Yahtzee Formulae

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Abstract

How to determine scores and other values in Yahtzee.
For all listed values:

number of dice = $n \geq 2$
number of sides per die = $d \geq 2$
bonus points needed = p
yahtzee score earned = y
set of current dice = $S \subset \mathbb{N} - \{0\}$

1 Scores

1.1 Rolls per turn

$$rollsperturn = \lceil \max(d, n) * 0.5 \rceil \tag{1}$$

1.2 Numbers

$a \in \{1, 2 \dots d\}$
c = number of occurrances of a in S

$$numscore_a = c * a \tag{2}$$

1.3 Kinds

$K = \{a \dots\} \subseteq S$

$$smallkindscore = \left\{ \begin{array}{ll} 0, & \text{if } |K| < \lceil n * 0.5 \rceil \\ \sum_{i \in S} i, & \text{if } |K| \geq \lceil n * 0.5 \rceil \end{array} \right\} \tag{3}$$

$$largekindscore = \left\{ \begin{array}{ll} 0, & \text{if } |K| < \lceil n * 0.75 \rceil \\ \sum_{i \in S} i, & \text{if } |K| \geq \lceil n * 0.75 \rceil \end{array} \right\} \tag{4}$$

1.4 Full House

$K = \{a \dots\} \subset S \; |K| = \lfloor n * 0.5 \rfloor$
$L = \{b \dots\} \subset S \; |L| = \lceil n * 0.5 \rceil$
$K \cap L = \emptyset$

$$fullhousescore = \left\{ \begin{array}{ll} 0, & \text{if } K \cup L \neq S \\ \lceil d * n * 0.83 \rceil, & \text{if } K \cup L = S \end{array} \right\} \tag{5}$$

1.5 Straights

Duplicates in set of straights are removed and not considered for this formula
$K = \{a, a+1 \dots\} \subseteq S$

$$smallstraightscore = \left\{ \begin{array}{ll} 0, & \text{if } |K| < \lceil n * 0.75 \rceil \\ d * n, & \text{if } |K| \geq \lceil n * 0.75 \rceil \end{array} \right\} \tag{6}$$

$$largestraightscore = \left\{ \begin{array}{ll} 0, & \text{if } |K| < n \\ \lceil d * n * 1.33 \rceil, & \text{if } |K| = n \end{array} \right\} \tag{7}$$

1.6 Yahtzee

$K = \{a \dots a\} = S$

$$y = \left\{ \begin{array}{ll} 0, & \text{if } |K| < n \\ \lceil d * n * 1.66 \rceil, & \text{if } |K| = n \end{array} \right\} \tag{8}$$

1.7 Chance

$K = \{a_0 \dots \; a_{n-1}\} = S$

$$chancescore = \sum_{i \in S} i \tag{9}$$

2 Bonus Scores

2.1 Number Score Bonus

$$p = \lceil n * 0.5 \rceil * \sum_{i=1}^{d} i \tag{10}$$

2.2 Bonus points to add

$$numbonusscore = \left\{ \begin{array}{ll} 0, & \text{if } \sum_{i=1}^{d} numscore_i < p \\ \lceil d * n * 1.162 \rceil, & \text{if } \sum_{i=1}^{d} numscore_i \geq p \end{array} \right\} \tag{11}$$

2.3 Yahtzee Bonus

$$yahzteebonuscore = \left\{ \begin{array}{ll} 0, & \text{if } y = 0 \\ \lceil d * n * 3.32 \rceil, & \text{if } y > 0 \end{array} \right\} \tag{12}$$

3 Future Formulae

3.1 Multiple Straights

Straights are impossible to achieve in the following situation:
Say we have $K = \{x, x+1 \dots \ x+d\} \subset S$ and $d < n$
Given the above property, a small straight is possible to get if $d \geq \lceil n * 0.75 \rceil$
A large straight is impossible to achieve in this scenario

A potential correction to this situation follows where
s = number of straights found and
$d < n$

$$straightscore = \left\{ \begin{array}{ll} s * d * (\lceil d * 0.75 \rceil), & \text{if } smallstraight \\ s * d * (\lceil d * 0.75 \rceil) * 1.33, & \text{if } largestraight \end{array} \right\} \tag{13}$$

If we wish to make straights more difficult to score, we can use the below formula

$$straightscore = \left\{ \begin{array}{ll} 0, & \text{if } s < \lceil n * 0.75 \rceil \\ s * d * n, & \text{if } s \geq \lceil n * 0.75 \rceil \text{ and } s < n \\ s * d * n * 1.33, & \text{if } s = n \end{array} \right\} \tag{14}$$