34 Aplicando la factorización de Cholesky, resolver el sistema:

$$\left.\begin{aligned} x-y+z&=4\\ -x+2y-z+2t&=-3\\ x-y+5z+2t&=16\\ 2y+2z+6t&=8 \end{aligned}\right\}$$

$$A=\begin{array}{c} \begin{array}{cccc} x & y & z & t \end{array}\\ \begin{pmatrix} 1 & -1 & 1 & 0\\ -1 & 2 & -1 & 2\\ 1 & -1 & 5 & 2\\ 0 & 2 & 2 & 6 \end{pmatrix}\end{array} \sim \begin{pmatrix} 1 & -1 & 1 & 0\\ 0 & 1 & 0 & 2\\ 0 & 0 & 4 & 2\\ 0 & 2 & 2 & 6 \end{pmatrix} \sim \quad F_4 \to F_4 - 2F_2$$

$F_2 \to F_2 + F_1$

$F_3 \to F_3 - F_1$

$$\sim \begin{pmatrix} 1 & -1 & 1 & 0\\ 0 & 1 & 0 & 2\\ 0 & 0 & 4 & 2\\ 0 & 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 & 0\\ 0 & 1 & 0 & 2\\ 0 & 0 & 4 & 2\\ 0 & 0 & 0 & 1 \end{pmatrix} = U$$

$F_4 \to F_4 - \frac{1}{2}F_3$

$$Q = L\cdot D^{\frac{1}{2}} = \begin{pmatrix} 1 & 0 & 0 & 0\\ -1 & 1 & 0 & 0\\ 1 & 0 & 1 & 0\\ 0 & 2 & \frac{1}{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0\\ -1 & 1 & 0 & 0\\ 1 & 0 & 2 & 0\\ 0 & 2 & 1 & 1 \end{pmatrix} \longrightarrow A = Q\cdot Q^t$$

$$AX = B \longrightarrow Q \cdot \underbrace{Q^t \cdot X}_{Z} = B \rightarrow Q \cdot \underset{?}{Z} = B$$

$$\underset{Q}{\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 1 & 1 \end{pmatrix}} \cdot \underset{Z}{\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}} = \underset{B}{\begin{pmatrix} 4 \\ -3 \\ 16 \\ 8 \end{pmatrix}} \rightarrow \left.\begin{array}{l} \boxed{z_1 = 4} \\ -z_1 + z_2 = -3 \\ z_1 + 2z_3 = 16 \\ 2z_2 + z_3 + z_4 = 8 \end{array}\right\}$$

$$z_2 = z_1 - 3 = \boxed{1} \qquad z_4 = 8 - 2z_2 - z_3 = \boxed{0}$$

$$z_3 = \frac{16 - z_1}{2} = \boxed{6}$$

$$\underset{\checkmark}{Q^t} \cdot \underset{?}{X} = \underset{\checkmark}{Z}$$

$$\underset{Q^t}{\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}} \cdot \underset{X}{\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}} = \underset{Z}{\begin{pmatrix} 4 \\ 1 \\ 6 \\ 0 \end{pmatrix}} \rightarrow \begin{array}{l} x - y + z = 4 \\ y + 2t = 1 \\ 2z + t = 6 \\ \boxed{t = 0} \end{array}$$

$$\boxed{z = 3}$$

$$\boxed{y = 1}$$

$$\boxed{x = 2}$$

35 Determinar, mediante algoritmo, la factorización de Cholesky de la matriz:

$$A = \begin{pmatrix} 16 & -12 & 8 & -16 \\ -12 & 18 & -6 & 9 \\ 8 & -6 & 5 & -10 \\ -16 & 9 & -10 & 46 \end{pmatrix}$$

A partir de la matriz Q obtenida, calcular el determinante de A.

$$Q = \begin{pmatrix} q_{11} & 0 & 0 & 0 \\ q_{21} & q_{22} & 0 & 0 \\ q_{31} & q_{32} & q_{33} & 0 \\ q_{41} & q_{42} & q_{43} & q_{44} \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -4 & -1 & -2 & 5 \end{pmatrix}}}$$

$$q_{11} = \sqrt{a_{11}} = \sqrt{16} = 4$$

$$q_{21} = \frac{a_{21}}{q_{11}} = \frac{-12}{4} = -3 \qquad q_{22} = \sqrt{a_{22} - q_{21}^2} = \sqrt{18-(-3)^2} = 3$$

$$q_{31} = \frac{a_{31}}{q_{11}} = \frac{8}{4} = 2 \qquad q_{32} = \frac{a_{32} - q_{31}\cdot q_{21}}{q_{22}} = \frac{-6-2\cdot(-3)}{3} = 0$$

$$q_{41} = \frac{a_{41}}{q_{11}} = \frac{-16}{4} = -4 \qquad q_{42} = \frac{a_{42} - q_{41}\cdot q_{21}}{q_{22}} = \frac{9-(-4)(-3)}{3} = -1$$

$$q_{33} = \sqrt{a_{33} - q_{32}^2 - q_{31}^2} = \sqrt{5-0^2-2^2} = 1$$

$$q_{43} = \frac{a_{43} - q_{42}\cdot q_{32} - q_{41}\cdot q_{31}}{q_{33}} = \frac{-10-(-1)\cdot 0-(-4)\cdot 2}{1} = -2$$

$$q_{44} = \sqrt{a_{44} - q_{43}^2 - q_{42}^2 - q_{41}^2} = \sqrt{46-(-2)^2-(-1)^2-(-4)^2} = 5$$

$$A = q \cdot q^t \longrightarrow |A| = |q \cdot q^t| = |q| \cdot |q^t| = |q| \cdot |q| = |q|^2$$

$$|A| = |q|^2 = (4 \cdot 3 \cdot 1 \cdot 5)^2 = 60^2 = \boxed{3600}$$

q triangular

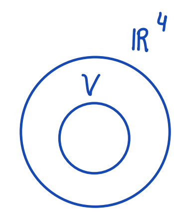
$\boxed{\text{TEMA 2}}$

1 Se considera el conjunto $V = \{(x, y, y, -x) \,/ x, y \in \mathbb{R}\}$ en el que se definen las siguientes operaciones:

$$(x, y, y, -x) + (z, t, t, -z) = (x + z, y + t, y + t, -(x + z))$$
$$\alpha\,(x, y, y, -x) = (\alpha x, \alpha y, \alpha y, -\alpha x)$$

Verificar que V es un espacio vectorial.

Sabemos que V es un subconjunto de $\mathbb{R}^4$.



$V \subseteq \mathbb{R}^4$

Por tanto si V es subespacio de $\mathbb{R}^4$, V también será espacio vectorial.

a) $\vec{0} \in V$? ✓ $\quad \vec{0} = (\underbrace{0}_{x}, \underbrace{0}_{y}, \underbrace{0}_{z}, \underbrace{0}_{t}) \to (0,0,0,-0) = (0,0,0,0)$ ✓

b) $\vec{u} + \vec{v} \in V$?

$$(x_1, y_1, y_1, -x_1) + (x_2, y_2, y_2, -x_2) =$$

$$= (\underbrace{x_1 + x_2}_{x}, \underbrace{y_1 + y_2}_{y}, \underbrace{y_1 + y_2}_{y}, \underbrace{-(x_1 + x_2)}_{-x}) \in V \quad ✓$$

$\boxed{V \text{ es e.V}}$

c) $\alpha \cdot \vec{u} \in V$?

$$\alpha\,(x_1, y_1, y_1, -x_1) = (\underbrace{\alpha x_1}_{x}, \underbrace{\alpha y_1}_{y}, \underbrace{\alpha y_1}_{y}, \underbrace{-\alpha x_1}_{-x}) \in V \quad ✓$$

2 Estudiar si los siguientes conjuntos son subespacios vectoriales:

a) $U = \{(x, y) \in \mathbb{R}^2 \,/\, x \geqslant 0, y \geqslant 0\}$

→ b) $U = \{(x, y, z) \in \mathbb{R}^3 \,/\, x^2 + y^2 + z^2 = 1\}$

c) $U = \{(x, y, z) \in \mathbb{R}^3 \,/\, z = x^2 + y^2\}$

→ d) $U = \{(x, y, z) \in \mathbb{R}^3 \,/\, x^2 + y^2 + z^2 = 0\}$

→ e) $U = \{(a - b, 2b + a, a, 0) \in \mathbb{R}^4 \,/\, a, b \in \mathbb{R}\}$

→ f) $U = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathcal{M}_{2\times 2} \,/\, a - b + c = 0, c - d = 0 \right\}$

g) $U = \{(\alpha, \beta, \gamma, \delta, \varepsilon) \in \mathbb{R}^5 \,/ \alpha, \beta, \gamma, \delta, \varepsilon \in \mathbb{R}\}$

h) $U = \{(x, y, z) \in \mathbb{R}^3 \,/\, |x| = |y|\}$

b) $U = \{(x,y,z) \in \overset{V}{\overset{\shortparallel}{\mathbb{R}^3}} \,/\, x^2+y^2+z^2 = 1\}$

1) $\vec{0} \in U$? $\vec{0} = (\underset{x}{0}, \underset{y}{0}, \underset{z}{0}) \rightarrow 0^2+0^2+0^2 = 1 \quad 0 \neq 1$ ✗

$\boxed{U \text{ no es subespacio v. de } \mathbb{R}^3}$

d) $U = \{(x,y,z) \in \mathbb{R}^3 \,/\, x^2+y^2+z^2 = 0\}$

1) $\vec{0} \in U$? $\rightarrow \vec{0} = (\underset{x}{0}, \underset{y}{0}, \underset{z}{0}) \rightarrow 0^2+0^2+0^2 = 0$ ✓

2) Vectores de U: $x^2+y^2+z^2 = 0$ $\qquad U = \{\vec{0}\}$ sub. trivial

$x = 0 \quad y = 0 \quad z = 0 \rightarrow$ Vector $\vec{0}$

$\boxed{U \text{ es el sub. trivial de } \mathbb{R}^3}$

e) $U = \{ (a-b, 2b+a, a, 0) \,/\, a, b \in \mathbb{R} \}$

1) $\vec{0} \in U$? $\vec{0} = (\underset{x}{0}, \underset{y}{0}, \underset{z}{0}, \underset{t}{0}) \to \vec{0} \in U$ ✓

$$\left.\begin{array}{l} a-b = 0 \\ 2b+a = 0 \\ a = 0 \\ 0 = 0 \;✓ \end{array}\right\} \begin{array}{l} \to 0-0 = 0 \;✓ \\ \to 2b = 0 \to b = 0 \\ \to a = 0 \end{array}$$

2) $(a_1 - b_1, 2b_1 + a_1, a_1, 0) + (a_2 - b_2, 2b_2 + a_2, a_2, 0) =$

$= (\underbrace{a_1 + a_2}_{a} \underbrace{-(b_1+b_2)}_{-b}, \underbrace{2(b_1+b_2)}_{2b} \underbrace{+ a_1 + a_2}_{+a}, \underbrace{a_1 + a_2}_{a}, 0) \in U$

3) $\alpha \cdot (a_1 - b_1, 2b_1 + a_1, a_1, 0) =$

$(\underbrace{\alpha a_1}_{a} \underbrace{- \alpha b_1}_{-b}, \underbrace{2\alpha b_1}_{2b} \underbrace{+ \alpha a_1}_{+a}, \underbrace{\alpha a_1}_{a}, 0) \in U$

$\boxed{U \text{ es subespacio v. de } \mathbb{R}^4}$

f) $U = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{M}_{2\times 2} \;/\; a - b + c = 0,\; c - d = 0 \right\}$

$0 - 0 + 0 = 0$ ✓ $\quad 0 - 0 = 0$ ✓

1) $\vec{0} \in U$? ✓ $\vec{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

2) Vectores de U: $\begin{pmatrix} a & a+c \\ c & c \end{pmatrix}$ $\quad$ 4 inc − 2 ec = 2 par

$\left.\begin{aligned} b &= a + c \\ d &= c \end{aligned}\right\}$

$$A + B = \begin{pmatrix} a_1 & a_1 + c_1 \\ c_1 & c_1 \end{pmatrix} + \begin{pmatrix} a_2 & a_2 + c_2 \\ c_2 & c_2 \end{pmatrix} =$$

$$= \begin{pmatrix} \overbrace{a_1 + a_2}^{a} & \overbrace{a_1 + a_2}^{a} + \overbrace{c_1 + c_2}^{c} \\ \underbrace{c_1 + c_2}_{c} & \underbrace{c_1 + c_2}_{c} \end{pmatrix} \in U$$

$$3)\ \alpha \cdot A = \begin{pmatrix} \overbrace{\alpha a_1}^{a} & \overbrace{\alpha a_1}^{a} + \overbrace{\alpha c_1}^{+c} \\ \underbrace{\alpha c_1}_{c} & \underbrace{\alpha c_1}_{c} \end{pmatrix} \in U$$

$\boxed{U \text{ es subespacio v. de } \mathbb{M}_{2\times 2}.}$

3 Determinar los valores de $a, b \in \mathbb{R}$ para que el vector $\vec{u} = (1, 2-a, b, 2b)$ sea combinación lineal de los vectores $\vec{v} = (1, 2, 0, 1)$ y $\vec{w} = (2, 1, 1, -1)$ de $\mathbb{R}^4$.

$$\underbrace{(1, 2-a, b, 2b)}_{\vec{u}} = \alpha_1 \underbrace{(1, 2, 0, 1)}_{\vec{v}} + \alpha_2 \underbrace{(2, 1, 1, -1)}_{\vec{w}}$$

$$\left.\begin{aligned} 1 &= \alpha_1 + 2\alpha_2 \\ 2-a &= 2\alpha_1 + \alpha_2 \\ b &= \alpha_2 \\ 2b &= \alpha_1 - \alpha_2 \end{aligned}\right\} \begin{aligned} &\longrightarrow \alpha_1 = 1 - 2\alpha_2 \\ &\longrightarrow \boxed{b = \frac{1}{5}} \\ &\longrightarrow 2\alpha_2 = \alpha_1 - \alpha_2 \rightarrow \alpha_1 = 3\alpha_2 = \frac{3}{5} \end{aligned}$$

$$1 - 2\alpha_2 = 3\alpha_2 \qquad 5\alpha_2 = 1 \rightarrow \alpha_2 = \frac{1}{5}$$

$$a = 2 - 2\alpha_1 - \alpha_2 = 2 - 2 \cdot \frac{3}{5} - \frac{1}{5} = 2 - \frac{6}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$$

4 Determinar x e y para que el vector $(1, x, 0, y)$ pertenezca al subespacio generado por $(1, 2, 1, 2)$ y $(1, -1, -1, 1)$.

El vector $(1, x, 0, y)$ debe ser C.L de $(1,2,1,2)$ y $(1,-1,-1,1)$:

$$(1, x, 0, y) = \alpha_1 (1,2,1,2) + \alpha_2 (1,-1,-1,1)$$

$$\left.\begin{array}{l} 1 = \alpha_1 + \alpha_2 \\ x = 2\alpha_1 - \alpha_2 \\ 0 = \alpha_1 - \alpha_2 \\ y = 2\alpha_1 + \alpha_2 \end{array}\right\} \begin{array}{l} \rightarrow 1 = \alpha_1 + \alpha_1 \rightarrow 2\alpha_1 = 1 \rightarrow \underline{\underline{\alpha_1 = \frac{1}{2}}} \\ \rightarrow x = 2 \cdot \frac{1}{2} - \frac{1}{2} = \boxed{\frac{1}{2}} \\ \rightarrow \alpha_2 = \alpha_1 \rightarrow \underline{\underline{\alpha_2 = \frac{1}{2}}} \\ \rightarrow y = 2 \cdot \frac{1}{2} + \frac{1}{2} = \boxed{\frac{3}{2}} \end{array}$$

5 Sea $U = \{(x, y, z, t) \ / \ x - y + z - t = 0\}$ un subconjunto del espacio vectorial $\mathbb{R}^4$:

a) Comprobar que U es un subespacio vectorial de $\mathbb{R}^4$.

b) Encontrar en U tres vectores $\vec{u}$, $\vec{v}$ y $\vec{w}$ linealmente independientes y verificar que cualquier vector de U se puede poner como combinación lineal de $\vec{u}$, $\vec{v}$ y $\vec{w}$.

a) 1) $\vec{0} \in U$? $\quad \vec{0} = (\underset{x}{0}, \underset{y}{0}, \underset{z}{0}, \underset{t}{0}) \qquad 0 - 0 + 0 - 0 = 0$ ✓

2) Vectores de U : $t = x - y + z \rightarrow (x, y, z, x - y + z)$

$$(x_1, y_1, z_1, x_1 - y_1 + z_1) + (x_2, y_2, z_2, x_2 - y_2 + z_2) =$$

$$= (\underbrace{x_1 + x_2}_{x}, \underbrace{y_1 + y_2}_{y}, \underbrace{z_1 + z_2}_{z}, \underbrace{x_1 + x_2}_{x} \underbrace{- (y_1 + y_2)}_{-y} \underbrace{+ z_1 + z_2}_{+z})$$

3) $\alpha (x_1, y_1, z_1, x_1 - y_1 + z_1)$

$$= (\underbrace{\alpha x_1}_{x}, \underbrace{\alpha y_1}_{y}, \underbrace{\alpha z_1}_{z}, \underbrace{\alpha x_1}_{x} \underbrace{- \alpha y_1}_{-y} \underbrace{+ \alpha z_1}_{+z}) \in U$$ ✓

$\boxed{U \text{ es sub. v. de } \mathbb{R}^4}$

ec. param.

b) $x - y + z - t = 0 \longrightarrow t = x - y + z \rightarrow$
$$\begin{cases} x = \alpha \\ y = \beta \\ z = \gamma \\ t = \alpha - \beta + \gamma \end{cases}$$

ec. imp.

$(\alpha, \beta, \gamma \in \mathbb{R})$

$$(x, y, z, t) = (\alpha, \beta, \gamma, \alpha - \beta + \gamma) =$$

$$= (\alpha, 0, 0, \alpha) + (0, \beta, 0, -\beta) + (0, 0, \gamma, \gamma) =$$

$$= \alpha \underline{\underline{(1, 0, 0, 1)}} + \beta \underline{\underline{(0, 1, 0, -1)}} + \gamma \underline{\underline{(0, 0, 1, 1)}}$$

sist. generador

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \rightarrow \text{son L. I} \qquad \vec{u}, \vec{v}, \vec{w} \text{ forman una base.}$$

Cualquier vector de U se puede expresar como C. L de $\vec{u}$, $\vec{v}$ y $\vec{w}$

y estos son L. I .

10 Dado el espacio vectorial $\mathbb{R}^4$, consideramos el subespacio:

$$V = \begin{cases} x_1 = \lambda \\ x_2 = \lambda + \mu \\ x_3 = \gamma \\ x_4 = \mu \end{cases} \quad (\lambda, \mu, \gamma \in \mathbb{R})$$

Hallar una base de V y calcular las coordenadas del vector $\vec{v} = (2, 4, 0, 2)$ en la base elegida.

$$(x_1, x_2, x_3, x_4) = (\lambda, \lambda+\mu, \gamma, \mu) =$$

$$= \lambda\,\underline{(1,1,0,0)} + \mu\,\underline{(0,1,0,1)} + \gamma\,\underline{(0,0,1,0)}$$

$S = \{(1,1,0,0), (0,1,0,1), (0,0,1,0)\}$ → son L.I

SIST. GEN.

$$\boxed{B_V = S}$$

coordenadas de $\vec{v}$ según la base B_V

$$\vec{v} = (2,4,0,2) = \lambda(1,1,0,0) + \mu(0,1,0,1) + \gamma(0,0,1,0)$$

$$\left.\begin{array}{l} 2 = \lambda \\ 4 = \lambda + \mu \\ 0 = \gamma \\ 2 = \mu \end{array}\right\} \begin{array}{l} \to \lambda = 2 \ ✓ \\ \to \lambda = 4 - 2 = 2 \\ \to \gamma = 0 \\ \to \mu = 2 \end{array}$$

$$\boxed{\vec{v} = (2,4,0,2) = (2,2,0)_B}$$