MCEN90017 Advanced Motion Control

Assignment 3 - Extremum Seeking

Submission instructions: This assignment is to be completed individually. All submissions must be uploaded to LMS by **11:59pm on Sunday 14 June 2020**. Your report should not exceed 8 pages, and should include all working, and clear explanations of your methodology and results. You will also need to submit your MATLAB code / Simulink model(s). The standard university policies on plagiarism apply, see `https://academicintegrity.unimelb.edu.au`.

A continuously stirred bioreactor is illustrated in the schematic of Figure 1. This bioreactor is used to grow a commercially valuable microbial biomass. An inflow feed transports a substrate concentration $\bar{\rho}$ into the reactor tank, which is consumed by the contained biomass, yielding growth. An outflow tap transports a mixture of the biomass and substrate from the reactor tank. The inflow is modelled by a nonnegative dilution rate u, while the outflow is modelled by a nonnegative production rate y. The bioreactor reactor tank is continuously stirred so as to promote homogenous microbial growth throughout the tank.

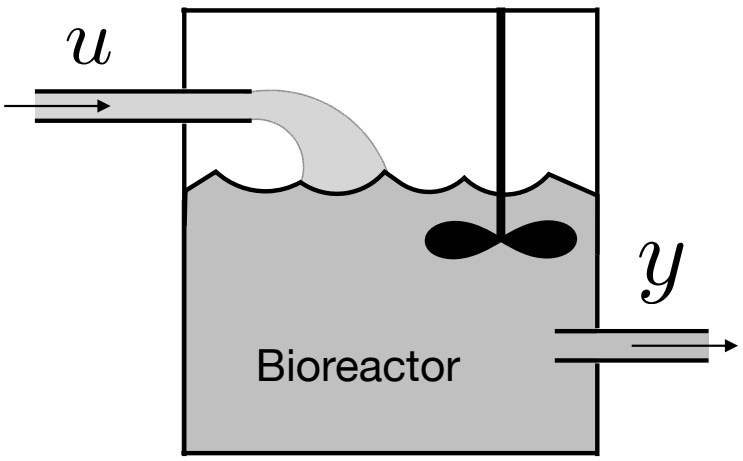

Figure 1: Continuously stirred bioreactor.

The microbial kinetics modelling biomass growth and outflow are described by a continuous time nonlinear state space model, given by

$$\begin{aligned} \dot{x}_1 &= \mu(x_2)\, x_1 - x_1\, u\,, \\ \dot{x}_2 &= -k_1\, \mu(x_2)\, x_1 + (\bar{\rho} - x_2)\, u\,, \\ y &= k_2\, \mu(x_2)\, x_1\,, \end{aligned} \tag{1}$$

in which $x_1 = x_1(t) \geq 0$ and $x_2 = x_2(t) \geq 0$ denote the biomass and substrate concentrations respectively, and $u = u(t) \geq 0$ and $y = y(t) \geq 0$ denote the inflow (dilution) and outflow

(production) rates, all at time $t \geq 0$. The reaction and production rate coefficients k_1 and k_2 have nominal values $k_1 \doteq 1.5$ and $k_2 \doteq 3.0$, while the concentration $\bar{\rho}$ of substrate in the inflow has the nominal value $\bar{\rho} \doteq 2.0$. The functional dependence $\mu(\cdot)$ appearing in the model is assumed to be nominally linear, i.e.

$$\mu(x_2) \doteq \bar{\eta}\, x_2\,, \tag{2}$$

in which $\bar{\eta} \in [0.25, 0.75]$ is a positive constant that is typically unknown. Pre-production preparation of the bioreactor is summarized by the initial biomass and substrate concentrations $x_1(0) \geq 0$ and $x_2(0) \geq 0$. A typical initialization is $x_1(0) = 0.1$ and $x_2(0) = 1.0$, although this may vary widely between production runs. All quantities are normalized, and may be assumed to be dimensionless.

The objective is to design an extremum seeking controller that asymptotically maximizes the outflow (production) rate $y = y(t)$ of the bioreactor, via manipulation of the inflow (dilution) rate $u = u(t)$, without reference to prior specification of all bioreactor model parameters.

1 Open-loop bioreactor analysis and simulation

Prior to designing or implementing an extremum seeking feedback controller, it is essntial to establish a qualitative and (where possible) a quantitative understanding of nominal open-loop behaviours that can be exhibited by the bioreactor, given a constant and positive inflow (dilution) rate $u = u(t) = \bar{u} > 0$.

Tasks

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1.1. Given constant $\bar{u}$, show that the bioreactor model (1), (2) exhibits an equilibrium at

$$(x_1,\, x_2) = (\bar{x}_1, \bar{x}_2) \doteq \left(\tfrac{1}{k_1}\,(\bar{\rho} - \tfrac{\bar{u}}{\bar{\eta}}),\, \tfrac{\bar{u}}{\bar{\eta}}\right). \tag{3}$$

1.2. By inspection of (3), write down the constraints on $\bar{u}$ required for this equilibrium to be located in the *open* positive orthant of the state space, i.e. in $\mathbb{R}^2_{>0} \doteq \mathbb{R}_{>0} \times \mathbb{R}_{>0}$. Confirm that there are no other equilibria in the open positive orthant. Are there any others in the *closed* positive orthant, i.e. in $\mathbb{R}^2_{\geq 0} \doteq \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}$?

1.3. By inspection of (1), (2), show that the open positive orthant of the state space is forward invariant; i.e. given any $(x_1(0),\, x_2(0)) \in \mathbb{R}^2_{>0}$, show that $(x_1(t),\, x_2(t)) \in \mathbb{R}^2_{>0}$ for all $t \geq 0$. (HINT: examine neighbourhoods of the boundary of the closed positive orthant.)

1.4. Without necessarily completing the calculations involved, discuss how asymptotical stability of the equilibrium (3) may be verified. Given any initial state in the open positive orthant, and any $\bar{u}$ satisfying the constraints derived in Task 1.2 above, describe (qualitatively) how you expect the open-loop bioreactor model (1), (2) to behave.

1.5. By inspection of (1), (2), (3), write down an expression for the corresponding equilibrium outflow (production) rate $y = \bar{y} = \bar{y}(\bar{u})$ as a function of the constant inflow (dilution) rate $\bar{u}$. Confirm that it exhibits a maximum for some constant inflow rate $\bar{u} = \bar{u}^*$, and derive explicit expressions for $\bar{u}^*$ and the corresponding maximum outflow rate $\bar{y}^*$.

1.6. Construct a Simulink simulation model of the bioreactor model (1), (2). Using the nominal values $\bar{\eta} \doteq 0.55$, $\bar{u} \doteq 0.275$, and the other values indicated, confirm that your simulation generates a transient response of the form illustrated in Figure 2, and a phase portrait of the form of Figure 3 (in which green circles denote the initial state). Also confirm that the steady state achieved is in agreement with (3).

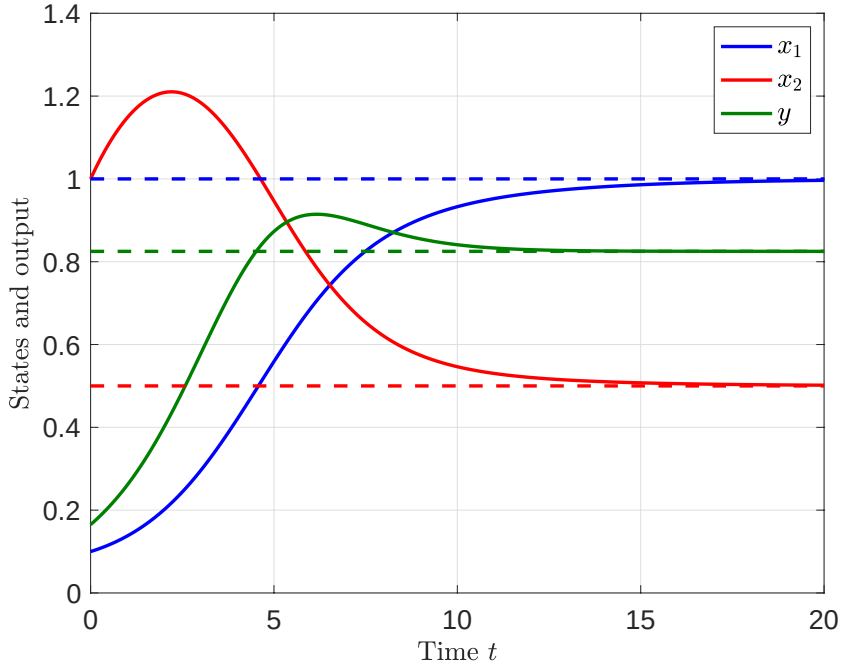

Figure 2: Open-loop transient response.

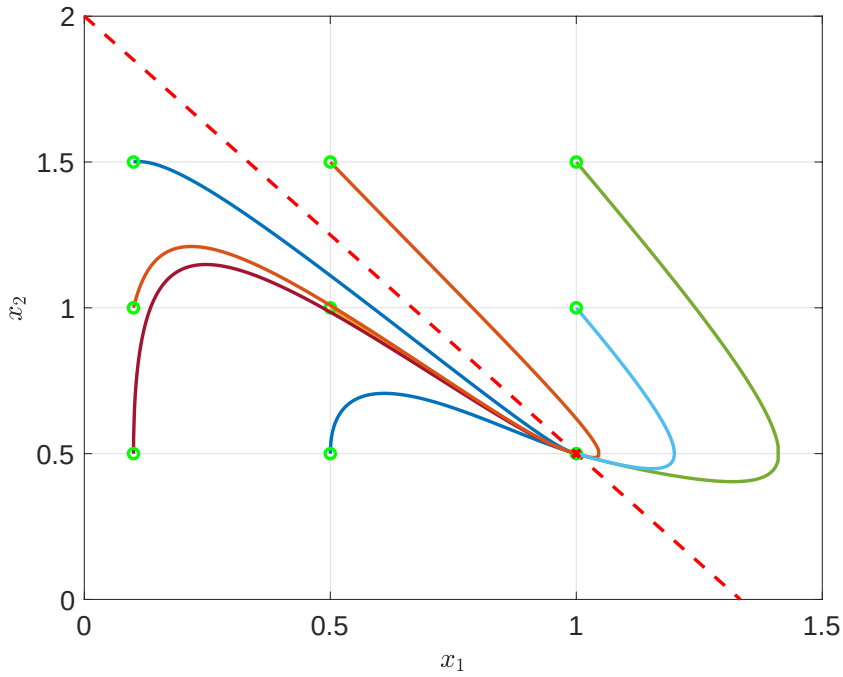

Figure 3: Open-loop phase portrait.

1.7. By simulating a variety of constant inflow (dilution) rates $\bar{u}$ in the vicinity of $\bar{u}^*$ from Task 1.5, confirm that the outflow (production) rate does appear to be maximized at $\bar{u} = \bar{u}^*$, with the expected maximum value.

2 Extremum seeking feedback control

As indicated, the objective is to design an extremum seeking feedback controller that maximizes the outflow (production) rate $y = y(t)$ of the bioreactor, as $t \to \infty$. The controller architecture to be employed is as per Figure 4. Note in particular that the outflow (production) rate is to be applied as the input to the controller, while the output of the controller is to be applied to the inflow (dilution) rate. The filter bandwidths, controller gain, dither parameters, etc, remain to be designed so as to achieve outflow (production) rate extremum seeking in closed loop.

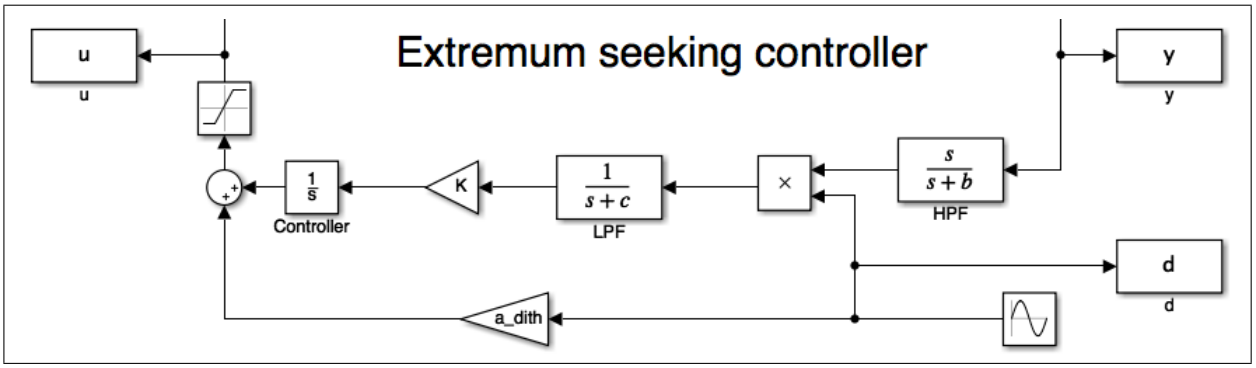

Figure 4: Extremum seeking controller architecture.

Tasks

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2.1. Recalling the lecture material on extremum seeking, list the properties of the open-loop bioreactor that are assumed in order to establish extremum seeking using the architecture of Figure 4. Using your answers obtained in Section 1, and employing the notation used in the lecture material, what are the explicit expressions for the maps $l(\cdot)$ and $Q(\cdot)$ in this application? Using the open-loop transient responses obtained in Section 1, and any other information you deem appropriate, select appropriate values for the filter bandwidths, controller gain, dither parameters, etc. Include your reasoning.

2.2. Implement the controller architecture of Figure 4 in your Simulink model of the bioreactor. Use your designed parameters from Task 2.1. By simulating the extremum seeker in feedback with the bioreactor, show that a typical closed loop response is of the form illustrated in Figure 5. Using your answers from the analysis of Section 1, confirm that the average outflow (production) rate is indeed maximized.

2.3. By varying the design parameters of Task 2.1, explore and discuss the tradeoffs between these parameters in terms of convergence rate, steady state error, etc.

2.4. Given the nominal parameters for the bioreactor and your controller, explore and discuss the robustness of the closed loop to variations in the initial state $(x_1(0), x_2(0))$ of the bioreactor, noting in particular the restriction indicated by Tasks 1.2 and 1.3. Document any cases where expected behaviour is not achieved; for example, where state trajectories appear to converge to boundary of the closed positive orthant, and / or where the outflow (production) rate is not maximized.

2.5. Given the nominal initial state $(x_1(0), x_2(0))$ of the bioreactor provided, explore and discuss the robustness of the closed loop to variations in the constant reaction rate parameter $\bar{\eta}$ of (2). Pay particular attention to the range of values of $\bar{\eta}$ indicated following (2).

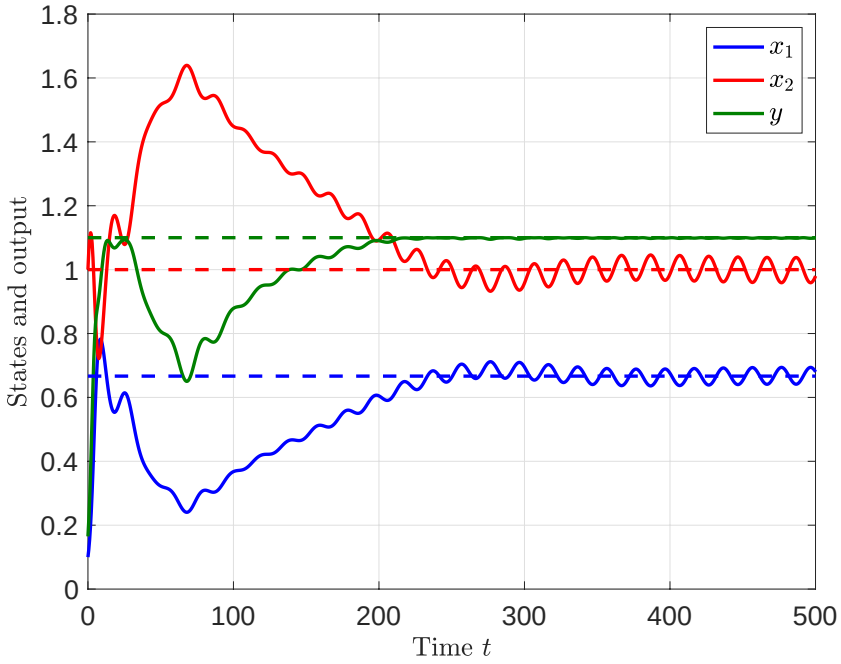

Figure 5: Typical closed loop transient response.

2.6. Suppose that the functional dependence $\mu(\cdot)$ appearing in (1) no longer satisfies the linear relationship (2), but rather is nonlinearly defined as per Figure 6 by

$$\mu(x_2) \doteq \eta(x_2)\, x_2\,, \quad \eta(x_2) \doteq \tfrac{1}{4} + \tfrac{1}{4}\left[1 + \tanh(4(x_2 - \beta))\right], \tag{4}$$

in which $\beta \doteq 1 - \frac{1}{4}\tanh^{-1}(\frac{1}{5})$. By modifying your simulation model for the bioreactor to replace (2) with (4), explore and discuss whether your extremum seeker can maximize the outflow (production) rate in this more general setting.

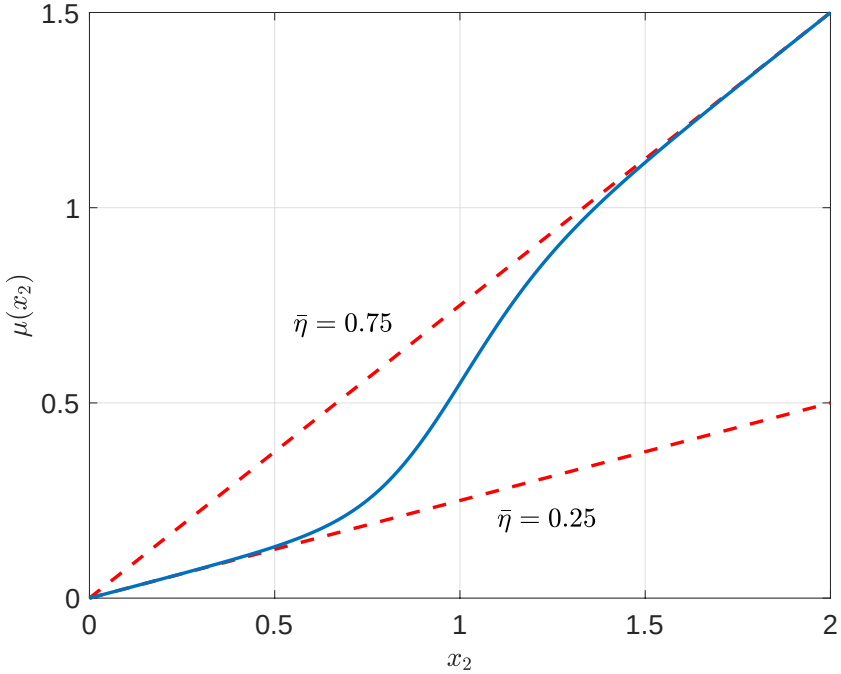

Figure 6: Nonlinear functional dependence $\mu(\cdot)$ of (4).

2.7. Summarize your findings and conclude whether extremum seeking can be considered to be a viable feedback control strategy for this particular application.