Lab: Non-linear Modeling

Jonathan Bryan

June 27, 2018

```r
knitr::opts_chunk$set(fig.width=8, fig.height=5)
library(ISLR)
attach(Wage)
```

7.8.1 Polynomial Regression and Step Functions

```r
#with orthogonal polynomials basis
fit = lm(wage ~ poly(age,4), data=Wage)
coef(summary(fit))
```

```
##                  Estimate Std. Error    t value     Pr(>|t|)
## (Intercept)     111.70361  0.7287409 153.283015 0.000000e+00
## poly(age, 4)1   447.06785 39.9147851  11.200558 1.484604e-28
## poly(age, 4)2  -478.31581 39.9147851 -11.983424 2.355831e-32
## poly(age, 4)3   125.52169 39.9147851   3.144742 1.678622e-03
## poly(age, 4)4   -77.91118 39.9147851  -1.951938 5.103865e-02
```

```r
fit2 = lm(wage ~ poly(age,4, raw=TRUE), data = Wage)
coef(summary(fit2))
```

```
##                                 Estimate   Std. Error   t value
## (Intercept)                -1.841542e+02 6.004038e+01 -3.067172
## poly(age, 4, raw = TRUE)1   2.124552e+01 5.886748e+00  3.609042
## poly(age, 4, raw = TRUE)2  -5.638593e-01 2.061083e-01 -2.735743
## poly(age, 4, raw = TRUE)3   6.810688e-03 3.065931e-03  2.221409
## poly(age, 4, raw = TRUE)4  -3.203830e-05 1.641359e-05 -1.951938
##                                Pr(>|t|)
## (Intercept)                0.0021802539
## poly(age, 4, raw = TRUE)1 0.0003123618
## poly(age, 4, raw = TRUE)2 0.0062606446
## poly(age, 4, raw = TRUE)3 0.0263977518
## poly(age, 4, raw = TRUE)4 0.0510386498
```

```r
fit2a = lm(wage ~ age +
             I(age^2) +
             I(age^3) +
             I(age^4), data = Wage)
coef(fit2a)
```

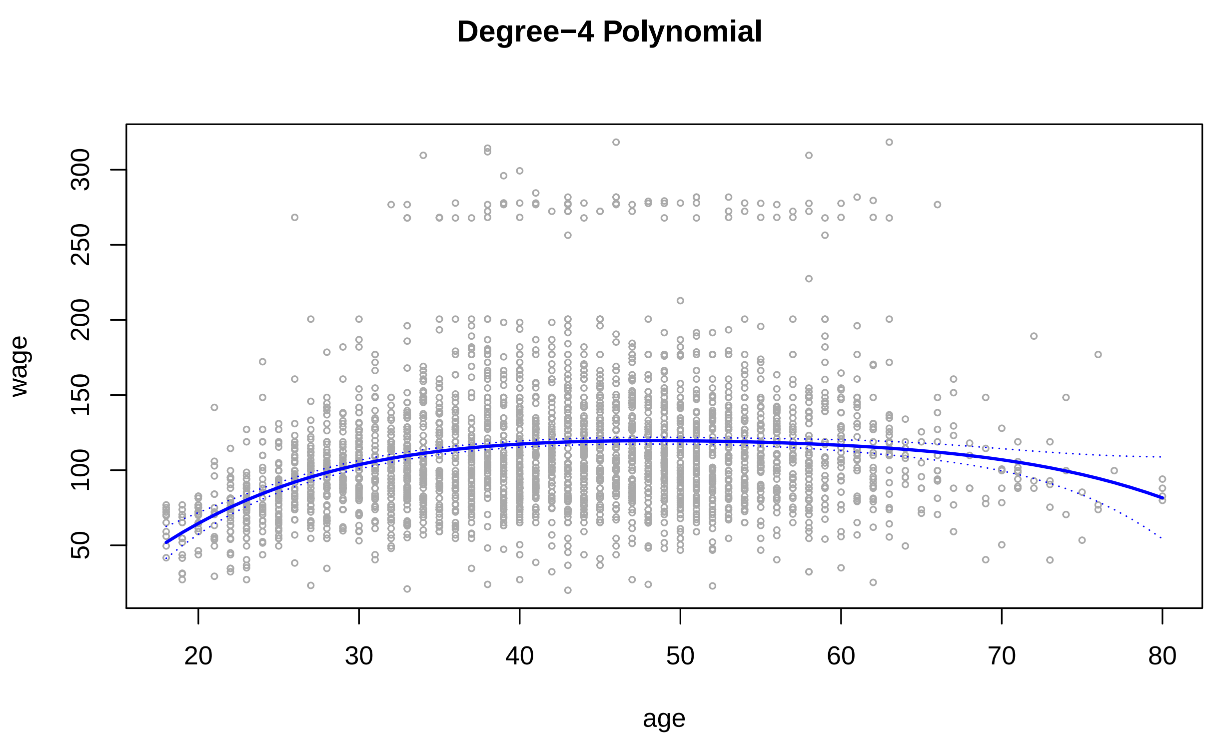
```
##   (Intercept)           age      I(age^2)      I(age^3)      I(age^4)
## -1.841542e+02  2.124552e+01 -5.638593e-01  6.810688e-03 -3.203830e-05
```

```r
fit2b = lm(wage ~ cbind(age,
                        age^2,
                        age^3,
                        age^4),
           data = Wage)
coef(fit2b)
```

```
##                         (Intercept) cbind(age, age^2, age^3, age^4)age
##                       -1.841542e+02                        2.124552e+01
##    cbind(age, age^2, age^3, age^4)    cbind(age, age^2, age^3, age^4)
##                       -5.638593e-01                        6.810688e-03
##    cbind(age, age^2, age^3, age^4)
##                       -3.203830e-05
```

```
#predicted wage values based on 4th degree poly
agelims = range(age)
age.grid = seq(from = agelims[1], to=agelims[2])
preds = predict(fit,newdata=list(age=age.grid), se=TRUE)
se.bands = cbind(preds$fit + 2*preds$se.fit, preds$fit - 2*preds$se)
```

```
#ploting predicted wage using 4th degree polynomial of age
par(mfrow = c(1,1),
    mar = c(4.5,4.5,1,1),
    oma=c(0,0,4,0))
plot(age, wage, xlim=agelims, cex = 0.5, col = "darkgrey")
title("Degree-4 Polynomial", outer = T)
lines(age.grid,preds$fit,lwd=2, col = "blue")
matlines(age.grid,se.bands, lwd=1,col="blue", lty = 3)
```




```
#difference between ortho polynomials and non-ortho polynomials
preds2 = predict(fit2,newdata=list(age=age.grid),se=TRUE)
max(abs(preds$fit - preds2$fit))
```

```
## [1] 7.81597e-11
```

```
#ANOVA testing to determine the best simpliest model
fit.1 = lm(wage ~ age, data = Wage)
fit.2 = lm(wage ~ poly(age,2), data = Wage)
```

```r
fit.3 = lm(wage ~ poly(age,3), data = Wage)
fit.4 = lm(wage ~ poly(age,4), data = Wage)
fit.5 = lm(wage ~ poly(age,5), data = Wage)
anova(fit.1,
      fit.2,
      fit.3,
      fit.4,
      fit.5)
```

```
## Analysis of Variance Table
##
## Model 1: wage ~ age
## Model 2: wage ~ poly(age, 2)
## Model 3: wage ~ poly(age, 3)
## Model 4: wage ~ poly(age, 4)
## Model 5: wage ~ poly(age, 5)
##   Res.Df     RSS Df Sum of Sq        F    Pr(>F)
## 1   2998 5022216
## 2   2997 4793430  1    228786 143.5931 < 2.2e-16 ***
## 3   2996 4777674  1     15756   9.8888  0.001679 **
## 4   2995 4771604  1      6070   3.8098  0.051046 .
## 5   2994 4770322  1      1283   0.8050  0.369682
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
coef(summary(fit.5))
```

```
##                  Estimate Std. Error     t value     Pr(>|t|)
## (Intercept)    111.70361  0.7287647 153.2780243 0.000000e+00
## poly(age, 5)1  447.06785 39.9160847  11.2001930 1.491111e-28
## poly(age, 5)2 -478.31581 39.9160847 -11.9830341 2.367734e-32
## poly(age, 5)3  125.52169 39.9160847   3.1446392 1.679213e-03
## poly(age, 5)4  -77.91118 39.9160847  -1.9518743 5.104623e-02
## poly(age, 5)5  -35.81289 39.9160847  -0.8972045 3.696820e-01
```

```r
fit.1 = lm(wage ~ education + age, data = Wage)
fit.2 = lm(wage ~ education + poly(age,2), data = Wage)
fit.3 = lm(wage ~ education + poly(age,3), data = Wage)
anova(fit.1,fit.2,fit.3)
```

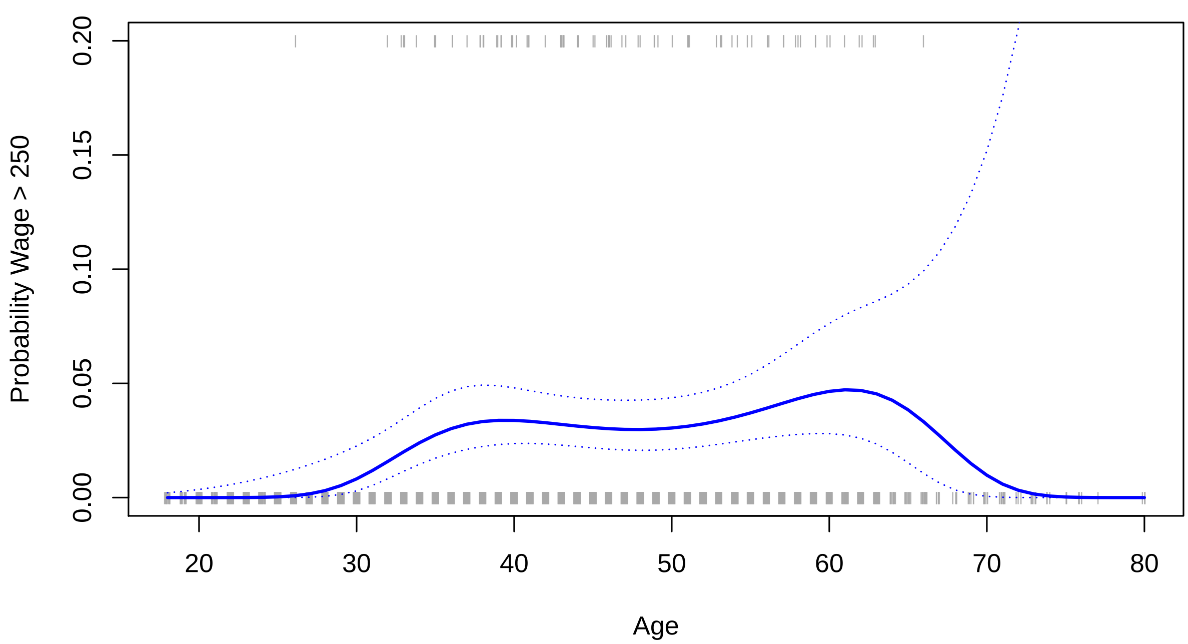
```
## Analysis of Variance Table
##
## Model 1: wage ~ education + age
## Model 2: wage ~ education + poly(age, 2)
## Model 3: wage ~ education + poly(age, 3)
##   Res.Df     RSS Df Sum of Sq        F Pr(>F)
## 1   2994 3867992
## 2   2993 3725395  1    142597 114.6969 <2e-16 ***
## 3   2992 3719809  1      5587   4.4936 0.0341 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
#logistic polynomial regression
fit = glm(I(wage>250) ~ poly(age,4), data=Wage, family = binomial)
preds = predict(fit,newdata=list(age=age.grid), se=TRUE)
pfit = exp(preds$fit)/(1+exp(preds$fit))
```

```r
se.bands.logit = cbind(preds$fit + 2*preds$se.fit,
                       preds$fit - 2*preds$se.fit)
se.bands = exp(se.bands.logit)/(1+exp(se.bands.logit))

preds = predict(fit,newdata=list(age=age.grid), type = "response", se = T)
```

```r
plot(age, I(wage>250),
     xlim = agelims,
     type = "n",
     ylim = c(0,0.2),
     ylab = "Probability Wage > 250",
     xlab = "Age")
points(jitter(age), I((wage>250)/5),
       cex=0.5,
       pch="|",
       col = "darkgrey")
lines(age.grid, pfit, lwd=2, col ="blue")
matlines(age.grid, se.bands,
         lwd=1,
         col = "blue",
         lty=3)
```




```r
table(cut(age,4))
```

```
##
## (17.9,33.5]   (33.5,49]   (49,64.5] (64.5,80.1]
##         750        1399         779          72
```

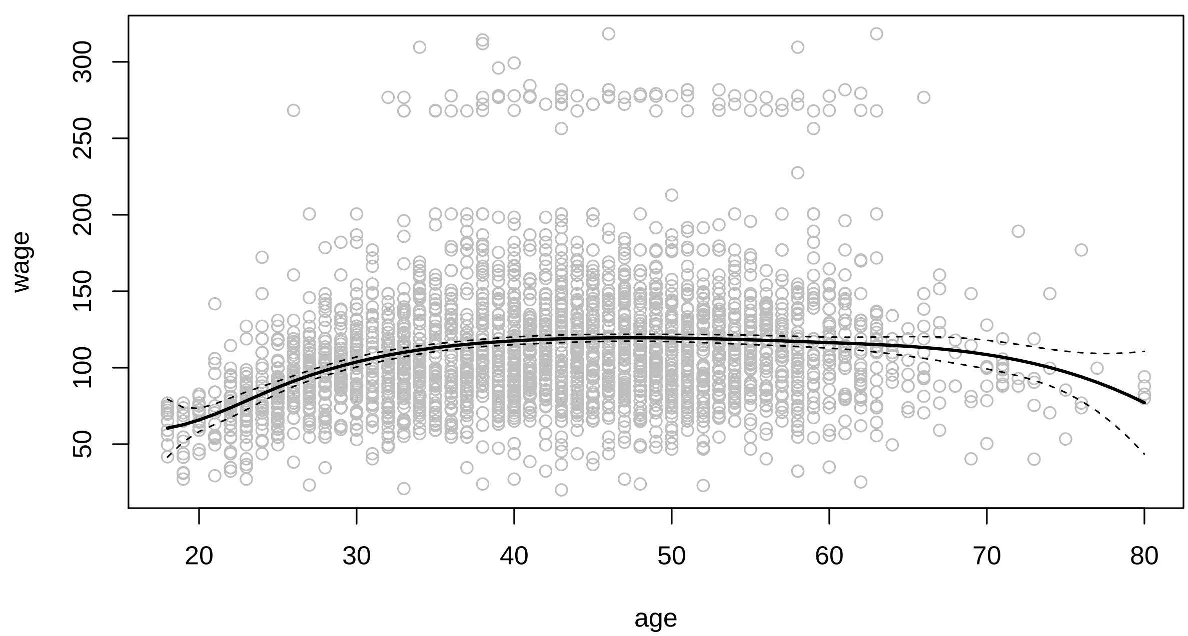
```r
fit=lm(wage~cut(age,4),data=Wage)
```

```r
coef(summary(fit))
```

```
##                         Estimate Std. Error   t value     Pr(>|t|)
## (Intercept)            94.158392   1.476069 63.789970 0.000000e+00
## cut(age, 4)(33.5,49]   24.053491   1.829431 13.148074 1.982315e-38
## cut(age, 4)(49,64.5]   23.664559   2.067958 11.443444 1.040750e-29
## cut(age, 4)(64.5,80.1]  7.640592   4.987424  1.531972 1.256350e-01
```

7.8.2 Splines

```r
#cubic regression spline with 3 knots
library(splines)
fit = lm(wage~bs(age,knots=c(25,40,60)),data=Wage)
pred = predict(fit, newdata = list(age=age.grid), se=TRUE)
plot(age,wage,col="gray")
lines(age.grid, pred$fit,lwd=2)
lines(age.grid, pred$fit+2*pred$se,lty="dashed")
lines(age.grid,pred$fit-2*pred$se,lty="dashed")
```




```r
dim(bs(age,knots=c(25,40,60)))
```

```
## [1] 3000    6
```

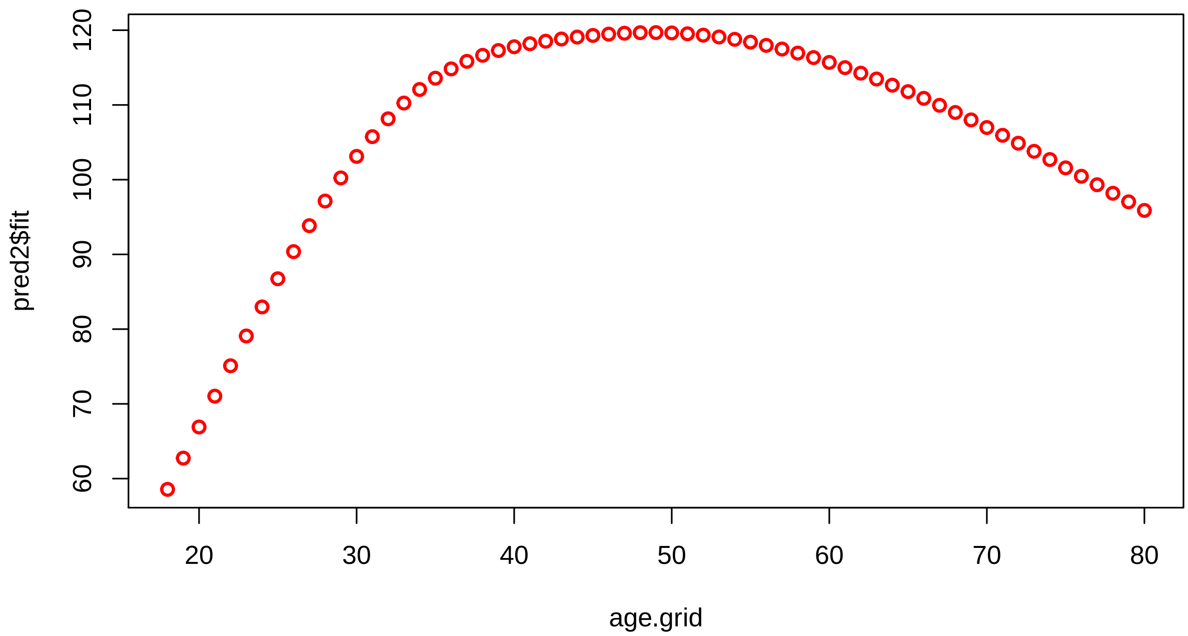
```r
dim(bs(age,df=6))
```

```
## [1] 3000    6
```

```r
attr(bs(age,df=6),"knots")
```

```
##   25%   50%   75% 
## 33.75 42.00 51.00
```

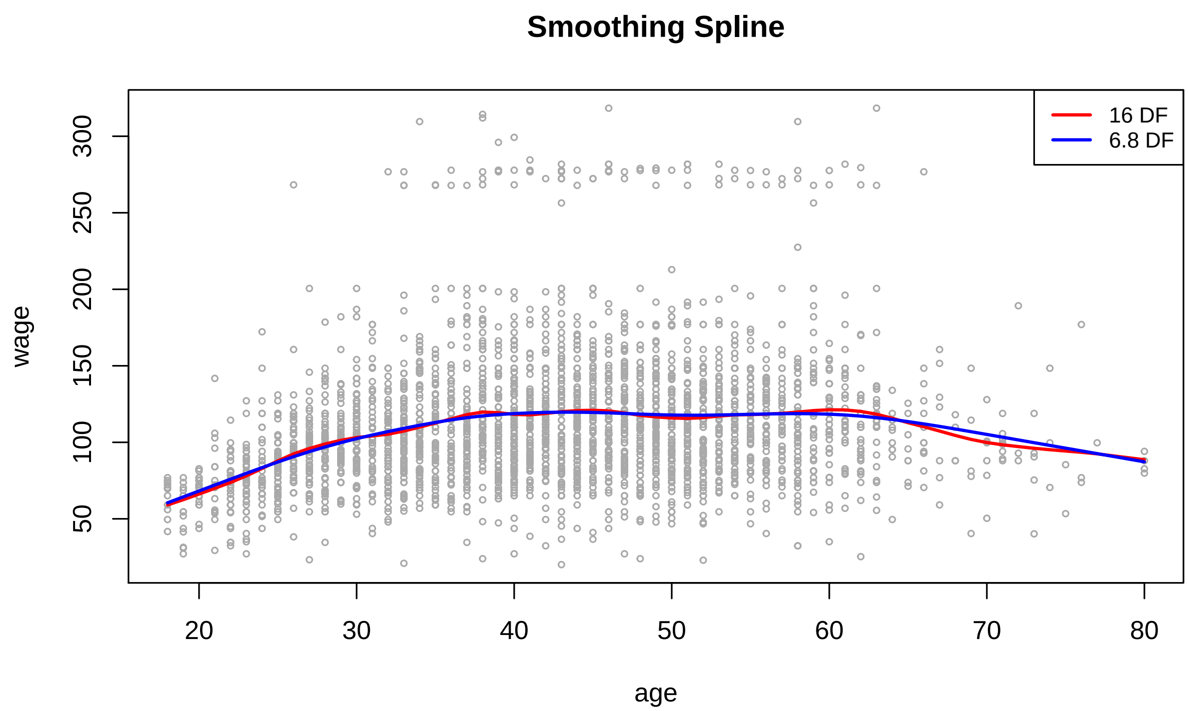
```r
#natural spline with 4 df
fit2 = lm(wage~ns(age,df=4),data=Wage)
pred2 = predict(fit2,newdata=list(age=age.grid),se=TRUE)
plot(age.grid, pred2$fit, col="red",lwd=2)
```



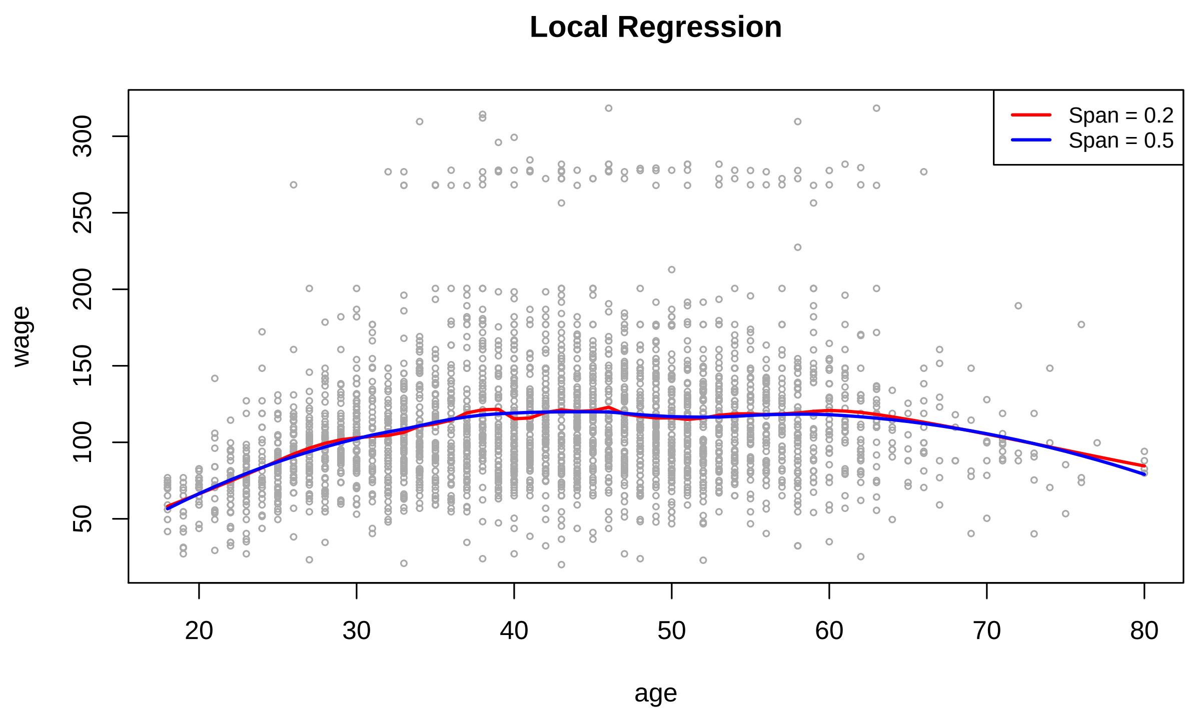

```r
#smoothing spline
plot(age, wage, xlim=agelims,
                cex = 0.5,
                col = "darkgrey")
title("Smoothing Spline")
fit = smooth.spline(age, wage, df=16)
fit2 = smooth.spline(age, wage, cv=TRUE)
fit2$df
```

```
## [1] 6.794596
```

```r
lines(fit,col="red",lwd=2)
lines(fit2,col="blue",lwd=2)
legend("topright", legend = c("16 DF",
                              "6.8 DF"),
       col = c("red","blue"),
       lty=1,
       lwd=2,
       cex = 0.8)
```




```
#local regression
plot(age,wage,
     xlim=agelims,
     cex = 0.5,
     col = "darkgrey")
title("Local Regression")
fit = loess(wage ~ age, span = 0.2, data = Wage)
fit2 = loess(wage ~ age, span = 0.5, data = Wage)
lines(age.grid, predict(fit,
                        data.frame(age=age.grid)),
      col="red",lwd=2)
lines(age.grid, predict(fit2,
                        data.frame(age=age.grid)),
      col="blue",lwd=2)
legend("topright", legend = c("Span = 0.2",
                              "Span = 0.5"),
       col = c("red", "blue"),
       lty = 1,
       lwd = 2,
       cex = 0.8)
```

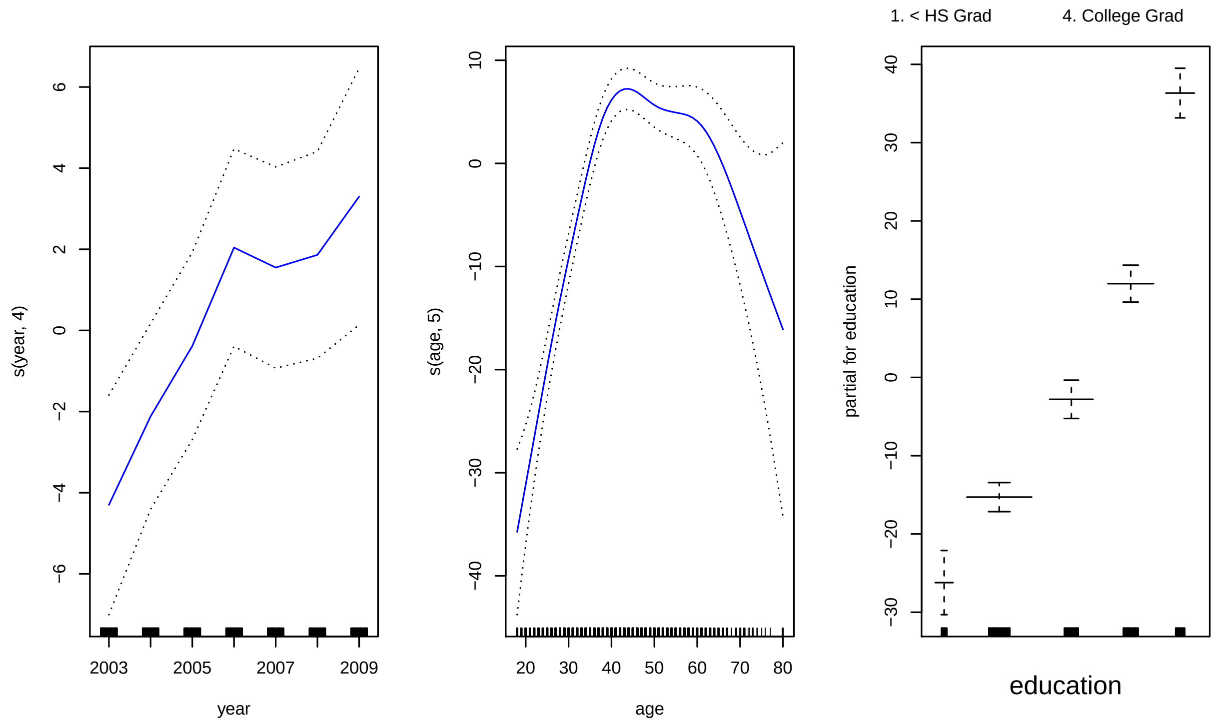


7.8.3 GAMs

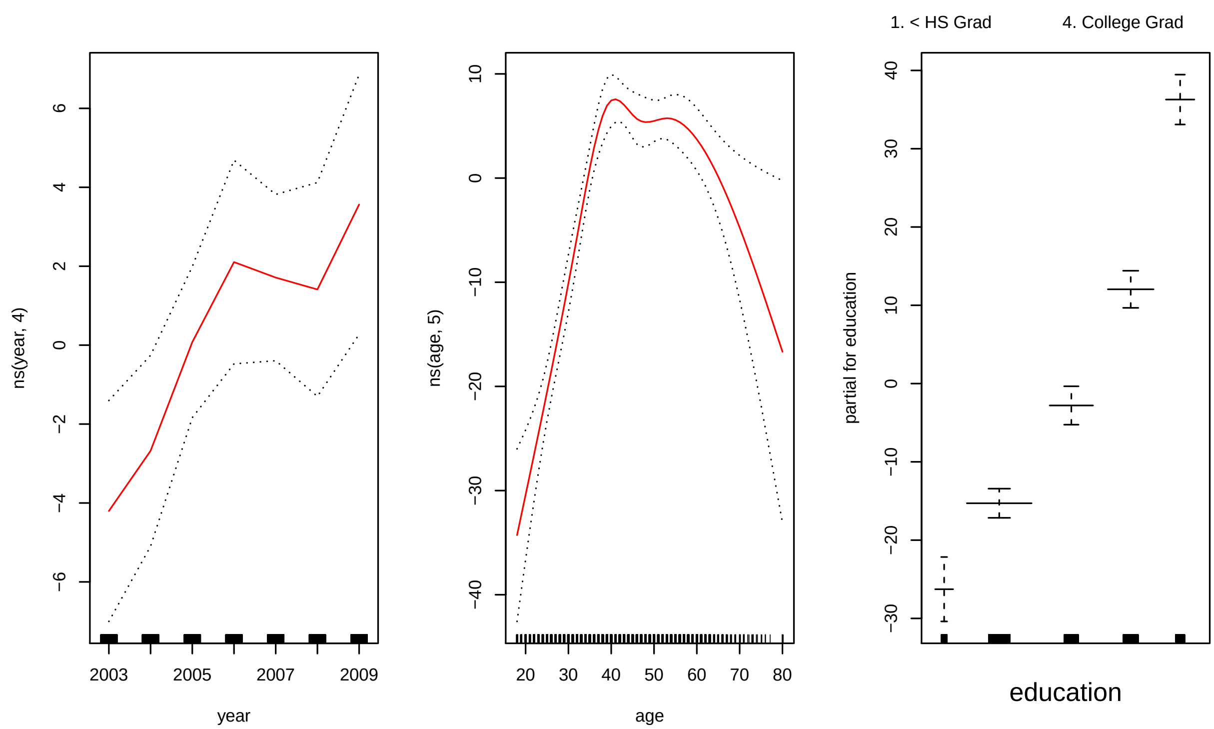
```
gam1 = lm(wage ~ ns(year,4) + ns(age,5) + education, data=Wage)

library(gam)
gam.m3 = gam(wage ~ s(year,4) + s(age,5) + education, data = Wage)

par(mfrow=c(1,3))
plot(gam.m3, se=TRUE, col="blue")
```




```r
plot.gam(gam1,se=TRUE, col="red")
```




```r
#ANOVA test to determine best functional form of year
gam.m1 = gam(wage ~ s(age,5) + education, data=Wage)
```

```r
gam.m2 = gam(wage~ year + s(age,5) + education, data= Wage)
anova(gam.m1, gam.m2, gam.m3, test = "F")
```

```
## Analysis of Deviance Table
##
## Model 1: wage ~ s(age, 5) + education
## Model 2: wage ~ year + s(age, 5) + education
## Model 3: wage ~ s(year, 4) + s(age, 5) + education
##   Resid. Df Resid. Dev Df Deviance       F    Pr(>F)
## 1      2990    3711731
## 2      2989    3693842  1  17889.2 14.4771 0.0001447 ***
## 3      2986    3689770  3   4071.1  1.0982 0.3485661
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

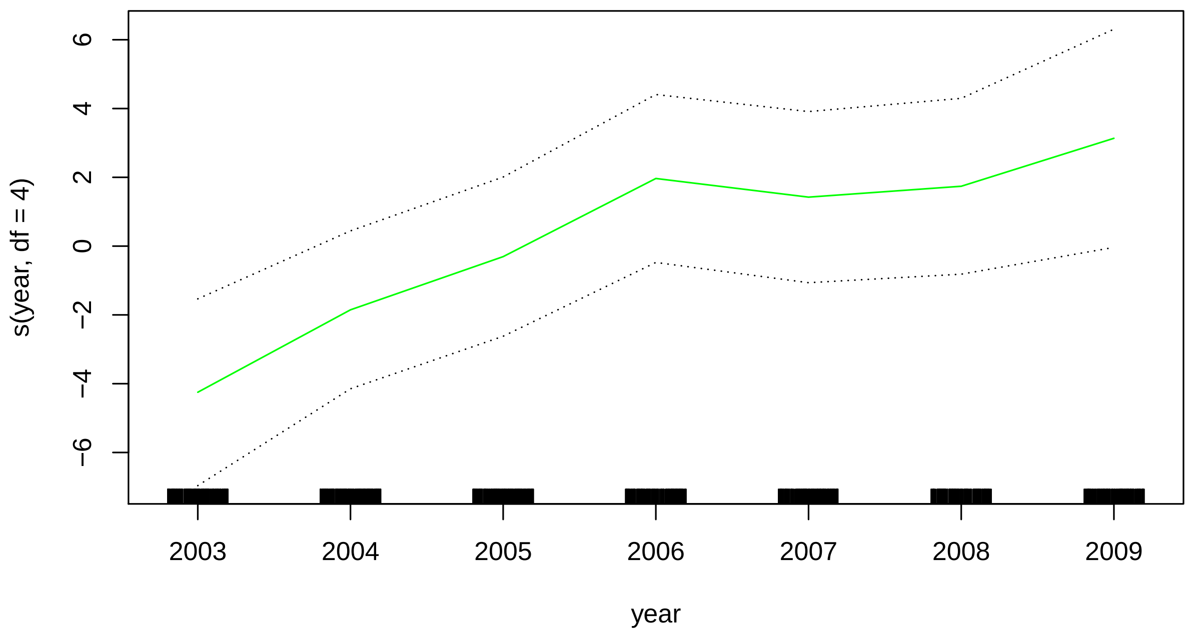
```r
#summary of GAM fit
summary(gam.m3)
```

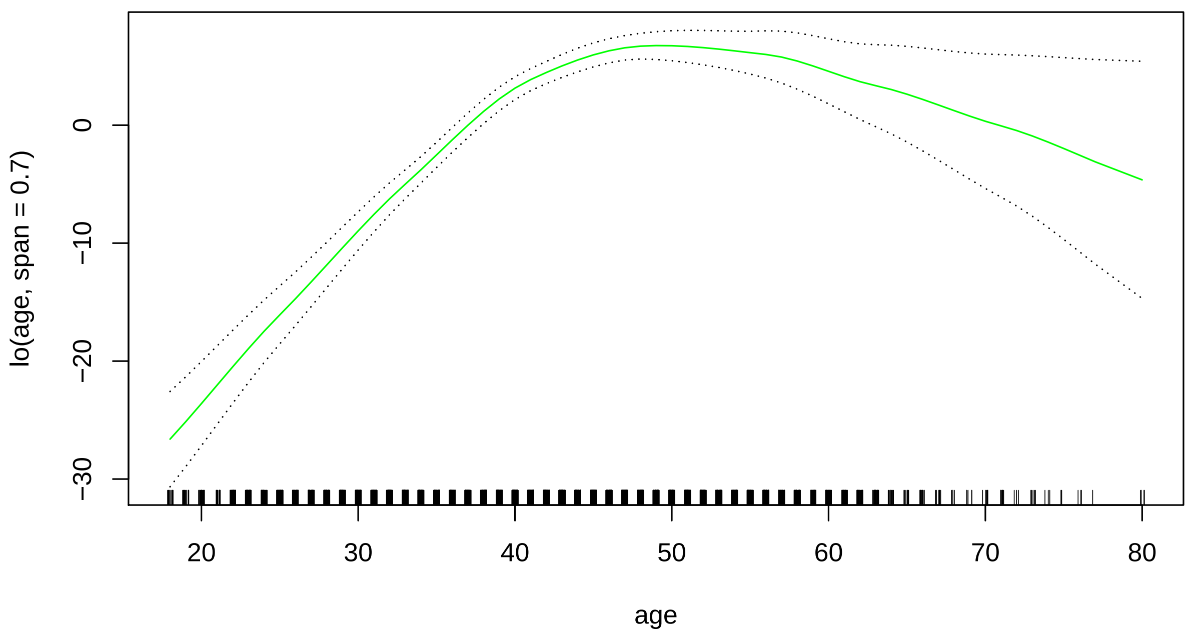
```
##
## Call: gam(formula = wage ~ s(year, 4) + s(age, 5) + education, data = Wage)
## Deviance Residuals:
##     Min      1Q  Median      3Q     Max
## -119.43  -19.70   -3.33   14.17  213.48
##
## (Dispersion Parameter for gaussian family taken to be 1235.69)
##
##     Null Deviance: 5222086 on 2999 degrees of freedom
## Residual Deviance: 3689770 on 2986 degrees of freedom
## AIC: 29887.75
##
## Number of Local Scoring Iterations: 2
##
## Anova for Parametric Effects
##               Df  Sum Sq Mean Sq F value    Pr(>F)
## s(year, 4)     1   27162   27162  21.981 2.877e-06 ***
## s(age, 5)      1  195338  195338 158.081 < 2.2e-16 ***
## education      4 1069726  267432 216.423 < 2.2e-16 ***
## Residuals   2986 3689770    1236
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##             Npar Df Npar F  Pr(F)
## (Intercept)
## s(year, 4)        3  1.086 0.3537
## s(age, 5)         4 32.380 <2e-16 ***
## education
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

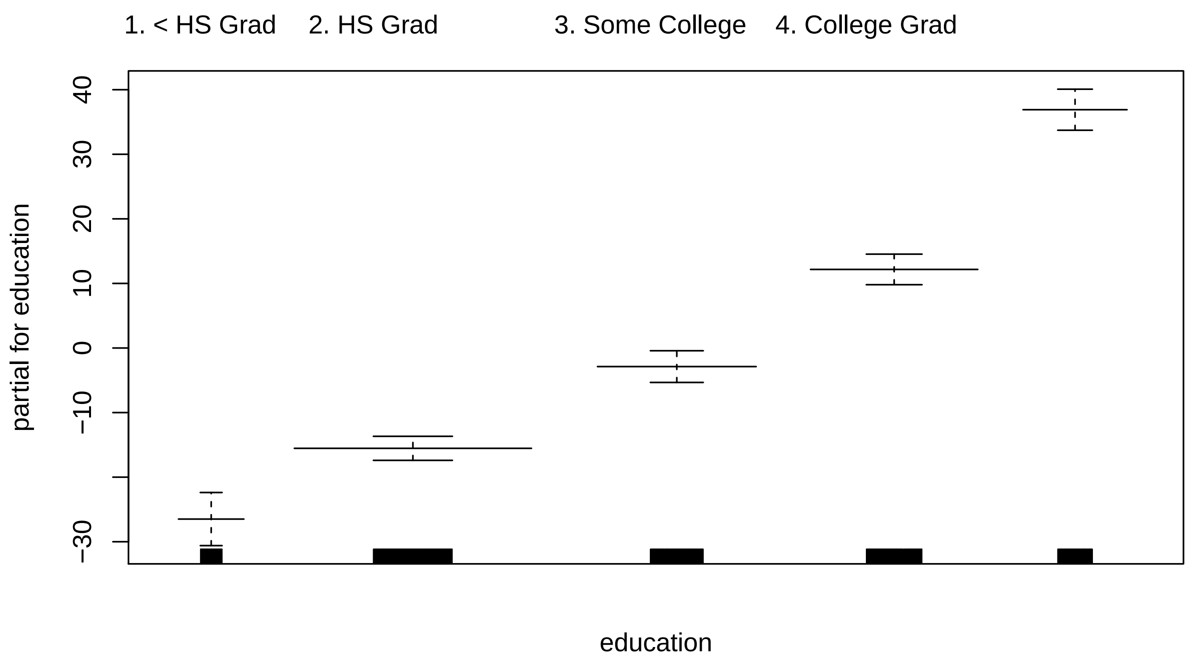
```r
#gam predictions
preds = predict(gam.m2, newdata = Wage)

#local regression pieces in gam
gam.lo = gam(wage ~ s(year, df = 4) + lo(age,span = 0.7) + education, data = Wage)
```

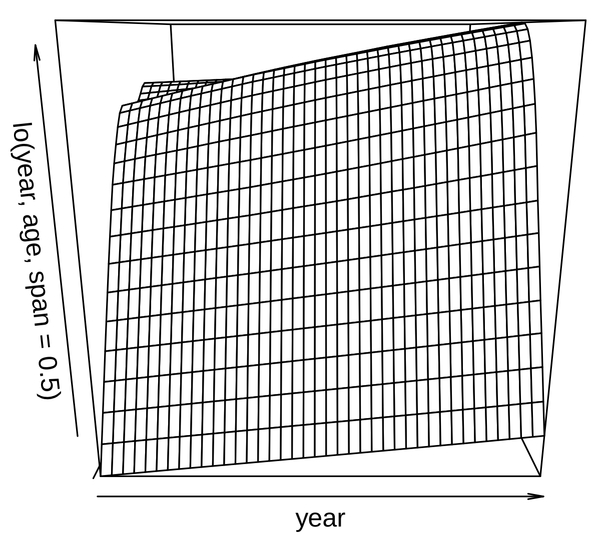
```r
plot.gam(gam.lo, se=TRUE, col = "green")
```

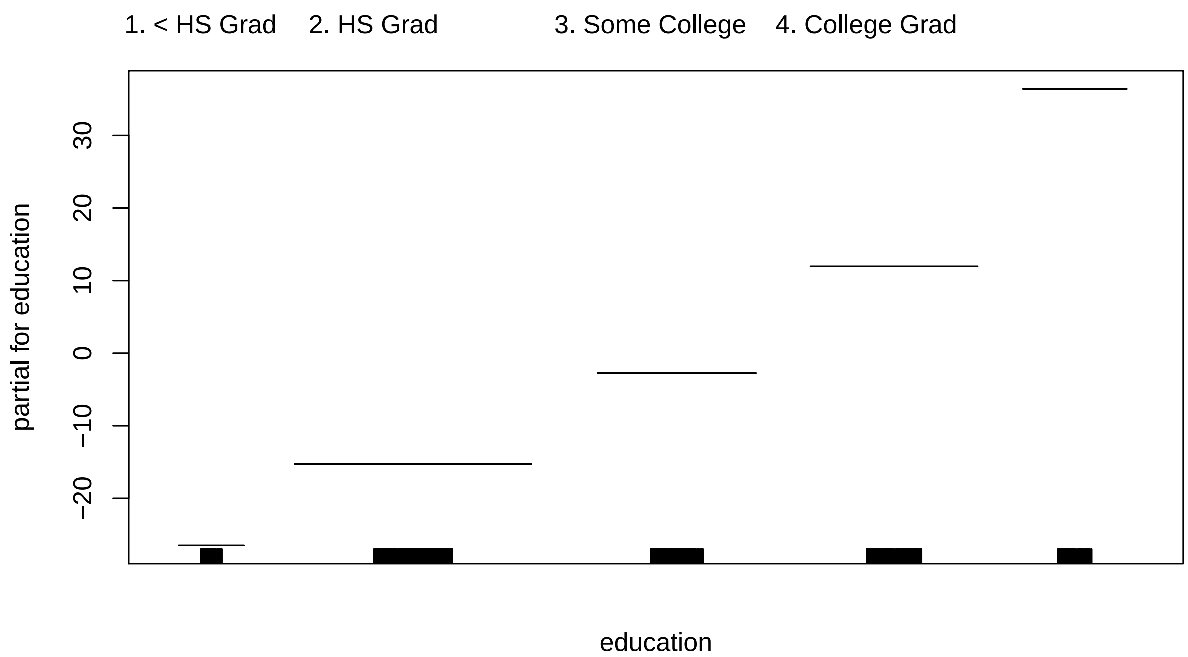




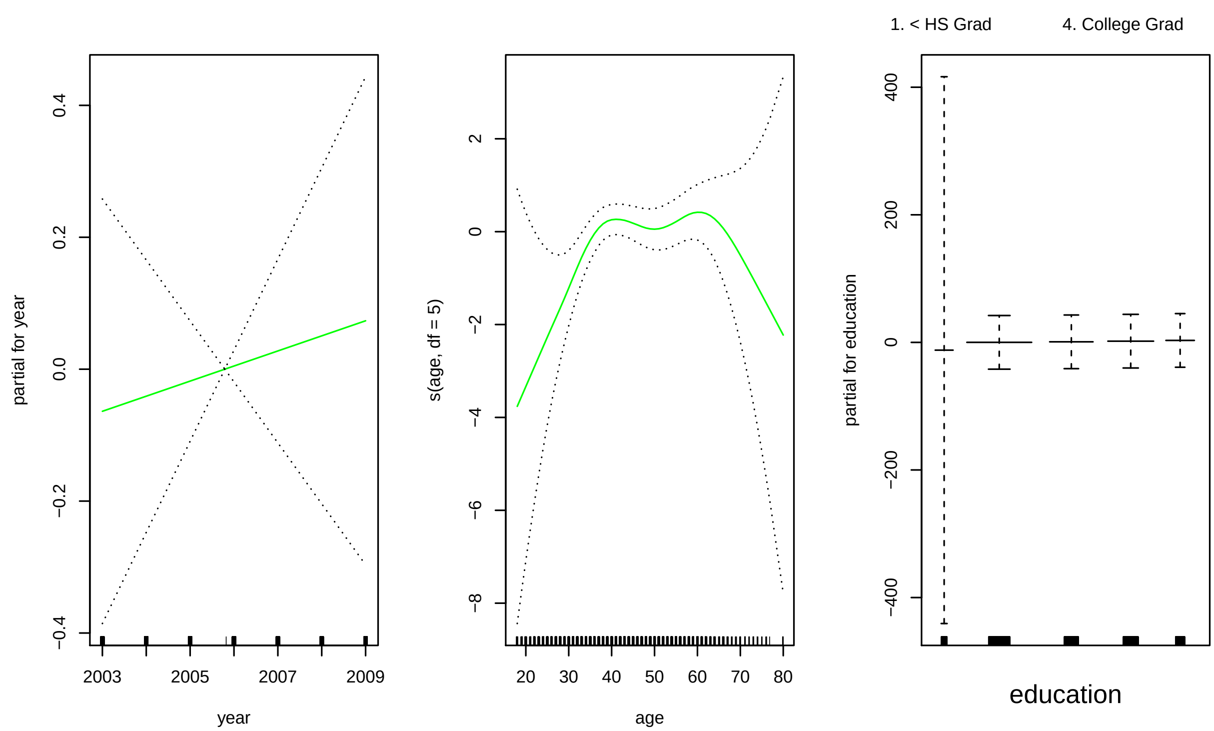



```r
gam.lo.i = gam(wage ~ lo(year, age, span = 0.5) + education, data = Wage)
library(akima)
plot(gam.lo.i)
```





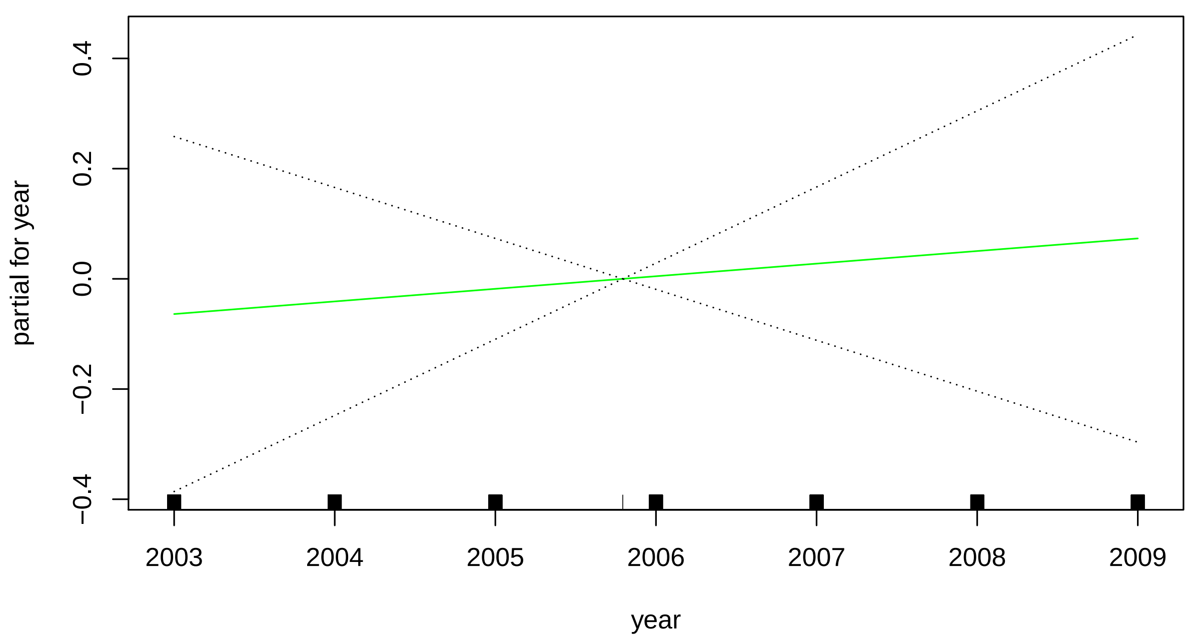

```r
#logistic regression gam
gam.lr = gam(I(wage) > 250 ~ year + s(age, df=5) + education, family = binomial, data = Wage)
par(mfrow=c(1,3))
plot(gam.lr, se=TRUE, col = "green")
```

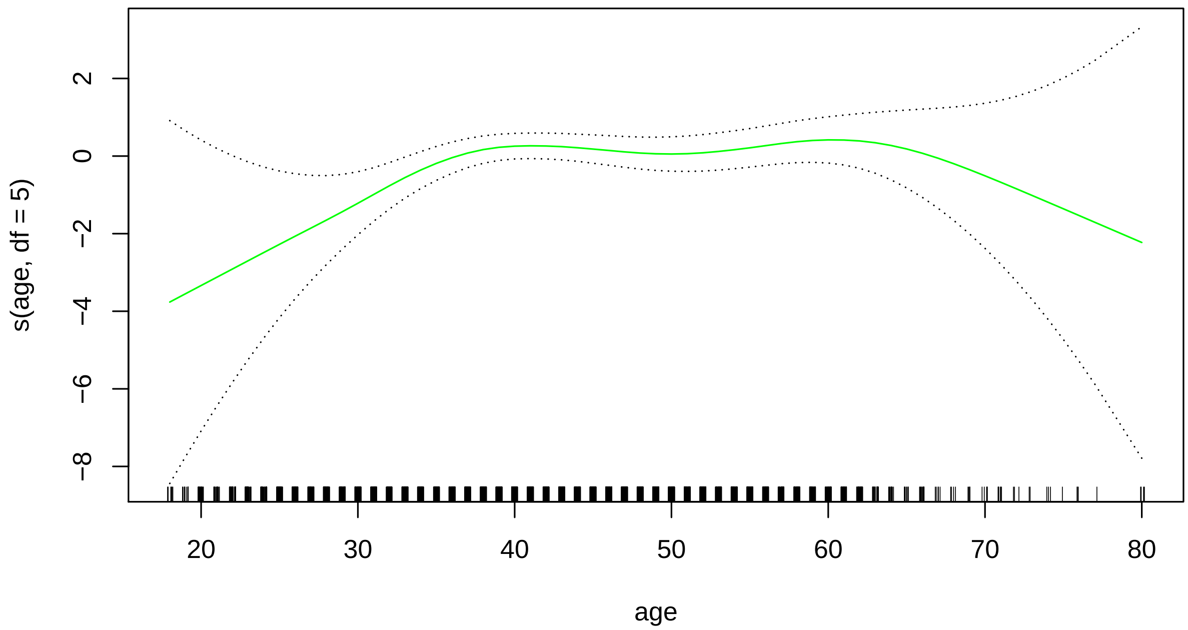


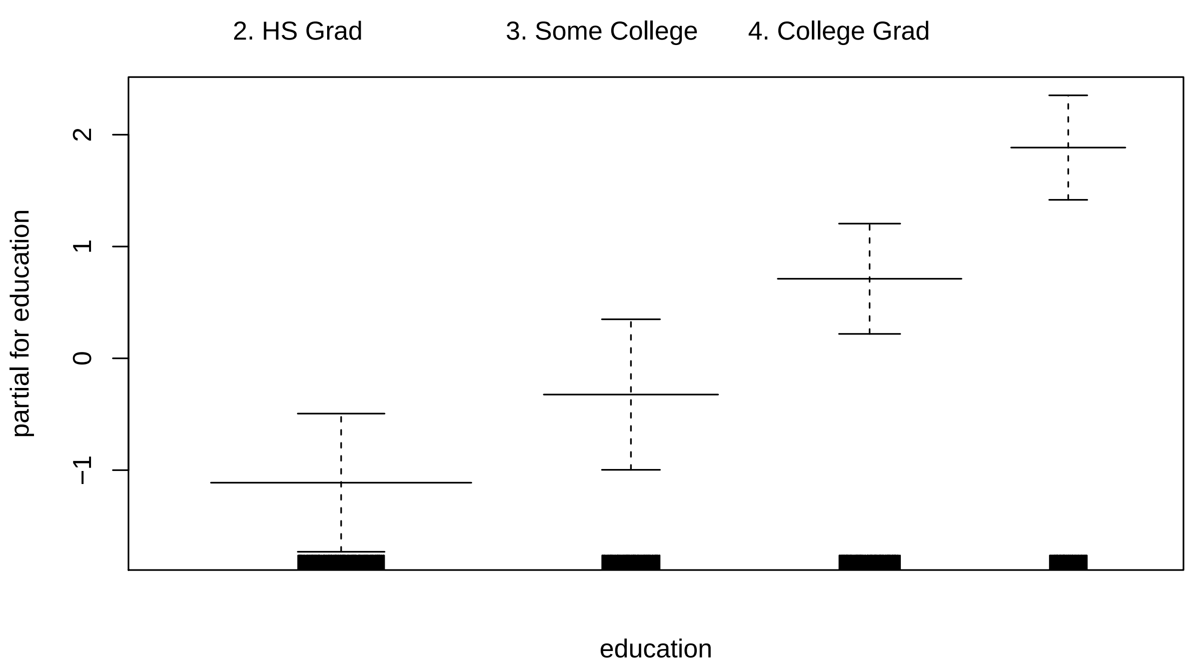
```r
table(education, I(wage > 250))
```

```
##
## education              FALSE TRUE
##   1. < HS Grad           268    0
##   2. HS Grad             966    5
##   3. Some College        643    7
##   4. College Grad        663   22
##   5. Advanced Degree     381   45
```

```r
#refit logistic gam removing those w/o HS degrees
gam.lr.s = gam(I(wage>250) ~ year +
                 s(age,df=5) +
                 education,
               family = binomial,
               data = Wage,
               subset= (education!="1. < HS Grad"))
plot(gam.lr.s, se=TRUE, col="green")
```




s(age, df = 5)
2
0
−2
−4
−6
−8
20
30
40
50
60
70
80
age


2. HS Grad
3. Some College
4. College Grad
partial for education
2
1
0
−1
education