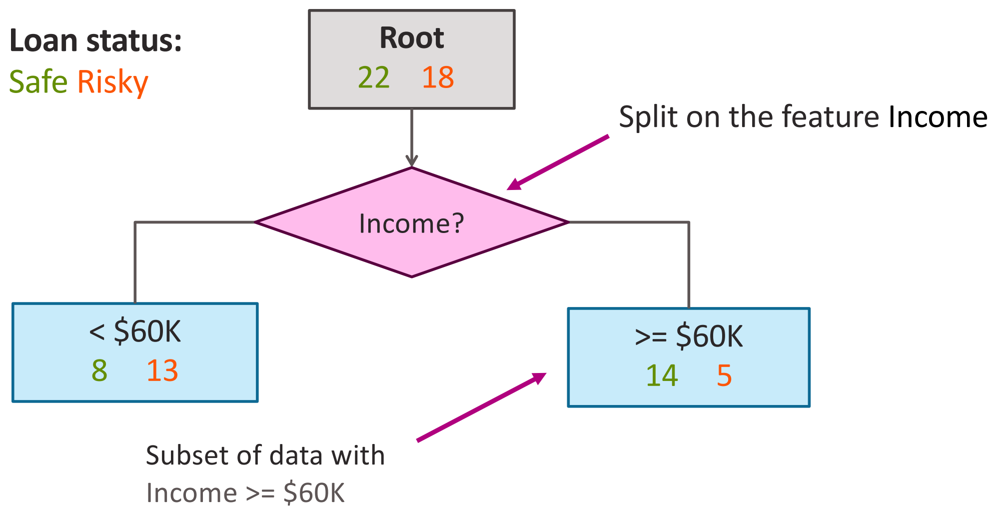


Decision tree learning:
Real valued features

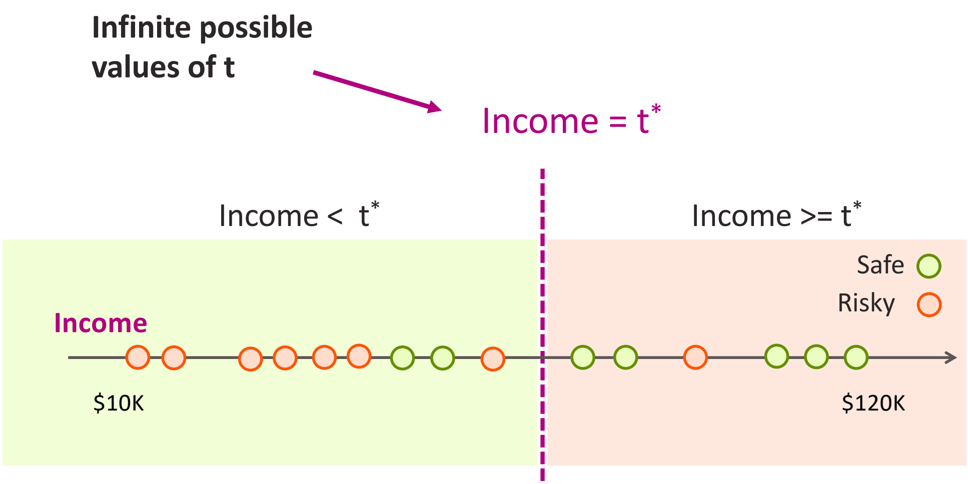
How do we use real values inputs?

Income	Credit	Term	y
$105 K	excellent	3 yrs	Safe
$112 K	good	5 yrs	Risky
$73 K	fair	3 yrs	Safe
$69 K	excellent	5 yrs	Safe
$217 K	excellent	3 yrs	Risky
$120 K	good	5 yrs	Safe
$64 K	fair	3 yrs	Risky
$340 K	excellent	5 yrs	Safe
$60 K	good	3 yrs	Risky

Threshold split

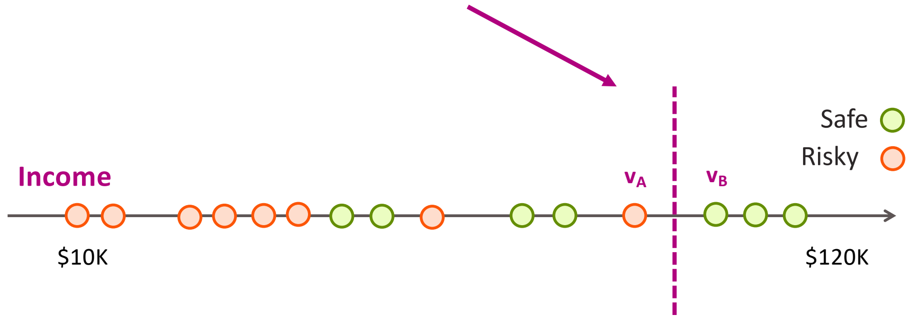


Finding the best threshold split

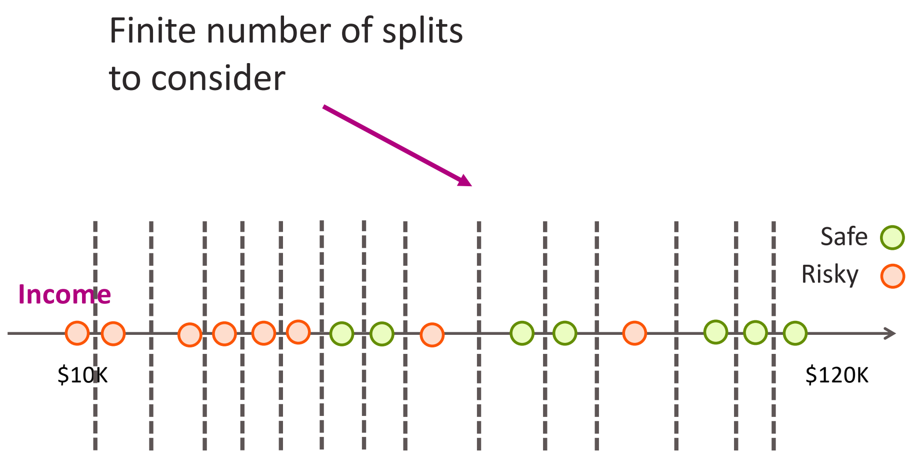


Consider a threshold between points

Same classification error for any threshold split between v_A and v_B



Only need to consider mid-points

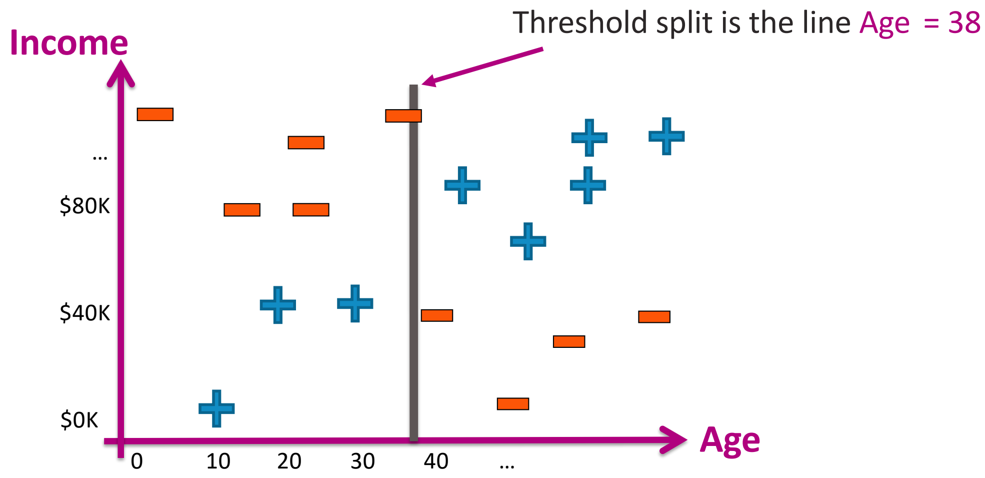


Threshold split selection algorithm

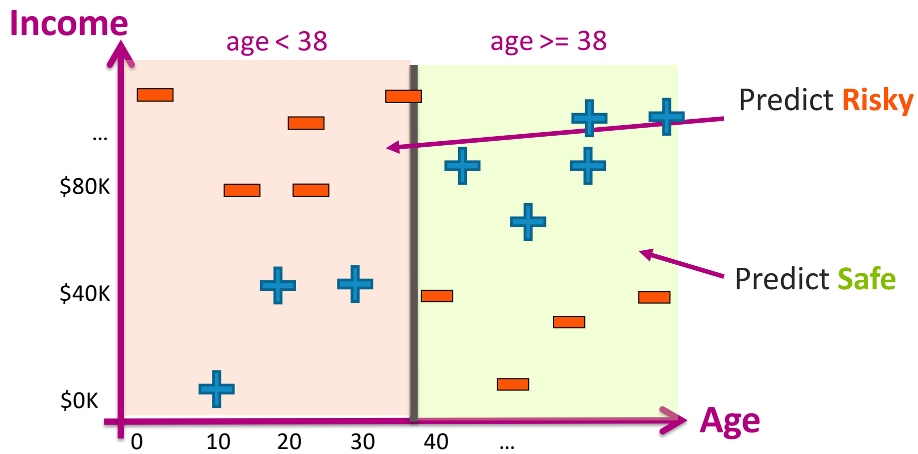
- Step 1: Sort the values of a feature $h_j(x)$:

  Let $\{\mathbf{v_1, v_2, v_3, \ldots\ v_N}\}$ denote sorted values
- Step 2:
  - For i = 1 … N-1
    - Consider split $t_i = (\mathbf{v_i} + \mathbf{v_{i+1}}) / 2$
    - Compute classification error for treshold split $h_j(x) >= \mathbf{t_i}$
  - Chose the $\mathbf{t}^*$ with the lowest classification error

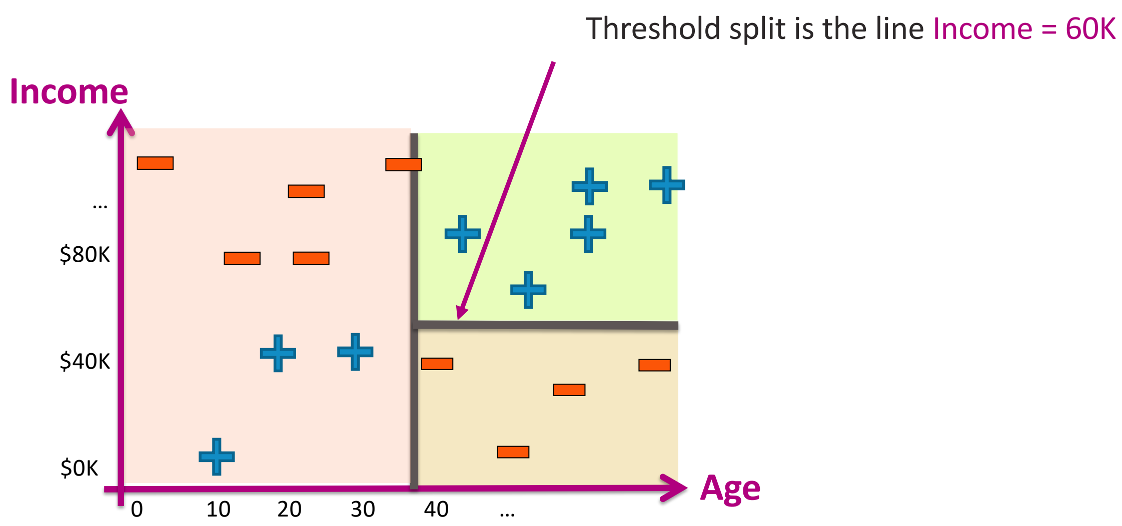
Visualizing the threshold split



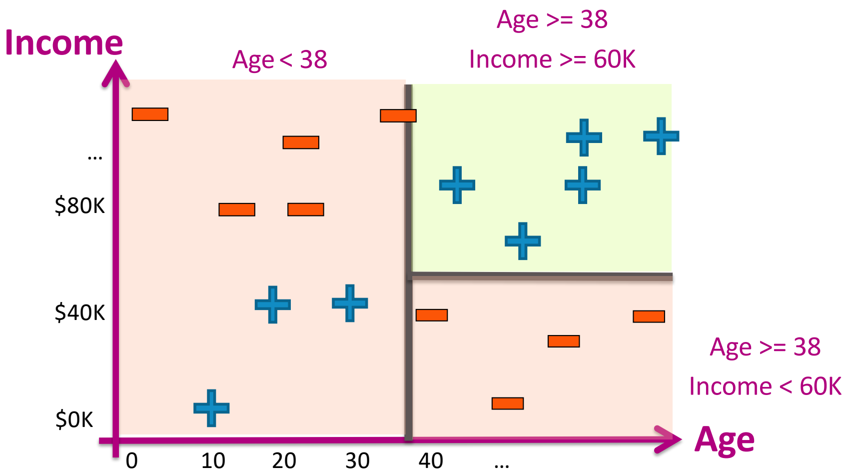
Split on Age >= 38



Depth 2: Split on Income >= $60K



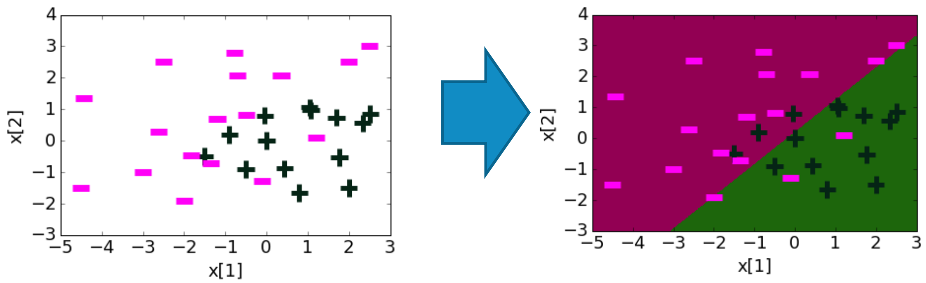
Each split partitions the 2-D space



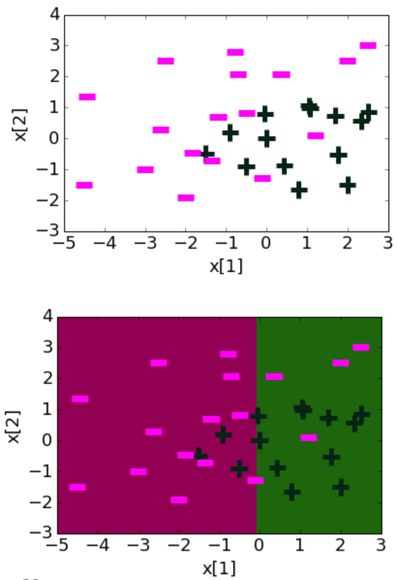
Decision trees vs logistic regression: *Example*

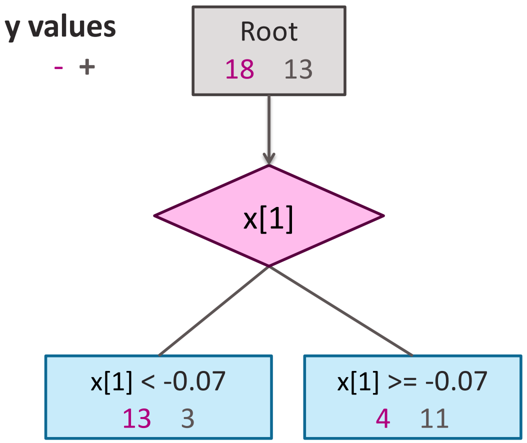
Logistic regression

Feature	Value	Weight Learned
$h_0(x)$	1	0.22
$h_1(x)$	x[1]	1.12
$h_2(x)$	x[2]	-1.07

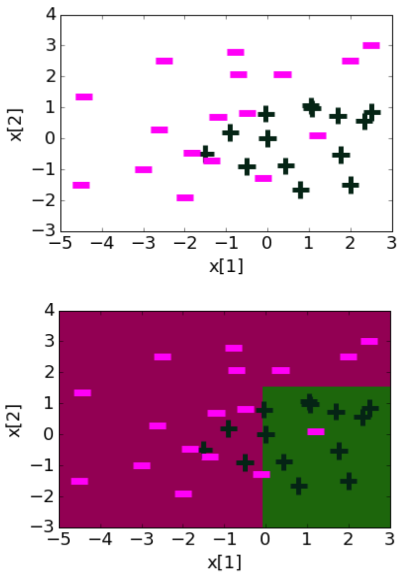


Depth 1: Split on x[1]

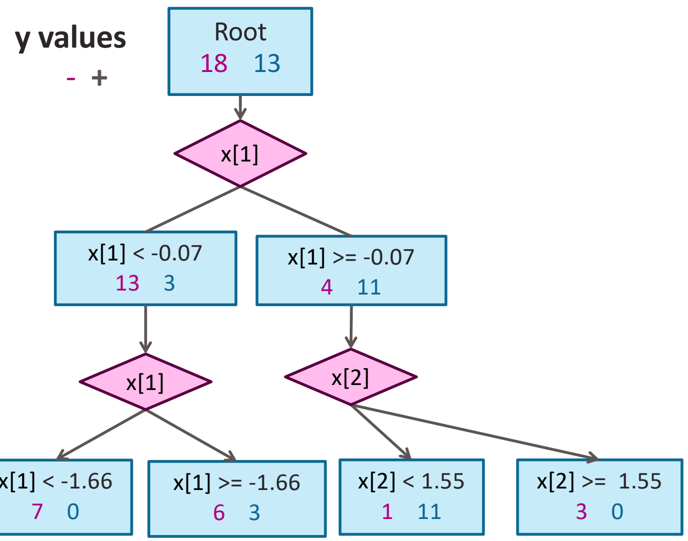




Depth 2



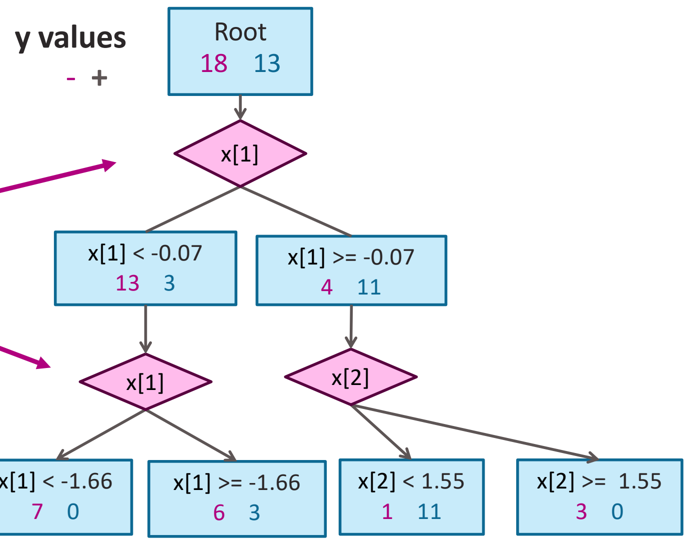
x[2]
x[1]



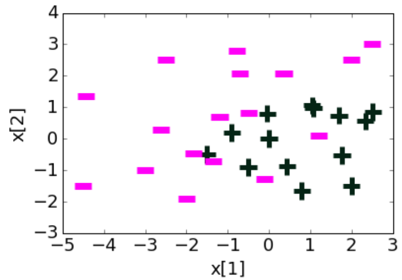
y values
- +
Root
18 13
x[1]
x[1] < -0.07
13 3
x[1] >= -0.07
4 11
x[1]
x[2]
x[1] < -1.66
7 0
x[1] >= -1.66
6 3
x[2] < 1.55
1 11
x[2] >= 1.55
3 0

Threshold split caveat

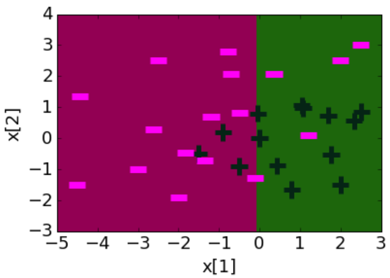
For threshold splits, same feature can be used multiple times



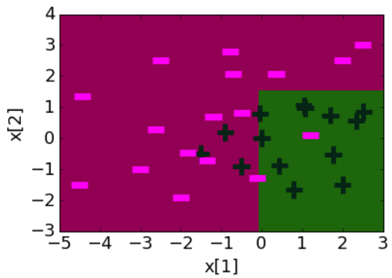
Decision boundaries



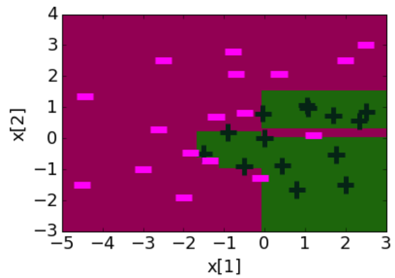
Depth 1



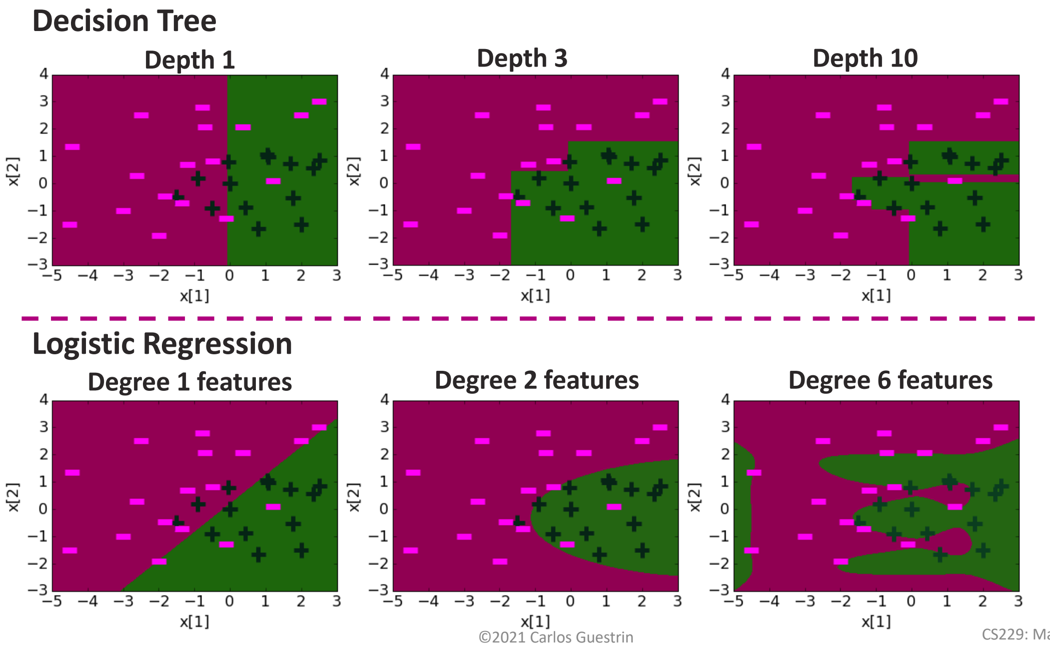
Depth 2



Depth 10



Comparing decision boundaries



Summary of decision trees

What you can do now

- Define a decision tree classifier
- Interpret the output of a decision trees
- Learn a decision tree classifier using greedy algorithm
- Traverse a decision tree to make predictions
  - Majority class predictions
- Tackle continuous and discrete features