Ejercicios

❶ Escribir una matriz $A \in \mathcal{M}_{3\times 2}$ que cumpla lo siguiente:

$$a_{ij} = (-1)^{i+j} \cdot (2j+1)$$

$$A = (a_{ij}) = \begin{array}{c} \begin{array}{cc} j=1 & j=2 \end{array} \\ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \\ 3\times 2 \end{array} = \begin{pmatrix} (-1)^{1+1}\cdot(2\cdot 1+1) \;\; 3 & (-1)^{1+2}\cdot(2\cdot 2+1) \;\; 5 \\ (-1)^{2+1}\cdot(2\cdot 1+1) & (-1)^{2+2}\cdot(2\cdot 2+1) \\ (-1)^{3+1}(2\cdot 1+1) & (-1)^{3+2}\cdot(2\cdot 2+1) \end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix} 3 & -5 \\ -3 & 5 \\ 3 & -5 \end{pmatrix}}}$$

$$(-1)^{i+j} \to \begin{pmatrix} + & - & + & \cdots \\ - & + & - & \cdots \\ + & - & + & \cdots \end{pmatrix}$$

Ejercicios

❷ Determinar x e y para que las matrices A y B sean iguales:

$$A = \begin{pmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{pmatrix} \quad B = \begin{pmatrix} x+3 & y^2+2 \\ 0 & -6 \end{pmatrix}$$

$A = B \Rightarrow$

$2x + 1 = x + 3 \to \boxed{x = 2}$

$3y = y^2 + 2 \to y^2 - 3y + 2 = 0$

$y = 3 : \cancel{9 - 9} + 2 \neq 0$ ✗

$y = 2 : 4 - 6 + 2 = 0$ ✓

$\cancel{0 = 0}$

$y^2 - 5y = -6 \to y^2 - 5y + 6 = 0$

$$y = \frac{5 \pm \sqrt{25-4\cdot 1\cdot 6}}{2\cdot 1} = \frac{5 \pm 1}{2} = \begin{cases} y = 3 \quad \times \\ \boxed{y = 2} \end{cases}$$

Ejercicios

❸ Considerando la matriz:

$$A = \begin{pmatrix} x & x+y & x-z \\ x-y & y & y+z \\ x+z-2 & z-y & z \end{pmatrix}$$

a) Calcular x, y, z para que la matriz A sea triangular superior.

b) Calcular x, y, z para que la matriz A sea triangular inferior.

c) Obtener las trazas de ambas matrices triangulares.

a) T sup → 0s bajo la diagonal.

$$\left.\begin{aligned} x - y &= 0 \\ z - y &= 0 \\ x + z - 2 &= 0 \end{aligned}\right\} \begin{aligned} &\to x = y \to \boxed{x = 1} \\ &\to z = y \to \boxed{z = 1} \\ &\to y + y - 2 = 0 \to 2y = 2 \end{aligned}$$

$$y = \frac{2}{2} = \boxed{1}$$

$$T_{sup} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

b) $T_{inf} \rightarrow$ 0s sobre la diagonal.

S.C.I (∞ sol).

$$\begin{aligned} x + y &= 0 &&\longrightarrow \quad \cancel{z + y = 0} \\ x - z &= 0 &&\longrightarrow \quad x = z \\ y + z &= 0 &&\longrightarrow \quad z + y = 0 \longrightarrow y = -z \end{aligned}$$

$$\left.\begin{aligned} x &= a \\ y &= -a \\ z &= a \end{aligned}\right\} \quad a \in \mathbb{R} \quad \Rightarrow \quad T_{inf} = \begin{pmatrix} a & 0 & 0 \\ 2a & -a & 0 \\ 2a-2 & 2a & a \end{pmatrix}$$

c) $tr(T_{sup}) = 3 \qquad tr(T_{inf}) = a$

Ejercicios

4. Calcular la matriz:

$$X = \frac{1}{6}(4A + 3B - 2I)$$

Donde:

$$A = \begin{pmatrix} -4 & 0 & 1 \\ 1 & -5 & 0 \\ -3 & 2 & 1 \end{pmatrix} \text{ y } B = \begin{pmatrix} 1 & 2 & 0 \\ -2 & 1 & -1 \\ 4 & 4 & 0 \end{pmatrix}$$

$$4A = 4 \cdot \begin{pmatrix} -4 & 0 & 1 \\ 1 & -5 & 0 \\ -3 & 2 & 1 \end{pmatrix} = \underset{3 \times 3}{\begin{pmatrix} -16 & 0 & 4 \\ 4 & -20 & 0 \\ -12 & 8 & 4 \end{pmatrix}}$$

$$3B = \underset{3\times 3}{\begin{pmatrix} 3 & 6 & 0 \\ -6 & 3 & -3 \\ 12 & 12 & 0 \end{pmatrix}} \qquad -2I = -2 \cdot \underset{3\times 3}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$X = \frac{1}{6}\left[\begin{pmatrix} -16 & 0 & 4 \\ 4 & -20 & 0 \\ -12 & 8 & 4 \end{pmatrix} + \begin{pmatrix} 3 & 6 & 0 \\ -6 & 3 & -3 \\ 12 & 12 & 0 \end{pmatrix} + \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}\right] =$$

$$= \frac{1}{6}\begin{pmatrix} -15 & 6 & 4 \\ -2 & -19 & -3 \\ 0 & 20 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} & 1 & \frac{2}{3} \\ -\frac{1}{3} & -\frac{19}{6} & -\frac{1}{2} \\ 0 & \frac{10}{3} & \frac{1}{3} \end{pmatrix}$$

Ejercicios

❺ Considerando las matrices:

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ y } B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

a) Verificar que: $(A+B)(A-B) \neq A^2 - B^2$

b) ¿Por qué sucede lo anterior?

$$(A+B)\cdot(A-B) = \underset{3\times 3}{\begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}} \cdot \underset{3\times 3}{\begin{pmatrix} 0 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}} =$$

$$= \begin{pmatrix} -1 & 1 & 0 \\ -1 & 2 & -2 \\ -1 & 0 & -1 \end{pmatrix}$$

$$A^2 = A\cdot A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^2 = B\cdot B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 - B^2 = \begin{pmatrix} 0 & 1 & 1 \\ -2 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

b) $(A+B)\cdot(A-B) = A^2 - \underbrace{AB}_{} + \underbrace{BA}_{} - B^2 \neq A^2 - B^2$

No puede cancelarse

porque en general $A\cdot B \neq B\cdot A$

Ejercicios

❻ Encontrar todas las matrices, X, simétricas de orden 2 que verifiquen: $X^2 = I$.

$$X = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

$$X^2 = I \longrightarrow \underset{2\times 2}{\begin{pmatrix} a & c \\ c & b \end{pmatrix}} \cdot \underset{2\times 2}{\begin{pmatrix} a & c \\ c & b \end{pmatrix}} = \underset{2\times 2}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$X^2 = X\cdot X$$

$$\begin{pmatrix} a^2+c^2 & ac+bc \\ ac+bc & b^2+c^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\left.\begin{array}{l} a^2+c^2 = 1 \\ a\underline{\underline{c}} + b\underline{\underline{c}} = 0 \\ b^2+c^2 = 1 \end{array}\right\} \longrightarrow c\,(a+b) = 0 \longrightarrow \boxed{c = 0}$$

$$\hookrightarrow a+b = 0 \rightarrow \boxed{b = -a}$$

$\boxed{c = 0}$

$$\begin{cases} a^2 = 1 \longrightarrow a = \pm\sqrt{1} = \boxed{\pm 1} \\ b^2 = 1 \longrightarrow b = \boxed{\pm 1} \end{cases}$$

$\boxed{b = -a}$

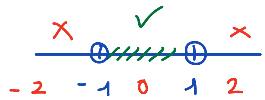
$$\begin{cases} \cancel{a^2+c^2 = 1} \\ (-a)^2+c^2 = 1 \longrightarrow a^2+c^2 = 1 \end{cases}$$

$$c^2 = 1-a^2 \rightarrow c = \pm\sqrt{1-a^2}$$

Matrices X : $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} a & \sqrt{1-a^2} \\ \sqrt{1-a^2} & -a \end{pmatrix}, \begin{pmatrix} a & -\sqrt{1-a^2} \\ -\sqrt{1-a^2} & -a \end{pmatrix} \quad -1 < a < 1$$

$$1-a^2 > 0 \longrightarrow 1-a^2 = 0 \longrightarrow a^2 = 1 \longrightarrow a = \pm 1$$



$a = 2: \ 1 - 2^2 = -3 > 0$? ✗

$a = 0: \ 1 > 0$ ✓

$a = -2: \ 1 - (-2)^2 = 1 - 4 = -3 > 0$? ✗

Ejercicios

❼ Calcular el rango de A según los valores del parámetro λ. Utilizar el método de Gauss:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ \lambda & -1 & 10 \\ -1 & \lambda & -6 \\ 2 & 5 & 1 \end{pmatrix}$$

como máximo $\longrightarrow \operatorname{rg}(A) = 3 \qquad \operatorname{rg}(A) = \operatorname{rg}(A^t)$

$$A^t = \begin{pmatrix} 1 & \lambda & -1 & 2 \\ 2 & -1 & \lambda & 5 \\ 1 & 10 & -6 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 & \lambda \\ 2 & 5 & \lambda & -1 \\ 1 & 1 & -6 & 10 \end{pmatrix} \sim$$

$C_2 \leftrightarrow C_4$

$$\sim \begin{pmatrix} 1 & 2 & -1 & \lambda \\ 0 & 1 & 2+\lambda & -1-2\lambda \\ 0 & -1 & -5 & 10-\lambda \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 & \lambda \\ 0 & 1 & 2+\lambda & -1-2\lambda \\ 0 & 0 & -3+\lambda & 9-3\lambda \end{pmatrix}$$

$F_2 \to F_2 - 2F_1$

$F_3 \to F_3 - F_1$

$F_3 \to F_3 + F_2$

$-3 + \lambda = 0 \longrightarrow \boxed{\lambda = 3}$

$9 - 3\lambda = 0 \qquad 3\lambda = 9 \qquad \boxed{\lambda = 3}$

- si $\lambda = 3$: $rg(A) = 2$ ✓

- si $\lambda \neq 3$: $rg(A) = 3$ ✓