Ejercicios

17 Resolver y clasificar los siguientes sistemas:

a) $$\left.\begin{array}{r} x-2y+3z=9 \\ -x+3y=-4 \\ 2x-5y+5z=17 \end{array}\right\}$$

b) $$\left.\begin{array}{r} x-3y+z=1 \\ 2x-y-2z=2 \\ x+2y-3z=-1 \end{array}\right\}$$

c) $$\left.\begin{array}{r} y-z=0 \\ x-3z=-1 \\ -x+3y=1 \end{array}\right\}$$

a) $$\left.\begin{array}{r} x-2y+3z=9 \\ -x+3y=-4 \\ 2x-5y+5z=17 \end{array}\right\} \begin{array}{l} \to 3y+4-2y+3z=9 \\ \to x=3y+4=\boxed{1} \\ \to 6y+8-5y+5z=17 \end{array}$$

$$\left.\begin{array}{r} y+3z=5 \\ y+5z=9 \end{array}\right\} \to y=5-3z=\boxed{-1}$$

$$(-)$$

$$-2z=-4 \to \boxed{z=2}$$

S.C.D

b) $$\left.\begin{array}{r} x-3y+z=1 \\ 2x-y-2z=2 \\ x+2y-3z=-1 \end{array}\right\} \begin{array}{l} \to x=1+3y-z \\ \to \cancel{2}+6y-2z-y-2z=\cancel{2} \\ \to 1+3y-z+2y-3z=-1 \end{array}$$

$$\left.\begin{array}{r} 5y-4z=0 \\ \ominus\ 5y-4z=-2 \end{array}\right\}$$

$$\diagup \quad \diagup \quad = 2 \rightarrow 0 \cancel{=} 2 \quad \text{S.I}$$

c) $\left.\begin{array}{l} y-z=0 \\ x-3z=-1 \\ \cancel{-x+3y=1} \end{array}\right\} \begin{array}{l} \rightarrow y=z \\ \rightarrow x-3z=-1 \\ \rightarrow -x+3z=1 \end{array}$

$x-3z=-1 \rightarrow x=3z-1$

$$\oplus \quad \diagup \quad \diagup \quad = \diagup \rightarrow 0=0 \quad \text{S.C.I}$$

las 2 ec son equivalentes

Nº par = Nº inc - Nº ec finales =

= 3 − 2 = 1 par (α).

$$\boxed{\begin{array}{l} x = 3\alpha - 1 \\ y = \alpha \qquad (\alpha \in \mathbb{R}) \\ z = \alpha \end{array}}$$

Ejercicios

18. Usando el método de Gauss, discutir y resolver los siguientes sistemas:

a) $\left.\begin{aligned} x+3y+z&=-3\\ 3x+9y+4z&=-7\\ 2x-y+z&=6\end{aligned}\right\}$ **b)** $\left.\begin{aligned} x+3y-z+t&=1\\ -2x+y+2z&=7\\ y-t&=0\end{aligned}\right\}$

c) $\left.\begin{aligned} x-y+z&=1\\ 2x+y+z&=0\\ 2x-2y+2z&=3\end{aligned}\right\}$

a) $A^* = \begin{array}{c} \begin{array}{cccc} x & y & z & T.I \end{array} \\ \left(\begin{array}{ccc|c} 1 & 3 & 1 & -3 \\ 3 & 9 & 4 & -7 \\ 2 & -1 & 1 & 6 \end{array}\right) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & 1 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & -7 & -1 & 12 \end{array}\right) \sim$

$F_2 \to F_2 - 3F_1$

$F_3 \to F_3 - 2F_1$

$F_2 \longleftrightarrow F_3$

$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 3 & 1 & -3 \\ 0 & -7 & -1 & 12 \\ 0 & 0 & 1 & 2 \end{array}\right) \end{array} \to \begin{aligned} x+3y+z&=-3 &\to \boxed{x=1}\\ -7y-z&=12 &\to -7y=14\\ \boxed{z=2}& & \boxed{y=-2}\end{aligned}$

- No hay filas $(0\ 0\ 0 | b)$
- $N^{\underline{o}}$ filas $= N^{\underline{o}}$ inc $= 3 \to$ S.C.D

b) $A^* = \begin{array}{c} \begin{array}{ccccc} x & y & z & t & T.I \end{array} \\ \left(\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 1 \\ -2 & 1 & 2 & 0 & 7 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right) \end{array} \sim \left(\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 1 \\ 0 & 7 & 0 & 2 & 9 \\ 0 & 1 & 0 & -1 & 0 \end{array}\right)$

$F_2 \to F_2 + 2F_1$

$$\sim \left(\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 1 \\ 0 & 7 & 0 & 2 & 9 \\ 0 & 0 & 0 & -9 & -9 \end{array}\right) \rightarrow \left.\begin{array}{r} x + 3y - z + t = 1 \\ 7y + 2t = 9 \\ -9t = -9 \end{array}\right\} \rightarrow \boxed{t = 1}$$

(columns: x, y, z, t, T.I)

$F_3 \rightarrow 7F_3 - F_2$

$7y = 7 \quad \boxed{y = 1}$

- No hay filas $(0\ 0\ 0\ 0 \mid b)$
- $N^{\underline{o}}$ filas $= 3 < N^{\underline{o}}$ inc $= 4 \rightarrow$ SCI

$$x = 1 - 3y + z - t = 1 - 3 + z - 1 = \boxed{z - 3}$$

$N^{\underline{o}}$ par $=$ 4 inc $-$ 3 ec $=$ 1 par (α).

$$\boxed{\begin{array}{l} x = \alpha - 3 \\ y = 1 \\ z = \alpha \\ t = 1 \end{array} \qquad (\alpha \in \mathbb{R})}$$

c) $$A^* = \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 0 \\ 2 & -2 & 2 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right) \rightarrow 0 = 1$$

SI

$F_2 \rightarrow F_2 - 2F_1$

$F_3 \rightarrow F_3 - 2F_1$ Hay filas $(0\ 0\ 0 \mid b)$

Ejercicios

⑲ Aplicando el método de Gauss, discutir y resolver el sistema en función del parámetro a:

$$\left.\begin{aligned} x+2y+z&=1\\ 2x+y+3z&=-4\\ x-y+(a+2)z&=-3a-5\\ 4x+2y+(a+6)z&=-3a^2-8 \end{aligned}\right\}$$

$$A^* = \begin{array}{c} \begin{matrix} x & y & z & & \text{T.I} \end{matrix} \\ \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & -4 \\ 1 & -1 & a+2 & -3a-5 \\ 4 & 2 & a+6 & -3a^2-8 \end{array}\right) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -6 \\ 0 & -3 & a+1 & -3a-6 \\ 0 & -6 & a+2 & -3a^2-12 \end{array}\right)$$

$F_2 \to F_2 - 2F_1$

$F_3 \to F_3 - F_1$

$F_4 \to F_4 - 4F_1$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -6 \\ 0 & 0 & a & -3a \\ 0 & 0 & a & -3a^2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -6 \\ 0 & 0 & a & -3a \\ 0 & 0 & 0 & -3a^2+3a \end{array}\right)$$

$F_3 \to F_3 - F_2$

$F_4 \to F_4 - 2F_2$

$F_4 \to F_4 - F_3$

$-3a^2+3a = 0 \longrightarrow -3a\cdot(a-1) = 0 \longrightarrow -3a = 0 \to a = 0$

$\hookrightarrow a-1 = 0 \longrightarrow a = 1$

- Si $a \neq 0$ y $a \neq 1$: SI

Última fila del tipo $(0\ 0\ 0 \mid b)$.

• Si $a = 0$:

$$\begin{array}{cccc|c} & x & y & z & T.I \\ & \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \end{array} \to \left.\begin{array}{l} x + 2y + z = 1 \\ -3y + z = -6 \end{array}\right\}$$

?

$$\downarrow$$

$$z = 3y - 6$$

No hay filas $(0\ 0\ 0 | b)$

$\to$ S.C.I $\to$ 1 par (α)

Nº filas < Nº inc

2 3

$$x + 2y + 3y - 6 = 1 \to x = 7 - 5y$$

$$\to \boxed{\begin{array}{l} x = 7 - 5\alpha \\ y = \alpha \qquad (\alpha \in \mathbb{R}) \\ z = 3\alpha - 6 \end{array}}$$

• Si $a = 1$:

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -6 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left.\begin{array}{l} x + 2y + z = 1 \\ -3y + z = -6 \\ \boxed{z = -3} \end{array}\right\} \begin{array}{l} \boxed{x = 2} \\ \boxed{y = 1} \end{array}$$

No hay filas $(0\ 0\ 0 | b)$

$\to$ SCD

Nº filas = Nº inc = 3

Ejercicios

20 Discutir los sistemas del ejercicio 18 mediante el Teorema de Rouché-Fröbenius.

Nota: Aprovechar la matriz final obtenida en los 3 sistemas por el método de Gauss.

a)
$$\underbrace{\left(\begin{array}{ccc|c} \multicolumn{3}{c|}{\overbrace{\begin{matrix}1 & 3 & 1\end{matrix}}^{A}} & -3 \\ 0 & -7 & -1 & 12 \\ 0 & 0 & 1 & 2 \end{array}\right)}_{A^*}$$

$$rg(A) = 3 = rg(A^*) = 3 = n = 3 \rightarrow \boxed{SCD}$$

b)
$$\left(\begin{array}{cccc|c} 1 & 3 & -1 & 1 & 1 \\ 0 & 7 & 0 & 2 & 9 \\ 0 & 0 & 0 & -9 & -9 \end{array}\right)$$

$$rg(A) = 3 = rg(A^*) = 3 \rightarrow \boxed{S.C.I}$$

$$n = 4$$

c)
$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$$rg(A) = 2 \neq rg(A^*) = 3 \rightarrow \boxed{SI}$$

21 Estudiar mediante menores y aplicando el Teorema de Rouché-Fröbenius la compatibilidad del sistema:

$$\left.\begin{aligned} x + 2y + 2z &= 3 \\ y + 2z &= 2 \\ 3x + 2y &= 1 \\ 5x + 7y + 6z &= 2 \end{aligned}\right\}$$

$$A^* = \left(\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 1 \\ \underbrace{5 \quad 7 \quad 6}_{A} & & & 2 \end{array}\right) \rightarrow \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$$

$$\begin{vmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 3 & 2 & 0 \end{vmatrix} = 12 - (6+4) = 2 \neq 0 \rightarrow \boxed{rg(A) = 3}$$

$$\begin{vmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 2 \\ 3 & 2 & 0 & 1 \\ 5 & 7 & 6 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -2 & -1 \\ 0 & 1 & 0 & 0 \\ 3 & 2 & -4 & -3 \\ 5 & 7 & -8 & -12 \end{vmatrix} = +1 \cdot \begin{vmatrix} 1 & -2 & -1 \\ 3 & -4 & -3 \\ 5 & -8 & -12 \end{vmatrix} =$$

$C_3 \rightarrow C_3 - 2C_2$

$C_4 \rightarrow C_4 - 2C_2$

$$= -14 \neq 0 \rightarrow \boxed{rg(A^*) = 4} \rightarrow \boxed{SI}$$

Ejercicios

22 Discutir y resolver el sistema homogéneo según el parámetro a:

$$\left.\begin{aligned} 2x - y - 2z &= 0 \\ x + y + z &= 0 \\ 4x - 5y + az &= 0 \end{aligned}\right\}$$

$$\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 + 1 = 3 \neq 0 \qquad A = \begin{pmatrix} 2 & -1 & -2 \\ 1 & 1 & 1 \\ 4 & -5 & a \end{pmatrix}$$

$$\begin{vmatrix} 2 & -1 & -2 \\ 1 & 1 & 1 \\ 4 & -5 & a \end{vmatrix} = 3a + 24 \rightarrow 3a + 24 = 0 \rightarrow \underline{\underline{a = -8}}$$

- $\underline{\text{Si } a \neq -8}$: $rg(A) = rg(A^*) = 3 = n \rightarrow$ SCD

SIST. HOMOG.

$$\boxed{x = y = z = 0} \rightarrow \text{solución trivial.}$$

- $\underline{\text{Si } a = -8}$: $rg(A) = rg(A^*) = 2 < n = 3 \rightarrow$ SCI

$$F_1 \leftrightarrow F_2 \quad \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -2 \\ 4 & -5 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -4 \\ \cancel{0} & \cancel{-9} & \cancel{-12} \end{pmatrix} \quad 1 \text{ par } (\alpha)$$

$F_2 \rightarrow F_2 - 2F_1$ $\qquad F_3 = 3F_2$

$F_3 \rightarrow F_3 - 4F_1$

$$\left.\begin{array}{l} x+y+z=0 \\ -3y-4z=0 \end{array}\right\} \begin{array}{l} \to x-\frac{4z}{3}+z=0 \quad x=\frac{z}{3} \\ \to 3y=-4z \to y=\frac{-4z}{3} \end{array} \to \boxed{\begin{array}{l} x=\frac{\alpha}{3} \\ y=-\frac{4\alpha}{3} \\ z=\alpha \\ (\alpha\in\mathbb{R}) \end{array}}$$

Ejercicios

23 Determinar la factorización LU de las matrices:

a) $\begin{pmatrix} 2 & 1 \\ 8 & 7 \end{pmatrix}$ **b)** $\begin{pmatrix} 1 & -3 & 5 \\ 0 & -4 & 7 \\ -1 & -2 & 1 \end{pmatrix}$ **c)** $\begin{pmatrix} 1 & 4 & -3 \\ 2 & 8 & 1 \\ -5 & -9 & 7 \end{pmatrix}$

a) $A_1 = 2 \neq 0 \qquad A_2 = |A| = \begin{vmatrix} 2 & 1 \\ 8 & 7 \end{vmatrix} = 14-8 = 6 \neq 0$

LU existe y es única. ✓

$$A = L\cdot U = \underbrace{\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}}_{L}\cdot\underbrace{\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}}_{U}$$

$$A = \begin{pmatrix} 2 & 1 \\ 8 & 7 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} = U$$

$F_2 \to F_2 - 4F_1$

$\uparrow$

k_{21}

b) $A_1 = 1 \neq 0 \qquad A_2 = -4 \neq 0 \qquad A_3 = |A| = 11 \neq 0$

LU existe y es única

$$A = L \cdot U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{5}{4} & 1 \end{pmatrix}}_{L} \cdot \underbrace{\begin{pmatrix} 1 & -3 & 5 \\ 0 & -4 & 7 \\ 0 & 0 & -\frac{11}{4} \end{pmatrix}}_{U} \quad ✓$$

$$A = \begin{pmatrix} 1 & -3 & 5 \\ 0 & -4 & 7 \\ -1 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & 5 \\ 0 & -4 & 7 \\ 0 & -5 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & 5 \\ 0 & -4 & 7 \\ 0 & 0 & -\frac{11}{4} \end{pmatrix} = U$$

$F_2 \to F_2 + 0 \cdot F_1$ ✗

$F_3 \to F_3 + \underset{k_{31}}{F_1}$

$F_3 \to F_3 - \frac{5}{4} F_2$

$$6 - \frac{5}{4} \cdot 7 = 6 - \frac{35}{4} = -\frac{11}{4}$$

c) $A_1 = 1 \neq 0 \qquad A_2 = \begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix} = 0 \leftarrow$

$A_3 = |A| = -77 \neq 0$

$\boxed{\text{LU no existe}}$