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   - $\rightarrow$ Class $\hat{\omega}$ is estimated by voting
3. **Multiple weight** vectors:
   - $\rightarrow$ Requires only one Perceptron
   - $\rightarrow$ Weight matrix with a weight vector per class: $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_{|\mathcal{W}|}$
   - $\rightarrow$ Only the weight vector associated to the class is updated while training
   - $\rightarrow$ Weight vector with the largest activation determines $\hat{\omega}$

Outline

1. Linear models
   - Binary
   - Multiclass
   - Parameter estimation
2. Perceptron
   - Introduction
   - Training
   - Limitations
   - Multiclass Perceptron
3. Multi-layer Perceptron
   - Introduction
   - Structure
   - Training
   - Training dynamics and regularization

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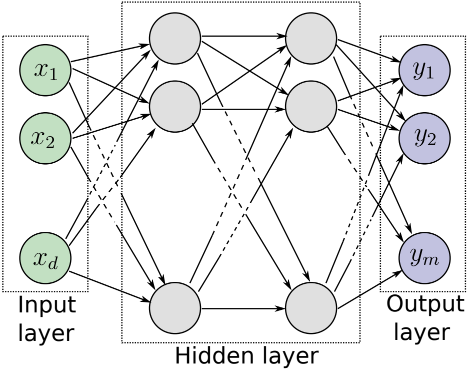
- **Universal Approximation Theorem**:

  A feedforward neural network with at least one hidden layer, using a nonlinear activation, can approximate any continuous function to any desired degree of accuracy, given sufficient neurons in the hidden layer

Structure



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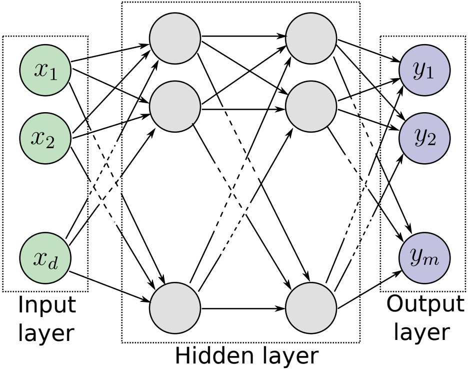

Input

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  $\Rightarrow$ d neurons

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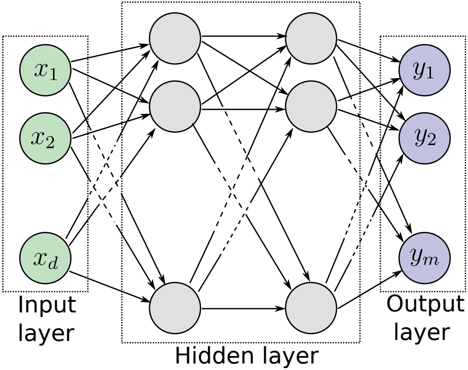

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- Non-linear activations
- Fully connected
- Configuration depends on the complexity of the function

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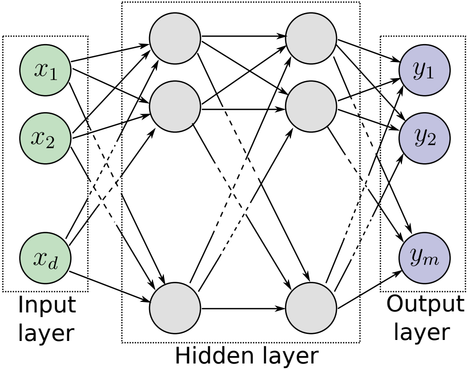

Input	Hidden	Output
- Representation space $\mathbb{R}^d$ $\Rightarrow$ d neurons	- Non-linear activations - Fully connected - Configuration depends on the complexity of the function	- Configuration depends on the task

Input layer

- Entrance point to the model for datum $\mathbf{x} \in \mathbb{R}$
- As many neurons as the dimensionality of $\mathbf{x} \Rightarrow d$ neurons
- There may be additional neurons $\Rightarrow$ additional biases

Hidden layer

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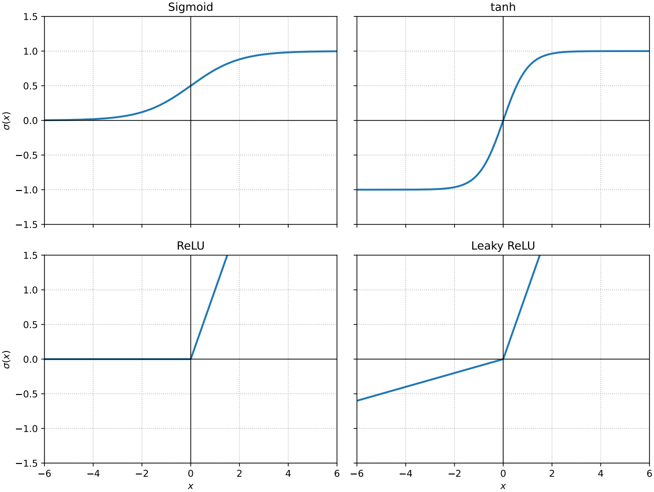
- In general, non-linear activation functions:

$$y_{h_{i,j}} = \sigma\left(\mathbf{h}_{i,j}^{T} \cdot \mathbf{y}_{h_{i-1}}\right)$$

Activation functions



Activation functions

- Sigmoid:

$$\sigma(x) = \frac{e^x}{1 + e^x}$$

- Hyperbolic tangent (tanh):

$$\sigma(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- Rectified Linear Unit (ReLU):

$$\sigma(x) = \max\{0, x\}$$

- Leaky Rectified Linear Unit (Leaky ReLU):

$$\sigma(x) = \begin{cases} x \text{ if } x \geq 0 \\ \alpha \cdot x \text{ if } x < 0 \end{cases} \qquad \text{with } \alpha \in [0.1, 0.3]$$

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$$\hat{P}(\omega|\mathbf{x}) = \frac{e^{y_\omega}}{\sum_{j=1}^{|\mathcal{W}|} e^{y_j}} \in [0,1]$$

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$$\hat{P}(\omega|\mathbf{x}) = \frac{e^{y_\omega}}{\sum_{j=1}^{|\mathcal{W}|} e^{y_j}} \in [0,1]$$

  → Binary case ($|\mathcal{C}| = 2$) may be modeled with a single neuron:

$$\hat{\omega} = \begin{cases} \omega_1 & \text{if } y \geq 0 \\ \omega_2 & \text{if } y < 0 \end{cases}$$

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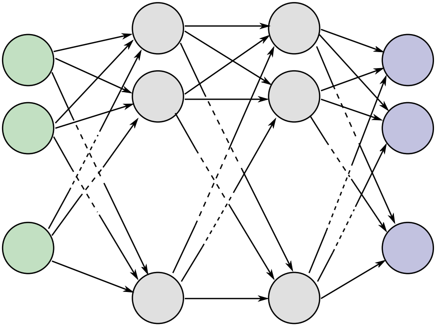

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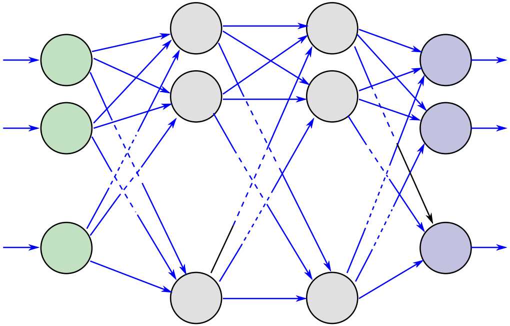

Contextualization

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- **Steps**:
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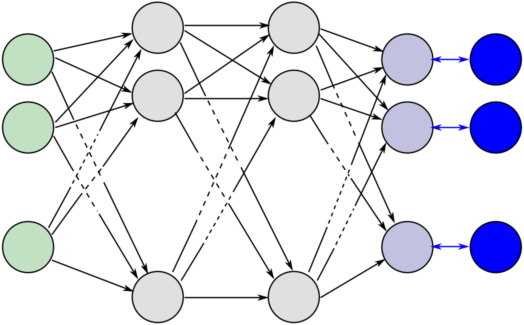

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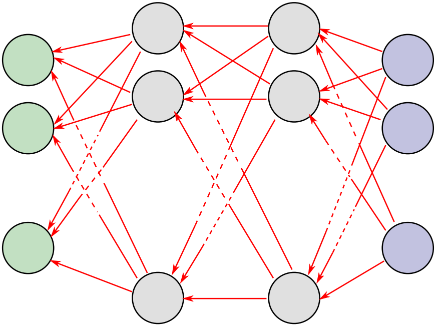

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  4. Weight update

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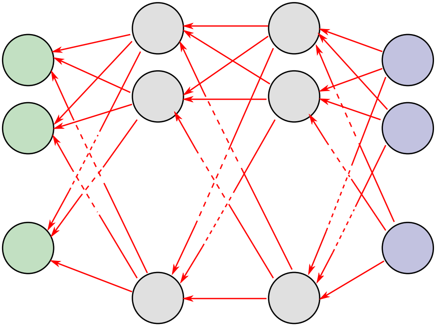
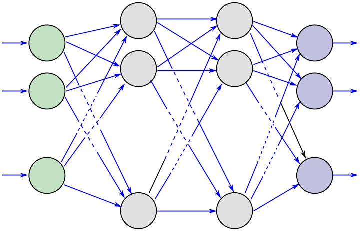


- The process is done until a convergence criterion is reached

Forward-backward training

1. **Forward pass:**
   - Datum **x** goes from input to output
   - Weights $\boldsymbol{\theta}$ modify **x** across the network



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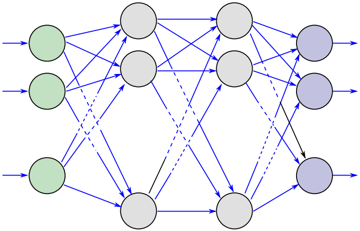
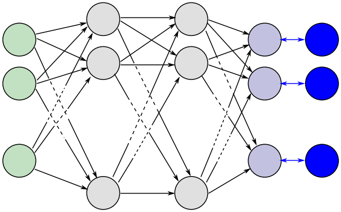


2. **Loss computation:**
   - Estimation $\hat{\mathbf{y}}$ is compared to reference $\mathbf{y}$
   - Depends on the task:
     - $\rightarrow$ Regression: MAE, MSE
     - $\rightarrow$ Classification: Categorical Cross Entropy



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3. **Backward pass** (backpropagation) **:**
   - Gradient of the loss function with respect to $\boldsymbol{\theta}$
   - Chain rule for the derivatives of the (nested) equation
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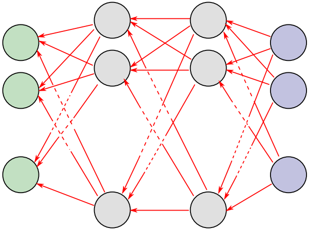

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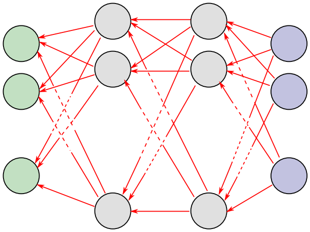

4. Weight update:

- Update parameters $\boldsymbol{\theta}$ (weights and biases)
- Optimization methods:
  - $\rightarrow$ Stochastic Gradient Descent
  - $\rightarrow$ Momentum, RMSProp, Adam, ...

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  $\rightarrow$ Mapping function: $\left[\omega_1, \ldots, \omega_{|\mathcal{W}|}\right] \rightarrow [0, 1]^{|\mathcal{W}|}$

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- **Categorical Cross Entropy** $\Rightarrow$ Typical loss function for classification

$$\mathcal{L}\left(\mathbf{y}, \hat{\mathbf{y}}\right) = -\sum_{j=1}^{|\mathcal{W}|} y_j \cdot \log\left(\hat{y}_j\right)$$

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The associated loss is:

$$\begin{aligned}\mathcal{L}(\mathbf{y}, \hat{\mathbf{y}}) &= -\sum_{j=1}^{3} y_j \cdot \log\left(\hat{y}_j\right) \\ &= -\left[0 \cdot \log(0.1) + 1 \cdot \log(0.7) + 0 \cdot \log(0.2)\right] = 0.357\end{aligned}$$

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2. **Vanishing/exploding gradients**: Gradients become negligible/excessively large producing stagnation/oscillations

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- **Gradient clipping**: Limiting gradient value above threshold

T5: Linear methods and Perceptron

Fundamentos del Aprendizaje Automático

Curso 2025/2026