The Interactions of a Basketball

Joshua Berg, Spencer Ader

Department of Physics, High Point University, High Point, NC 27262

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Abstract

With each basketball player comes a unique shooting technique. Understanding how a basketball interacts with a rim and backboard can be beneficial to shooters who would like to enhance their foul shot percentages and rebounding. We studied how the coefficient of restitution affects the direction of the bounce of a 29.5-inch regulation basketball after a collision with a stationary rim and backboard. The program Tracker was used to analyze experimental videos for changes in velocity and angular momentum due to coefficient of restitution. From the experimental data we acquired values for the coefficient of restitution and kinetic and static friction, which showed how altering the angular frequency and collision point changes the interaction of the basketball with the rim and backboard. Results from the experiment were compared to a computational model of the interaction written in vPython. We used the velocity after contact to compare among the interactions. It was determined that there is a range of optimal shooting conditions that result in made baskets.

I. INTRODUCTION

Many studies have been conducted regarding the physics of a basketball. Of these studies, the main focus has been the optimum arc of a basketball shot that leads to the most successful foul shot percentage.[1] However, to our knowledge no studies have been conducted examining the interaction between a basketball and a rim. The collision point of the basketball with a rim and backboard plays a pivotal role in determining where the basketball will travel following contact. The angular frequency at which the basketball rotates is a major factor contributing to the post-interaction effects. Studies regarding the physics of the spin on a basketball discuss the value of backspin to a basketball shot. A diagram of which is shown in Figure 1, examines the effects of initial front spin, initial backspin, and no initial spin on the ball's interaction with a stationary object.[1]

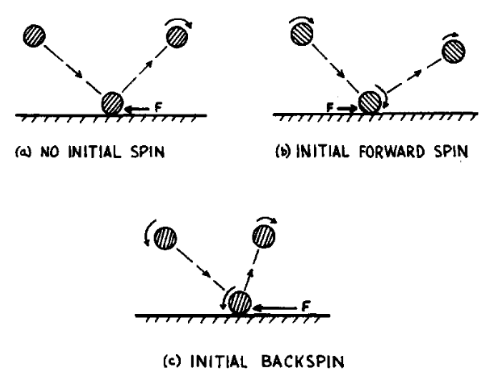


FIG. 1. The velocity of a ball after it collides with a rigid surface having different initial angular frequencies.[2]

Previous studies have suggested that there are different optimal shooting conditions. To achieve optimal shooting percentages, Okubo and Hubbard (2006) used computational evidence to recommend a shot with backspin of two revolutions per second and a floor angle of thirty degrees between the board surface and the horizontal projection of the line from the hoop center to the release point. Brancazio (1981) analyzed the optimum-shooting angle through mathematically modeling a shot that went directly through the hoop without contacting the rim, which requires a smaller launching force. Garwin (1968) discussed the kinetic energy behind a bouncing ball on parallel surfaces and how a ball with initial spin will have a lowered velocity after a collision with a parallel surface. Additional papers from Okubo (2002, 2003, 2004a, 2004b) discussed how variation in different situations–such as ball flight, ball and rim collision, and ball and backboard collision affected the percentages of shots that were successfully made. This paper presents computational evidence gathered using experimentally calculated constants (coefficient of restitution and friction forces) for the innumerable range of shooting conditions that result in made baskets.

Our study found the horizontal and vertical coefficients of restitution for a basketball's interaction with a rim and backboard. We used these experimental values to calculate kinetic and static friction for both the backboard and rim. These experimental values were used to computationally simulate how alterations in shot angle, angular momenta and initial velocity affect the likelihood of a made basket. We expected to see a trend that showed shots made under varying shot conditions and techniques. To find this trend we analyzed the angle between the initial and final velocity vectors using the center of mass as a determinant for the vertical and horizontal coefficients of restitution.[2]

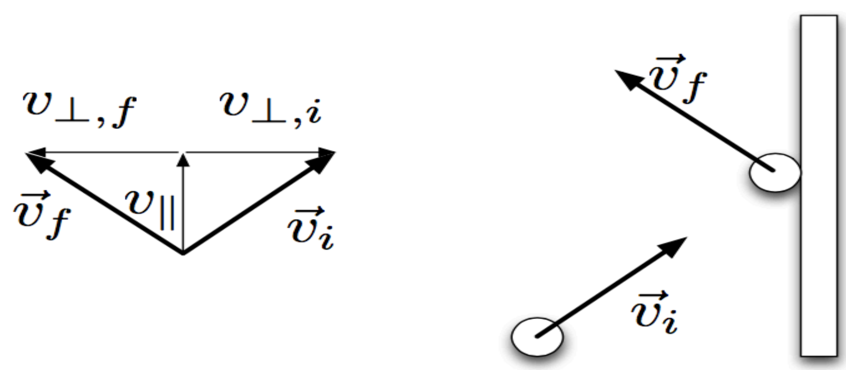


FIG. 2. The collision with the wall maintains the same y-component but the x-velocity of the object changes its direction. The perpendicular component of velocity following the interaction is equivalent to the coefficient of restitution.[2]

Additionally, the initial and final angular momentums of the basketball were needed to calculate the coefficients of restitution. The two velocities are results that provide information regarding the effects of the interaction between the rim and backboard with the ball.

Disparities in angular frequency, coefficient of restitution and kinetic and static friction for a particular shooter affects a basketball's interaction with a rim, which determines whether enters the basket due to the coefficient of restitution. We validated the calculations done by Okubo (2006) with a simpler model–created in vPython– and compared the results. We present results from our computational model and discuss the conditions that resulted in successful baskets (see Figure 4).

II. MATERIALS AND METHODS

We created a vPython model of a basketball being shot into a basketball hoop to find conditions where baskets were made and theoretical values for ending position of the basketball for the missed shots. Video of the interaction between the rim and backboard with a 0.75-meter Wilson Evolution indoor basketball was taken with a Casio EXFH100 camera at 300 frames per second. The camera was placed on a tripod so that the center of focus on the camera was even with the middle of the front piece of the rim. The camera was placed three meters from the center of the rim (see Figure 3).

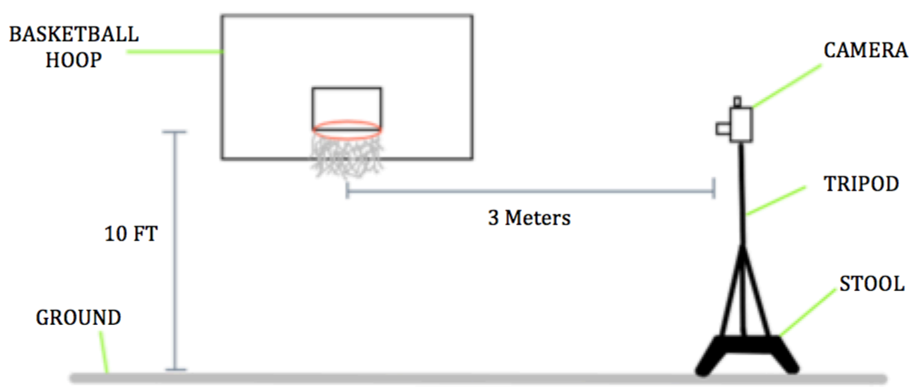


FIG. 3. A schematic of the experimental set up to use video analysis to calculate the coefficient of restitution and friction using center of mass, angular momentum, and initial and final velocity.

The videos from the experiment were analyzed using the program Tracker to determine the coefficient of restitution using angular velocity and initial velocities for each shot. Each shot was taken from the same position three meters from the front of the rim. Data from the videos were compared to the theoretical values in vPython. In total there were four different shot styles used to test for the coefficient of restitution. These shot types were "hard"- a fast velocity shot; "soft"- a shot with a low velocity and minimal spin; "lots of spin"- one with excessive amounts of spin and "normal"- a shot with an average velocity and normal spin. The initial and final velocities in both the x and y directions was found using Tracker. Additionally, the initial and final angular momenta for each trial were calculated with Tracker. To calculate the vertical coefficient of restitution direction we used the equation:

$$e_y = -\frac{V_{yf}}{V_{yi}}$$

where the subscripts y_i and y_f denote the value of the velocity in the vertical direction before and after the collision. Additionally, to solve for the horizontal coefficient of restitution we used the equation:

$$e_x = -\frac{V_{xf} - rw_f}{V_{xi} - rw_i}$$

where $V_x - rw$ is the net horizontal speed of a point at the bottom of the ball.

III. RESULTS AND DISCUSSION

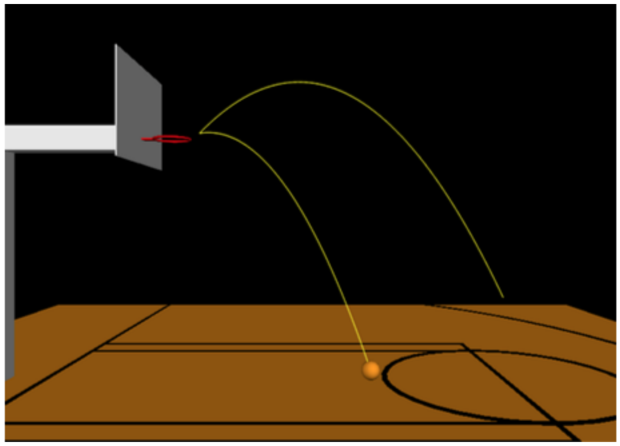


FIG. 4. A 3D rendering of a basketball court simulation made in vPython.

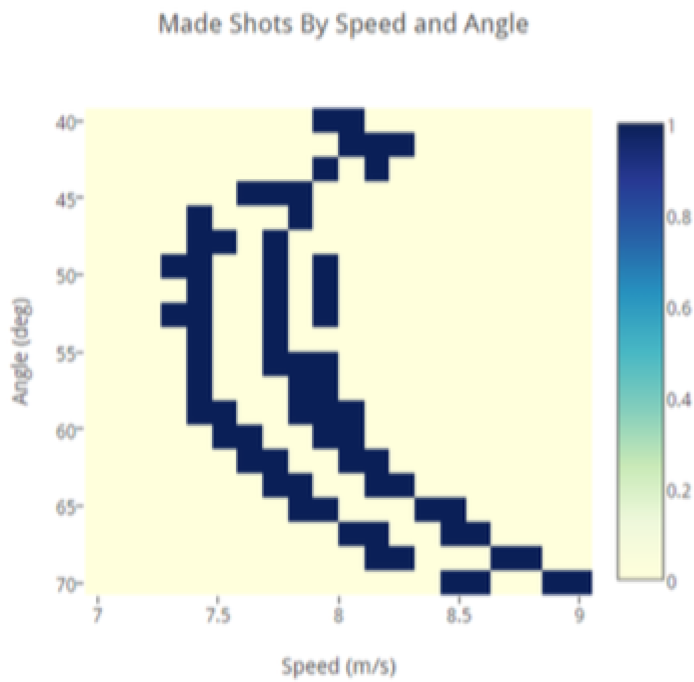
After solving for the horizontal and vertical coefficient of restitution constants, they were put into a computational vPython model. With these constants in the program we were then able to create a loop that ran through varying shot conditions and output whether or not the shot was made (see Figure 4). The horizontal and vertical coefficient of restitution for the rim was determined using video analysis with the program Tracker. We know that it is possible for the horizontal coefficient of restitution (e_x) to have either a negative or a positive value. From our results, the horizontal coefficient of restitution was negative because the basketball slid throughout the interaction with the rim instead of rolling due to friction and the coefficient of restitutions. The vertical coefficient of restitution is expected to be between the values zero and one. For each trial it can be seen that the values were in between zero and one; the higher coefficients of restitution represent little kinetic energy lost during the interaction while lower values represent lots of energy transfer (see TABLE 1).

TABLE I. Horizontal and Vertical Coefficient of Restitutions

Type of Shot	**Trial**	**Horizontal**	**Vertical**
	1	-5.12E-01	0.8396226415
	2	2.17E+00	0.8052325581
Normal	3	-4.60E+00	0.5894736842
	4	-1.55E+00	0.7532808399
	5	-4.22E+00	0.7663316583
	1	-2.00E+00	0.6449086162
	2	-1.02E+01	0.05322580645
Hard	3	-1.79E+00	0.09424242424
	4	-2.01E+00	0.4523809524
	5	-1.60E+00	0.7613882863
	1	-7.13E-01	0.6535211268
	2	-8.67E-01	0.8301282051
Lots of Spin	3	-7.02E-02	0.5186170213
	4	-1.02E+00	0.5511811024
	5	1.43E+00	0.7202216066
	1	-1.32E+00	0.7624113475
	2	-1.33E+00	0.8235294118
Soft	3	-1.54E+00	0.5055350554
	4	-1.16E+00	0.8344370861
	5	-2.84E-01	0.696369637
AVERAGES:		**-1.66E+00**	**0.6328019534**

One of the biggest concerns we had during our research was the effectiveness of having a single coefficient of restitution acting as a constant in our vPython program. When quantitatively analyzing sports balls there is a high dependency on the bounce properties of both the ball and the surface on which it interacts. This is why the coefficient of restitution is not completely considered to be a constant value. Its value is dependent on underlying variables such as velocity, shot angle and spin of the ball.

When looking at the differences in angular momentum between the trials, greater angular velocities before the interaction of the ball and rim resulted in greater differences between initial and final angular momentums. This is because after a collision with the rim, a basketball is slowed down due to the force of friction on the rim acting in the opposite direction of the balls rotation. Additionally, during the interaction there is a transfer of kinetic energy between the basketball and rim as some energy from the basketball's movement remains on the rim. This represents the inelasticity of collisions and is the coefficient of restitution. Shots that had very small angular velocities prior to the collision resulted in a complete change of angular rotation direction because the force of friction was much greater than that of rotation. With more initial spin of the basketball came an increase in the coefficient of friction mu (μ). In conclusion, with more initial rotation comes more opportunity for the transfer of kinetic energy. This transfer of kinetic energy results in a greater likelihood that the shot will be made because when a ball with lots of energy interacts with a rim, it transfer the energy to the rim, which reduces its movement following this interaction. increased chance kinetic energy transfer directly leads to larger coefficients of restitution as well as a greater change in the basketball's final rotation and position.



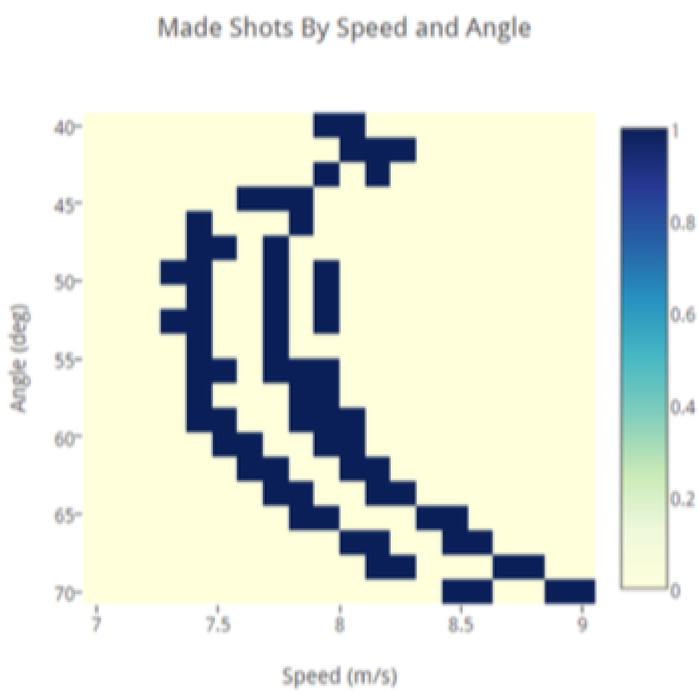


FIG. 5. A comparison of two graphs representing baskets made (dark blue) for shots with varying angular momenta.

In Figure 5, the graphs depict shots that traveled through the hoop with a blue mark. We used our vPython program to loop through a combination of shot angles, initial speeds and angular momenta. The shot angles range that we tested for was between forty and seventy degrees above the horizontal tangent. The initial speeds that were tested during the vPython loop ranged from seven to nine meters per second. The first graph is a loop with a consistent angular momentum of 9.99e-08 radians per second, while the second graph is a loop with a consistent angular momentum of -1.65 radians per second. Despite the significant difference in angular momenta for the two graphs, they appear to fit the same curve for conditions where the basketball went through the hoop. From this, concluded that there is a range of shot types that result in made baskets. This means that there are different optimal shooting conditions that result in made baskets.

We believe there may be a proportional relationship between shot angle, angular momenta and initial velocity from specified distances that can be manipulated to determine whether a shot will be made. We would like to possibly research this hypothesis with further research. Additionally, there is an opportunity to look further into the conditions that affect a shots likelihood of going into the hoop.

ACKNOWLEDGMENTS

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