Classical Mechanics

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Chapter 1

Degrees of Freedom and Dynamical Variables

1.1 Dynamical variables

Dynamical variables are aset of variables that describe a system that changes over time. Equations of motion describe how these variables change. With Newtonian physics we relied on knowing about precise initial conditions before we had a clue what was going on.

1.2 Degrees of Freedom

Point masses have three degrees of freedom, in the x, y, z planes. A system of M point masses have:

$$N = 3M$$

degrees of freedom (3 for each particle). However, the existence of j independent constraints (rigid 'bars' between the masses) reduces this to:

$$N = 3M - j$$

This leads to the conclusion that any rigid body with more than two point masses connected with rigid rods has exactly 6 degrees of freedom - x, y, z of the centre of mass, along with three angles to describe the orientation. Another way of specifying this is simply to give two points within the plane.

1.2.1 Constraints and their types

The existance of these j constraints means that the masses, M, are no longer independent. To describe the system fully (as long as we know what the system looks like), we only need as many co-ordinates as there are degrees of freedom.

If the constraints are *holonomic* then it is possible to express the full position and orientation of the system, $\boldsymbol{r}$ as a function of these co-ordinates q_k:

$$\boldsymbol{r} = \boldsymbol{r}(q_1, q_2, \ldots, q_N, t).$$

There are two types of holonomic constraints:

- Rheonomic - time dependent constraint
- Scleronomous - no *explicit* time dependence

If we can express the constraints as a function:

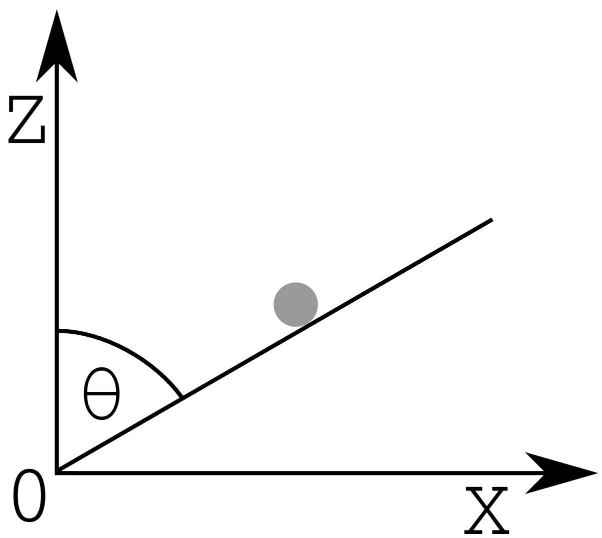
$$\boldsymbol{r} = \boldsymbol{r}(q_1, q_2, \ldots, q_N, t) = 0,$$

Then they are all holonomic.

Constraints are non-holonomic if the state depends on the path taken to achieve it - differential constraints or inequalities. These are usually velocities.

1.2.2 Examples

Example 1: point mass on an inclined plane. Here, we can find that all constraints are



holonomic if we say:

$$\tan\theta = \frac{x}{z}; \;\rightarrow\; z\tan\theta - x = 0$$

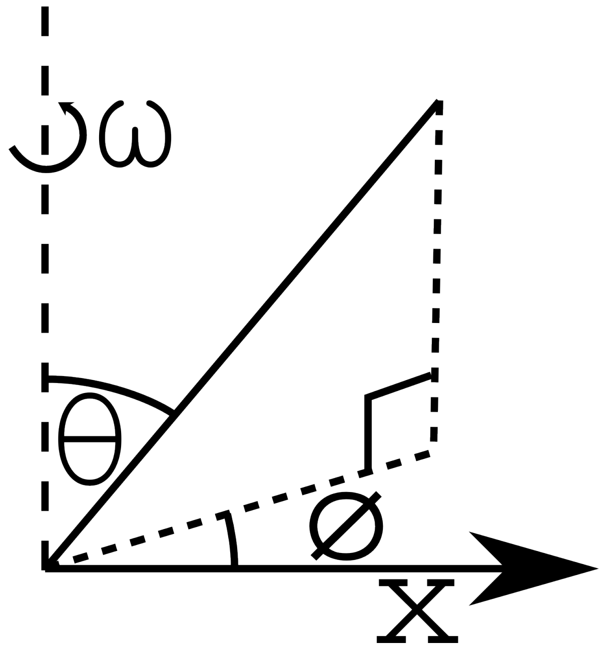
This describes the system fully.

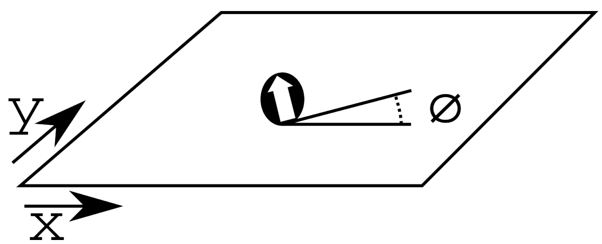
Example 2: bead on a rotating wire. For the rotating wire, we find two equations that both have time dependence and hence are *rheonomic*:

$$x = z\tan\theta\cos\omega t$$

$$y = z\tan\theta\sin\omega t$$

These constraints mean that the particle always lies on the wire.





Example 3: coin, rolling on a plane. This is slightly more complicated, and is also non-holonomic:

$$\mathrm{d}x = a\mathrm{d}\theta\cos\phi$$

$$\mathrm{d}x = a\mathrm{d}\theta\sin\phi$$

If we say that the angle that the coin has turned is θ. We can reach any point in this 4D space - i.e. we cannot separate the constraints from the dynamics.