T5: Linear methods and Perceptron

Fundamentos del Aprendizaje Automático

Curso 2025/2026

Structure

Problem statement

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  $\rightarrow$ $\boldsymbol{\theta} \Rightarrow$ weights of the model
  $\rightarrow$ $J(\boldsymbol{\theta}) \Rightarrow$ criterion function to estimate $\boldsymbol{\theta}$

Outline

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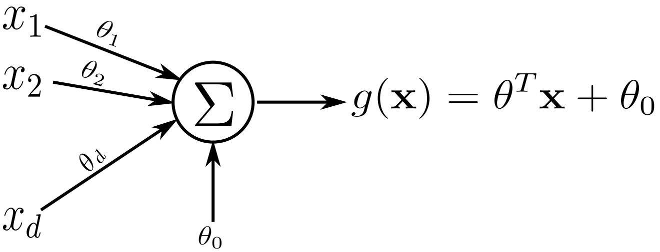
- Two-category case ($\mathcal{W} = \{\omega_1, \omega_2\}$):

$$g(\mathbf{x}) = \boldsymbol{\theta}^T \mathbf{x} + \theta_0$$

$\mathbf{x} \in \mathbb{R}^d$ feature vector
$\boldsymbol{\theta} \in \mathbb{R}^d$ weight vector
$\theta_0 \in \mathbb{R}$ weight threshold



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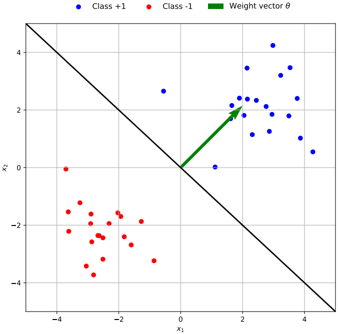
- Geometric interpretation:
  - $\rightarrow$ Vector $\boldsymbol{\theta}$: normal to surface $\mathcal{H}$ and defines its *orientation*
  - $\rightarrow$ Bias θ_0: Shifts surface $\mathcal{H}$ along $\boldsymbol{\theta}$



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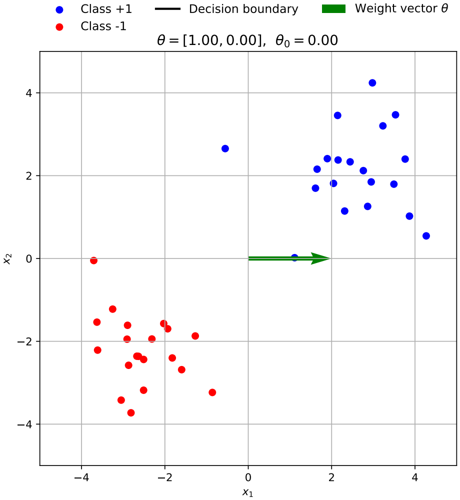

Formalization

- Multiclass: Generalizes to one linear function per label:

$$g_k(\mathbf{x}) = \boldsymbol{\theta}_k^T \mathbf{x} + \theta_{0k}, \quad k = 1, 2, ..., |\mathcal{W}|$$

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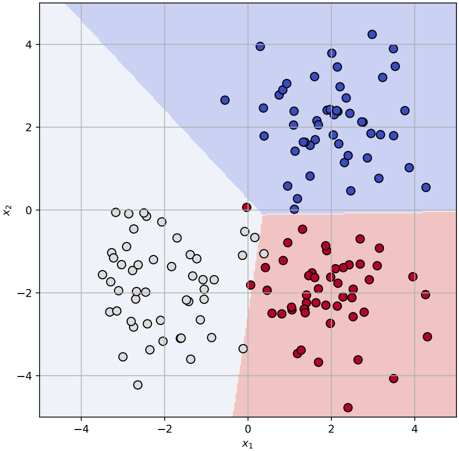
- Decision boundaries: Pairs $< i, j > \in \{1, \ldots, |\mathcal{W}|\}$ have equal scores:

$$g_i(\mathbf{x}) = g_j(\mathbf{x}) \Rightarrow (\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)^T \mathbf{x} + (\theta_{0i} - \theta_{0j}) = 0$$

Decision frontiers



Introduction

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  $\rightarrow$ Minimize discrepancy between predictions and ground truth (ω) on $\mathcal{D}$:

$$\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} \frac{1}{|\mathcal{D}|} \sum_{i=1}^{|\mathcal{D}|} \mathcal{L}\left(g(\mathbf{x}_i; \boldsymbol{\theta}), \omega_i\right)$$

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- This minimization process may be solved analytically in some cases
  $\rightarrow$ Practical scenarios: iterative optimization process

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Gradient descent algorithm

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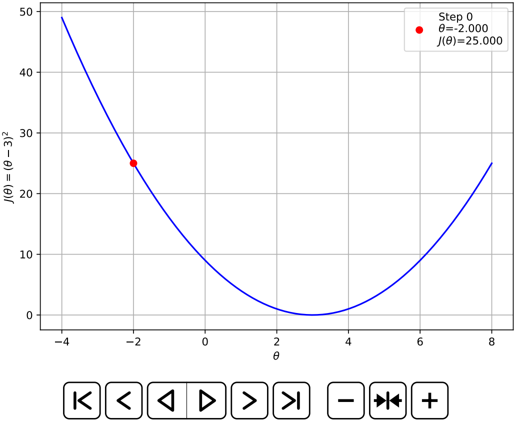
     - $\rightarrow$ Example: $\left|\eta(k)\boldsymbol{\nabla} J\left(\boldsymbol{\theta}^{(k)}\right)\right| < \delta$

Gradient descent

Example:

- Function $J(\theta) = (\theta - 3)^2$
- Initial value $\theta_0 = -2$
- Learning rate $\eta(k) = 0.1$



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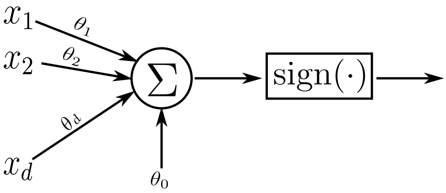

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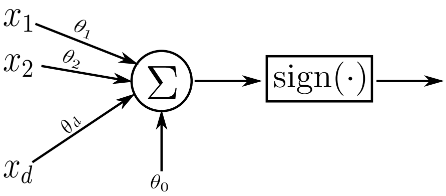
$$\hat{\omega} = \text{sign}(\boldsymbol{\theta}^T \mathbf{x} + \theta_0) \qquad \text{sign}(\cdot) = \begin{cases} +1 \text{ if } \boldsymbol{\theta}^T \mathbf{x} + \theta_0 \geq 0 \\ -1 \text{ if } \boldsymbol{\theta}^T \mathbf{x} + \theta_0 < 0 \end{cases}$$



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- We may define vector $\mathbf{z} = [z_1, \ldots, z_{|\mathcal{D}|}]$ such that:

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- This way, all samples in $\mathcal{D}$ will be correctly classified if and only if:

$$z_i \boldsymbol{\theta}^T \mathbf{x}_i > 0 \; \forall i$$

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$\rightarrow$ The corresponding gradient is:

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The Perceptron Criterion

Batch Perceptron algorithm

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} - \eta(k) \cdot \sum_{i:z_i\boldsymbol{\theta}^T\mathbf{x}_i \leq 0} (-z_i\mathbf{x}_i)$$

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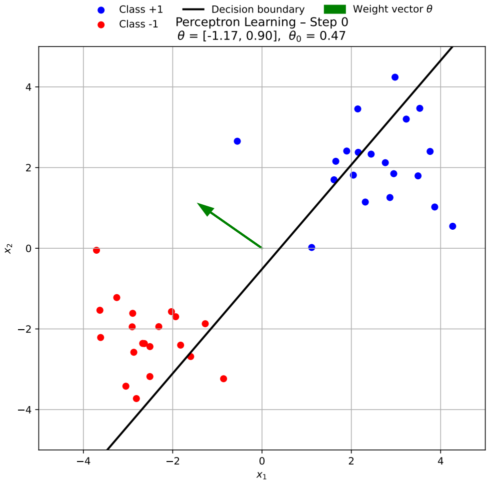
Single-sample Perceptron algorithm

$$\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} + \eta(k) z_i\mathbf{x}_i$$

The Perceptron Criterion

Approach	Update frequency	Pros	Cons
Single-sample	Every sample	Fast reaction to new data stochasticity	Noisy updates
Batch	Accumulation	Smoother Stable convergence	Slower

The Perceptron Criterion

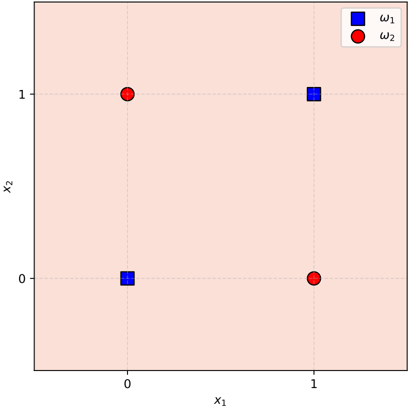


The XOR problem

The *Exclusive OR* (XOR) operation $\Rightarrow$ binary classification task:

- Feature space: $\mathbf{x} \in \{0,1\}^2$
- Label space: $\mathcal{W} = \{0,1\}$

- Function $f_{xor}(\mathbf{x}) = \begin{cases} \omega_1 \text{ if } x_1 = x_2 \\ \omega_2 \text{ if } x_1 \neq x_2 \end{cases}$



The XOR problem

- Showcases the main limitation of the percepton mechanism

- Unable to address non-linearly separable labels
  - → Need for non-linearities in the scheme

- Two related mechanisms:
  - → Stacking perceptrons into layers
  - → Using non-linear activation functions