JAN 17, 2018

SLIDE DECKS POSTED IN MYCOURSES (NOT EXACT)
CLASS VIDEOS POSTED " "
MATLAB

4-5 EXAMS — NO FINAL
↳ PROJECT DUE FINALS WEEK

WHAT IS A SIGNAL?
↳ CARRIES INFORMATION
↳ PATTERNS OR VARIATIONS THAT REPRESENT OR ENCODE INFORMATION
↳ DSP = INFORMATION PROCESSING

START BY MATHEMATICALLY REPRESENTING AS FUNCTIONS
EX: TUNING FORK
$$x(t) = e^{\sigma t} A \cos(2\pi(440)t + \phi)$$
↳ $A \cos(\omega t + \varphi)$

JAN 22, 2018

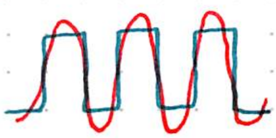
SIGNALS → REPRESENTED AS MATHEMATICAL FUNCTIONS
↳ EG. $A \cos(\omega t + \varphi)$

AMPLITUDE

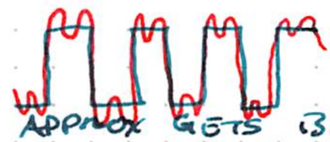
(ANGULAR) FREQ. [RAD/S]
$f = \omega/2\pi$ [Hz] $= \frac{1}{T}$

WHY USE COS FUNCTION?
↳ DEVELOP OUR TOOLS BASED ON STD. SINUSOIDS

EX: SQUARE WAVE DEFINED BY 1 COS
(NOT GREAT)



EX: SQ. WAVE DEFINED BY LIN. COMBINATION OF COS



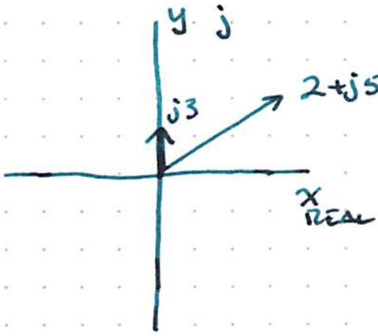
BY ADDING MORE COS, APPROX GETS BETTER
$$x(t) = 1/2 + \sum_{n=0}^{6} \frac{2}{(2n+1)\pi} \cos((2n+1)\omega_0 t - \pi/2)$$

MANY SIGNALS WILL REQUIRE BOTH SIN & COS, SO WE NEED TO USE THE COMPLEX PLANE
$$z(t) = X e^{j\omega t}$$

Jan 24, 2018

Complex Numbers

↳ Geometry: Rectangular Form



Addition: Add real & j separately

Multiplication: $z_1 z_2 = (ac-bd) + j(ad+bc)$

Trig Conversions

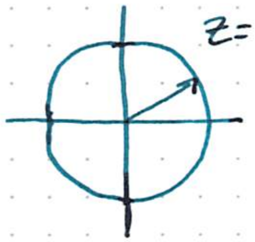
$$\cos\theta = \frac{x}{r}$$
$$\sin\theta = \frac{y}{r}$$
$$\theta = \tan^{-1}(y/x)$$

$x^2 + y^2 = r^2(\sin^2 + \cos^2) = r^2$

$$\boxed{r = \sqrt{x^2+y^2}}$$

↳ Geometry: Polar Form

$r =$ length; $\theta =$ angle from (+) x



$z = x + jy = r\cos\theta + jr\sin\theta$

Euler's Formula: $e^{j\theta} = \cos\theta + j\sin\theta$

$z = re^{j\theta} = r\cos\theta + jr\sin\theta$

Common Cases

$z = 1 = e^{j0} = e^{jk2\pi}$ — $k \equiv (+)$ int

$(r=1;\ \theta=0)$

$z = x + jx = x\sqrt{2}\, e^{j\pi/4}$

$z = -1 = e^{\pm j\pi}$

$(r=1;\ \theta = \pm\pi)$

$z = j = e^{j\frac{\pi}{2}} = e^{-j\frac{3\pi}{2}}$

Jan 26, 2018

$z_1 = r_1 e^{j\theta_1} = \overbrace{r_1\cos\theta_1}^{x} + j\overbrace{r\sin\theta_1}^{y}$; $z_2 = r_2 e^{j\theta_2} = r_2\cos\theta_2 + jr_2\sin\theta_2$

$r_1 = \sqrt{x^2+y^2}$

$\theta_1 = \tan^{-1} \cancel{(x/y)}\ (y/x)$

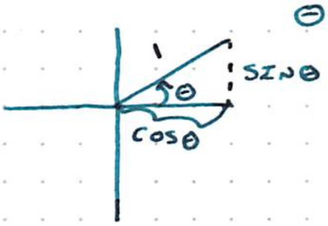
$z_1 + z_2 = r_1 e^{j\theta_1} + r_2 e^{j\theta_2} = (r_1\cos(\theta_1) + r_2\cos(\theta_2)) + j(r_1\sin(\theta_1) + r_2\sin(\theta_2))$

$z_1 z_2 = r_1 r_2 e^{j(\theta_1+\theta_2)}$

$z_1^* = r_1 e^{-j\theta}$

Complex Exponential: $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$

$\theta = \omega t$



Inverse Euler's Identities

$$\begin{aligned} & e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \\ + \quad & e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t) \\ \hline & e^{j\omega t} + e^{-j\omega t} = 2\cos(\omega t) \end{aligned}$$

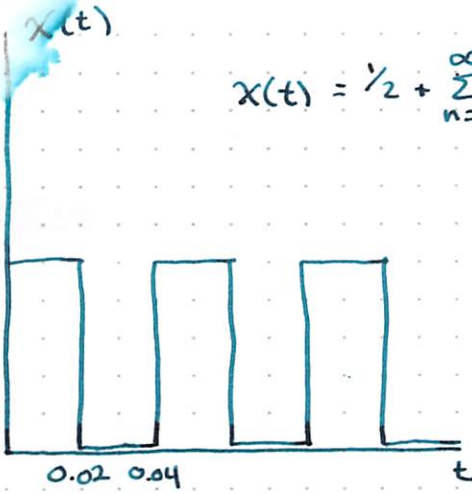
$\rightarrow \boxed{\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}}$

$$\begin{aligned} & e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \\ - \quad & e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t) \\ \hline & e^{j\omega t} - e^{-j\omega t} = 2j\sin(\omega t) \end{aligned}$$

$\rightarrow \boxed{\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}}$

$z(t) = Xe^{j\omega t}$

magnitude



$$x(t) = \frac{1}{2} + \sum_{n=0}^{\infty} \frac{2}{(2n+1)\pi} \cos\left((2n+1)\omega_0 t - \frac{\pi}{2}\right)$$

$\frac{2\pi}{T}$ "FUNDAMENTAL ANGULAR FREQUENCY"

$f_0 = \frac{1}{T_0}$ "FUNDAMENTAL FREQUENCY"

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_n \cos(2\pi k f_0 t + \varphi_k)$$

ANY PERIODIC SIGNAL CAN BE REPRESENTED AS A LINEAR COMBINATION OF HARMONICALLY RELATED SINUSOIDS

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jw kt}$$

FOURIER SERIES REPRESENTATION OF $x(t)$

$$X_k = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-jk\omega_0 t} \, dt$$

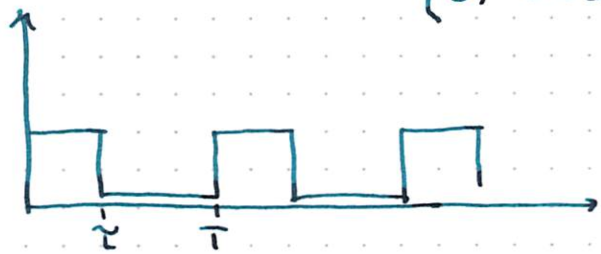
FOURIER SERIES

SYNTHESIS FORMULA

$$x(t) \xleftarrow{x(t) = \sum_{k=-\infty}^{\infty} x[k] e^{j\omega_0 kt}} \xrightarrow{x[k] = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-jk\omega_0 t}} X[k]$$

ANALYSIS FORMULA

EXAMPLE: $x(t) = \begin{cases} 1, & 0 < t \leq \tau \\ 0, & \tau < t \leq T \end{cases}$



$$x[k] = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} \, dt$$

$$= \frac{1}{T} \int_0^{\tau} e^{-j2\pi kt/T} \, dt$$

$$= \frac{1}{T} \left. \frac{e^{-j2\pi kt/T}}{-j2\pi k/T} \right|_0^{\tau}$$

$$\boxed{= \frac{1 - e^{-j2\pi k\tau/T}}{j2\pi k}} \quad k \neq 0$$

JAN 29, 2018

HW 2 - NEXT MONDAY
EXAM 1 - NEXT WED || FRIDAY

$$x(t) = x(t+T) \xleftarrow{x(t) = \sum_{k=-\infty}^{\infty} x[k] e^{jk\omega_0 t}} \xrightarrow{x[k] = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-jk\omega_0 t} dt} x[k]$$

FOURIER SERIES

$$x[k] = \frac{1}{T_0} \int_0^{\tau} x(t) e^{-j2\pi kt/T} \, dt$$

$$= \frac{1 - e^{-j2\pi k\tau/T}}{2j\pi k} \quad (k \neq 0)$$

EULER'S DEF. FOR SINE

$$= \frac{e^{-j\pi k\tau/T}}{\pi k} \frac{\left(e^{j\pi k\tau/T} - e^{-j\pi k\tau/T}\right)}{2j}$$

$\frac{\tau}{T} = \alpha$ (DUTY CYCLE)

$$= \frac{\tau e^{-j\pi k\tau/T}}{T} \frac{\sin(\pi k\tau/T)}{\pi k\tau/T}$$

$\text{SINC}(x) = \frac{\sin(x)}{x}$

$$\boxed{= \alpha e^{-j\pi k\tau/T} \, \text{SINC}(\pi k\tau/T)} \quad (k \neq 0)$$

FOR $K=0$

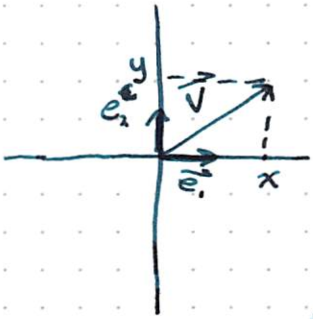
$$X[K] = X[0] = \frac{1}{T_0}\int_0^{\tau} x(t)\, e^{-j2\pi K t/T}\, dt$$

$$= \frac{1}{T_0}\int_0^{\tau} x(t)\, dt$$

$$= \frac{1}{T_0}\int_0^{\tau} dT = \tau/T = \alpha$$

$\therefore$ FOR ALL K $\quad x[K] = \alpha e^{-j\pi K \tau/T}\, \mathrm{SINC}(\pi K \tau/T)$

$$\boxed{x(t) = \sum_{K=-\infty}^{\infty} \alpha e^{-j\pi K \tau/T}\, \mathrm{SINC}(\pi K \tau/T)\, e^{j2\pi K t/T}}$$



- COORDINATE SYSTEM

$\downarrow$

BASIS

$\downarrow$

$$\left.\begin{array}{l}\vec{e}_1 = [1, 0] \\ \vec{e}_2 = [0, 1]\end{array}\right\} \vec{V} = x\vec{e}_1 + y\vec{e}_2$$

DOT PRODUCT: $\langle a, b\rangle = a_1 b_1 + a_2 b_2$

$\vec{a} = [a_1, a_2]$

$\vec{b} = [b_1, b_2]$

A COLLECTION OF SIGNALS MAKE A "SPACE". EACH SIGNAL IS A POINT IN THE SPACE.

↳ NEED A BASIS, A COORDINATE SYSTEM

↳ INNER PRODUCT FOR SIGNALS

$$\langle V(t), w(t)\rangle = \frac{1}{T_0}\int_{\langle T_0\rangle} V(t)\, w^*(t)\, dt$$

$\langle V(t), w(t)\rangle = 0 \rightarrow$ ORTHOGONAL

BASIS $\rightarrow$ FUNCTIONS $e^{jK\omega_0 t}$ $\quad K = 0, \pm1, \pm2 \ldots$

$x(t) \rightarrow$ COORDINATES ARE $\langle x(t), e^{jK\omega_0 t}\rangle$

$V(t) = e^{jK\omega_0 t}$; $w(t) = e^{jn\omega_0 t}$

$$\langle e^{jK\omega_0 t}, e^{jn\omega_0 t}\rangle = \frac{1}{T_0}\int_{\langle T_0\rangle} e^{jK\omega_0 t}\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{\langle T_0\rangle} e^{j(K-n)\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\, \frac{e^{j(K-n)\omega_0}}{j(K-n)\omega_0}\Bigg|_0^{T_0} \quad (K \neq n)$$

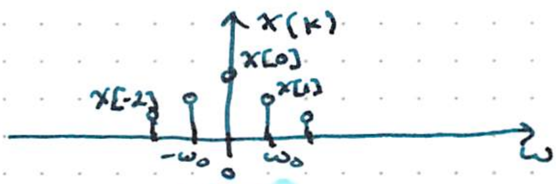
$$= \begin{cases} 1, & K = n \\ 0, & K \neq n \end{cases}$$

JAN 31, 2018

$$x(t) = \sum_{K=-\infty}^{\infty} x[K]\, e^{jK\omega_0 t} \; ; \quad x[K] = \frac{1}{T_0}\int_{\langle T_0\rangle} x(t)\, e^{-jK\omega_0 t}\, dt$$

SPECTRUM $\rightarrow x[K]$

↳ GRAPH OF $x[K]$ AS A FUNCTION OF f



~~K[0]~~ $K=0 \rightarrow x[0]$

$K=1 \rightarrow x[1] \leftrightarrow (\omega_0)$

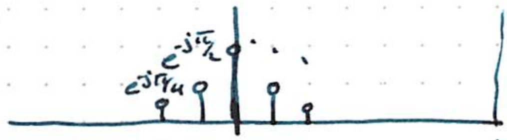
$K=-1 \rightarrow x[-1] \leftrightarrow (-\omega_0)$

$$x(t) = \begin{cases} 1, & 0 < t < \tau \\ 0, & \tau < t < T \end{cases}$$

$$X[K] = \alpha e^{-j\pi K\alpha}\, \mathrm{SINC}(\pi K \alpha)$$

2 SPECTRUM PLOTS
↳ AMPLITUDE
↳ $|X[k]| = |\alpha| \, |e^{-j\pi k\alpha}| \, |\text{sinc}(\pi k\alpha)| = \alpha |\text{sinc}(\pi k\alpha)|$
↳ PHASE
↳ $\angle(X[k]) = \angle(\alpha) + \angle(e^{-j\pi k\alpha}) + \angle(\text{sinc}(\pi k\alpha)) = -\pi k\alpha + 0/\pi$

COMPLEX SPECTRUM PLOT:



$x(t) = \cos(\omega_0 t)$

$X[k] = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-jk\omega_0 t} dt$

$= \frac{1}{T_0} \int_0^{T_0} \cos(\omega_0 t) e^{-jk\omega_0 t} dt$

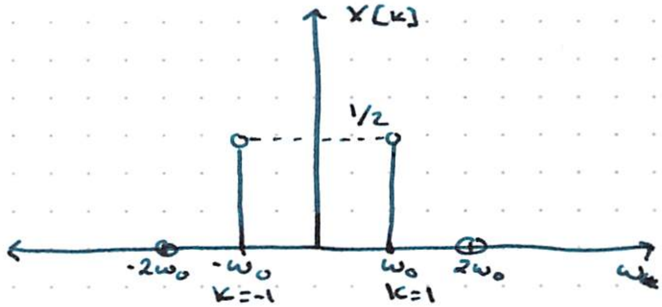
$= \frac{1}{T_0} \int_0^{T} \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) e^{-jk\omega_0 t} dt$

$= \frac{1}{T_0} \int_0^{T_0} \frac{e^{j\omega_0 t}}{2} e^{-jk\omega_0 t} dt + \frac{1}{2T_0} \int_0^{T_0} e^{-j\omega_0 t} e^{-jk\omega_0 t} dt$

$\frac{1}{T_0} \int_0^{T_0} e^{j\omega_0 t} e^{-jk\omega_0 t} dt = \begin{cases} 0 & k \neq 1 \\ 1 & k = 1 \end{cases}$

$\frac{1}{T_0} \int_0^{T_0} e^{-j\omega_0 t} e^{-jk\omega_0 t} = \begin{cases} 0 & k \neq -1 \\ 1 & k = -1 \end{cases}$

$X[k] = \begin{cases} 1/2 & k = 1 \text{ OR } k = -1 \\ 0 & k \neq \pm 1 \end{cases}$



$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}$

$x(t) = \cos(\omega_0 t) = \underset{X[-1]}{\underset{\uparrow}{1/2}} e^{j\omega_0 t} + \underset{X[1]}{\underset{\uparrow}{\tfrac{1}{2}}} e^{-j\omega_0 t}$

$x(t) = \sin(3t + \pi/4)$

$= \frac{e^{j(3t + \pi/4)} - e^{-j(3t + \pi/4)}}{2j}$

$= 1/2 \, \frac{1}{j} e^{j\pi/4} e^{j3t} - 1/2 \, \frac{1}{j} e^{-j\pi/4 - j3t}$

$\frac{1}{j} = -j$

$= \frac{1}{2} e^{-j\pi/2} e^{j3t} + e^{j\pi/2} e^{-j3t}$

FEB 02, 2018

$\boxed{\text{EXAM 1 NEXT FRIDAY.} \quad \text{↳ EVERYTHING UP TO AND INCLUDING TODAY}}$

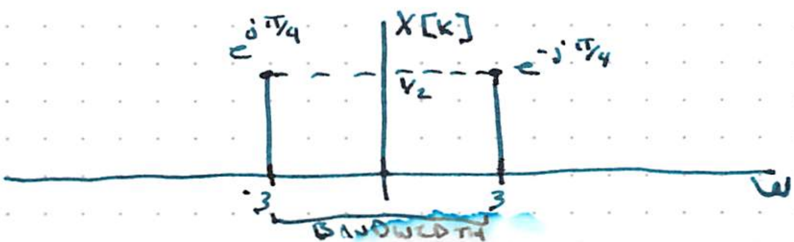
$x(t) = \sin(3t + \pi/4)$

$X[k] = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-j\omega_0 k t} dt \; ; \; x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}$

$x(t) = \sin(3t + \pi/4) = \frac{e^{j(3t + \pi/4)} - e^{-j(3t + \pi/4)}}{2j} = \underbrace{\underbrace{1/2 \; 1/j \; e^{j\pi/4}}_{X[1]}}_{\frac{1}{j} = -j = e^{-j\pi/2}} \underset{\omega_0}{e^{j3t}} - \underbrace{\underbrace{1/2 \; 1/j \; e^{-j\pi/4}}_{X[-1]}}_{-1/j = j = e^{j\pi/2}} \underset{-\omega_0}{e^{-j3t}}$

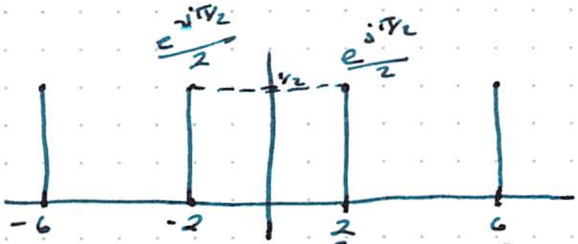
$= \frac{1}{2} e^{j(\pi/4 - \pi/2)} e^{j3t} + 1/2 e^{j(\pi/2 - \pi/4)} e^{-j3t}$

$= \frac{1}{2} e^{-j\pi/4} e^{j3t} + \frac{1}{2} e^{j\pi/4} e^{-j3t} \rightarrow X[k] = \begin{cases} \frac{e^{-j\pi/4}}{2}, & k = 1 \\ \frac{e^{j\pi/4}}{2}, & k = -1 \\ 0, & k \neq \pm 1 \end{cases}$



RECONSTRUCTING $x(t)$ FROM SPECTRUM

$$x_N(t) = \sum_{k=-N}^{N} x[k] e^{jk\omega_0 t}$$ → N^{TH} PARTIAL SUM



$x(t) = ?$

$$x(t) = \sum_{k=-\infty}^{\infty} x[k] e^{jk\omega_0 t}$$

$k=1$, $\omega_0 = 2$

$\omega = 3 \times \omega_0$, $k=3$

$$x(t) = \underbrace{\underbrace{\frac{e^{j\pi/2}}{2} e^{j2t}}_{k=1} + \underbrace{\frac{e^{-j\pi/2}}{2} e^{-j2t}}_{k=-1}}_{\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}} + \underbrace{\underbrace{\frac{1}{2} e^{j3\cdot 2t}}_{k=3} + \underbrace{\frac{1}{2} e^{-j3\cdot 2t}}_{k=-3}}_{\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}}$$

$$x(t) = \cos(2t + \pi/2) + \cos(6t)$$

FOURIER TRANSFORMATIONS
↳ WHAT IS THE SPECTRUM OF A NON-PERIODIC SQUARE PULSE?
↳ IDEA: START W/ PERIODIC & STRETCH $T \to \infty$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \; ; \quad X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

FEB 05, 2018
- STUDY FOURIER SYNTHESIS (RECONSTRUCTING $x(t)$ FROM SPECTRUM)

TO GO FROM PERIODIC ($x(t) = x(t+T)$) TO NON-PERIODIC, THE IDEA IS TO THINK OF A "PERIODIC" SIGNAL W/ $T \to \infty$

	PERIODIC $x(t)=x(t+T)$ ‖ FOURIER SERIES	NON-PERIODIC ‖ FOURIER TRANSFORM
SYNTHESIS	$x(t) = \sum_{k=-\infty}^{\infty} x[k] e^{jk2\pi t/T}$	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$
ANALYSIS	$x[k] = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk2\pi t/T} dt$	$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

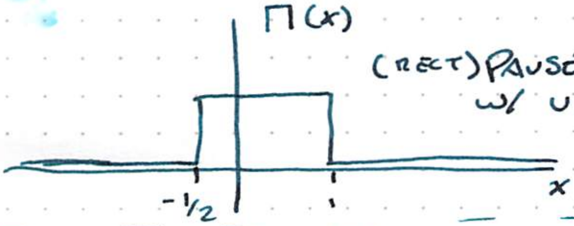
$X(j\omega)$ ← FOURIER TRANSFORM OF $x(t)$

$$X(j\omega) \leftarrow \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$x(t)$ ← WE GET FROM INVERSE FOURIER TRANSFORM OF $X(j\omega)$

$$x(t) = \mathcal{F}^{-1}\{X(j\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

DETOUR → SOME SPECIAL FUNCTIONS

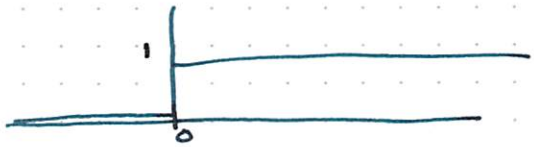


(RECT) PULSE SIGNAL w/ UNIT AREA

$$\Pi(x) = \begin{cases} 1, & -\tfrac{1}{2} \le x < 1 \\ 0, & \text{ELSE} \end{cases}$$

UNIT STEP (HEAVISIDE FUNCTION)

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases} \quad (\text{NOT DEFINED @ } t=0)$$



$$u(t-t_0) = \begin{cases} 1, & t > t_0 \\ 0, & t < t_0 \end{cases}$$

IMPULSE (DELTA, DIRAC'S DELTA) FUNCTION

SIFTING PROPERTY → $\int \delta(t-t_0) f(t)\,dt = f(t_0)$

$f(t)$ IS A SIGNAL CONTINUOUS @ t_0

$$\delta(t) = \frac{du(t)}{dt} \qquad u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

FOURIER TRANSFORM ~~EXPONENTIAL~~ EXAMPLES

→ DECAYING EXPONENTIAL → $h(t) = e^{-at} u(t)$, $a > 0$

$$H(j\omega) = \mathcal{F}\{h(t)\} = \mathcal{F}\{e^{-at} u(t)\}$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\,dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t}\,dt = \int_{0}^{\infty} e^{-(a+j\omega)t}\,dt$$

$$= -\frac{e^{-(a+j\omega)t}}{a+j\omega}\Bigg|_{0}^{\infty} = \frac{1}{a+j\omega} = H(j\omega)$$

$$\lim_{t\to\infty} \frac{e^{-at} e^{-j\omega t}}{a+j\omega} \to 0$$

$$|H(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\phi(H(j\omega)) = \phi\left(\frac{1}{a+j\omega}\right) = \phi(1)^{\to 0} - \phi(a+j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

Feb 12, 2018 HW 3 - Due next Monday

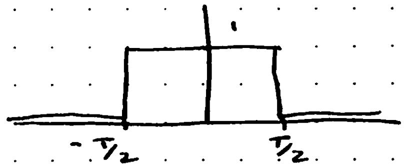
$$X(j\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \mathcal{F}^{-1}\{X(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Example from last class: $x(t) = e^{-at}\mu(t)$

$X(j\omega) = \frac{1}{a+j\omega}$

Ex 2: Pulse of duration T $\rightarrow$ $x(t) = \sqcap\left(\frac{t}{T}\right)$

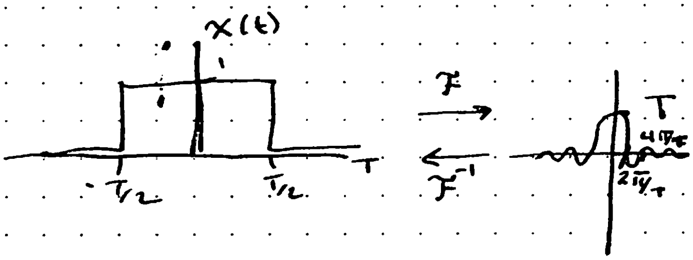


$$X(j\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

$$= \int_{-T/2}^{T/2} (1)\, e^{-j\omega t}\, dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-T/2}^{T/2}$$

$$= \frac{e^{-j\omega T/2}}{-j\omega} - \frac{e^{j\omega T/2}}{-j\omega}$$

$$= \frac{2\left(e^{j\omega T/2} - e^{-j\omega T/2}\right)}{2\,(j\omega)}$$



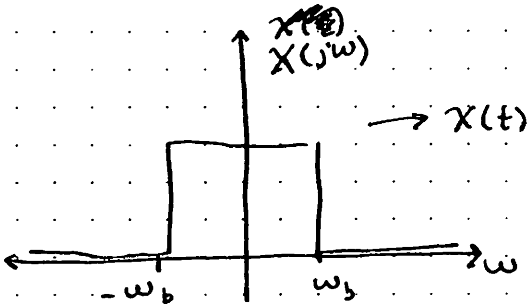
$$= \frac{2\sin(\omega T/2)}{\omega}$$

$$= T\,\frac{\sin(\omega T/2)}{T\,\omega/2}$$

Example 3: Ideal Low Pass Filter (LPF)

$$= T\,\mathrm{sinc}(\omega T/2) = \mathcal{F}\{x(t)\}$$

Given $X(j\omega)$ $\rightarrow$ what is $x(t)$?



$$x(t) = \mathcal{F}^{-1}\{X(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

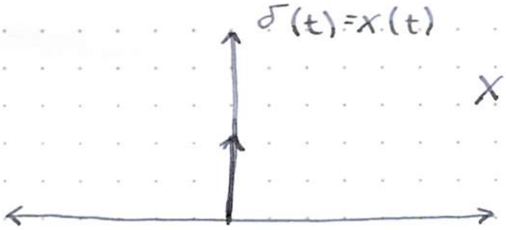
$$= \frac{1}{2\pi}\int_{-\omega_b}^{\omega_b} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\left.\frac{e^{j\omega t}}{jt}\right|_{-\omega_b}^{\omega_b}$$

$$= \frac{1}{2\pi}\,\frac{e^{j\omega_b t} - e^{-j\omega_b t}}{jt}$$

$$= \frac{\omega_b}{\pi}\,\mathrm{sinc}(\omega_b t) = x(t)$$

EXAMPLE: IMPULSE @ ORIGIN



$X(j\omega) = ?$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$$

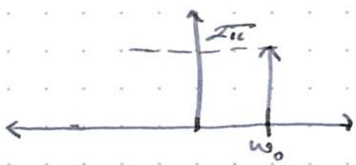
$t_0 = 0;\ f(t) = e^{-j\omega t}$

SIFTING PROPERTY $\int_{-\infty}^{\infty} \delta(t-t_0) f(t) dt = f(t_0)$

$$X(j\omega) = e^{-j\omega t}\Big|_{t=0} = 1$$

Ex: $X(j\omega) = 2\pi\,\delta(\omega - \omega_0)$

$x(t)$?
↳ SIFTING



FOURIER TRANSFORM PROPERTIES

PROPERTY 1: LINEARITY

$x(t) \overset{\mathcal{F}}{\leftrightarrow} X(j\omega) \qquad y(t) \overset{\mathcal{F}}{\leftrightarrow} Y(j\omega)$

$a\,x(t) + b\,y(t) \overset{\mathcal{F}}{\longleftrightarrow} a X(j\omega) + b\,Y(j\omega)$

(a & b ARE NOT DEPENDANT ON t)

$$\mathcal{F}\{a x(t) + b y(t)\} = \mathcal{F}\{a x(t)\} + \mathcal{F}\{b y(t)\} = a\,\mathcal{F}\{x(t)\} + b\,\mathcal{F}\{y(t)\}$$

$$5\,\frac{\omega_b}{\pi}\,\mathrm{sinc}(\omega_b t) + 10\,\frac{\omega_2}{\pi}\,\mathrm{sinc}(\omega_2 t) = z(t)$$

$$\frac{\omega_b}{\pi}\,\mathrm{sinc}(\omega_b t) \overset{\mathcal{F}}{\longleftrightarrow} \Pi\left(\frac{\omega}{2\omega_b}\right)$$

$$Z(j\omega) = a\,\Pi\left(\frac{\omega}{2\omega_b}\right) + b\,\Pi\left(\frac{\omega}{2\omega_2}\right)$$

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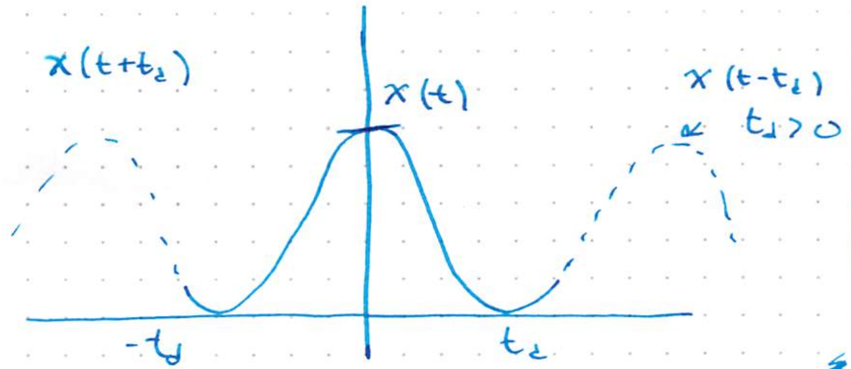
PROPERTY 2: $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega)$

$$\boxed{x(t-t_d) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_d} X(j\omega)}$$

$$\mathcal{F}\{x(t-t_d)\} = \int_{-\infty}^{\infty} x(t-t_d) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(\tau) e^{-j\omega(\tau + t_d)} d\tau$$

$\tau = t - t_d$

$$= e^{-j\omega t_d} \int_{-\infty}^{\infty} x(\tau) e^{-j\omega t} d\tau = e^{-j\omega t_d} X(j\omega)$$



$$x(t-t_d) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_d} X(j\omega)$$

$$\left| e^{-j\omega t_d} X(j\omega) \right| = \underbrace{\left| e^{-j\omega t_d} \right|}_{=1} \underbrace{\left| X(j\omega) \right|}_{\text{SAME AS BEFORE}}$$

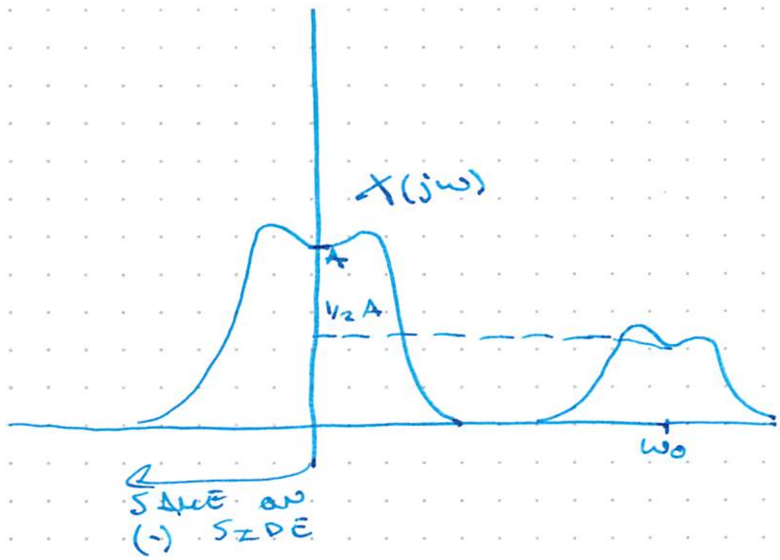
$$\angle \left(e^{-j\omega t_d} X(j\omega) \right) = \underbrace{\angle \left(e^{-j\omega t_d} \right)}_{-t_d \omega} \underbrace{\angle \left(X(j\omega) \right)}_{\text{SAME AS BEFORE}}$$

PROPERTY 3: FREQUENCY SHIFT

$$e^{j\omega_0 t} x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j(\omega - \omega_0))$$

MODULATION

$$x(t) \cos(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \tfrac{1}{2} X(j(\omega + \omega_0)) + \tfrac{1}{2} X(j(\omega - \omega_0))$$



$$\mathcal{F}\{x(t)\cos(\omega_0 t)\} = \mathcal{F}\left\{ \frac{x(t) e^{j\omega_0 t}}{2} + \frac{x(t) e^{-j\omega_0 t}}{2} \right\}$$

$$= \tfrac{1}{2} \mathcal{F}\{x(t) e^{j\omega_0 t}\} + \tfrac{1}{2} \mathcal{F}\{x(t) e^{-j\omega_0 t}\}$$

$$= 1/2\, X(j(\omega - \omega_0)) + 1/2\, X(j(\omega + \omega_0))$$

PROPERTY 4: CONVOLUTION

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) \, d\tau \quad \leftarrow \text{IN TIME}$$

$$X(j\omega) * Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\vartheta) Y(j\omega - j\vartheta) \, d\vartheta \quad \leftarrow \text{IN FREQ}$$

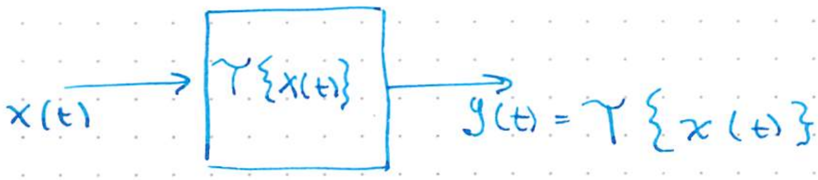
$$x(t) * y(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega) Y(j\omega)$$

$$x(t) y(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega) * Y(j\omega)$$

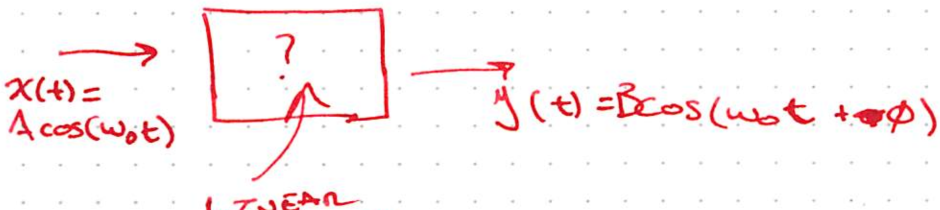
SYSTEMS

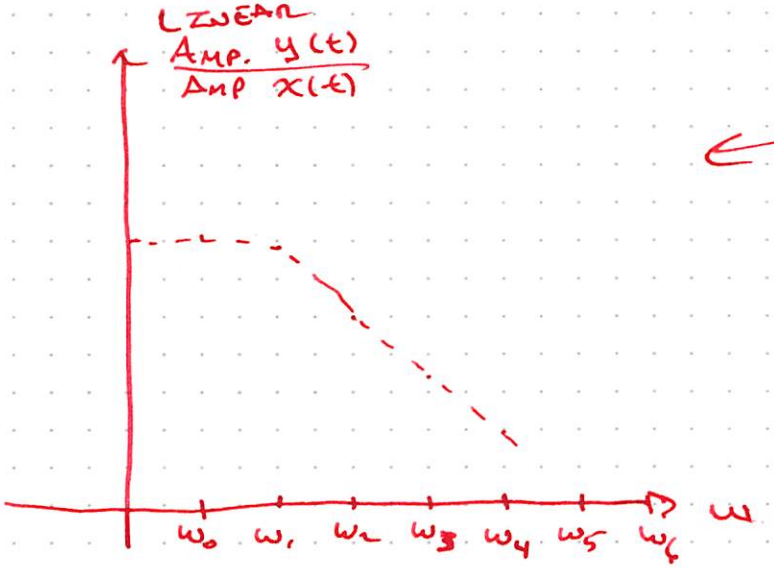
SYSTEM: A DEVICE WHICH CAN MANIPULATE, CHANGE, RECORD, OR TRANSMIT SIGNALS

↳ MATHEMATICALLY, A SYSTEM IS REPRESENTED AS A TRANSFORMATION



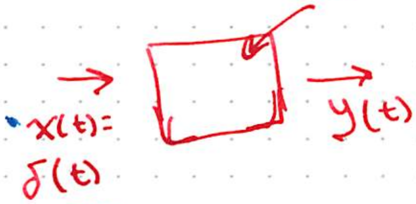
FEB 16, 2018 - FROM RECORDING

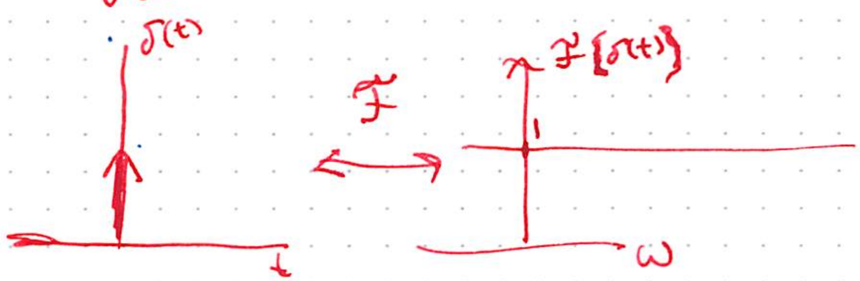




SAME FOR PHASES: $\angle y(t) - \angle x(t)$

LINEAR SYSTEM → SUPERPOSITION THEOREM





IF INPUT IS $x(t)$ & OUTPUT IS $y(t) = T\{x(t)\}$

IF INPUT IS $x(t) = \sum_i \alpha_i x_i(t)$

↳ OUTPUT IS $y(t) = T\{x(t)\}$

$= T\{\sum_i \alpha_i x_i(t)\}$

$= \sum_i \alpha_i T\{x_i(t)\}$

$= y(t) = \sum_i \alpha_i y_i(t)$

$$x(t) = A\cos\omega_0 t + A\cos\omega_1 t \longrightarrow \boxed{} \longrightarrow y(t)$$

$$x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$y(t) = x(t) * h(t) \xleftrightarrow{\mathcal{F}} Y(j\omega) = X(j\omega)H(j\omega)$$

IF GIVEN BLACK BOX, HOW DO WE FIND $h(t)$?
↳ HAVE $x(t) = \delta(t)$

GET OUTPUT $y(t) = x(t) * h(t)$

FIRST GET $Y(j\omega) = X(j\omega)H(j\omega)$

$X(j\omega) = \mathcal{F}\{\delta(t)\} = 1$; $Y(j\omega) = H(j\omega)$

$h(t) = \mathcal{F}^{-1}\{H(j\omega)\}$

THE OUTPUT (RESPONSE) OF THE SYSTEM WHEN THE INPUT IS AN IMPULSE
↳ $h(t)$ = "IMPULSE RESPONSE"
$H(j\omega)$ = "FREQ. RESPONSE"

GIVEN $h(t)$ & $x(t)$ → WHAT IS THE OUTPUT?

$$\underset{H(j\omega)}{\overset{h(t)}{\downarrow \mathcal{F}}} \cdot \underset{X(j\omega)}{\overset{x(t)}{\downarrow \mathcal{F}}} = Y(j\omega)$$

$$y(t) = \mathcal{F}^{-1}\{Y(j\omega)\}$$

GENERALIZED FOURIER TRANSFORM:

$$e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} 2\pi\delta(\omega-\omega_0)$$

PERIODIC FUNCTION $x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}$

$$\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}\right\} = \sum_{k=-\infty}^{\infty} X[k]\, \mathcal{F}\{e^{jk\omega_0 t}\}$$

WRITE AS FOURIER

LINEARITY

$$\mathcal{F}\{x(t)\} = 2\pi \sum_{k=-\infty}^{\infty} X[k]\,\delta(\omega - k\omega_0)$$

$$\mathcal{F}\{\cos(\omega_0 t)\} = \mathcal{F}\left\{\frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2}\right\} = \frac{1}{2}\mathcal{F}\{e^{j\omega_0 t}\} + \frac{1}{2}\mathcal{F}\{e^{-j\omega_0 t}\}$$

$$= \frac{\cancel{2}\pi}{\cancel{2}}\delta(\omega-\omega_0) + \frac{\cancel{2}\pi}{\cancel{2}}\delta(\omega+\omega_0)$$

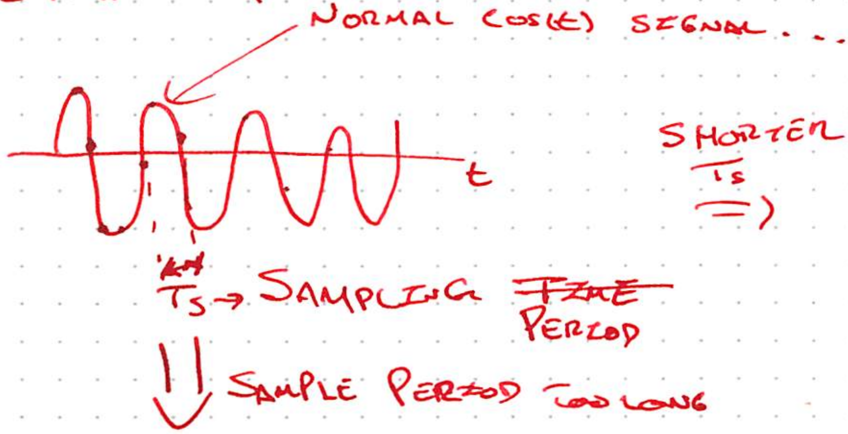
GENERALIZED FOURIER TRANSFORM:
↳ FOURIER TRANSFORM FOR PERIODIC SIGNALS

$$x(t) = \sum_{-\infty}^{\infty} X[k] e^{-j\omega_0 t}$$

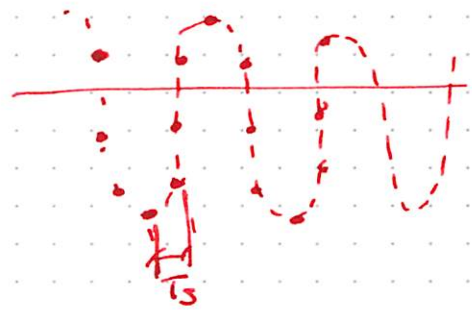
$$\mathcal{F}\{x(t)\} = \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} X[k] e^{-j\omega_0 t}\right\} = \sum_{k=-\infty}^{\infty} X[k]\, \mathcal{F}\{e^{-jk\omega_0 t}\}$$

$$= 2\pi \sum_{k=-\infty}^{\infty} X[k]\, \delta(\omega - k\omega_0)$$

SAMPLING:

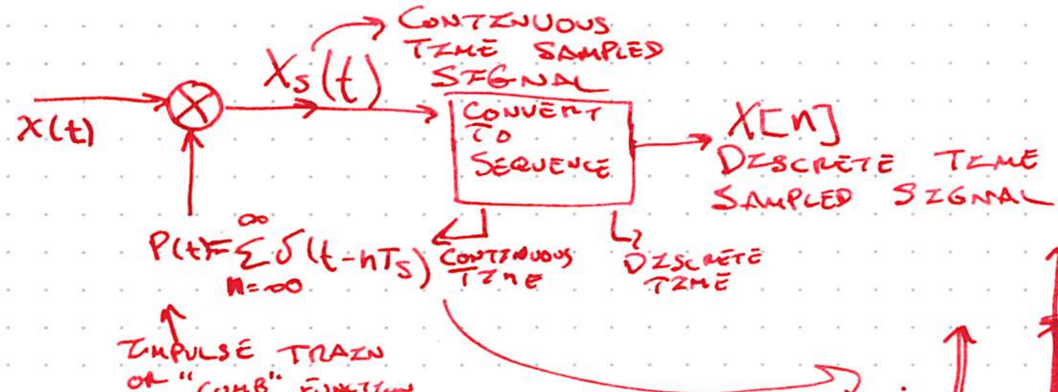


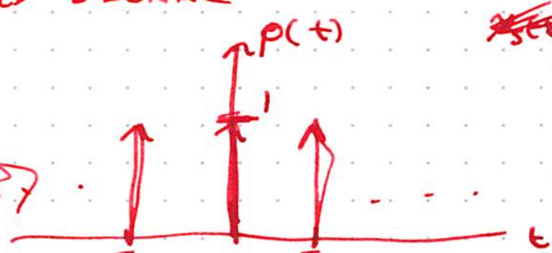
SHORTER T_S ⟹



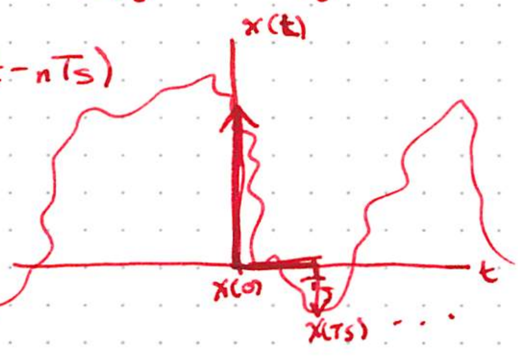
... LIKE IDE

MODEL OF IDEAL SAMPLER





$$x_s(t) = x(t)p(t) = \sum_{n=-\infty} x(t)\delta(t - nT_s)$$



$x[n] \rightarrow$ SEQUENCE OF SAMPLES
$n \rightarrow$ INDEX (DISCRETE TIME) $\rightarrow$ INDICATES ~~RELATIONSHIP~~ RELATIVE ORDER OF SAMPLES

$x[0]$ COMES B4 $x[1]$ ... $x[2]$...

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} \underset{\uparrow\, x(nT_s)}{x[n]}\delta(t-nT_s)$$

WE NEED $\mathcal{F}\{x_s(t)\} = \mathcal{F}\{\sum_{n=-\infty} x[n]\underbrace{\delta(t-nT_s)}_{f(t)}\}$

$$P(j\omega) = \mathcal{F}\{p(t)\} = \mathcal{F}\{\sum_{k=-\infty}^{\infty} p[k]e^{jk\omega_s t}\}$$

$$\omega_s = 2\pi f_s = \frac{2\pi}{T_s}$$

$p[k] = ?$

$$= \frac{1}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \delta(t)e^{-jk\omega_s t}dt = \frac{1}{T_s}\int_{-\infty}^{\infty} \delta(t)e^{-jk\omega_s t}dt = \frac{1}{T_s}e^{-jk\omega_s t}\Big|_{t=0}$$

$$p[k] = \frac{1}{T_s} \rightarrow p(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s}e^{jk\omega_s t}$$

$$P(j\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s}\mathcal{F}\{e^{jk\omega_s t}\} = \sum_{k=-\infty}^{\infty}\frac{1}{T_s}\delta(\omega-k\omega_s)$$

$$\mathcal{F}\{x_s(t)\} = \mathcal{F}\{x(t)p(t)\} = \mathcal{F}\{\sum_{k=-\infty}^{\infty}\frac{x(t)}{T_s}e^{jk\omega_s t}\} \underset{\text{LINEARITY}}{=} \frac{1}{T_s}\sum_{k=-\infty}^{\infty}\mathcal{F}\{\underbrace{x(t)e^{jk\omega_s t}}_{X(j(\omega-k\omega_s))}\}$$

$$= \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega-k\omega_s))$$

~~FEB~~ FEB 21, 2018 - EXAM 2: FRIDAY, MARCH 2

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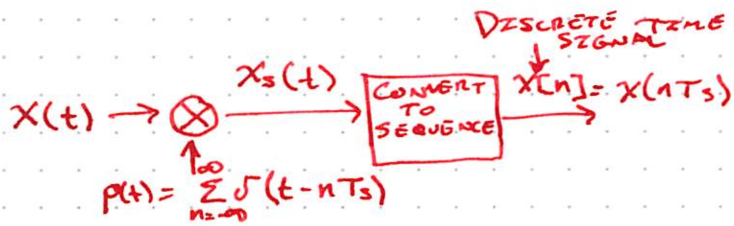


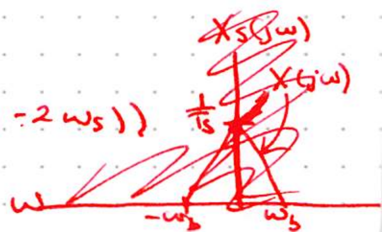
$$x_s(t) = x(t)p(t) = \sum_{n=-\infty}^{\infty}x(t)\delta(t-nT_s)$$

$$X_s(j\omega) = \mathcal{F}\{x_s(t)\} = \mathcal{F}\{\frac{1}{T_s}\sum_{k=-\infty}^{\infty} x(t) e^{jk\omega_s t}\} \qquad \omega_s = \frac{2\pi}{T_s}$$

$$p(t) = \sum_{-\infty}^{\infty}\delta(t-nT_s) \underset{\text{F.S}}{=} \frac{1}{T_s}\sum_{k=-\infty}^{\infty}e^{jk\omega_s t}$$

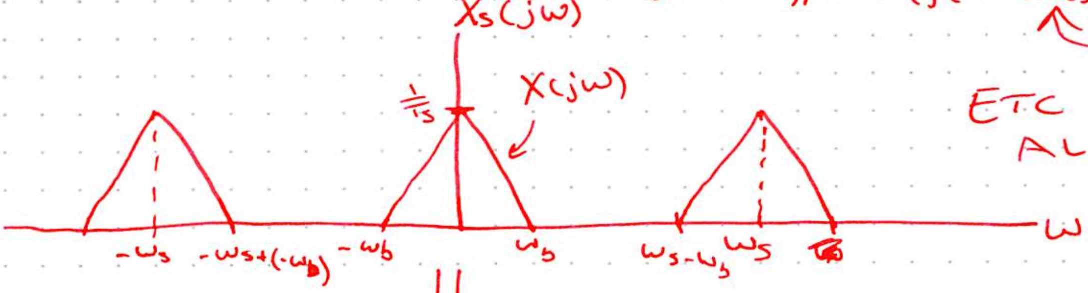
$$X_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty}\mathcal{F}\{x(t)e^{jk\omega_s t}\} = \frac{1}{T_s}\sum_{k=-\infty}^{\infty}X(j(\omega-k\omega_s))$$

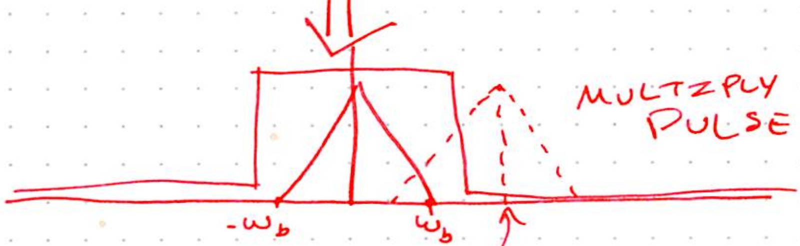
$$X_s(j\omega) = \frac{1}{T_s}\left(X(j\omega) + X(j(\omega-\omega_s)) + X(j(\omega+\omega_s)) + X(j(\omega-2\omega_s))\right)$$



$$X_s(j\omega) = \frac{1}{T_s}\left(X(j\omega) + X(j(\omega-\omega_s)) + X(j(\omega+\omega_s)) + X(j(\omega-2\omega_s))\ldots.\right)$$

ETC FOR ALL





MULTIPLY BY PULSE (LOW-PASS FILTER)

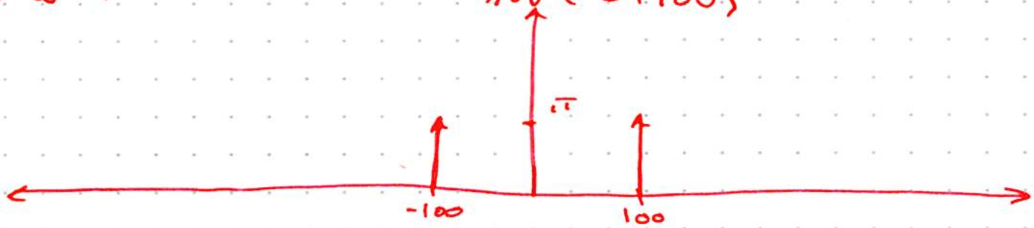
↳ IF WE MAKE ω_s TO SMALL, THE SPECTRUMS CAN OVERLAP

$\boxed{\omega_s > 2\omega_b}$ (SHANNON)

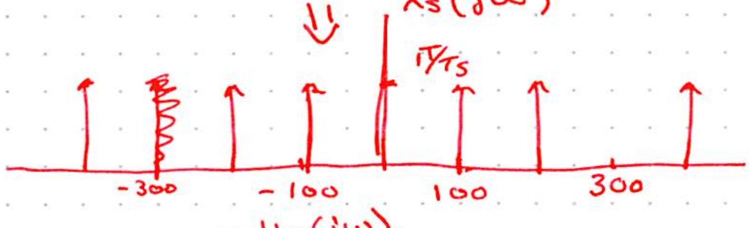
↳ SAMPLING / NYQUIST THEOREM

$x(t) = \cos(100t)$

$X(j\omega) = \pi\delta(\omega-100) + \pi\delta(\omega+100)$



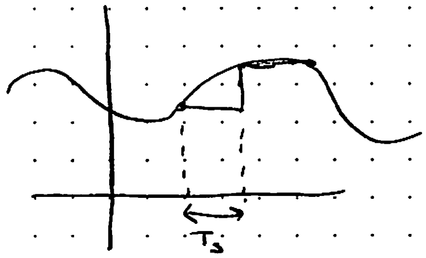
SAMPLE $x(t)$ AT $\omega_s = 300$ ← $>2\omega_b = >2(100)$



~~FRIDAY 23~~ FEB 23, 2018
↳ EXAM - ESSENTIALLY HW #3
↳ FOURIER & FILTERS

SAMPLE & HOLD:

SAMPLING $f > 2 f_{m}$



FEB 26, 2018

↳ EXAM FRIDAY - FOURIER TRANSFORM, FOURIER SERIES, ~~FILTER~~

SEQUENCE OF SAMPLES

x[n]

↑

INDEX

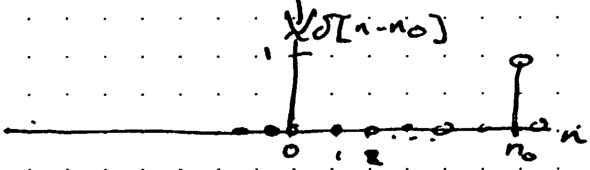
(SORT VALUES IN TIME)

$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n\neq 0 \end{cases}$

UNIT IMPULSE

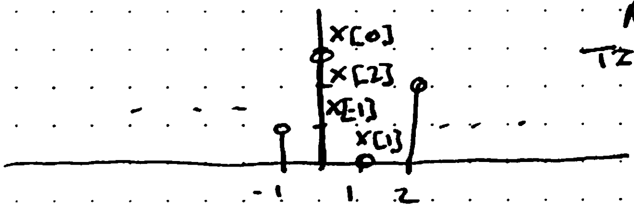
KRONECKER DELTA

$\delta[n-n_0] = \begin{cases} 1, & n=n_0 \\ 0, & n\neq n_0 \end{cases}$



$x[n] = \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots$

↑ TIME-SHIFTED IMPULSES



FILTERS

→ $\ldots\ x[-2],\ x[-1],\ \boxed{x[0],\ x[1],\ x[2]},\ x[3]\ \ldots$ ← n=1

↑ n=0

$y[0] = \tfrac{1}{3}x[0] + \tfrac{1}{3}x[1] + \tfrac{1}{3}x[2]$

$y[1] = \tfrac{1}{3}x[1] + \tfrac{1}{3}x[2] + \tfrac{1}{3}x[3]$

GIVEN x[n]

↳ FILTER IS 3-POINT MOVING AVG.

↳ FIND OUTPUT OF FILTER

[FIND OUTPUT @ EACH POINT & PLOT SPECTRUM]

↑

NON-CAUSAL

B/C ~~~~ y[n] RELIES ON $y[n],\ \underbrace{y[n+1],\ y[n+2]}_{\text{FUTURE EVENTS}}$

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$y[n] = \overset{b_0}{\tfrac{1}{3}}x[n] + \overset{b_1}{\tfrac{1}{3}}x[n-1] + \overset{b_2}{\tfrac{1}{3}}x[n-2]$

↳ $y[n] = \sum_{k=0}^{M} b_k x[n-k]$ ← CAUSAL, FINITE IMPULSE RESPONSE (FIR) FILTER

↑ COEFF

$M = 2$

$b_k = \{\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\}$

FEB 28, 2018

FIR FILTER
↳ $y[n] = \sum_{k=0}^{M} b_k x[n-k]$ ← DIFF. EQ. FOR A CAUSAL FIR FILTER

$y[n] = \frac{1}{4} x[n] + \frac{1}{4} x[n-1] + \frac{1}{4} x[n-2] + \frac{1}{4} x[n-3]$

TO GET IMPULSE WE SET $x[n] = \delta[n]$

$y[n] = h[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$

$y[n] = \{ \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0 \ldots \}$

$y[n] = \{y[0], y[1], y[2], y[3] \ldots\} = y[0]\delta[n] + y[1]\delta[n-1] + y[2]\delta[n-$

FOR A GENERAL ORDER-3 FIR FILTER

$y[n] = \sum_{k=0}^{3} b_k x[n-k] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-3]$

$h[n] = y[n]$ FOR $x[n] = \delta[n]$

$y[n] = b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2] + b_3\delta[n-3] = h[n]$

WHY IS $h[n]$ IMPORTANT?
↳ FOR A LTI (LINEAR TIME-INVARIANT) FILTER THE OUTPUT IS $y[n] = h[n] * x[n]$
↳ CONVOLUTION OF IMPULSE RESPONSE & INPUT

$y[n] = h[n] * x[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$

$\Updownarrow$

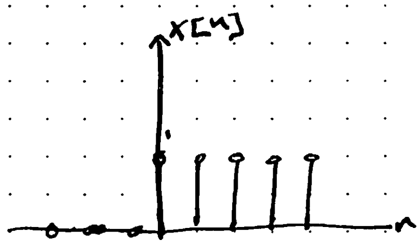
$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$ ← DIFFERENCE EQ FOR FIR

FOR FIR, IT TURNS OUT THAT $h[k] = \{b_k\}$

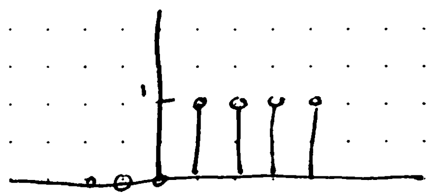
Ex: $y[n] = x[n] - x[n-1]$ ← 1^{st} DIFFERENCE

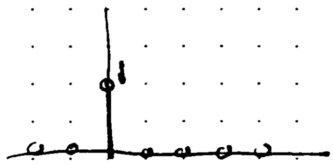
INPUT: UNIT STEP

$x[n] = u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



$y[n] = u[n] - u[n-1]$
$= \delta[n]$





PROPERTIES OF CONVOLUTION SUM

$y[n] = h[n] * x[n]$

LINEARITY $\rightarrow$ $z[n] * (\alpha x[n] + \beta y[n]) = \alpha z[n] * x[n] + \beta z[n] * y[n]$

COMMUTATIVITY $\rightarrow$ $x[n] * y[n] = y[n] * x[n]$

$$y[n] = \sum_{k=0}^{M} h_k x[n-k]$$

ASSOCIATIVITY $\rightarrow$ $x[n] * y[n] * z[n] = (x[n] * y[n]) * z[n]$ $\quad \sum_{k=0}^{M} x[k] h_{n-k}$

IDENTITY ELEMENT $\rightarrow$ $x[n] * \delta[n] = x[n]$

TIME DELAY $\rightarrow$ $x[n] * \delta[n-n_0] = x[n-n_0]$

Example

$y[n] = x[n] - x[n-1]$

$x[n] = u[n] \qquad h[n] = \delta[n] - \delta[n-1]$

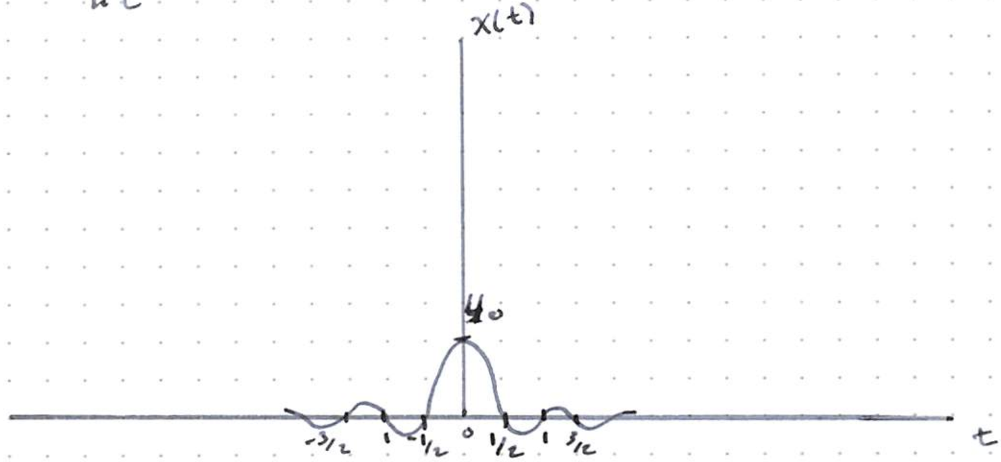
$$\begin{aligned} y[n] &= h[n] * x[n] = (\delta[n] - \delta[n-1]) * u[n] \\ &= u[n] * \delta[n] - u[n] * \delta[n-1] \\ &= u[n] - u[n-1] \end{aligned}$$

~~Jason Blocklove~~
~~HW # 4~~

~~1) a) $x(t) = \frac{\omega_b}{\pi} \frac{\sin(\omega_b t)}{\omega_b t}$ $\omega_s = 3\omega_b$~~

EXAM 2 STUDYING SINC

1) a) $x(t) = \frac{20\sin(2\pi t)}{\pi t} = \frac{40\sin(2\pi t)}{2\pi t}$



b) $y(t) = x(t-2)$

$Y(j\omega) = X(j\omega)e^{-j\omega 2}$

c) $T = 10$

$p(t) = \frac{1}{4} + \frac{4}{\pi^2}\cos(\omega_0 t) + \frac{4}{9\pi^2}\cos(3\omega_0 t)$

$P(j\omega) = \boxed{1/4} + \frac{4}{\pi} e^{j}\delta(\omega-\omega_0) + \frac{4}{\pi} e^{-j}\delta(\omega+\omega_0) + \frac{4}{9\pi} e^{j}\delta(\omega-3\omega_0) + \frac{4}{9\pi} e^{-j}\delta(\omega+3\omega_0$

HW3

1) a) $x(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

$X(j\omega) = e^{+j\omega}\cancel{X(j\omega)} + 2\cancel{X(j\omega)} + e^{-j\omega}\cancel{X(j\omega)}$

b) $x(t) = \frac{100\sin(100\pi(t-2))}{100\pi(t-2)}$ $t_d = 2$

SINC

$\frac{\omega_b}{\pi}\operatorname{sinc}(\omega_b t) \xleftrightarrow{\mathcal{F}} \Pi\left(\frac{\omega}{2\omega_b}\right)$

$X(j\omega) = \underbrace{\Pi\left(\frac{\omega}{200\pi}\right)}_{\text{SINC}} \underbrace{e^{-j\omega 2}}_{\text{DELAY}}$

c) $x(t) = e^{-t}u(t) - e^{-t}u(t-4)$

$= e^{-t}u(t) - e^{-4}e^{-(t-4)}u(t-4)$

$X(j\omega) = \frac{1}{1+j\omega} - e^{4}\left(e^{-j4\omega}\frac{1}{1+j\omega}\right)$

HW3

2) a) $X(j\omega) = \dfrac{e^{-j\omega 0.2}}{0.1 + j\omega} = e^{-j\omega 0.2} \dfrac{1}{0.1 + j\omega}$

$$x(t) = e^{-0.1(t-0.2)} u(t-0.2)$$

b) $X(j\omega) = 2 + 2\cos(\omega)$

$= 2 + e^{j\omega} + e^{-j\omega}$

$\downarrow \quad \downarrow$

$\delta(t+1) \; \delta(t-1)$

$= 2\delta(t) + \delta(t+1) + \delta(t-1)$

c) $X(j\omega) = \dfrac{1}{1+j\omega} - \dfrac{1}{2+j\omega}$

$x(t) = e^{-t}u(t) - e^{-2t}u(t)$

d) $X(j\omega) = j\delta(\omega - 100\pi) - j\delta(\omega + 100\pi)$

$= j\frac{2\pi}{2\pi}\delta(\omega - 100\pi) - j\frac{2\pi}{2\pi}\delta(\omega + 100\pi)$

$= \frac{j}{2\pi} 2\pi\delta(\omega - 100\pi) - \frac{j}{2\pi} 2\pi\delta(\omega + 100\pi)$

$x(t) = \frac{j}{2\pi}\delta(t)e^{j100\pi t} - \frac{j}{2\pi}\delta(t)e^{-j100\pi t}$

$= \frac{-1}{\pi}\left(\dfrac{e^{j100\pi t} - e^{-j100\pi t}}{2j}\right)$

$= -\frac{1}{\pi}\sin(100\pi t)$

3) $h(t) = \dfrac{4\sin(\omega_c t)}{\pi t}$ $\qquad x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$

a) $X(j\omega) = 2\pi \sum_{k-\infty}^{\infty} \delta(\omega - 2\pi k)$

b) $H(j\omega) =$

c) $Y(j\omega) = X(j\omega)H(j\omega)$

4) $H(j\omega) \begin{cases} 10e^{-j0.0025\omega} & |\omega| < 1000\pi \\ 0 & |\omega| > 1000\pi \end{cases}$

Ex2

a) $x(t) = 3\delta(t-1) + e^{-2t}u(t-1)$

$+ e^{-2}e^{-2(t-1)}u(t-1)$

$X(j\omega) = 3e^{-j\omega} + e^{-2}e^{-j\omega}\frac{1}{2+j\omega}$

b) $v(t) = \begin{cases} 1 & 3 \le t < 7 \\ 0 & \text{ELSE} \end{cases}$

FV1-3

$x(t) = \sum_{k=-4}^{4} a_k e^{j200\pi kt}$ $\quad a_k = \begin{cases} \frac{1}{\pi|k|^2} & k \neq 0 \\ 1 & k = 0 \end{cases}$

a) $X(j\omega) = \sum_{k=-4}^{4} 2\pi a_k \delta(\omega - 200\pi k)$

b) $H(j\omega) = \begin{cases} 10 & |\omega| \le 300\pi \\ 0 & |\omega| > 300\pi \end{cases}$

c) $y(t) = ?$

$Y(j\omega) = H(j\omega)X(j\omega)$

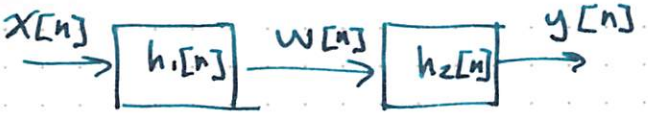
$= 20\pi\delta(\omega) + 20\pi\delta(\omega - 200\pi) + 20\pi\delta(\omega + 200\pi)$

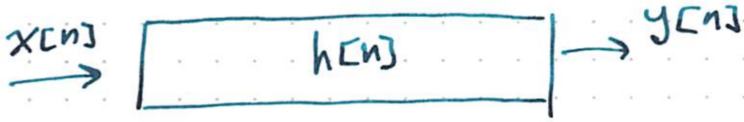
$y(t) = 10 + \frac{10}{\pi}e^{j200\pi t} + \frac{10}{\pi}e^{-j200\pi t}$

Ex4

MAR 05, 2018
NEXT HW – DUE AFTER BREAK

CASCADING SYSTEM – SERIES





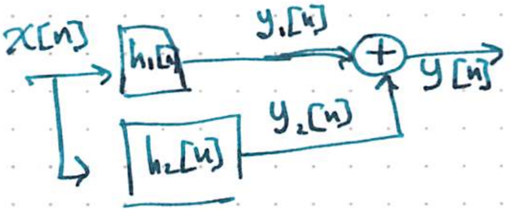
$y[n] = h[n] * x[n]$

$w[n] = h_1[n] * x[n]$, $y[n] = \cancel{w[n] *}\ h_2[n] * w[n]$

$$y[n] = h_2[n] * h_1[n] * x[n]$$
$$= h[n] * x[n]$$

$h[n] = h_2[n] * h_1[n] = h_1[n] * h_2[n]$ } IMPULSE RESPONSE FOR CASCADE CONFIGURATION

PARALLEL



$\Longrightarrow$

$y[n] = y_1[n] + y_2[n]$

$y[n] = h_1[n] * x[n] + h_2[n] * x[n]$

$y[n] = (h_1[n] + h_2[n]) * x[n]$

$y[n] = h[n] * x[n]$

FREQUENCY RESPONSE OF DISCRETE TIME FILTERS

↳ WORK W/ COMPLEX EXPONENTIAL

$$\underbrace{\cos(\omega_0 t)}_{\text{CONTINUOUS TIME}} \xrightarrow{T_s} \underbrace{\underbrace{\cos(\omega_0 n T_s)}_{\text{SAMPLE VALUES}} = \overbrace{\cos[\hat{\omega}_0 n]}^{\text{DISCRETE TIME}}}_{x[n] = x(nT_s)}$$

$\hat{\omega}_0 = \omega_0 T_s$

DIGITAL FREQ.

$$\hat{\omega}_0 = \omega_0 T_s = \frac{2\pi f_0}{f_s}$$

↑ SOMETIMES WRITTEN AS Ω

$f_s \geq 2 f_0 \rightarrow \frac{f_s}{2} \geq f_0$

$-\pi \leq \hat{\omega} \leq \pi$

FREQ RESPONSE
↳ COMPLEX EXPONENTIAL AS INPUT

$x[n] = \underbrace{A e^{j\varphi} e^{j\hat{\omega} n}}_{\text{MOSTLY THIS FORM}} = A e^{j(\hat{\omega} n + \varphi)}$

$y[n] = ?$ FOR INPUT $x[n] = A e^{j\varphi} e^{j\hat{\omega} n}$

$$y[n] = \underbrace{\sum_{k=0}^{M} h[k]x[n-k]}_{h[n] * x[n]} = \sum_{k=0}^{M} h[k]Ae^{j\varphi}e^{j\hat{\omega}(n-k)}$$

$$y[n] = \left(\sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k}\right)\underbrace{Ae^{j\varphi}e^{j\hat{\omega}n}}_{x[n]}$$

$$= \underbrace{\left(\sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k}\right)}_{H(e^{j\hat{\omega}})} x[n]$$

FREQ. RESPONSE OF DISCRETE-TIME FILTER W/ IMPULSE RESPONSE $h[n]$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k} = H(\hat{\omega}) = H(\Omega)$$

$\cancel{H(\varphi)}$ ← F.T.

WHEN INPUT IS COMPLEX EXPONENTIAL $x[n] = Ae^{j\varphi}e^{j\hat{\omega}n}$

THE OUTPUT IS $y[n] = H(e^{j\hat{\omega}})Ae^{j\varphi}e^{j\hat{\omega}n} = H(e^{j\hat{\omega}})x[n] = y[n]$

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})|e^{j\angle H(e^{j\hat{\omega}})}$$

$$y[n] = |H(e^{j\hat{\omega}})|e^{j\angle H(e^{j\hat{\omega}})}Ae^{j\varphi}e^{j\hat{\omega}n} = \underbrace{\left(|H(e^{j\hat{\omega}})|A\right)}_{A}\underbrace{e^{j(\varphi+\angle H(e^{j\hat{\omega}}))}}_{e^{j\varphi}}\underbrace{e^{j\hat{\omega}n}}_{e^{j\hat{\omega}n}} = x[n]$$

$x[n] = A\cos[\hat{\omega}_1 n + \theta]$ ← $y[n] = ?$

$$x[n] = A\left(\frac{e^{j(\hat{\omega}_1 n+\theta)} + e^{-j(\hat{\omega}_1 n+\theta)}}{2}\right) = \underbrace{\frac{A}{2}e^{j\theta}e^{j\hat{\omega}_1 n}}_{x_1[n]} + \underbrace{\frac{A}{2}e^{-j\theta}e^{-j\hat{\omega}_1 n}}_{x_2[n]}$$

$y[n] = y_1[n] + y_2[n]$ $\qquad y_1[n] =$ $\qquad y_2[n] =$

$$x[n] = A\cos[\hat{\omega}_1 n + \theta]$$

$$y[n] = A|H(e^{j\hat{\omega}_1})|\cos[\hat{\omega}_1 n + \theta + \angle H(e^{j\hat{\omega}_1})]$$

MAR 07, 2018

FREQ RESPONSE

$$H(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\hat{\omega}k}$$

$x[n] = A\cos[\hat{\omega}_1 n + \theta]$ ← GIVEN

$y[n] = ?$

$$x[n] = \underbrace{\frac{A}{2}e^{j\phi}e^{j\hat{\omega}_1 n}}_{x_1[n]} + \underbrace{\frac{A}{2}e^{-j\phi}e^{-j\hat{\omega}_1 n}}_{x_2[n]}$$

$$y[n] = y_1[n] + y_2[n]$$

$$y_1[n] = H(e^{j\hat{\omega}_1})\frac{A}{2}e^{j\phi}e^{j\hat{\omega}_1 n}$$

$$H(e^{j\hat{\omega}_1 n}) = |H(e^{j\hat{\omega}_1 n})|e^{\angle H(e^{j\hat{\omega}_1 n})}$$

$$y_1[n] = \frac{A}{2}|H(e^{j\hat{\omega}_1})|e^{j(\hat{\omega}_1 n + \phi + \angle(H(e^{j\hat{\omega}_1})))}$$

$$y_2[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\frac{A}{2}e^{-j\phi}e^{-j\hat{\omega}_1(n-k)}$$

$$y_2[n] = \left(\sum_{k=-\infty}^{\infty} h[k]e^{j\hat{\omega}k}\right)\frac{A}{2}e^{-j\phi}e^{-j\hat{\omega}_1 n}$$

$$y[n] = \underbrace{\frac{A}{2}|H(e^{j\hat{\omega}_1})|e^{j(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))}}_{y_1[n]} + \underbrace{\frac{A}{2}|H(e^{j\hat{\omega}_1})|e^{-j(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))}}_{y_2[n]}$$

$$y[n] = A|H(e^{j\hat{\omega}_1})|\cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

$h[n] = \delta[n - n_0] \rightarrow$ FREQ RESPONSE

$$H(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} h[k]e^{-jk\hat{\omega}}$$

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

$$= \sum_{k=-\infty}^{\infty} \delta[k-n_0]e^{-jk\hat{\omega}} = \delta[0-n_0]e^{-j0\hat{\omega}} + \dots + \delta[n_0-n_0]e^{-jn_0\hat{\omega}} + \dots$$

$$\delta[k-n_0] = \begin{cases} 1 & k = n_0 \\ 0 & \text{ELSE} \end{cases}$$

$$H(e^{j\hat{\omega}}) = e^{-jn_0\hat{\omega}}$$

DIFFERENCE EQ. $y[n] = x[n] + 2x[n-1] + x[n-2]$

$\{b_k\} = \{1, 2, 1\} = h[k]$ B/C FIR FILTER

$$H(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} h[k]e^{-jk\hat{\omega}}$$

$$1 = e^{j\hat{\omega}}e^{-j\hat{\omega}} \qquad = \underbrace{1e^{-j(0)\hat{\omega}}}_{k=0} + \underbrace{2e^{-j(1)\hat{\omega}}}_{k=1} + \underbrace{1e^{-j(2)\hat{\omega}}}_{k=2}$$

$$= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}})$$

COS

$$H(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}}(1 + \cos(\hat{\omega}))$$

MAR 09, 2018

↳ 1^{ST} PROJECT DUE IN 3 WEEKS

↳ MATLAB PROJECT

$$\{b_k\} = \{1, 2, 1\}$$

b_0, b_1, b_2

$y[n] = x[n] + 2x[n-1] + x[n-2] \rightarrow$ DIFF. EQUATION

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

FOR FIR

$$h[k] = b_k$$

$$~~H(e^{j}~~ H(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = e^{-j(0)\hat{\omega}} + 2e^{-j(1)\hat{\omega}} + e^{-j(2)\hat{\omega}}$$

$$= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}})$$

COS

$$= 2e^{-j\hat{\omega}}(1 + \cos(\hat{\omega}))$$

PLOT $H(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}}(1 + \cos(\hat{\omega}))$

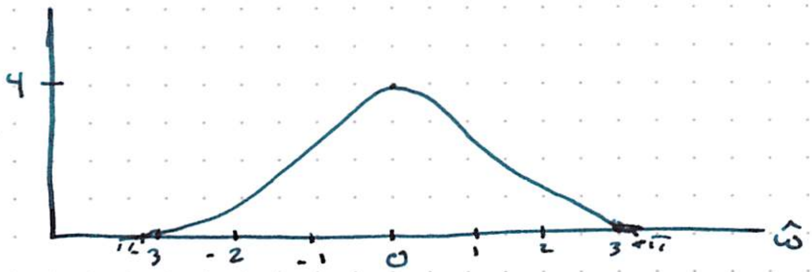
↳ $|H(e^{j\hat{\omega}})|$

↳ $\angle H(e^{j\hat{\omega}})$

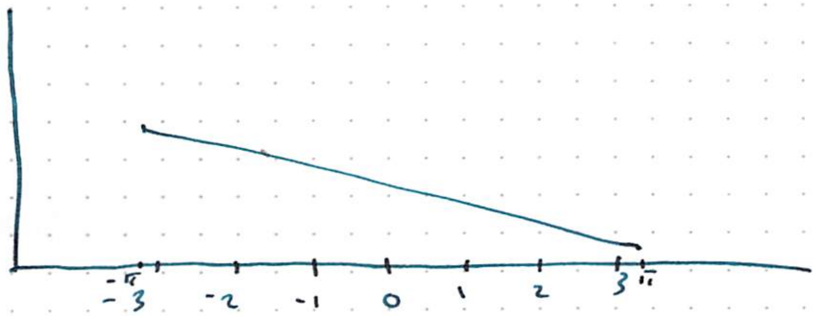
$$|H(e^{j\hat{\omega}})| = 2(1+\cos(\hat{\omega}))$$

$$\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$$

MAG:



PHASE



$b_k = \{4, -2, 1, -2, 4\}$ $M=4$

$$H(e^{j\hat{\omega}}) = 4 - 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - 2e^{-j3\hat{\omega}} + 4e^{-j4\hat{\omega}}$$

~~$e^{j\hat{\omega}}$ AS A COMMON FACTOR~~

$$= e^{-j2\hat{\omega}}(4e^{j2\hat{\omega}} - 2e^{j\hat{\omega}} + 1 - 2e^{-j\hat{\omega}} + 4e^{-j2\hat{\omega}})$$

$$= 2e^{-j2\hat{\omega}}\left(\tfrac{1}{2} - \cos(\hat{\omega}) + 4\cos(2\hat{\omega})\right)$$

$\{b_k\} = \{1, 2, 1\}$

FIND $y[n] = ?$ WHEN $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$ → $A=2$, $\phi = \pi/4$, $\hat{\omega} = \pi/3$

↳ $H(e^{j\hat{\omega}}) = (2 + 2\cos(\hat{\omega}))e^{-j\hat{\omega}}$

$$y[n] = \underbrace{H(e^{j\hat{\omega}})}_{\text{EVAL @ } \hat{\omega} = \hat{\omega}_1 = \pi/3} 2e^{j\pi/4}e^{j(\pi/3)n}$$

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\pi/3} = (2 + 2\cos(\pi/3))e^{-j\pi/3} = 3e^{-j\pi/3}$$

$$y[n] = 3e^{-j\pi/3}\,2e^{j\pi/4}e^{j(\pi/3)n}$$

$$= 6e^{-j\pi/12}e^{j(\pi/3)n}$$

$H(e^{j\hat{\omega}}) = (2 + 2\cos(\hat{\omega}))e^{-j\hat{\omega}}$

$x[n] = 2\cos(\pi/3 n + \pi/4)$ → $A = 2$, $\phi = \pi/4$, $\hat{\omega} = \pi/3$

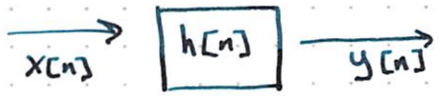
$$y[n] = A|H(e^{j\hat{\omega}_1})|\cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

$$= 2(3)\cos(\pi/3 n + \pi/4 + (-\pi/3))$$

$$= 6\cos(\pi/3 n - \pi/12)$$

MAR 21, 2018

FROM "MON" LECTURE



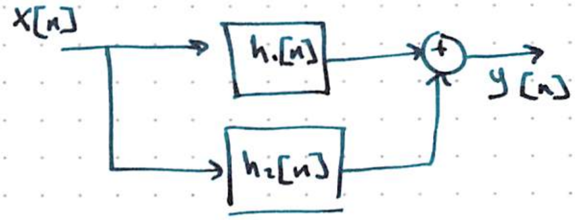
$y[n] = h[n] * x[n]$

CASCADE

$y[n] = h_1[n] * h_2[n] * x[n]$

$h[n] = h_1[n] * h_2[n]$

PARALLEL

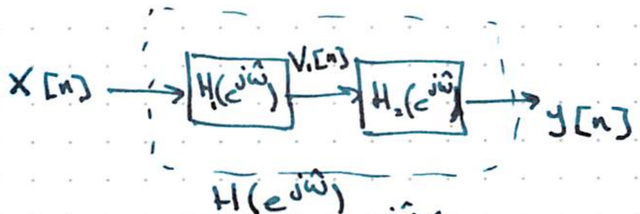


$h[n] = h_1[n] + h_2[n]$

$y[n] = (h_1[n] + h_2[n]) * x[n]$

$x[n] = A e^{j\varphi} e^{j\hat{\omega} n}$

$y[n] = H(e^{j\hat{\omega}_1}) x[n] = H(e^{j\hat{\omega}_1}) A e^{j\varphi} e^{j\hat{\omega}_1 n}$



$x[n] = A e^{j\varphi} e^{j\hat{\omega}_1 n}$

$v_1[n] = H_1(e^{j\hat{\omega}_1}) A e^{j\varphi} e^{j\hat{\omega}_1 n}$

$v_1[n] = A |H(e^{j\hat{\omega}_1})| \underbrace{e^{j\varphi} e^{j\angle(H(e^{j\hat{\omega}_1}))}}_{e^{j(\varphi + \angle(H(e^{j\hat{\omega}_1})))}} e^{j\hat{\omega}_1 n}$

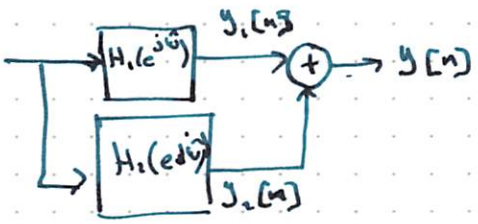
COMPLEX EXPONENTIAL

$y[n] = H_2(e^{j\hat{\omega}}) v_1[n] = H_2(e^{j\hat{\omega}_1}) H_1(e^{j\hat{\omega}_1}) x[n]$

$y[n] = H(e^{j\hat{\omega}_1}) x[n] =$

FOR CASCADED FILTERS:

$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}}) H_2(e^{j\hat{\omega}})$



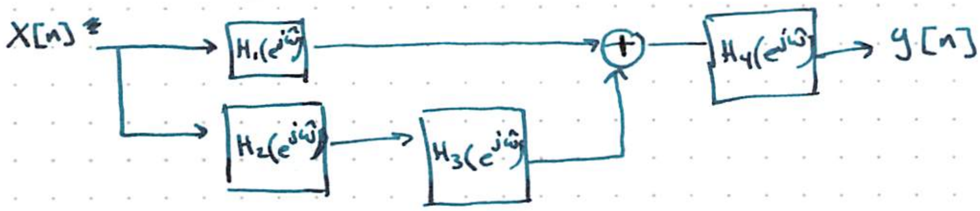
$y_1[n] = H_1(e^{j\hat{\omega}}) x[n]$

$y_2[n] = H_2(e^{j\hat{\omega}}) x[n]$

$y[n] = y_1[n] + y_2[n] = (H_1(e^{j\hat{\omega}}) + H_2(e^{j\hat{\omega}})) x[n]$

$y[n] = \underbrace{H(e^{j\hat{\omega}})} x[n]$

$$\boxed{H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}}) + H_2(e^{j\hat{\omega}}) \quad \text{FOR PARALLEL FILTERS}}$$



$$y[n] = \left(H_1(e^{j\hat{\omega}}) + \left(H_2(e^{j\hat{\omega}})\, H_3(e^{j\hat{\omega}})\right)\right) H_4(e^{j\hat{\omega}})\; x[n]$$

FREQ RESPONSE:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k] e^{-jk\hat{\omega}} \overset{\text{FIR}}{=} \sum_{k=0}^{M} b_k e^{-jk\hat{\omega}}$$

$h[k] \leftarrow$ SOME OTHER FILTER (MAY OR MAY NOT BE FIR)

$$\boxed{H(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} h[k] e^{-jk\hat{\omega}}} \longleftarrow$$ FREQ RESPONSE FOR ANY LTI FILTER

FOURIER TRANSFORMS SO FAR:

SAMPLING

CONT. TIME (CONT. FREQ.)

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t}\, dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$$

DISCRETE TIME - PERIODIC FREQ

$$X(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} x[k] e^{-jk\hat{\omega}}$$

$$x[k] = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(e^{j\hat{\omega}}) e^{jk\hat{\omega}}\, d\hat{\omega}$$

DISCRETE TIME FOURIER TRANSFORM

DISCRETE FREQ - PERIODIC TIME

$$f(t) = \sum_{n=-\infty}^{\infty} F[n] e^{jn\omega_0 t}$$

$$F[n] = \frac{1}{T_0} \int_{\langle T_0 \rangle} f(t) e^{-jn\omega_0 t}\, dt$$

THIS COMES LATER...

- THE DISCRETE TIME FOURIER TRANSFORM (DTFT) ALLOWS US TO CALCULATE THE SPECTRUM OF A SIGNAL.
- THE DTFT OF A FILTER'S IMPULSE RESPONSE IS ITS FREQUENCY RESPONSE

EX.

$$x[n] = \underbrace{\delta[n]}_{x[0]=1} + \underbrace{2\,\delta[n-1]}_{x[1]=2} + \underbrace{\delta[n-2]}_{x[2]=1}$$

$$X(e^{j\hat{\omega}}) = ?$$

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\hat{\omega}} = (1)e^{-j(0)\hat{\omega}} + (2)e^{-j(1)\hat{\omega}} + (1)e^{-j(2)\hat{\omega}}$$

$$= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}\right)$$

$$X(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}}(1+\cos(\hat{\omega}))$$

EX $\quad x[n] = \delta[n-n_0] \xleftrightarrow{DTFT} X(e^{j\hat{\omega}}) = e^{-jn_0\hat{\omega}}$

$$x[n] = \delta[n] - \delta[n-1] \xleftrightarrow{DTFT} X(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}}$$

$$x[k] = \alpha^k u[k]$$

$$X(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} x[k]e^{-jk\hat{\omega}} = \alpha\sum_{k=0}^{\infty} e^{-jk\hat{\omega}}$$

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \quad |r| \leq 1$$

$r = \alpha e^{-j\hat{\omega}}$ $\quad$ ASSUME $|\alpha|<1$

$$|r| = |\alpha|\underbrace{|e^{-j\hat{\omega}}|}_{=1} = |\alpha| < 1$$

$$X(e^{j\hat{\omega}}) = \frac{1}{1-\alpha e^{-j\hat{\omega}}}$$

WED LECTURE $\quad$ WEDNESDAY

↳ EXAM NEXT WEEK - SAMPLING, FILTERS THROUGH HW 5

↳ $y[n] = A|H(e^{j\hat{\omega}_1})|\cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$

DTFT:

ANALYSIS → $X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\hat{\omega}}$ → SPECTRUM IS CONTINUOUS ON $\hat{\omega}$ $\quad -\pi \leq \hat{\omega} \leq \pi$

↳ PERIODIC W/ $T = 2\pi$

SYNTHESIS → $x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\hat{\omega}})e^{jn\hat{\omega}}\,d\hat{\omega}$

PROPERTIES OF DTFT

LINEARITY

↳ $\alpha x[n] + \beta y[n] \xleftrightarrow{DTFT} \alpha X(e^{j\hat{\omega}}) + \beta Y(e^{j\hat{\omega}})$

TIME SHIFT

↳ $x[n-m] \longleftrightarrow e^{-jm\hat{\omega}}X(e^{j\hat{\omega}})$

FREQ SHIFT

↳ $e^{jn\hat{\omega}_0}x[n] \longleftrightarrow X(e^{j(\hat{\omega}-\hat{\omega}_0)})$

CONVOLUTION/MULT.

☆ ↳ $x[n] * y[n] \longleftrightarrow X(e^{j\hat{\omega}})Y(e^{j\hat{\omega}})$; $x[n]y[n] \leftrightarrow X(e^{j\hat{\omega}}) * Y(e^{j\hat{\omega}})$

RELATING DTFT AND FT

$x_s(t) \rightarrow$ CONT. TIME EQUIVALENT SAMPLE SIGNAL

$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t-kT_s)$$

$$x(t) \longrightarrow \otimes \longrightarrow x_s(t)$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$$

$$FT \rightarrow X_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t-kT_s)$$

$$\mathcal{F}\{x_s(t)\} = \sum_{k=-\infty}^{\infty} x(kT_s)\,\mathcal{F}\{\delta(t-kT_s)\}$$

$$X_s(j\omega) = \sum_{k=-\infty}^{\infty} x(kT_s)e^{-jkT_s\omega} = \sum_{k=-\infty}^{\infty} x[k]e^{-jkT_s\omega} = X_s(j\omega)$$

$$FT \rightarrow X_s(j\omega) = \sum_{k=-\infty}^{\infty} x[k]e^{-jkT_s\omega}$$

$$DTFT \rightarrow X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\hat{\omega}}$$

$$\underbrace{X_s(e^{j\hat{\omega}})}_{DTFT} = \underbrace{X_s\left(j\frac{\hat{\omega}}{T_s}\right)}_{FT}$$

$$X_s(e^{j\hat{\omega}}) = X_s\left(j\frac{\hat{\omega}}{T_s}\right) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} X\left(j\,\frac{\hat{\omega}-2\pi k}{T_s}\right)$$

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Z TRANSFORM

$$X(z) = \sum_{k=-\infty}^{\infty} x[k]z^{-k}$$

$$x[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$Z(x[n]) = Z(u[n]) = \sum_{k=-\infty}^{\infty} u(k)z^{-k} = \sum_{k=0}^{\infty} z^{-k} + \cancel{\sum 0 \times z^{-k}} = \sum_{k=0}^{\infty} z^{-k}$$

$$\sum_{k=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad |\alpha| < 1$$

$$\alpha = z^{-1} \rightarrow \sum_{k=0}^{\infty} z^{-k} = \frac{1}{1-z^{-1}} \rightarrow \frac{z}{z-1} \quad |z^{-1}| < 1$$

$$\hookrightarrow \left|\frac{1}{z}\right| < 1 = \boxed{|z| > 1}$$

REGION OF CONVERGENCE

$$z = r e^{j\omega}$$

$|z|$, $\angle z$

$$\begin{cases} |z| = 1, \quad z = e^{j\omega} \\ \hookrightarrow X(z), X(e^{j\omega}) = \underbrace{\sum x(k)e^{-j\omega k}}_{DTFT} \end{cases}$$

$$x[n] = a^n u[n]$$

$$Z(x(n)) = \sum_{k=-\infty}^{\infty} a^k u[k] z^{-k} = \sum_{k=0} a^k z^{-k} = \sum_{k=0}^{\infty} (\underbrace{a z^{-1}}_{\alpha})^k \rightarrow \frac{1}{1-az^{-1}} \quad |az^{-1}| < 1$$

$$\frac{1}{1-az^{-1}} \rightarrow \frac{z}{z-a} = \frac{z/a}{z/a - 1}$$

$$\frac{|a|}{|z|} < 1$$

$$\underbrace{|z| > |a|}_{RoC}$$

Z PROPERTIES:

- $\hookrightarrow x[n] \rightarrow X(z)$
- $\hookrightarrow a^n x[n] \rightarrow X(z/a)$

Z-TRANSFORM PROPERTIES

$x[n] \rightarrow X(z)$

LINEARITY: $\alpha x[n] + \beta y[n] \rightarrow \alpha X(z) + \beta Y(z)$

FREQUENCY SCALING: $a^n x[n] \rightarrow X(z/a)$

MULT BY "TIME" $n x[n] \rightarrow -z \frac{dX(z)}{dz}$

TIME DELAY $x[n-n_0] \rightarrow z^{-n_0} X(z)$

TIME INV. $x[-n] \rightarrow X(1/z)$

$\sin(\hat{\omega}_0 n) u[n] \xrightarrow{z} \frac{z \sin(\hat{\omega}_0)}{z^2 - 2\cos(\hat{\omega}_0)}$

CONVOLUTION TO MULT.

$x[n] \longrightarrow \underset{\text{LTI}}{\boxed{h[n]}} \longrightarrow y[n]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$Y(z) = X(z) H(z)$$ — TRANSFER FUNCTION

IIR FILTERS

$$y[n] = \underbrace{\sum_{k=0}^{M} a_k y[n-k]}_{\text{FEEDBACK PART}} + \underbrace{\sum_{k=0}^{N} b_k x[n-k]}_{\text{FEED-FORWARD PART}}$$

(ALSO, EQ FOR FIR)

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$$Z(x[n]) = X(z) = \sum_{k=-\infty}^{\infty} x[k] z^{-k}$$

$z = x + yj = r e^{j\hat{\omega}}$; $r \rightarrow |z|$; $\hat{\omega} \rightarrow \varphi$

IF $|z| = 1 \rightarrow$ DTFT

$x[n] \longrightarrow \underset{h[n]}{\boxed{\text{LTI}}} \longrightarrow y[n]$

① $y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$

② $Y(z) = X(z) \cdot \underbrace{H(z)}_{\text{TRANSFER FUNCTION}}$

$Z^{-1}[Y(z)] = y[n]$

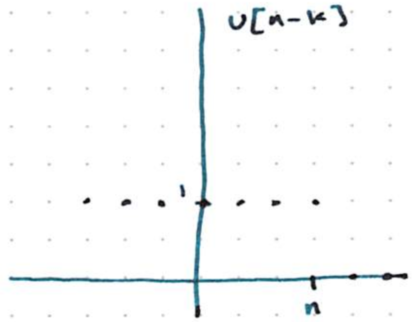
Ex. FILTER W/ IMPULSE RESPONSE $h[n] = 5(-0.8)^n u[n]$

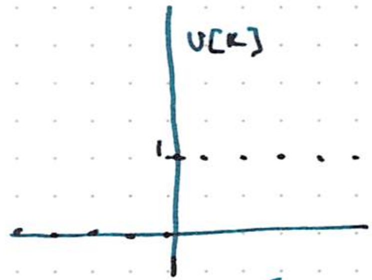
$x[n] = u[n]$

FIND $y[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} u[k]\, 5(-0.8)^{n-k}\, u[n-k] = \sum_{k=-\infty}^{\infty} 5(-0.8)^{n-k} \underbrace{u[k]\, u[n-k]}_{\text{GET RID OF}}$$





$$u[k]\,u[n-k] = \begin{cases} 1 & 0 \le k \le n \\ \text{ELSE } 0 \end{cases}$$

$$y[n] = \sum_{k=0}^{n} 5(-0.8)^{n-k}$$

$Y(z) = X(z) \cdot H(z)$

$x[n] = u[n] \longrightarrow X(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$

$H(z), \quad a^n u[n] \longrightarrow \frac{1}{1-az^{-1}} = \frac{z}{z-a}$

$h[n] = 5(-0.8)^{n-k} u[n] \longrightarrow H(z) = 5\left[\frac{1}{1-(-0.8)z^{-1}}\right] = \frac{5}{1+0.8z^{-1}}$

$Y(z) = X(z)\, H(z)$

$= \frac{z}{z-1} \cdot \frac{5}{1+0.8z^{-1}}$

STEPS $z^{-1}(Y(z))$

① $Y(z) = \frac{z}{z-1} \; \frac{5z}{z+0.8}$

② $\tilde{Y}(z) = \frac{Y(z)}{z} = \frac{5z^2}{z^2-0.2z-0.8} \cdot \frac{1}{z} = \frac{5z}{z^2-0.2z-0.8}$

③ FACTOR $D(z)$ & PARTIAL FACTOR EXPANSION

$\tilde{Y}(z) = \frac{A}{z-1} + \frac{B}{z+0.8}$

④ USE "COVER UP METHOD":

$A = (z-1)\,\tilde{Y}(z)\big|_{z=1}$

$B = (z+0.8)\,\tilde{Y}(z)\big|_{z=-0.8}$

$$A = (z-1)\tilde{Y}(z)\Big|_{z=1} = \cancel{(z-1)}\frac{5(z)}{\cancel{(z-1)}(z+0.8)} = \frac{5}{1.8} = 2.78$$

$$B = (z+0.8)\tilde{Y}(z)\Big|_{z=-0.8} = \cancel{(z+0.8)}\frac{5(z)}{(z-1)\cancel{(z+0.8)}} = \frac{5(0.8)}{-1.8} = \frac{-4}{-1.8} = 2.22$$

$$\tilde{Y}(z) = \frac{2.78}{(z-1)} + \frac{2.22}{(z+0.8)} = \frac{Y(z)}{z}$$

$$Y(z) = \frac{2.78z}{(z-1)} + \frac{2.22z}{(z+0.8)} = 2.78\frac{z}{(z-1)} + 2.22\frac{z}{(z+0.8)}$$

FROM TABLE ↳ $y[n] = 2.78\,y_1[n] + 2.22\,y_2[n]$

$= 2.78\,u[n] + 2.22\,(-0.8)^n u[n]$

EXAM 3 STUDYING

1) $y[n] = x[n] - \beta x[n-1]$

a) $x[n] = \begin{cases} 0 & n<0 \\ \beta^n & n = 0,1,2,3,4,5,6 \\ 0 & n>6 \end{cases}$

$x[n] = \sum_{k=0}^{6} \beta^k \delta[n-k]$

$y[n] = \sum_{k=0}^{6} \beta^k \delta[n-k] - \beta \sum_{k=0}^{6} \beta^k \delta[n-1-k]$

OR MAKE A TABLE OF VALUES & USE SOME ARITHMATIC

$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

a) $x(t) = 7 + 2\cos(20\pi t) + 3\cos(60\pi t + \pi/2)$

↳ $X(j\omega)$ FROM TABLE

$X(j\omega) = 2\pi(7)\delta(\omega) + 2\pi\delta(\omega \pm 20\pi) + 3\pi e^{\pm j\pi/2}\delta(\omega \pm 60\pi)$

b) GREATEST $\neq 0$ VAL @ $\omega_b = 60\pi$

NYQUIST $= 2\omega_b = 120\pi$

MAX $T_s = \frac{2\pi}{120\pi} = \frac{1}{60}$ s

$T_{s\,MAX} = \frac{2\pi}{\text{NYQUIST}}$

c) $\omega_s = \frac{2\pi}{T_s} = 100\pi$ RAD/S

$x(t) = 7 + 2\cos(20\pi t) + 3\cos(60\pi t + \pi/2)$

↳ ALIASING OCCURS

PLOT $X_s(j\omega)$

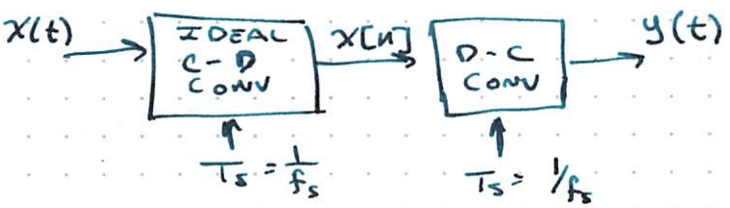
$\omega_s = 100\pi$ $\quad P(j\omega) = \underbrace{\frac{2\pi}{T_s}}_{\omega_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s) = 100\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 100\pi k)$

$\omega_s \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$

$X_s(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) = 50 \sum_{k=-\infty}^{\infty} X(j(\omega - 100\pi k))$

d) FIND $X_r(j\omega)$ IF $H(j\omega) = \begin{cases} 200 & |\omega| \le \pi/T_s = 50\pi \\ 0 & |\omega| > \pi/T_s \end{cases}$ $\quad \frac{2\pi}{T_s} = 100\pi$

IMPULSES @ $|\omega| \le 50\pi$ ARE MULT BY 200, ALL OTHERS → 0 $\quad \frac{1}{T_s} = 50$



$x(t) = 6\cos(2\pi(8000)t) + 8\sin(2\pi(6000)t)$ $\leftarrow f_{MAX}$

$2\pi f = \omega$

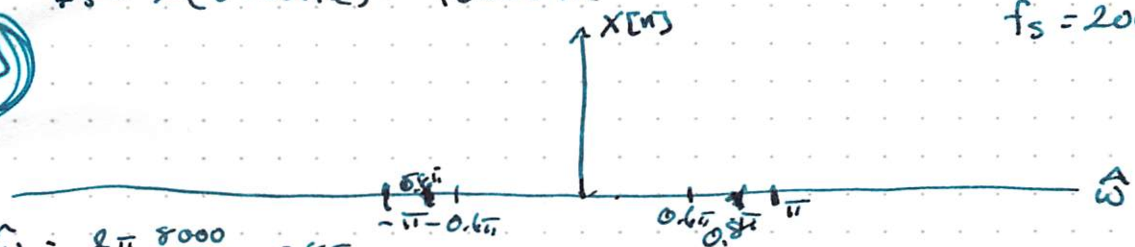
a) MIN f_s SUCH THAT $y(t) = x(t)$

SAMPLING ~~THM~~ $\Rightarrow f_s \ge 2f_{MAX}$

$f_{MAX} = 8000\text{Hz}$

$f_s = 2(8000\text{Hz}) = 16000\text{ Hz}$

b) $\quad f_s = 20000\text{ Hz}$



$\hat{\omega}_1 = 2\pi \frac{8000}{20000} = 0.8\pi$

$\hat{\omega}_2 = 2\pi \frac{6000}{20000} = 0.6\pi$

c) MAX f_s SUCH THAT SPECTRUM OF $x[n]$ WILL HAVE A NON-ZERO DC COMPONENT

NEED $\hat{\omega} = 2\pi$, IF $f_s = 8000$, $\hat{\omega} = 2\pi \frac{8000}{8000}$ ← MAX $\therefore f_{sMAX} = 8000$

f_s RATIO NEEDS TO $= 1$

DC VAL @ $f_s = 8000\text{Hz}$

$6\cos(2\pi \frac{8000}{8000} n) = 6\cos(2\pi n)$ $\boxed{6}$

$$x[n] \rightarrow \boxed{\begin{matrix}\text{LTI 1}\\ h_1[n]\end{matrix}} \xrightarrow{y_1[n]} \boxed{\begin{matrix}\text{LTI 2}\\ h_2[n]\end{matrix}} \rightarrow y[n]$$

SYSTEM 1 : 7 POINT RUNNING AVG. FILTER

$$h_1[n] = \begin{cases} 0 & n<0 \\ 1/7 & n = 0,1,2,3,4,5,6 \\ 0 & n>6 \end{cases}$$

SYSTEM 2:

$$y_2[n] = y_1[n] - y_1[n-1]$$

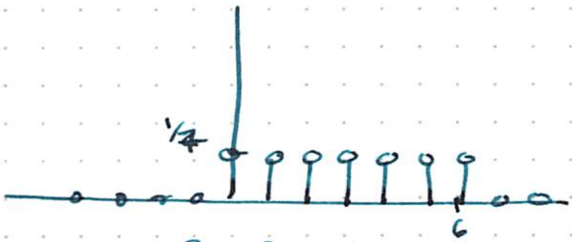
a) FIND FILTER COEFFS FOR BOTH SYSTEMS

1: $$b_n = \begin{cases} 0 & n<0 \\ 1/7 & n = 0,1,2,3,4,5,6 \\ 0 & n>6 \end{cases}$$

2: $$b_n = \begin{cases} 1 & n=0 \\ -1 & n=1 \\ 0 & \text{ELSE} \end{cases}$$

b) ~~$x[n] = \delta[n]$~~
WHEN $x[n] = \delta[n]$; FIND $y_1[n]$ AND PLOT

$$y_1[n] = h_1[n] = 1/7 \sum_{k=0}^{6} \delta[n-k]$$



c) DETERMINE IMPULSE RESPONSE OF OVERALL SYSTEM:
FIND $y[n]$ WHEN $x[n] = \delta[n]$

$$y[n] = y_1[n] - y_1[n-1]$$
$$= 1/7 \sum_{k=0}^{6} \delta[n-k] - 1/7 \sum_{k=0}^{6} \delta[n-1-k]$$

n	$y[n]$
$n=0$	$y[n] = 1/7$
1	0
2	0
3	0
4	0
5	0
6	0
7	$-1/7$
	0
	⋮

$$y[n] = \begin{cases} 1/7 & n=0 \\ -1/7 & n=7 \\ 0 & \text{ELSE} \end{cases}$$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

FIND $y[n]$ WHEN $x[n] = \delta[n] - \delta[n-2]$

$$\{b_k\} = \{\underset{n=0}{1}, \underset{1}{2}, \underset{2}{4}, \underset{3}{3}, \underset{4}{1.5}\}$$

~~y[n] x[n]~~

$$y[n] = y[n+1] - y[n-1]$$

CHART #5

$$y[n] = \sum_{k=0}^{4} \underbrace{(k+1)}_{b_k} x[n-k]$$

$$x[n] = u[n]$$

a) $\{b_k\} = \{1, 2, 3, 4, 5\}$

$$y[n] = x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 5x[n-4]$$

b) $h[n] = 1\delta[n] + 2\delta[x-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]$

c) FIND $y[n]$: $-5 \le n \le \infty$, WHEN INPUT IS $u[n]$

$$y[n] = \sum_{k=0}^{4} h[k]\, u(n-k)$$

OR

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= u[n] * h[n] \\ &= u[n] * (\delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]) \\ &= u[n] + 2u[n-1] + 3u[n-2] + 4u[n-3] + 5u[n-4] \\ &= \delta[n] + \underbrace{3}_{b_0+b_1}\delta[n-1] + \underbrace{6}_{b_0+b_1+b_2}\delta[n-2] \ldots \end{aligned}$$

$$x[n] \to \boxed{h_1[n]} \xrightarrow{y_1[n]} \boxed{h_2[n]} \to y[n]$$

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 2^n & n = 0, 1, 2, 3, 4, 5 \\ 0 & n > 5 \end{cases}$$

a) $\{b_{1_k}\} = \begin{cases} 0 & n < 0 \\ 2^n & n = 0 .. 5 \\ 0 & n > 5 \end{cases}$

$$y_2[n] = y_1[n] - 2y_1[n-1]$$

$\{b_{2_k}\} = \{1, -2\}$

b) $y_1[n] = \delta[n] + 2\delta[n-1] + 4\delta[n-2] + 8\delta[n-3] + 16\delta[n-4] + 32\delta[n-5]$

c)

$x[n] = (0.5)^n \underbrace{(u[n] - u[n-5])}_{}$

$\underbrace{1 \text{ FROM } n=0 \text{ TO } n=4}_{\text{MULT EACH BY } 0.5^n}$

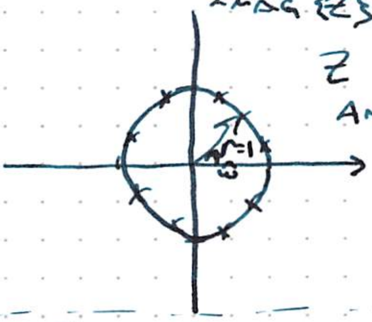
c)

~~MAR~~ APR 2, 2018
↳ SOLUTIONS TO EXAM UPLOADED
↳ NEXT EXAM - CURRENT HW: Z-TRANSFORMS, ETC.
↳ FRI, APR 13 ↳ SOME DTFT

$\mathcal{Z}$ = Z-TRANSFORM Z

$$\mathcal{Z}\{x[n]\} = \sum_{k=-\infty}^{\infty} x[k] z^{-k} = X(z)$$

$$X(z)\Big|_{z=e^{j\hat{\omega}}} = \sum_{k=-\infty}^{\infty} x[k] e^{-jk\hat{\omega}} = \underbrace{X(e^{j\hat{\omega}})}_{\text{DTFT of } x[n]}$$



Z CAN LAY ON CIRCUMFERENCE; $r = 1$
ANGLE IS $\hat{\omega}$ $\quad -\pi \leq \hat{\omega} \leq \pi$

IIR FILTERS
↳ INFINITE IMPULSE RESPONSE

DIFFERENCE EQ FOR FIR: $y[n] = \sum_{k=0}^{M} b_k x[n-k]$

" " " IIR: $y[n] = \sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k]$

WHAT IS H(Z), THE TRANSFER FUNC?

$y[n] = h[n] * x[n]$

↓ Z-TRANSFORM PROPERTY

$$\mathcal{Z}\{y[n]\} = Y(z) = \mathcal{Z}\{h[n] * x[n]\} = H(z)X(z)$$

$$Y(z) = H(z)X(z)$$

↳ $\boxed{H(z) = \frac{Y(z)}{X(z)}}$

$$\underbrace{y[n] = \sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k]}_{\text{TAKE Z-TRANSFORM OF BOTH SIDES}}$$

$$\mathcal{Z}\{y[n]\} = Y(z) = \mathcal{Z}\{\sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k]\}$$

$$= \mathcal{Z}\{\sum_{l=1}^{N} a_l y[n-l]\} + \mathcal{Z}\{\sum_{k=0}^{M} b_k x[n-k]\}$$

$$= \sum_{l=1}^{N} a_l \mathcal{Z}\{y[n-l]\} + \sum_{k=0}^{M} b_k \mathcal{Z}\{x[n-k]\}$$

TIME SHIFT: $\mathcal{Z}\{x[n-k]\} = z^{-k} \underbrace{X(z)}_{\mathcal{Z}\{x[n]\}}$

$$Y(z) = \sum_{l=1}^{N} a_l z^{-l} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z) = Y(z) \sum_{l=1}^{N} a_l z^{-l} + X(z) \sum_{k=0}^{M} b_k z^{-k}$$

$$Y(z)\left[1+\sum_{l=1}^{N} a_l z^{-l}\right] = X(z)\left[\sum_{k=0}^{M} b_k z^{-k}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{l=1}^{N} a_l z^{-l}} \underset{a_0=1}{=} \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{l=0}^{N} a_l z^{-l}} = H(z)$$

$$\cancel{H(z) = \frac{1-z^{-1}}{(1+2z^{-1})(1+\frac{1}{2}z^{-1})}}$$

APRIL 4, 2018

IIR $\rightarrow$ $y[n] = \sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k]$

$$y[n] = h[n] * x[n] \xrightarrow{Z} Y(z) = H(z)X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{l=0}^{N} a_l z^{-l}}$$

TIME SHIFT PROPERTY

FIR $\rightarrow$ $y[n] = \sum_{k=0}^{M} b_k x[n-k]$

$$Z\{y[n]\} = Y(z) = Z\left\{\sum_{k=0}^{M} b_k x[n-k]\right\} = \sum_{k=0}^{M} b_k z^{-k} X(z) = Y(z)$$

$$Y(z) = X(z)\sum_{k=0}^{M} b_k z^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k} \rightarrow \text{FIR}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{l=0}^{N} a_l z^{-l}}$$

FEED FORWARD

FEEDBACK

FOR AN LTI FILTER (FIR OR IIR), WHAT IS THE IMPULSE RESPONSE?

$$h[n] = Z^{-1}\{H(z)\}$$

SUPPOSE TYPICAL CASE: $H(z) = \dfrac{b_1 z^{-1} - b_0 z^{-2}}{(1-pz^{-1})(1-p^* z^{-1})}$ $\longleftarrow$ $h[n] = ?$

COMPLEX $p = re^{j\theta}$ COMPLEX CONJ. $p^* = re^{-j\theta}$

1) WRITE $H(z)$ USING (+) POWER OF z

$$H(z) = \frac{b_1 z - b_0}{(z-p)(z-p^*)}$$

2) $\tilde{H}(z)$ BY DIVIDING $\frac{H(z)}{z}$

PARTIAL FRACTION EXPANSION

$$\tilde{H}(z) = \frac{H(z)}{z} = \frac{b_1 z - b_0}{z(z-p)(z-p^*)} = \frac{A}{z} + \frac{B}{z-p} + \frac{C}{z-p^*}$$

$$A = z\tilde{H}(z)\Big|_{z=0} = \frac{\cancel{z}(b_1 z - b_0)}{\cancel{z}(z-p)(z-p^*)}\Big|_{z=0} = \frac{-b_0}{pp^*} = \frac{-b_0}{re^{j\theta}\,re^{-j\theta}} = \frac{-b_0}{r^2}$$

$p = re^{j\theta}$

$$A = \frac{-b_0}{|p|^2}$$

ETC... $B = \dfrac{b_1 p - b_0}{p(p-p^*)}$; $C = \dfrac{b_1 p^* - b_0}{p^*(p^*-p)} = -\dfrac{b_1 p^* - b_0}{p^*(p-p^*)}$

$p = re^{j\theta}$

$$B = \frac{b_1 p - b_0}{p(p-p^*)} = \frac{b_1 re^{j\theta} - b_0}{re^{j\theta}(re^{j\theta} - re^{-j\theta})} = \frac{b_1 re^{j\theta} - b_0}{r^2 e^{j\theta}\underbrace{(e^{j\theta} - e^{-j\theta})}_{\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}} 2j/2j}$$

$$B = \frac{b_1 re^{j\theta} - b_0}{r^2 e^{j\theta} 2j \sin(\theta)}$$

$$= \frac{b_1 r - b_0 e^{-j\theta}}{r^2 2j \sin\theta} = \frac{-j b_1 r + j b_0 e^{-j\theta}}{r^2 2 \sin\theta}$$

$$C = -\frac{b_1 p^* - b_0}{p^*(p-p^*)} = \frac{b_1 re^{-j\theta} - b_0}{re^{-j\theta}(re^{j\theta} - re^{-j\theta})} \cdots = \frac{b_1 rj - jb_0 e^{j\theta}}{r^2 2 \sin\theta}$$

$$\boxed{C = B^*}$$

$$\tilde{H}(z) = \frac{A}{z} + \frac{B}{z-p} + \frac{B^*}{z-p^*} \qquad H(z) = \frac{b_1 z - b_0}{\underbrace{(z-p)}_{\substack{\text{ROOT} \\ z=p}}\underbrace{(z-p^*)}_{\substack{\text{ROOT} \\ z=p^*}}}$$

$$H(z) = \tilde{H}(z)z = A + \frac{Bz}{z-p} + \frac{B^* z}{z-p^*}$$

ROOTS APPEAR AS COMPLEX CONJUGATE PAIRS

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = A\delta[n] + Bp^n u[n] + B^*(p^*)^n u[n]$$

FROM TABLE

$B = r_B e^{j\theta_B}$

$$= A\delta[n] + r_B e^{j\theta_B} r^n e^{jn\theta} u[n] + r_B e^{-j\theta_B} r^n e^{-jn\theta} u[n]$$

$$= A\delta[n] + r_B r^n 2\left(\underbrace{\frac{e^{j(n\theta+\theta_B)} + e^{-j(\theta n+\theta_B)}}{2}}_{\cos(\theta n + \theta_B)}\right) u[n]$$

$$= A\delta[n] + 2r_B r^n \cos(\theta n + \theta_B) u[n]$$

$r > 1 \rightarrow h[n]$ WILL OSCILLATE & AMP. WILL INCREASE EXPONENTIALLY OVER TIME

$r < 1 \rightarrow h[n]$ WILL DECREASE ASYMPTOTICALLY GOING TO ZERO

Apr 6, 2018

$$y[n] = \sum_{l=1} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{l=0}^{N} a_l z^{-l}} \qquad a_l = \begin{cases} 1, & l=0 \\ -a_l', & l>0 \end{cases}$$

IF $N=2$

$$H(z) = \frac{\sum_{k=0}^{M} b_k y(n-k)}{\underbrace{1 + a_1 z^{-1} + a_2 z^{-2}}}$$

2 ROOTS
↳ COMPLEX #S
↳ COMPLEX CONJ. PAIRS → p, p^*

$$H(z) = \frac{b_1 z^{-1} - b_0 z^{-2}}{(1-pz^{-1})(1-p^* z^{-1})} \rightarrow (1-pz^{-1})(1-p^*z^{-1}) = 1 - (p+p^*)z^{-1} + pp^*$$

$p = re^{j\theta}$

$$= 1 - r(e^{j\theta} + e^{-j\theta})z^{-1} + r^2 z^{-2} = 1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}$$

$$H(z) = \frac{N(z)}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}} = \frac{N(z)}{(1 - pz^{-1})(1 - p^* z^{-1})}$$

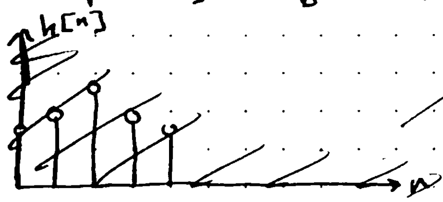
$$H(z) = A + \frac{Bz}{z-p} + \frac{B^* z}{z - p^*}, \quad A = -b_0 / r^2$$

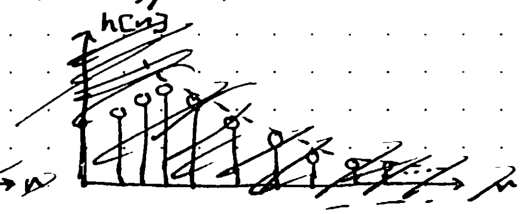
$$B = \frac{j(b_0 e^{-j\theta} - b_1 r)}{2r^2 \sin\theta} = r_B e^{j\theta_B}$$

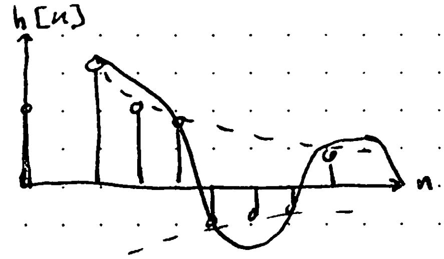
$$\boxed{p = re^{j\theta}}$$

~~$h[n] = A\delta[n]$~~

$$h[n] = \frac{-b_0}{r^2}\delta[n] + 2 r_B r^n \cos(\theta n + \theta_B) u[n]$$







h[n] HAS NON-ZERO VALUES FOREVER (ASYMPTOTICALLY → 0)
IIR

$$H(z) = \frac{N(z)}{D(z)} \leftarrow \text{RATIONAL FORM}$$

ZERO - POLE - GAIN FORM → BY FACTORING POLYNOMIALS FROM RATIONAL FORM

$$H(z) = K \frac{\prod_{i=1}^{M} (z - z_i)}{\prod_{j=1}^{N} (z - p_j)}$$

GAIN, ZEROES, POLES

IF WE MAKE $z = z_2$

↳ $$H(z) = K \frac{(z - z_1)(z - z_2) \cdots (z - z_M)}{\prod_{j=1}^{N} (z - p_j)}$$

$H(z_2) = 0$, ⇒ z_2 IS A ZERO OF $H(z)$

ZEROS z_i ARE WHERE $H(z)\big|_{z=z_i} = 0$

$Y(z) = H(z)X(z)$, WHAT VALUES OF z MAKE $Y(z) = 0$?

⇓

ZEROES OF $H(z)$

$$H(z) = \frac{K\prod_{i=1}^{M}(z - z_i)}{\prod_{j=1}^{N}(z - p_j)} = K \frac{(z - z_1)(z - z_2)\ldots(z - z_M)}{(z - p_1)(z - p_2)(z - p_3)\ldots(z - p_N)}\Bigg|_{z = p_j} \to \infty$$

POLES (p_j) ARE VALUES WHICH MAKE $H(z)\big|_{z=p_j} \to \infty$

FIR?

~~FIR~~ (ZD) $y[n] = \sum_{k=0}^{M} \underbrace{b_k}_{h[k]} x[n-k] = \sum_{k=0}^{M} h[k]\, x[n-k]$

$$Y(z) = \mathcal{Z}\{y[n]\} = \mathcal{Z}\Big\{\sum_{k=0}^{M} h[k]\, x[n-k]\Big\} = \sum_{k=0}^{M} h[k]\, z^{-k} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{M} h[k] z^{-k} = N(z)$$

$$h[k] = \delta[k] - 2\delta[k-1] + 2\delta[k+2] - \delta[k-3]$$

$$H(z) = \frac{(1 - 2z^{-1} + 2z^{-2} - z^{-3})z^3}{z^3} = \frac{z^3 - 2z^2 + 2z - 1}{z^3}$$

3 POLES @ ORIGIN

$p_j = 0$

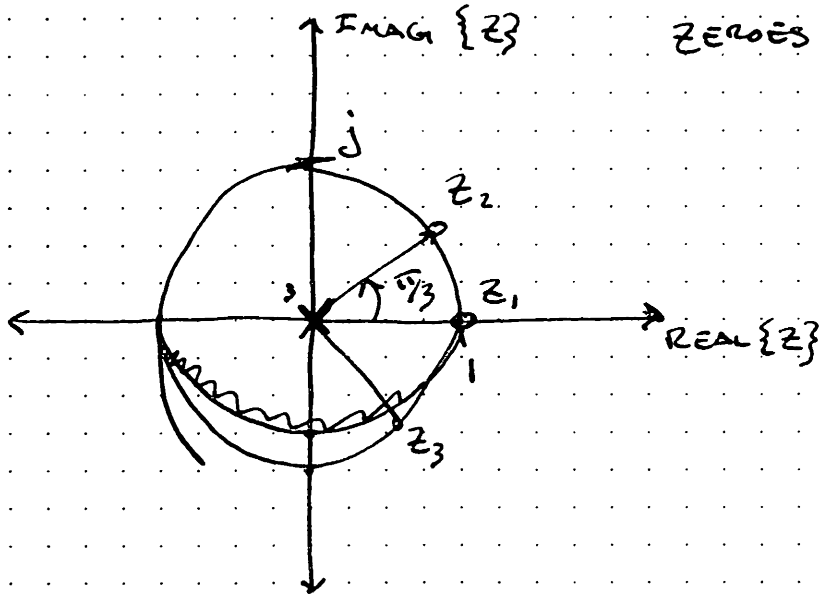
> IN GENERAL, FOR AN FIR FILTER OF ORDER M, THERE ARE M ZEROES W/ SOME VALUE, AND M POLES <u>ALL</u> AT THE ORIGIN

$H(z) =$
- ZEROES $\rightarrow H(z_i) = 0$
- POLES $\rightarrow H(p_j) \rightarrow \infty$

POLE - ZERO MAPS/PLOTS

$$H(z) = \frac{z^3 - 2z^2 + 2z - 1}{z^3} = \frac{(z-1)(z^2 - z + 1)}{z^3} = \frac{(z-1)(z-e^{j\pi/3})(z-e^{-j\pi/3})}{z^3}$$

ZEROES $\begin{cases} z_1 = 1 \\ z_2 = e^{j\pi/3} \\ z_3 = e^{-j\pi/3} \end{cases}$



APR 9, 2018

EXAM 3 REDO - NEXT WED.

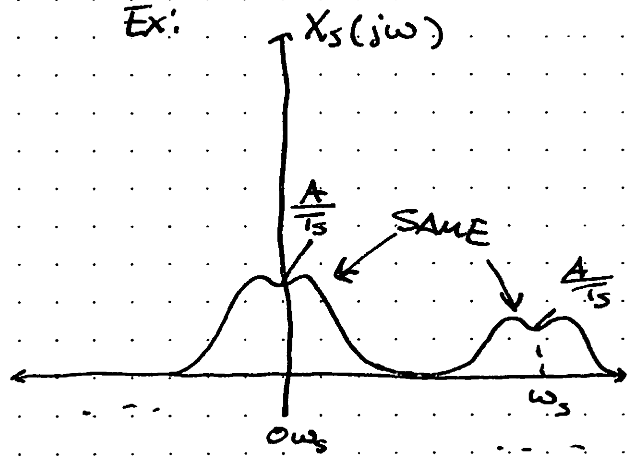
↳ REPLACE GRADE NO MATTER WHAT

EXAM 4 - Z-TRANSFORM (HW6)

EXAM 3:

1) $x(t) = A\cos(9\pi t)\cos(15\pi t)$; $X_s(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$ $\quad \omega_s = 2\pi f_s$, $f_s = \frac{1}{T_s}$

Ex:



a) $\mathcal{F}\{\underbrace{A\cos(9\pi t)}\cos(15\pi t)\}$ MODULATION ; $\omega_s = 60\pi$

$f(t)\cos(15\pi t) \longrightarrow \frac{1}{2}F(j(\omega - 15\pi)) + \frac{1}{2}F(j(\omega + 15\pi))$

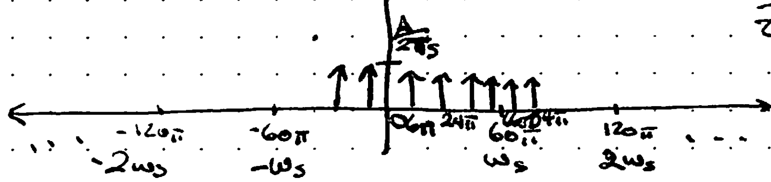
$F(j\omega) = \mathcal{F}\{f(t)\} = \mathcal{F}\{A\cos(9\pi t)\} = A\pi\delta(\omega - 9\pi) + A\pi\delta(\omega + 9\pi)$

$X(j\omega) = \frac{1}{2}\left(A\pi\delta(\overbrace{\omega - 9\pi - 15\pi}^{-24\pi}) + A\pi\delta(\overbrace{\omega + 9\pi - 15\pi}^{-6\pi})\right)$
$+ \frac{1}{2}\left(A\pi\delta(\underbrace{\omega - 9\pi + 15\pi}_{6\pi}) + A\pi\delta(\underbrace{\omega + 9\pi + 15\pi}_{24\pi})\right)$

$X(j\omega) = \frac{A\pi}{2}\left(\delta(\omega - 24\pi) + \delta(\omega - 6\pi) + \delta(\omega + 6\pi) + \delta(\omega + 24\pi)\right)$

$\omega_s = 60\pi$
$T_s = 1/30$

REPEAT PLOT CENTERED @ ZERO AT MULTIPLES OF ω_s



b) SAME IDEA, NEW ω_s

c) $X_r(j\omega) = H_r(j\omega)X_s(j\omega)$ ONLY FROM -15 TO 15
$= A\pi(\delta(\omega - 6\pi) + \delta(\omega + 6\pi))$

$x_r(t) = A\cos(6\pi t)$

APRIL 11, 2018

$$y[n] = \sum_{l=1}^{N} a_l\, y[n-l] + \sum_{k=0}^{M} b_k\, x[n-k]$$

$\downarrow$ Z TRANSFORM BOTH SIDES

$$Y(z) = \sum_{l=1}^{N} a_l z^{-l} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = [1 + \sum_{l=1}^{N} a_l z^{-l}] = X(z) \sum_{k=0}^{M} b_k z^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{l=1}^{N} a_l z^{-l}} = \frac{N(z)}{D(z)} = \frac{K \prod_{i=1}^{M} (z - z_i)}{\prod_{j=1}^{N} (z - p_j)}$$

FIR FILTER $\quad b_k = \{1, -2, 2, -1\} \qquad y[n] = \sum_{k=0}^{3} \underbrace{b_k}_{h[k]} x[n-k]$

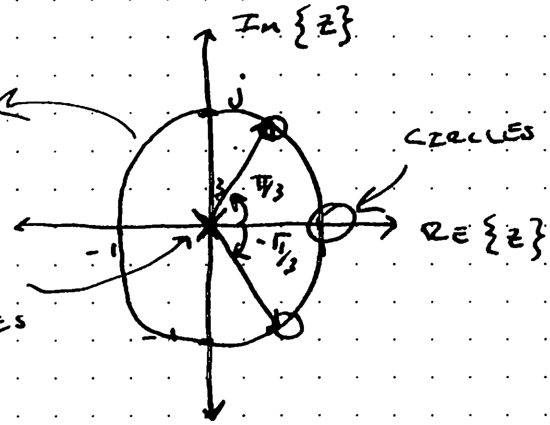
$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3} = \frac{z^3 - 2z^2 + 2z - 1}{z^3}$$

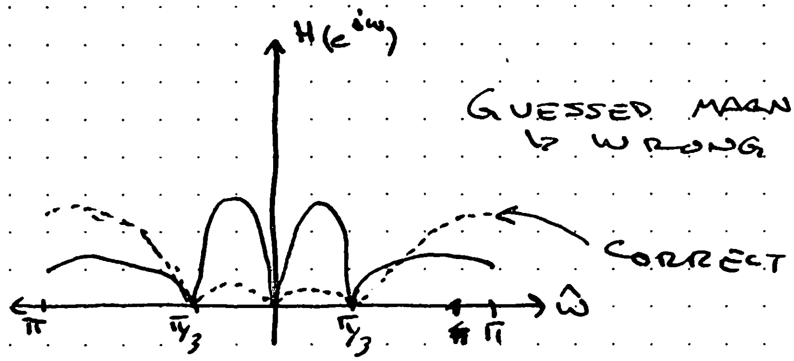
POLES $\rightarrow$ $p_{1,2,3} = 0$

ZEROS $\rightarrow$ $z_1 = 1, \quad z_{2,3} = \frac{1}{2} \pm j\frac{\sqrt{3}}{2} = e^{\pm j\pi/3}$

ZEROS - POLES PLOT

$$H(z) = \frac{K \prod_{i=1}^{M} (z - z_i)}{\prod_{j=1}^{N} (z - p_j)}$$





APRIL 13, 2018

$$H(z) = \frac{b_1 z^{-1} - b_0 z^{-2}}{(1 - p z^{-1})(1 - p^* z^{-1})} = \frac{N(z)}{(z-p)(z-p^*)}$$

$$\tilde{H}(z) = \frac{H(z)}{z} = A(z) + \frac{B}{z-p} + \frac{C}{z-p^*} \qquad C = B^*$$

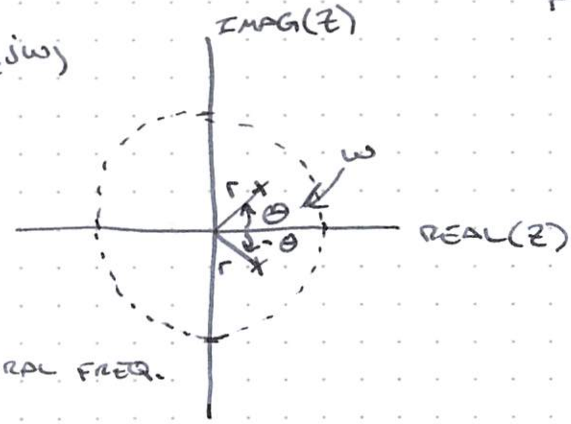
$$H(z) = zA(z) + \frac{Bz}{z-p} + \frac{B^* z}{z-p^*} \qquad B = r_B e^{j\theta_B} \quad p = re^{j\theta}$$

$$h[n] = \ldots + \left(r_B r^n e^{j\theta_B} e^{jn\theta} + r_B r^n e^{-j\theta_B} e^{-jn\theta}\right) u[n]$$

$$h[n] = \ldots + 2 r_B r^n \Bigg(\underbrace{\frac{e^{j(\theta n + \theta_B)} + e^{-j(\theta n + \theta_B)}}{2}}_{\cos(\theta n + \theta_B)} \Bigg) u[n]$$

$$= \ldots + 2 r_B r^n \cos(\theta n + \theta_B) u[n] \qquad p = re^{j\theta} \quad p^* = re^{-j\theta}$$

$$H(z) \longrightarrow H(e^{j\omega}) \qquad e^{j\omega} = z$$



$|p| = r < 1$
↳ POLES SHRINK OVER TIME

$|p| = r > 1$
↳ POLE OSCILLATIONS GET LARGER

$|p| = r = 1$
↳ STAYS SAME

θ IS NATURAL FREQ.

EXAM 4 STUDYING

$$H(z) = \frac{z^2 + z}{(z-3/4)^2(z-1/2)} \quad \tilde{H}(z) = \frac{A}{(z-3/4)} + \frac{B}{(z-3/4)^2} + \frac{C}{(z-1/2)} = \frac{z+1}{(z-3/4)^2(z-1/2)}$$

$$A = \frac{d}{dz}\left((z-3/4)^2 \tilde{H}(z)\right)\Big|_{z=3/4} = \frac{d}{dz}\left(\frac{z+1}{z-1/2}\right)\Big|_{z=3/4} = \frac{-6}{(2(z)-1)^2} = \cancel{-12}\ -24$$

$$\frac{d}{dz}\left(\frac{z+1}{z-1/2}\right) = \frac{(z-1/2)'(z+1) - (z+1)'(z-1/2)}{(z-1/2)^2} = \frac{\cancel{z}+1-\cancel{z}+1/2}{(z-1/2)^2}$$

$$= \frac{3/2}{(z-1/2)^2} = \frac{6}{(2z-1)^2} \quad \left(\tfrac{3}{2}-1\right)^2$$

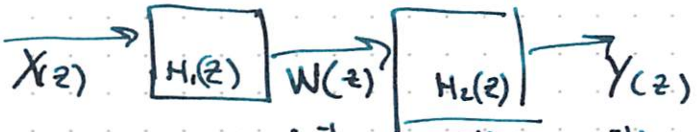
$$B = (z-3/4)^2 \tilde{H}(z)\Big|_{z=3/4} = \frac{z+1}{z-1/2}\Big|_{z=3/4} = 7$$

$$C = (z-1/2)\tilde{H}(z)\Big|_{z=1/2} = \frac{z+1}{(z-3/4)^2}\Big|_{z=1/2} = \frac{3/2}{(-1/4)^2} = 24$$

$$\tilde{H}(z) = \frac{-24}{(z-3/4)} + \frac{7}{(z-3/4)^2} + \frac{24}{(z-1/2)}$$

$$H(z) = \frac{24z}{(z-1/2)} + \frac{7}{(z-3/4)^2} - \frac{24}{z-3/4}$$

$\cancel{H(z) = \frac{1+0.64z^{-1}}{1+0.64z^{-2}}}$



$H_1(z) = (1-jz^{-1})(1+jz^{-1})(1+z^{-1}) \qquad h[n] = \delta[n] + \delta[n-4]$

$$H(z) = H_1(z)H_2(z) = (1-jz^{-1})(1+jz^{-1})(1+z^{-1})(1+z^{-4})$$
$$= (1+z^{-2})(1+z^{-1})(1+z^{-4})$$
$$= (1+z^{-2}+z^{-1}+z^{-3})(1+z^{-4})$$
$$= 1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}+z^{-6}+z^{-7}$$
$$= \sum_{k=0}^{7} z^{-k}$$

$h[n] = \delta[n] - 7\delta[n-2] - 3\delta[n-3]$

$H(z) = 1 - 7z^{-2} - 3z^{-3}$

3b) $X_b(z) = \dfrac{1+z^{-2}}{1+0.9z^{-1}+0.81z^{-2}}$

$$= \frac{z^2+1}{z^2+0.9z+0.81} \Rightarrow \text{ROOTS: } -0.45 \pm 0.7794j = 0.9\,e^{\pm j\,2.0943}$$

$$\tilde{X}_b(z) = \frac{X_b(z)}{z} = \frac{z^2+1}{z(0.9e^{j\frac{2\pi}{3}})(0.9e^{-j\frac{2\pi}{3}})}$$

$$= \frac{A}{z} + \frac{B}{0.9e^{j\frac{2\pi}{3}}} + \frac{C=B^*}{0.9e^{-j\frac{2\pi}{3}}}$$

APRIL 20, 2018

$$H(z) = \frac{N(z)}{D(z)} = \frac{N(z)}{A(z)(z-p)(z-p^*)}$$

$$p = re^{j\theta}$$

$h[n] = \ldots + 2r_B r^n \cos(\theta n + \theta_B)\,u[n]$

$r \geq 1 \rightarrow$ FILTER NOT STABLE
$r < 1 \rightarrow$ FILTER STABLE

MATLAB & DIGITAL FILTERS SLIDES

April 23, 2018

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}$$

$$X[k] = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t) e^{-jk\omega_0 t}\, dt$$

Fourier Series

Use rectangular approximation

$$dt \rightarrow \Delta t = \frac{T_0}{N}$$

$$\omega_0 = \frac{2\pi}{T_0} = \cancel{2\pi N \Delta t}\ \frac{2\pi}{N\Delta t}$$

$$X[k] = \frac{T_s}{T_0} \sum_{n=0}^{N-1} x(nT_s) e^{-jk2\pi n \frac{T_s}{T_0}}$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x(nT_s) e^{-j2\pi k n/N}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j2\pi k n/N}$$

$$x[n] = \sum_{n=0}^{N-1} X[k] e^{jk\hat{\omega}_0 n} \qquad \hat{\omega}_0 = \frac{2\pi}{N}$$

April 25, 2018