Outline

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- Sources of error:
  1) **Bias** → Systematic assumptions by the model (*underfitting*)
  2) **Variance** → Sensitivity to training data (*overfitting*)

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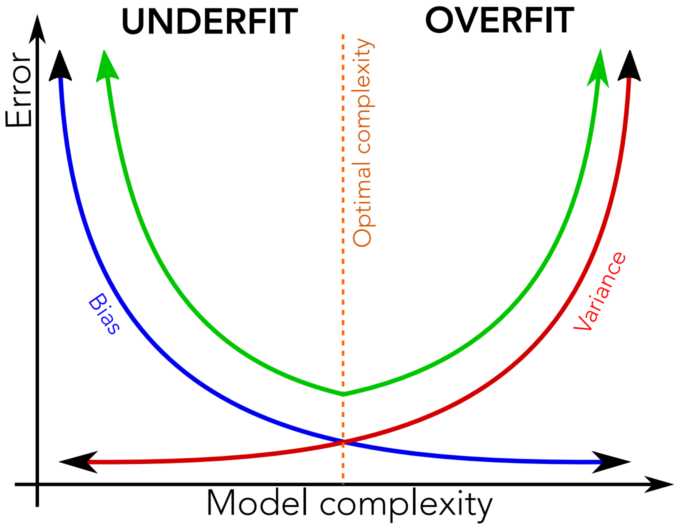
- **Bias**: Systematic deviation from $f(\mathbf{x})$
- **Variance**: Dependence on the data sampling in $\mathcal{D}$

Bias-variance tradeoff



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  - $\rightarrow$ **Model regularization**: Add constraints/penalties to the loss function

T2: Computational learning

Fundamentos del Aprendizaje Automático

Curso 2025/2026