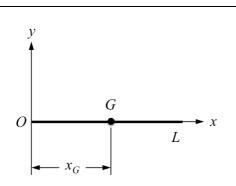
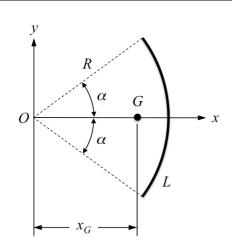
# CENTROS DE GRAVEDAD DE FIGURAS GEOMÉTRICAS SIMPLES

Elementos con densidad lineal de masa  $\lambda$  uniforme



Varilla delgada de longitud L

$$x_G = \frac{1}{2}L$$
$$y_G = 0$$

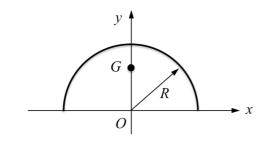


Arco circular de longitud L y radio R

$$L = 2R\alpha$$

$$x_G = \frac{R}{\alpha} \operatorname{sen} \alpha = \frac{2R^2}{L} \operatorname{sen} \left(\frac{L}{2R}\right)$$

$$y_G = 0$$

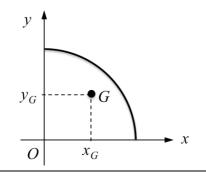


Semicircunferencia de radio R

$$L = \pi R$$

$$x_G = 0$$

$$y_G = \frac{2R}{\pi}$$



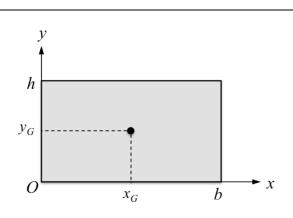
Cuadrante de radio *R* 

$$L = \frac{1}{2}\pi R$$

$$x_G = \frac{2R}{\pi}$$

$$y_G = \frac{2R}{\pi}$$

# Elementos con densidad superficial de masa $\sigma$ uniforme

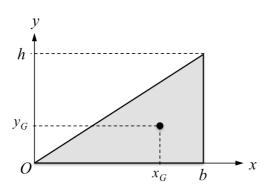


Rectángulo

$$S = bh$$

$$x_G = \frac{b}{2}$$

$$y_G = \frac{h}{2}$$

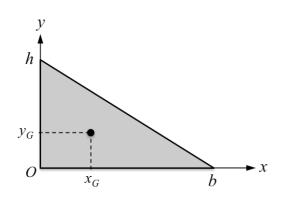


Triángulo rectángulo

$$S = \frac{1}{2}bh$$

$$x_G = \frac{2}{3}t$$

$$y_G = \frac{1}{3}h$$

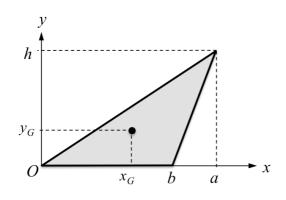


Triángulo rectángulo

$$S = \frac{1}{2}bh$$

$$x_G = \frac{1}{3}t$$

$$y_G = \frac{1}{3}h$$

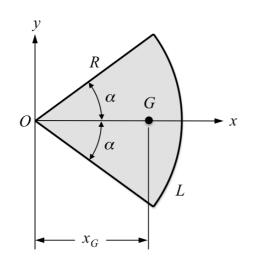


Triángulo

$$L = \frac{1}{2}bh$$

$$x_G = \frac{1}{3}(a+b)$$

$$y_G = \frac{1}{3}h$$

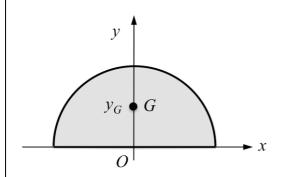


Sector circular de radio R

$$S = R^{2}\alpha$$

$$x_{G} = \frac{2R}{3\alpha} \operatorname{sen}\alpha = \frac{2R^{3}}{3S} \operatorname{sen}\left(\frac{S}{R^{2}}\right)$$

$$y_{G} = 0$$

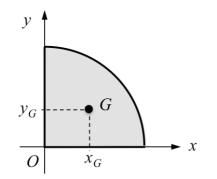


Semidisco de radio R

$$S = \frac{1}{2}\pi R^2$$

$$x_G = 0$$

$$y_G = \frac{4R}{3\pi}$$

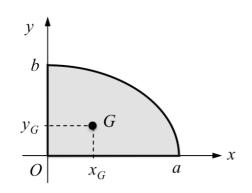


Cuadrante de radio R

$$S = \frac{1}{4}\pi R^2$$

$$x_G = \frac{4R}{3\pi}$$

$$y_G = \frac{4R}{3\pi}$$

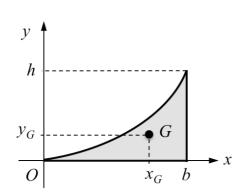


Cuadrante de elipse

$$S = \frac{1}{4}\pi a$$

$$x_G = \frac{4a}{3\pi}$$

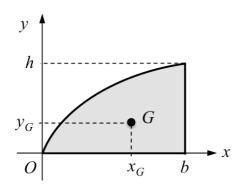
$$y_G = \frac{4b}{3\pi}$$



Enjuta parabólica

$$S = \frac{1}{3}bh$$

$$x_G = \frac{3}{4}b$$



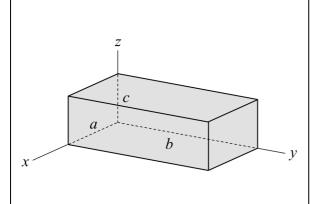
Cuadrante de parábola

$$S = \frac{2}{3}bh$$

$$x_G = \frac{5}{8}b$$

$$y_G = \frac{2}{5}h$$

# Elementos con densidad volumétrica de masa $\rho$ uniforme



Paralelepípedo rectangular

$$V = abc$$

$$x_G = \frac{a}{2}$$

$$y_G = \frac{b}{2}$$

$$z_G = \frac{c}{2}$$

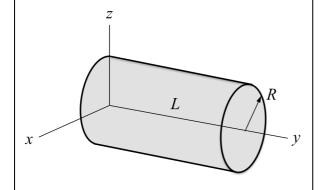
Tetraedro regular

$$V = \frac{1}{6}abc$$

$$x_G = \frac{a}{4}$$

$$y_G = \frac{b}{4}$$

$$z_G = \frac{c}{4}$$



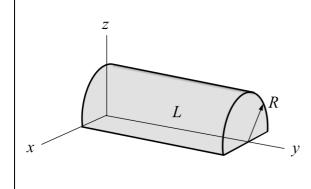
Cilindro de revolución

$$V = \pi R^2 L$$

$$x_G = 0$$

$$y_G = \frac{L}{2}$$

$$z_G = 0$$



Semicilindro

$$V = \frac{\pi R^2 L}{2}$$

$$x_G = 0$$

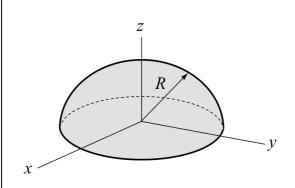
$$y_G = \frac{L}{2}$$

$$z_G = \frac{4R}{3\pi}$$

$$x_G = 0$$

$$y_G = \frac{L}{2}$$

$$z_G = \frac{4R}{3\pi}$$



## Semiesfera

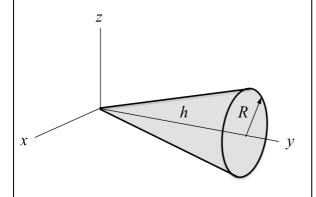
$$V = \frac{2}{3}\pi R^3$$

$$x_G = 0$$

$$y_G = 0$$

$$z_G = \frac{3}{8}R$$

$$z_G = \frac{3}{8}R$$



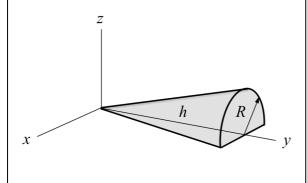
# Cono de revolución

$$V = \frac{1}{3}\pi R^2 h$$

$$x_G = 0$$

$$y_G = \frac{3}{4}h$$

$$z_G = 0$$



## Semicono

$$V = \frac{1}{6}\pi R^2 h$$

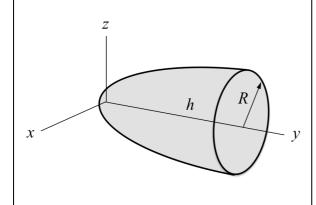
$$x_G = 0$$

$$y_G = \frac{3}{4}h$$

$$x_G = 0$$

$$y_G = \frac{3}{4}h$$

$$z_G = \frac{R}{\pi}$$



# Paraboloide

$$V = \frac{1}{2}\pi R^2 h$$
$$x_G = 0$$
$$y_G = \frac{2}{3}h$$

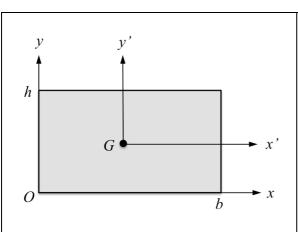
$$x_G = 0$$

$$y_G = \frac{2}{3}h$$

$$z_G = 0$$

# MOMENTOS DE INERCIA DE FIGURAS GEOMÉTRICAS SIMPLES

Elementos con densidad superficial de masa  $\sigma$  uniforme



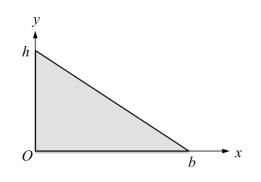
Rectángulo

$$I_{x} = \frac{1}{3}\sigma bh^{3} = \frac{1}{3}Mh^{2} \qquad I_{y} = \frac{1}{3}\sigma hb^{3} = \frac{1}{3}Mb^{2}$$

$$I_{xG} = \frac{1}{12}\sigma bh^{3} = \frac{1}{12}Mh^{2}$$

$$I_{yG} = \frac{1}{12}\sigma hb^{3} = \frac{1}{12}Mb^{2}$$

$$I_{xy} = \frac{1}{4}\sigma b^{2}h^{2} = \frac{1}{4}Mbh \qquad I_{xyG} = 0$$

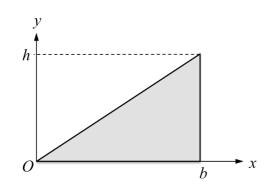


Triángulo rectángulo

$$M = \frac{1}{2}\sigma bh$$

$$I_x = \frac{1}{12}\sigma bh^3 = \frac{1}{6}Mh^2 \quad I_y = \frac{1}{12}\sigma hb^3 = \frac{1}{6}Mb^2$$

$$I_{xy} = \frac{1}{12}\sigma b^2 h^2 = \frac{1}{6}Mbh$$

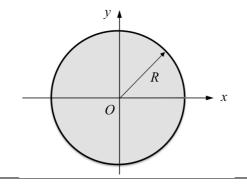


Triángulo rectángulo

$$M = \frac{1}{2}\sigma bh$$

$$I_{x} = \frac{1}{12}\sigma bh^{3} = \frac{1}{6}Mh^{2} \quad I_{y} = \frac{1}{4}\sigma hb^{3} = \frac{1}{2}Mb^{2}$$

$$I_{xy} = \frac{1}{8}\sigma b^{2}h^{2} = \frac{1}{4}Mbh$$

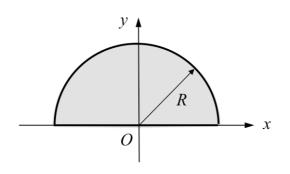


Círculo

$$M = \sigma \pi R^{2}$$

$$I_{O} = \frac{1}{2} \sigma \pi R^{4} = \frac{1}{2} M R^{2}$$

$$I_{x} = I_{y} = \frac{1}{2} I_{O} = \frac{1}{4} \sigma \pi R^{4} = \frac{1}{4} M R^{2}$$

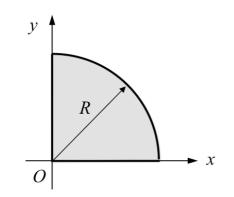


Semicírculo

$$M = \frac{1}{2}\sigma\pi R^{2}$$

$$I_{O} = \frac{1}{4}\sigma\pi R^{4} = \frac{1}{2}MR^{2}$$

$$I_{x} = I_{y} = \frac{1}{2}I_{O} = \frac{1}{8}\sigma\pi R^{4} = \frac{1}{4}MR^{2}$$



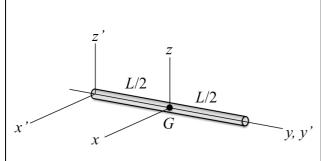
Cuadrante circular

$$M = \frac{1}{4}\sigma\pi R^{2}$$

$$I_{O} = \frac{1}{8}\sigma\pi R^{4} = \frac{1}{2}MR^{2}$$

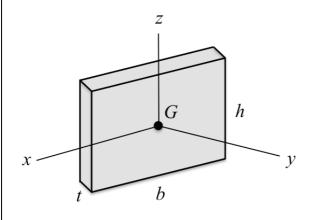
$$I_{x} = I_{y} = \frac{1}{2}I_{O} = \frac{1}{16}\sigma\pi R^{4} = \frac{1}{4}MR^{2}$$

# Volúmenes



Varilla

$$I_{yG} = 0$$
  $I_{xG} = I_{zG} = \frac{1}{12}ML^2$   
 $I_{y'} = 0$   $I_{x'} = I_{z'} = \frac{1}{3}ML^2$ 



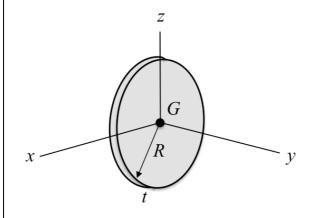
Placa rectangular

$$I_{xG} = \frac{1}{12}Mh^{2}$$

$$I_{yG} = \frac{1}{12}M(b^{2} + h^{2})$$

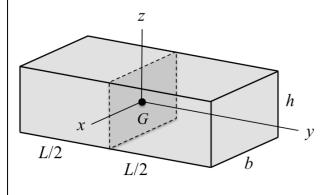
$$I_{zG} = \frac{1}{12}Mb^{2}$$

$$I_{xyG} = I_{yzG} = I_{xzG} = 0$$



Placa circular

$$I_{yG} = \frac{1}{2}MR^2$$
 
$$I_{xG} = I_{zG} = \frac{1}{4}MR^2$$

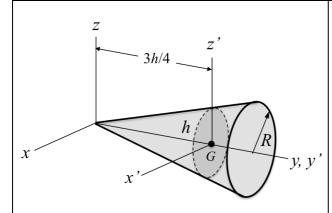


Prisma rectangular

$$I_{xG} = \frac{1}{12}M(h^2 + L^2)$$

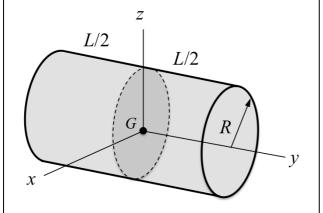
$$I_{yG} = \frac{1}{12}M(b^2 + h^2)$$

$$I_{zG} = \frac{1}{12}M(b^2 + L^2)$$



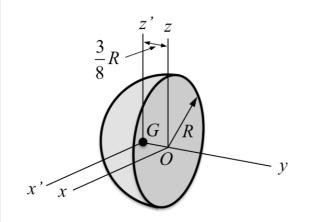
#### Cono de revolución

$$\begin{split} x_G &= 0 \quad y_G = \frac{3}{4}h \quad z_G = 0 \\ I_x &= I_z = \frac{3}{20}M(R^2 + 4h^2) \quad I_y = \frac{3}{10}MR^2 \\ I_{xG} &= I_{zG} = \frac{3}{80}M(4R^2 + h^2) \quad I_{yG} = \frac{3}{10}MR^2 \end{split}$$



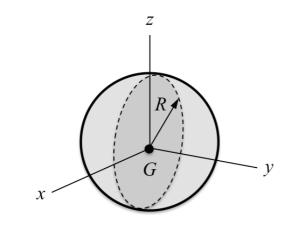
### Cilindro de revolución

$$I_{xG} = I_{zG} = \frac{1}{12}M(3R^2 + L^2)$$
$$I_{yG} = \frac{1}{2}MR^2$$



#### Semiesfera

$$x_G = 0$$
  $y_G = -\frac{3}{8}R$   $z_G = 0$  
$$I_{xG} = I_{zG} = \frac{83}{320}MR^2$$
  $I_{yG} = \frac{2}{5}MR^2$ 



#### Esfera

$$I_{xG} = I_{yG} = I_{zG} = \frac{2}{5}MR^2$$