The Role of Space, Density and Migration in Social Dilemmas - Supplementary

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1 EXTRA FIGURES

1.1 Two Defector Pollution

In Figure 1 we see the contour lines of pollution due to two defectors at a distance $r = 10$ away from each other. Pollution clouds here are of size $R = 10$. Each contour line, or *isopleth*, is a line of constant pollution.

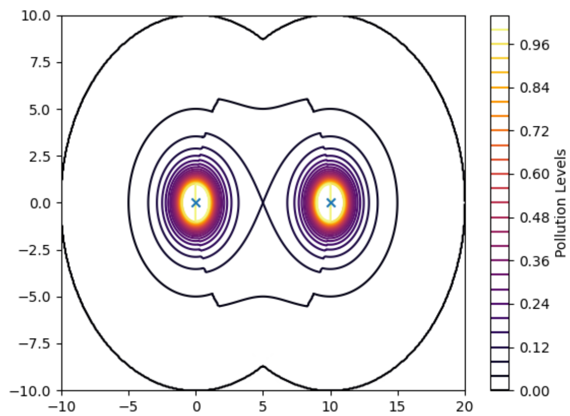

Figure 1: For $R = 10$, the contour plot of pollution in space due to two defectors at $x = 0$ and $x = 10$ is shown with colours indicating pollution levels. Each isopleth is a line of constant pollution.

1.2 Fixed Strategy

In Figure 2 we plot the time evolution for $N = 50$ fixed-strategy agents for a variety of initial number of defectors D and values of migration M. We find that the change in per-capita pollution (PCP) decreases over time, meaning migration does minimise pollution felt (and hence personal expense) over time even without strategy changes. Furthermore, not only does higher migration equilibriate the system faster but also decreases the long-term level of pollution more. That is, comparing the blue $(M, D) = (2, 10)$ line to the equivalent orange $(10, 10)$ line, where both are initialised with 10 defectors, we see that the higher level of $M = 10$ has a significantly lower change in PCP than for $M = 2$.

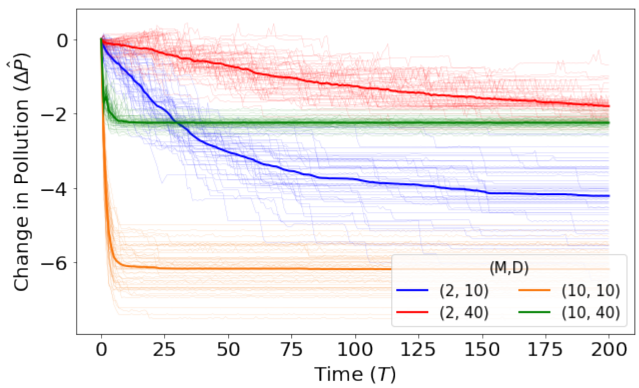

Figure 2: For four combinations of M and D, we plot the evolution of change in per-capita pollution $\hat{P}(t) - \hat{P}(0)$ for individual runs in faint lines and as an ensemble average in thicker lines. Agents here have fixed-strategies and only migrate.

Dependence on cleaning ϕ. In Figure 3 we see that the general trends in pollution due to number of defectors D and migration M (that pollution decreases as D decreases or as M increases) remains regardless of the cleaning ϕ. However, increasing ϕ does tend to accentuate these affects in absolute terms.

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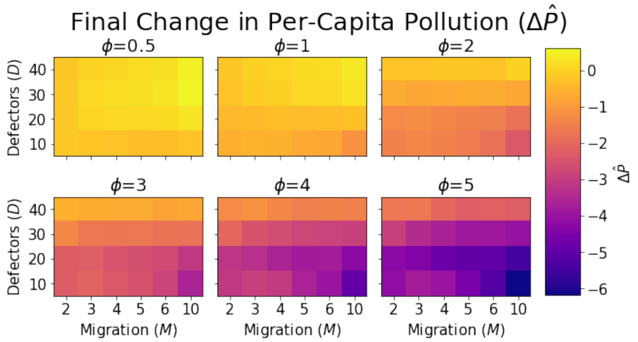


Figure 3: Final per-capita pollution (PCP) for the fixed-strategy experiment after $T = 201$ timesteps against for different values of migration M, initial number of defectors D and cleaning ϕ. Darker colours denote greater reduction in pollution.

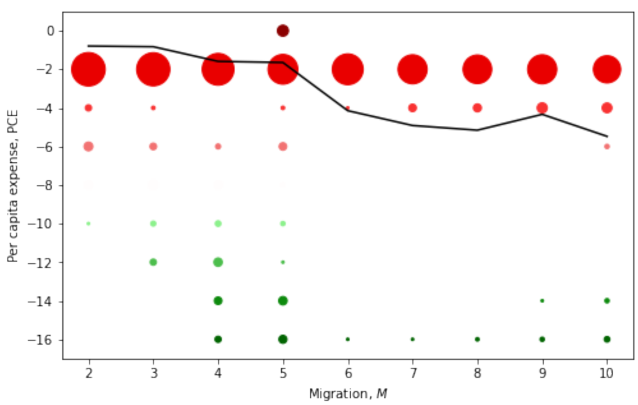


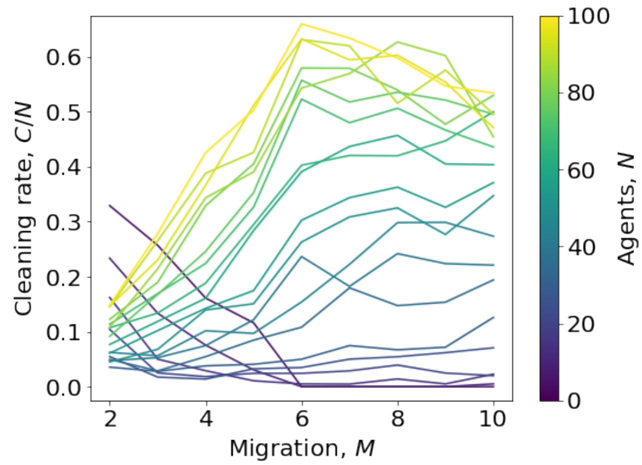
Figure 4: Final per-capita expense (PCE) after $T = 201$ timesteps against different values of migration M. The black line represents the ensemble mean while the vertical line of coloured circles represent the empirical distribution as a histogram over 300 runs. The colour and center of a circle denotes the location of the bin while size denotes the bin count.

1.3 Cooperation Stability

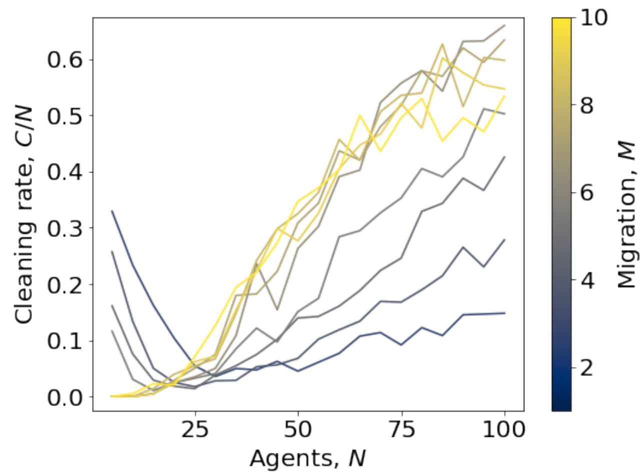
In Figure 4 we show the final per-capita expense (PCE), analogously to the cleaning rate in the main text. We see once again that migration leads to coexistence of very low expense and high expense when $M < R$ and that it leads to polarisation for $M \geq R$.

1.4 Density Dependence

In Figure 5 we see, effectively, the cross-sections of the heatmap in the main text. In other words keeping number of agents fixed (Figure 5a) we see how final cleaning rate changes with migration, or keeping migratory radius fixed (Figure 5b) how the rate changes with density.



(a) Migratory-dependence



(b) Density-dependence

Figure 5: Trends in final cleaning rate for a range of different migratory distances M (left figure) and different number of agents N - or in other words different urban density - (right figure). Each line represents a specific value of M or N with colour denoting values (low values are darker, high values lighter).

	Notation	Explanation
Game parameters	ϕ	*Cleaning.* The amount of pollution removed by a cooperator at each site within a radius 1 of themselves.
	f	*Cooperation cost.* The cost incurred by a cooperator.
	g	*Defection gain.* The gain for a defector.
City parameters	L	*City size.* The size of the doubly-periodic lattice city.
	N	*Population size.* The number of agents in a city. Sometimes also referred to as *density*
	$D = \delta N$	*Number of defectors.* Number of defectors in the city, typically meaning the initial number of defectors (as a fraction δ of the population size). When meaning the number of defectors at time t, D will be shorthand for $D(t)$.
	T	*Total timesteps.* The total number of timesteps during which simulations are ran.
Radii	R	*Cloud size.* Radius of a defector's pollution cloud.
	M	*Migratory radius.* The furthest distance within which agents consider empty sites for possible relocation.
	M_ν	*Neighbourhood radius.* The size of an agent's neighbourhood, from which the agent imitates the strategy of the best performing neighbour. For all simulations, $M_\nu = 1$ is fixed.
Densities	$N_* \equiv \frac{L^2}{\delta A}$	*Critical density.* The critical number of agents above which a city is considered *high-density* and below which is *low-density*.
	$N_*^{\pm}$	*Critical density bounds.* The bounds on the critical density due to being in discretised space.
	N_0	*Midpoint density.* The inferred midpoint of a sigmoid, fitted to dynamical data.
Site variables	$\boldsymbol{r} = (x, y)$	*Position.* Position of a site $\boldsymbol{r}$ in terms of x and y coordinates, where $\boldsymbol{r} \in [1, L]^2$.
	$P(\boldsymbol{r})$	*Pollution.* Pollution at site $\boldsymbol{r}$.
Agent variables	i	*Agent.* Label of agent i where $i \in \{1, \cdots, N\}$.
	$\boldsymbol{r}_i$	*Agent position.* Position of agent i.
	σ_i	*Strategy.* Strategy of agent i.
	E_i	*Expense.* Expense of agent i.
Global variables	$\hat{P}(t)$	*PCP.* Per-capita pollution at time t, i.e. mean of pollution over occupied sites at time t.
	$\hat{E}(t)$	*PCE.* Per-capita expense at time t, i.e. mean of expense over all agents at time t.

Table 1: Table of parameters and variables.