Lab-Linear-Regression

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3.6.1 Libraries

```
library(MASS)
library(ISLR)
```

3.6.2 Simple Linear Regression

```
boston = Boston
fix(boston)
names(boston)
```

```
##  [1] "crim"    "zn"      "indus"   "chas"    "nox"     "rm"      "age"
##  [8] "dis"     "rad"     "tax"     "ptratio" "black"   "lstat"   "medv"
```

Linear regression using medv as response and lstat as predictor

```
lm.fit = lm(medv ~ lstat, data=boston)
lm.fit
```

```
## 
## Call:
## lm(formula = medv ~ lstat, data = boston)
## 
## Coefficients:
## (Intercept)        lstat  
##       34.55        -0.95
```

```
summary(lm.fit) #Run summary statistics on linear model
```

```
## 
## Call:
## lm(formula = medv ~ lstat, data = boston)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -15.168  -3.990  -1.318   2.034  24.500 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 34.55384    0.56263   61.41   <2e-16 ***
## lstat       -0.95005    0.03873  -24.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 6.216 on 504 degrees of freedom
## Multiple R-squared:  0.5441, Adjusted R-squared:  0.5432 
## F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
names(lm.fit) #Determine the names of data stored in lm.fit
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

```r
coef(lm.fit) #Get the coefficient names and values of lm.fit
```

```
## (Intercept)       lstat
##  34.5538409  -0.9500494
```

```r
confint(lm.fit) #Get 95% confidence intervals for the coefficients
```

```
##                 2.5 %     97.5 %
## (Intercept) 33.448457 35.6592247
## lstat       -1.026148 -0.8739505
```

```r
new_data = data.frame(lstat = c(5,10,15))
predict(lm.fit, new_data, interval ="confidence")
```

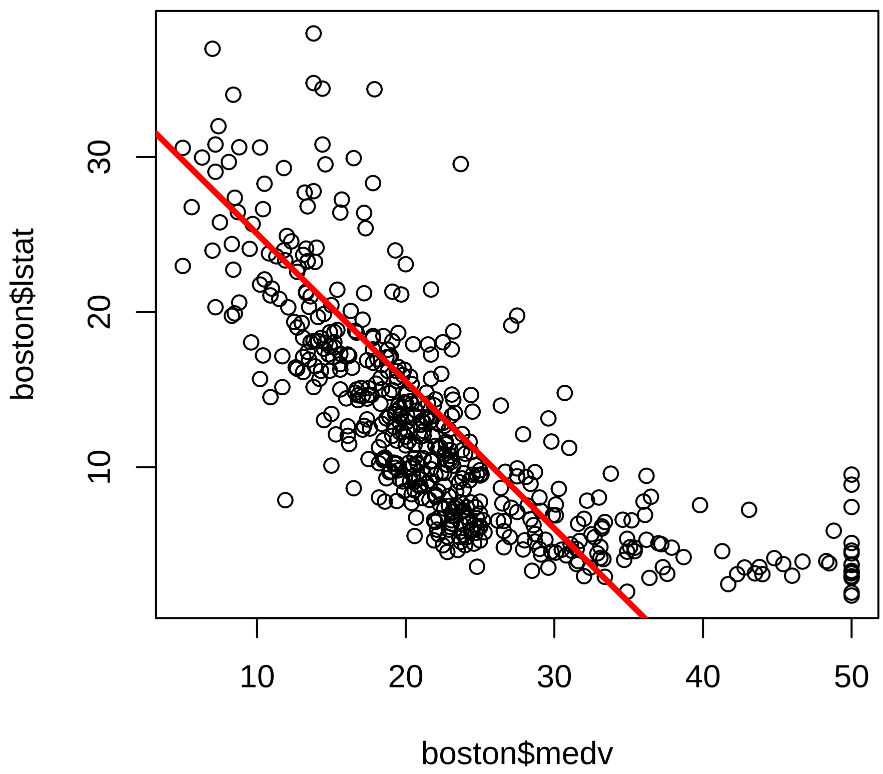
```
##        fit      lwr      upr
## 1 29.80359 29.00741 30.59978
## 2 25.05335 24.47413 25.63256
## 3 20.30310 19.73159 20.87461
```

```r
predict(lm.fit,new_data,interval = "prediction")
```

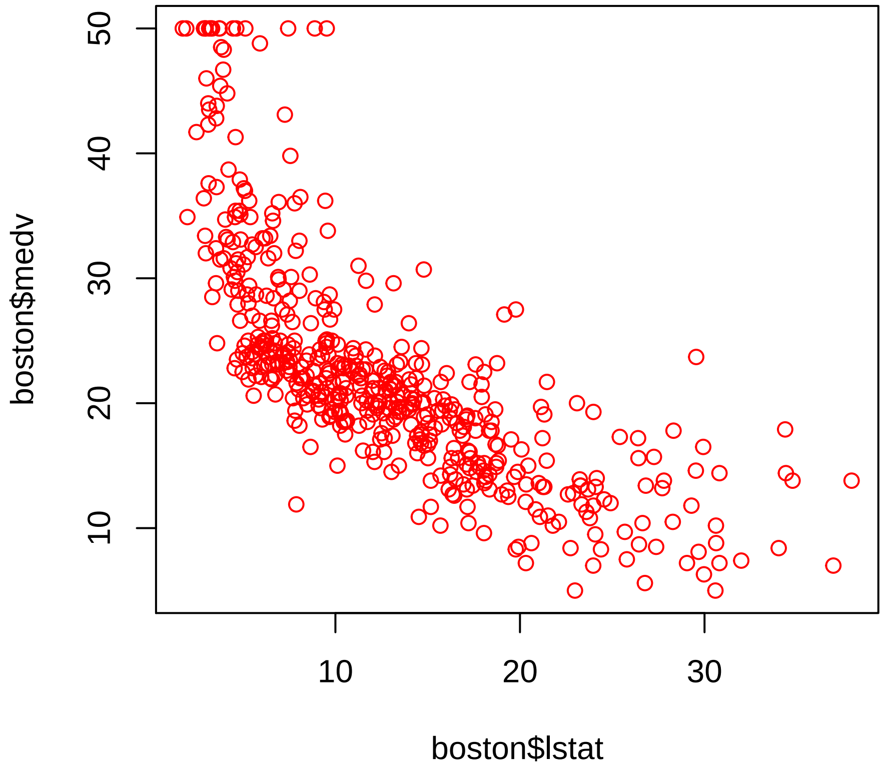
```
##        fit       lwr      upr
## 1 29.80359 17.565675 42.04151
## 2 25.05335 12.827626 37.27907
## 3 20.30310  8.077742 32.52846
```

Graphical plots

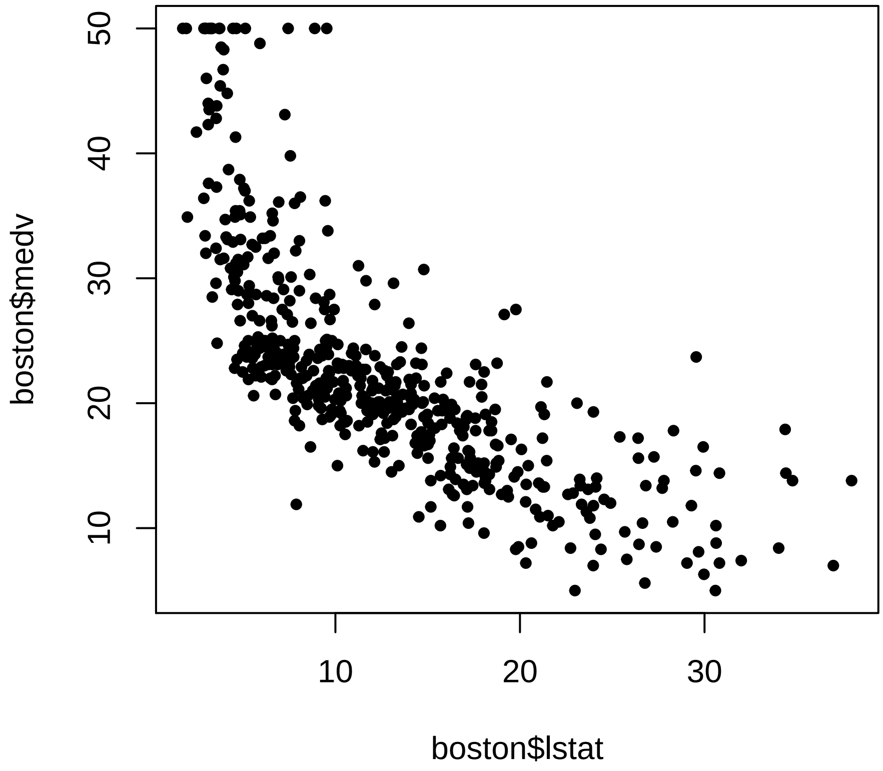
```r
plot(boston$medv, boston$lstat)
abline(lm.fit)

#Different plots with adjusted settings
abline(lm.fit, lwd = 3, col ="red")
```



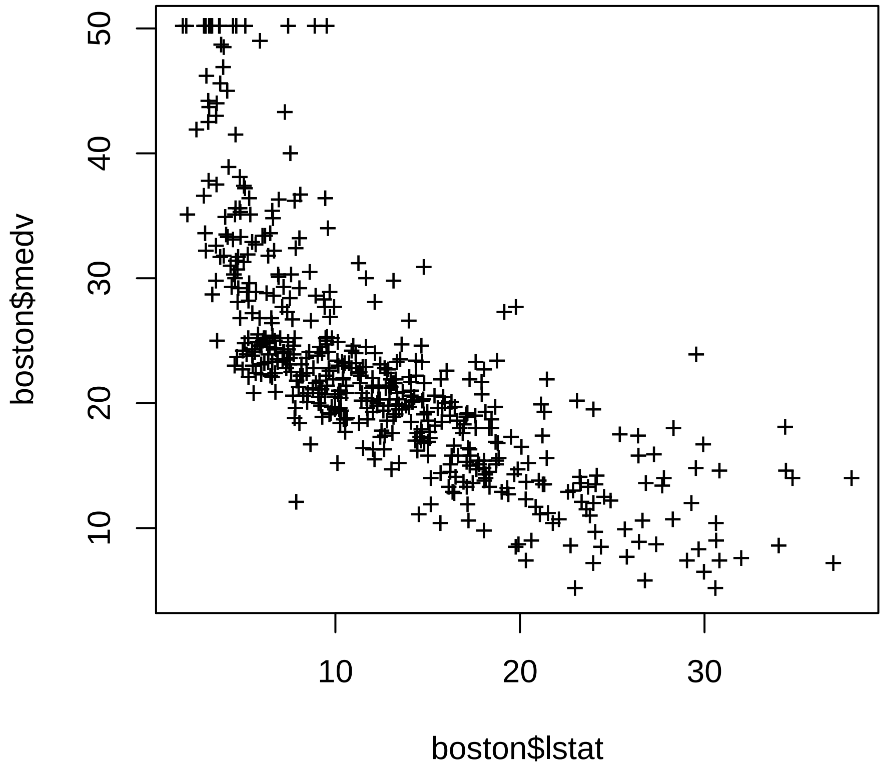
```
plot(boston$lstat, boston$medv, col = "red")
```



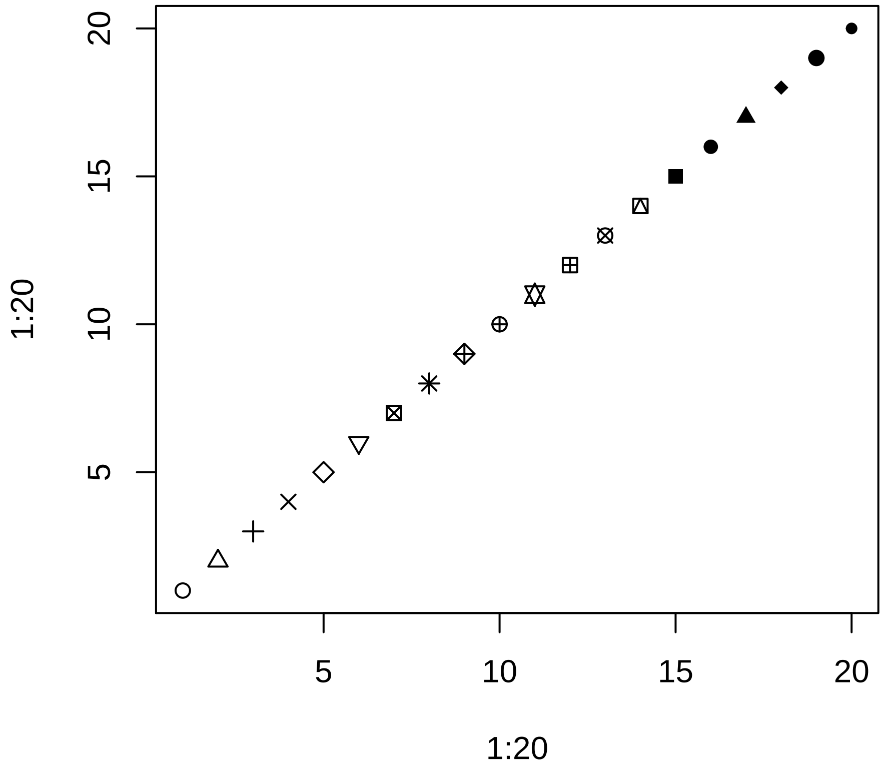
```r
plot(boston$lstat, boston$medv, pch = 20)
```



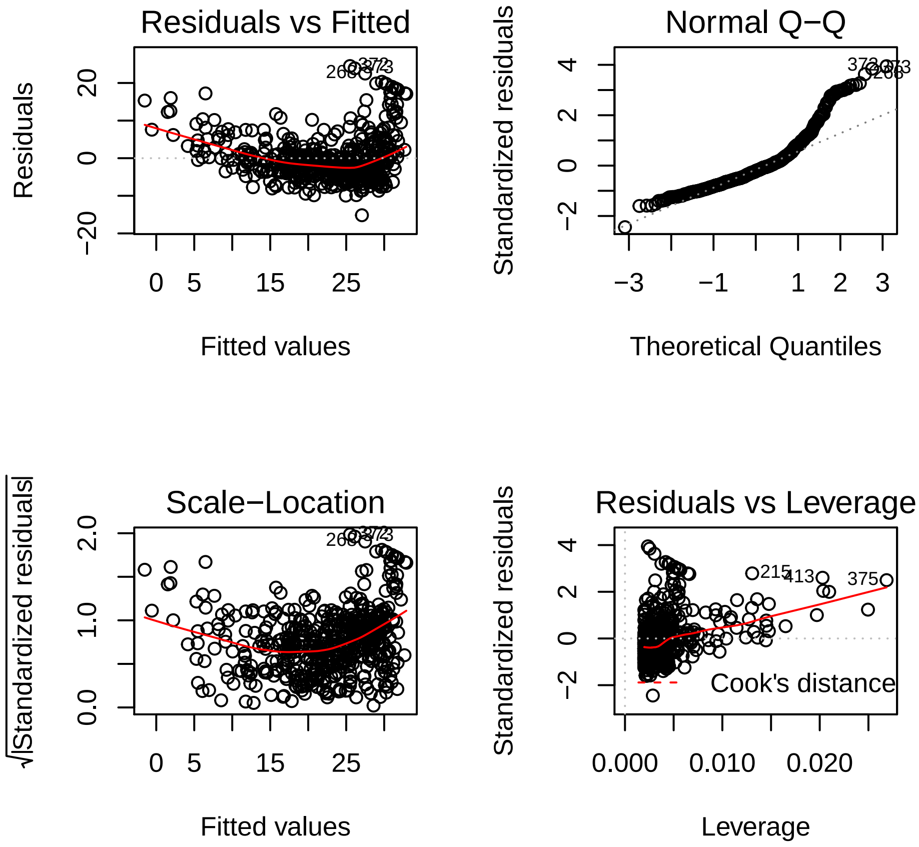
```r
plot(boston$lstat, boston$medv, pch = "+") #Using plus (+) symbol for scatter plot
```



```
plot(1:20, 1:20, pch = 1:20) #Different possible symbols
```

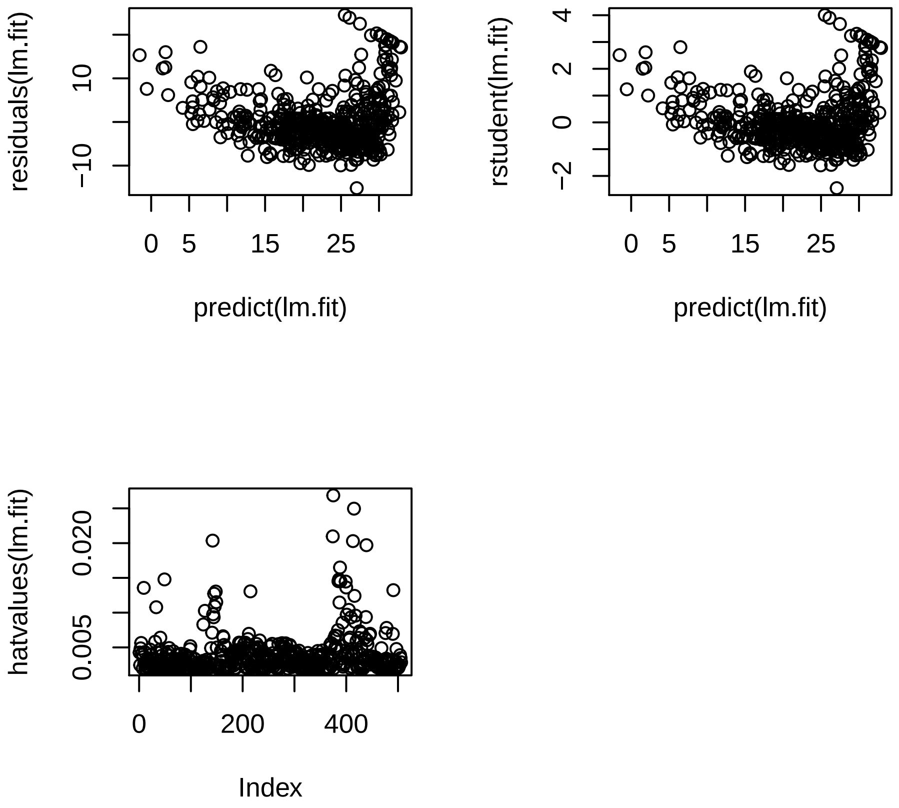


```
#Diagnostic plots
par(mfrow= c(2,2))
plot(lm.fit)
```



```
plot(predict(lm.fit), residuals(lm.fit))
plot(predict(lm.fit), rstudent(lm.fit))
plot(hatvalues(lm.fit)) #Leverage diagnostic
which.max(hatvalues(lm.fit)) #Index with largest hat value
```

```
## 375
## 375
```



3.6.3 Multiple Linear Regression

```r
#MLR with just lstat and age as predictors
lm.fit = lm(medv ~ lstat + age, data = boston)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat + age, data = boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.981  -3.978  -1.283   1.968  23.158
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.22276    0.73085  45.458  < 2e-16 ***
## lstat       -1.03207    0.04819 -21.416  < 2e-16 ***
## age          0.03454    0.01223   2.826  0.00491 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.173 on 503 degrees of freedom
## Multiple R-squared:  0.5513, Adjusted R-squared:  0.5495
## F-statistic:   309 on 2 and 503 DF,  p-value: < 2.2e-16
```

```
#MLR with all 13 predictors from the Boston dataset
lm.fit_all = lm(medv ~ ., data = boston)
summary(lm.fit_all)
```

```
##
## Call:
## lm(formula = medv ~ ., data = boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.595  -2.730  -0.518   1.777  26.199
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.646e+01  5.103e+00   7.144 3.28e-12 ***
## crim        -1.080e-01  3.286e-02  -3.287 0.001087 **
## zn           4.642e-02  1.373e-02   3.382 0.000778 ***
## indus        2.056e-02  6.150e-02   0.334 0.738288
## chas         2.687e+00  8.616e-01   3.118 0.001925 **
## nox         -1.777e+01  3.820e+00  -4.651 4.25e-06 ***
## rm           3.810e+00  4.179e-01   9.116  < 2e-16 ***
## age          6.922e-04  1.321e-02   0.052 0.958229
## dis         -1.476e+00  1.995e-01  -7.398 6.01e-13 ***
## rad          3.060e-01  6.635e-02   4.613 5.07e-06 ***
## tax         -1.233e-02  3.760e-03  -3.280 0.001112 **
## ptratio     -9.527e-01  1.308e-01  -7.283 1.31e-12 ***
## black        9.312e-03  2.686e-03   3.467 0.000573 ***
## lstat       -5.248e-01  5.072e-02 -10.347  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.745 on 492 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7338
## F-statistic: 108.1 on 13 and 492 DF,  p-value: < 2.2e-16
```

Variance inflation factors

```
library(car)
vif(lm.fit_all)
```

```
##     crim       zn    indus     chas      nox       rm      age      dis
## 1.792192 2.298758 3.991596 1.073995 4.393720 1.933744 3.100826 3.955945
##      rad      tax  ptratio    black    lstat
## 7.484496 9.008554 1.799084 1.348521 2.941491
```

Exclude age predictor

```
#MLR excluding age
lm.fit1 = lm(medv ~ . -age, data = boston)
summary(lm.fit1)
```

```
##
## Call:
## lm(formula = medv ~ . - age, data = boston)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.6054  -2.7313  -0.5188   1.7601  26.2243
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  36.436927   5.080119   7.172 2.72e-12 ***
## crim         -0.108006   0.032832  -3.290 0.001075 **
## zn            0.046334   0.013613   3.404 0.000719 ***
## indus         0.020562   0.061433   0.335 0.737989
## chas          2.689026   0.859598   3.128 0.001863 **
## nox         -17.713540   3.679308  -4.814 1.97e-06 ***
## rm            3.814394   0.408480   9.338  < 2e-16 ***
## dis          -1.478612   0.190611  -7.757 5.03e-14 ***
## rad           0.305786   0.066089   4.627 4.75e-06 ***
## tax          -0.012329   0.003755  -3.283 0.001099 **
## ptratio      -0.952211   0.130294  -7.308 1.10e-12 ***
## black         0.009321   0.002678   3.481 0.000544 ***
## lstat        -0.523852   0.047625 -10.999  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.74 on 493 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7343
## F-statistic: 117.3 on 12 and 493 DF,  p-value: < 2.2e-16
```

3.6.4 Interaction Terms

```
lm.fit_interaction = lm(medv ~ lstat*age, data = boston) #The * is equivalent to lstat+age+lstat:age
summary(lm.fit_interaction)
```

```
##
## Call:
## lm(formula = medv ~ lstat * age, data = boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.806  -4.045  -1.333   2.085  27.552
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 36.0885359  1.4698355  24.553  < 2e-16 ***
## lstat       -1.3921168  0.1674555  -8.313 8.78e-16 ***
## age         -0.0007209  0.0198792  -0.036   0.9711
## lstat:age    0.0041560  0.0018518   2.244   0.0252 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.149 on 502 degrees of freedom
## Multiple R-squared:  0.5557, Adjusted R-squared:  0.5531
## F-statistic: 209.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

3.6.5 Non-linear Transformations of the Predictors

```
lm.fit_non_linear = lm(medv ~ lstat + I(lstat^2), data = boston)
summary(lm.fit_non_linear)
```

```
##
## Call:
## lm(formula = medv ~ lstat + I(lstat^2), data = boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.2834  -3.8313  -0.5295   2.3095  25.4148
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 42.862007   0.872084   49.15   <2e-16 ***
## lstat       -2.332821   0.123803  -18.84   <2e-16 ***
## I(lstat^2)   0.043547   0.003745   11.63   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.524 on 503 degrees of freedom
## Multiple R-squared:  0.6407, Adjusted R-squared:  0.6393
## F-statistic: 448.5 on 2 and 503 DF,  p-value: < 2.2e-16
```

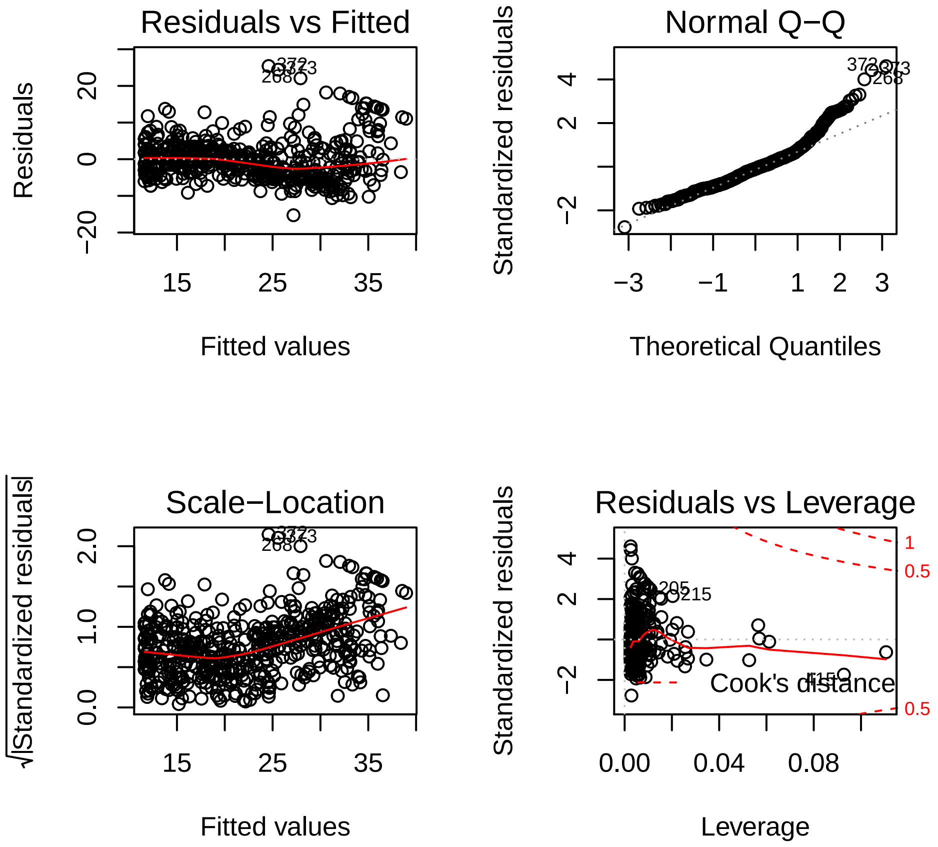
Analysis of variance

```
lm.fit = lm(medv ~ lstat, data = boston)
anova(lm.fit,lm.fit_non_linear)
```

```
## Analysis of Variance Table
##
## Model 1: medv ~ lstat
## Model 2: medv ~ lstat + I(lstat^2)
##   Res.Df   RSS Df Sum of Sq     F    Pr(>F)
## 1    504 19472
## 2    503 15347  1    4125.1 135.2 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual analysis

```
par(mfrow=c(2,2))
plot(lm.fit_non_linear)
```



Polynomial transformation

```
lm.fit_polynomial = lm(medv ~ poly(lstat,5), data = boston)
summary(lm.fit_polynomial)
```

```
##
## Call:
## lm(formula = medv ~ poly(lstat, 5), data = boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.5433  -3.1039  -0.7052   2.0844  27.1153
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)         22.5328     0.2318  97.197  < 2e-16 ***
## poly(lstat, 5)1 -152.4595     5.2148 -29.236  < 2e-16 ***
## poly(lstat, 5)2   64.2272     5.2148  12.316  < 2e-16 ***
## poly(lstat, 5)3  -27.0511     5.2148  -5.187 3.10e-07 ***
## poly(lstat, 5)4   25.4517     5.2148   4.881 1.42e-06 ***
## poly(lstat, 5)5  -19.2524     5.2148  -3.692 0.000247 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 5.215 on 500 degrees of freedom
## Multiple R-squared:  0.6817, Adjusted R-squared:  0.6785 
## F-statistic: 214.2 on 5 and 500 DF,  p-value: < 2.2e-16
```

Log transformation

```
lm.log = lm(medv ~ log(lstat), data = boston)
summary(lm.log)
```

```
## 
## Call:
## lm(formula = medv ~ log(lstat), data = boston)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -14.4599  -3.5006  -0.6686   2.1688  26.0129 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  52.1248     0.9652   54.00   <2e-16 ***
## log(lstat)  -12.4810     0.3946  -31.63   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 5.329 on 504 degrees of freedom
## Multiple R-squared:  0.6649, Adjusted R-squared:  0.6643 
## F-statistic:  1000 on 1 and 504 DF,  p-value: < 2.2e-16
```

3.6.6 Qualitative Predictors

```
fix(Carseats)
names(Carseats)
```

```
##  [1] "Sales"       "CompPrice"   "Income"      "Advertising" "Population" 
##  [6] "Price"       "ShelveLoc"   "Age"         "Education"   "Urban"      
## [11] "US"
```

```
carseats = Carseats
```

```
lm.fit_dummy = lm(Sales ~ . + Income:Advertising + Price:Age, data = Carseats)
summary(lm.fit_dummy)
```

```
## 
## Call:
## lm(formula = Sales ~ . + Income:Advertising + Price:Age, data = Carseats)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -2.9208 -0.7503  0.0177  0.6754  3.3413 
## 
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)    
## (Intercept)          6.5755654  1.0087470   6.519 2.22e-10 ***
## CompPrice            0.0929371  0.0041183  22.567  < 2e-16 ***
## Income               0.0108940  0.0026044   4.183 3.57e-05 ***
## Advertising          0.0702462  0.0226091   3.107 0.002030 ** 
## Population           0.0001592  0.0003679   0.433 0.665330    
## Price               -0.1008064  0.0074399 -13.549  < 2e-16 ***
## ShelveLocGood       4.8486762  0.1528378  31.724  < 2e-16 ***
## ShelveLocMedium     1.9532620  0.1257682  15.531  < 2e-16 ***
## Age                -0.0579466  0.0159506  -3.633 0.000318 ***
## Education          -0.0208525  0.0196131  -1.063 0.288361
## UrbanYes            0.1401597  0.1124019   1.247 0.213171
## USYes              -0.1575571  0.1489234  -1.058 0.290729
## Income:Advertising  0.0007510  0.0002784   2.698 0.007290 **
## Price:Age           0.0001068  0.0001333   0.801 0.423812
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.011 on 386 degrees of freedom
## Multiple R-squared:  0.8761, Adjusted R-squared:  0.8719
## F-statistic:   210 on 13 and 386 DF,  p-value: < 2.2e-16
```

```
attach(Carseats)
contrasts(ShelveLoc)
```

```
##        Good Medium
## Bad       0      0
## Good      1      0
## Medium    0      1
```

3.6.7 Writing Functions

```
LoadLibraries = function(){
  library(ISLR)
  library(MASS)
  print("The libraries have been loaded.")
}
```