Solution to Q2: Market Share Prediction using Markov Chain

Part 1: Model Setup

We are given a scenario where person X chooses between two preferences. We can model this behavior using a Markov Chain.

States:

- State A: First Preference (starts with 55% share)
- State B: Second Preference (starts with 45% share)

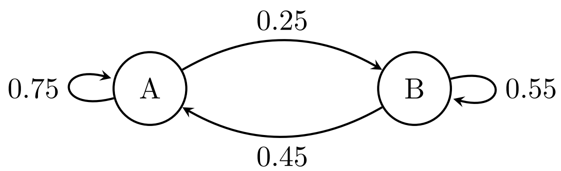
Transition Probabilities: The problem gives us the probabilities of sticking to a choice (retention). We can derive the switching probabilities from this since the row sum must be 1.

- $P(A \to A) = 0.75 \implies P(A \to B) = 0.25$
- $P(B \to B) = 0.55 \implies P(B \to A) = 0.45$

So, the Transition Matrix P is:

$$P = \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix}$$

Choice Changes Diagram: Below is the transition diagram showing the movement between preferences:



Part 2: Market Share Prediction

We need to find the market share after 2 years.

- Frequency: 2 times per year.
- Duration: 2 years.
- Total steps (n): 4.

Let $V_0 = [0.55, \quad 0.45]$. We calculate the future states using $V_{next} = V_{current} \times P$.

Step 1 (6 months):

$$V_1 = [0.55, \quad 0.45] \times \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix} = [0.615, \quad 0.385]$$

Step 2 (1 year):

$$V_2 = [0.615, \quad 0.385] \times \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix} = [0.6345, \quad 0.3655]$$

Step 3 (1.5 years):

$$V_3 = [0.6345, \quad 0.3655] \times \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix} = [0.64035, \quad 0.35965]$$

Step 4 (2 years):

$$V_4 = [0.64035, \quad 0.35965] \times \begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix}$$

Calculating the final values:

$$A_{final} = (0.64035 \times 0.75) + (0.35965 \times 0.45) = 0.48026 + 0.16184 = 0.6421$$

$$B_{final} = 1 - 0.6421 = 0.3579$$

Final Answer

After 2 years, the predicted market shares are:

Preference	**Probability**	%
First Preference (A)	0.6421	64.21%
Second Preference (B)	0.3579	35.79%

The system shows a drift towards the first preference (State A) due to its higher retention probability (0.75) compared to State B (0.55).