Biomechanica_Notebook

November 4, 2022

1 General information

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- Date: Fri, 4th Nov.
- About: Data processing script for high-speed camera.

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2 Example: Analysis of a pendulum (SHO) using a high speed camera

1. Model description. 2. Load packages. 3. Import data. 4. Plot data. 5. Fit data to model. 6. Calculate derivatve. 7. Results. 8. Filter noise method 1. 9. Filter noise method 2. 10. Discussion.

2.1 Model

In this experiment, I took data from a simple (harmonic) pendulum using a high speed camera. The data shows us the x-coordinate of the position of the mass. We can describe the x-position using Newton's second law of motion and performing a force analysis.

Using a force analysis we arrive to the following differential equation:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin(\theta) = 0 \tag{1}$$

If we assume that our (initial) angle is very small we can approximate $\sin\theta \approx \theta$ and we arrive at

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0. \tag{2}$$

By integrating the differential equation (twice) we arrive to the solution, which has the form:

$$\theta(t) = A\sin(\omega t + \phi), \tag{3}$$

with $\omega^2 = \frac{g}{l}$. Finally, in the reference frame of the camera, the equation becomes:

$$x(t) = A\sin(\omega t + \phi) + x_0. \tag{4}$$

A is the amplitude of the pendulum (in the x-direction), ω is the frequency of the pendulum. The frequency depends on the ratio of the gravitational acceleration g and the length of the cord l (0.58 ± 0.01 m). ϕ is an arbitrary phase of the oscillation and x_0 is the offset.

Note that we assumed $\sin\theta \approx \theta$, this is something that we should treat with care during the experiment. Futhermore, we assumed that we release the pendulum with with zero initial speed ($\dot{\theta} = 0$), check this!

2.2 Load packages

I load the following packages: * numpy; * matplotlib.pyplot; * scipy.optimize * pandas;

We need these packages to analyze, plot and fit our data. These packages are in genereal very useful in doing numerical calculations with Python.

[ ]:
```
# %matplotlib widget #requires package ipympl installed, for interactive plots.
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import os
np.set_printoptions(precision=4,threshold =9,suppress=True)  #Compact display␣
 ↪of arrays.

cwd = os.getcwd() #get current working directory.
cwd= os.path.split(os.getcwd())[0]
```

2.3 Load, read and filter data from high speed camera

In this section we load, read and filter the data from the high speed camera. The output data is a .mqa extension, this is a program specific extension used by the tracking software. This extension is not a problem for Python 3 to handle.

- Line 1: make dataframe 'df' that contains all data.
- Line 3-5: change the numerical separator from a comma to a dot (In the Netherlands we use comma for numbers, international standard is a dot.). This is optional.
- Line 6-7: remove "Not-a-Number", nan, values from the list.
- Line 9-12: make a subselection of the data. Handling large datafiles can be very slow and annoying. This is optional ofcourse. Lastly, I need to rescale my camera-data from pixels to meters.

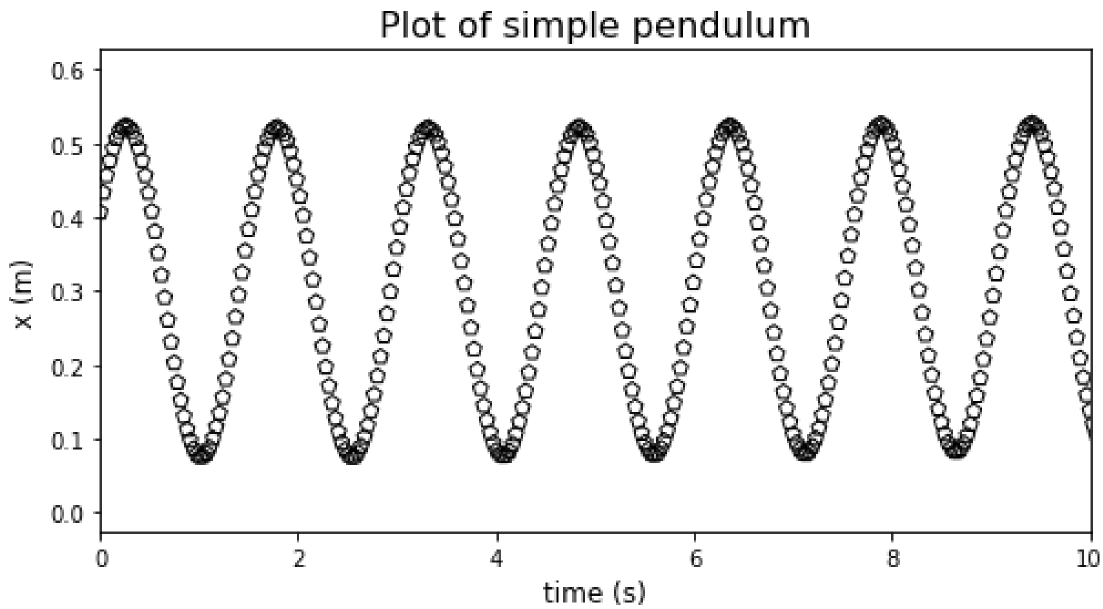
[ ]:
```
df=pd.read_csv(cwd+'/data/Slingerdata_1.mqa', sep='\t',header=0)

Time = df['Time'].str.replace(',', '.').astype(float).to_list()
Xpos = df['Point #1.X'].str.replace(',', '.').astype(float).to_list()
Ypos = df['Point #1.Y'].str.replace(',', '.').astype(float).to_list()

# Time = df['Time'] #comment if data is comma delimited.
# Xpos = df['Point #1.X'] #comment if data is comma delimited.
```

```
# Ypos = df['Point #1.Y'] #comment if data is comma delimited.

Xpos = [x for x in Xpos if str(x) != 'nan']
Ypos = [x for x in Ypos if str(x) != 'nan']

scaling = 1/1010 #scaling factor from pixel to [m]
t=np.asarray(Time[:15000])
x = np.asarray(Xpos[0:15000])*scaling
y = np.asarray(Ypos[0:15000])*scaling
```

2.4 Plot the data

Here I plot the raw pendulum data. On the y-axis I plot the x-position (m) of pendulum, on the x-axis I plot the time (s). I also added labels, limits and a title. These options are assumed to be self-explanatory.

[ ]:
```
fig, axes = plt.subplots(1,1,figsize=(8,4)) #Creates a single figure with␣
↪dimensions (8 by 4 (inch))
axes.scatter(t[::10],x[::10] ,color = 'black', s = 50, marker␣
↪='p',ec='black',fc='none') #plot every tenth datapoint t[::10], can be␣
↪modified.

#plot axis labels,title and adjust limits
axes.set_title('Plot of simple pendulum',fontsize=16)
axes.set_xlabel('time (s)',fontsize=12)
axes.set_ylabel('x (m)',fontsize=12)
axes.set_xlim(0,10)
axes.set_ylim(np.min(x)-0.1,np.max(x)+0.1)

plt.show()
```



2.5 Fit data to model

- Line 1-2: Here we define our model (sinus_model()) using a Python function: a function starts with def and ends with return.

Sinus_model() is our user defined model based on the theory. We vary the time coordinate (the independent variable, t) and the parameters (x_0, A, ω and ϕ) are determined by a fit. Be careful that the independent variable (t) must appear before the parameters, otherwise an error message will appear.

- Line 4: Apply a fit using the curve_fit function. Curve_fit documentation

Curve_fit asks for a minimum of three input arguments (model, independent variable (t), dependent variable (x)), more input arguments are optional. In our case, I provide an initial guess of the optimal parameters (p_0) and I provide bounds to the parameters. Other options include: weights, method, etc. Note that providing a good initial guess of p_0 and apply parameter bounds significantly reduces the complexity of finding the optimal parameters, if possible, provided them! The output of curve_fit are the optimal parameters popt and the covariance matrix $\mathcal{K}_{p_i p_j}$ pcov, where $\mathcal{K}_{p_i p_j}$ is defined as:

$$\mathcal{K}_{p_i p_j} = \begin{bmatrix} s_{p_a p_a} & s_{p_b p_a} \\ s_{p_a p_b} & s_{p_b p_b} \end{bmatrix} \tag{5}$$

For example, $s_{p_a p_a}$ is the (co)variance of parameter a. To obtain the standard deviation σ_a of parameter a we take the square root of the variance

$$\sigma_a = \sqrt{s_{p_a p_a}} \tag{6}$$

- Line 6-8: Show the optimal values and corresponding standard deviations.

[ ]:
```python
from scipy.optimize import curve_fit

def sinus_model(t,x0,A,omega,phi):
    return x0+A*np.sin(omega*t+phi)

popt, pcov = curve_fit(sinus_model,t,x,p0=[0.3,0.23,4.12,0.5],bounds = ([0.1,0.2,2,0],[0.5,0.5,10,2*np.pi]))

parameter=['Offset (m)', 'Amplitude (m)', 'Frequency (rad/s)', 'phase (rad)']
for i in range (4):
    print("The optimal value for ", parameter[i], 'is:', "{:.2f}".format(popt[i]), '\u00B1', "{:.5f}".format(np.sqrt(pcov[i,i])))
```

```
The optimal value for  Offset (m) is: 0.30 ± 0.00003
The optimal value for  Amplitude (m) is: 0.22 ± 0.00005
The optimal value for  Frequency (rad/s) is: 4.12 ± 0.00002
The optimal value for  phase (rad) is: 0.48 ± 0.00043
```

- Line 10-15: Make figure, plot the data (black) and the fit (orange).

We plot the fit by making a new dataset, called tfit. We evaluate the the function using the optimal parameters at the values of tfit.

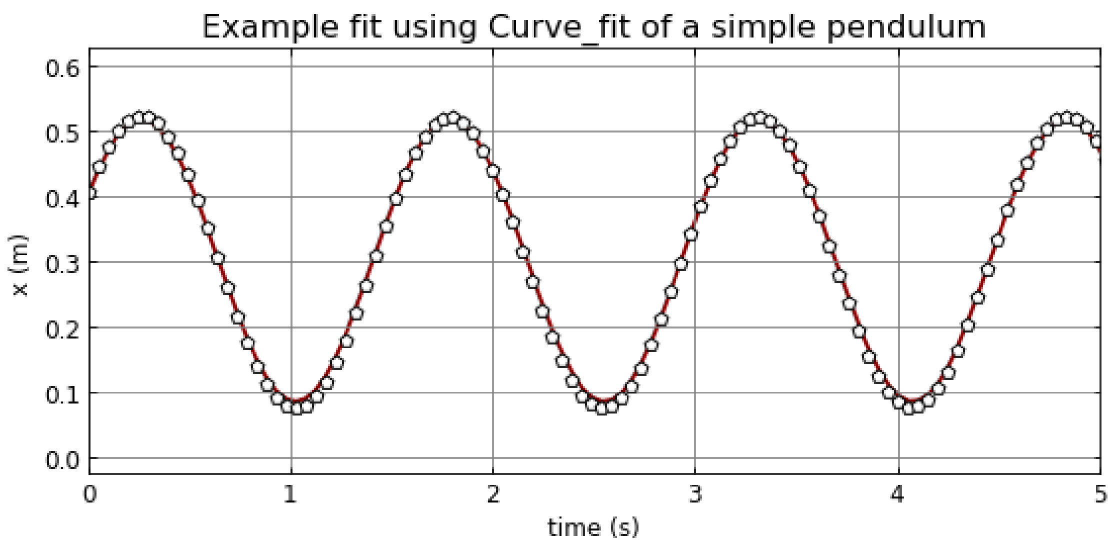
- Line 17-25: Optional commands.
- Line 27-28: Save the figure.

Note that the figure is saved as a .svg extension. A .svg extension stands for Scalable Vector Image, the image is saved as an object (and not as a collection of pixels). One can import .svg picture in e.g. inkscape, powerpoint or adobe photoshop to manipulate these, try it!.

- Line 35-37: Calculation of the gravitational acceleration, the error on g is calculated using the general rules for error propagation.

[ ]:
```python
xfit = np.linspace(0,max(t),1000) #create new data for fit.

#Plot data + fit
fig,axes=plt.subplots(1,1,figsize=(8,4),sharex=True,sharey=True)
axes.plot(xfit,sinus_model(xfit,*popt), color ='darkred', lw = 2,zorder =1) #plot fit
axes.scatter(t[::15],x[::15] ,color = 'black', s = 50, marker ='p',ec='black',fc='white',zorder=2) #plot every 15th datapoint

#plot axis labels and limits
axes.set_title('Example fit using Curve_fit of a simple pendulum',fontsize=16)
axes.set_xlabel('time (s)',fontsize=12)
axes.set_ylabel('x (m)',fontsize=12)
axes.set_xlim(0,5)
axes.set_ylim(np.min(x)-0.1,np.max(x)+0.1)
```

```
#Some plot settings
axes.tick_params(direction="in",labelsize=12,bottom = True,top = True,left=␣
↪True,right=True) #inward direction of tick-lines
axes.grid(True,color='gray')
plt.tight_layout() #creates optimal padding levels for figure (especially␣
↪usefull for side-by-side figures)

# location='Path' #Path to your prefered location, e.g. User/Downloads/
# plt.savefig(location+'simple_pendulum.svg') #Extension can be .png/.jpg/.etc␣
↪or .svg/.pdf (Vector Image)

plt.show()

#Calculate g + error
l,omega,dl,domega=0.58,popt[2],0.01,np.sqrt(pcov[2][2])
print('The gravitational acceleration g [$m/s^2$] = ', "{:.2f}".
↪format(omega**2*l), "\u00B1",
      "{:.2f}".format(np.sqrt((2*omega*l*domega)**2+(omega**2*dl)**2)))
```




```
The gravitational acceleration g [$m/s^2$] =  9.84 ± 0.17
```

2.5.1 Optional: Residuals

In the previous section we fit a model to our data. We observe that the sinusoidal fit captures the data very well. However, we are also interested in the limitations of the fit. To spot these limitations can be cumbersome, to simplify our life, we calculate the difference between the data and the fit: the residuals. To visualize the residuals I will make a grid using gridspec. Using gridspec we can make two subfigures, a main figure and a sub (residuals) figure. In the main figure

I plot the data and optimal fit. In the residuals plot I plot the difference of the data and the fit, the residuals ϵ of the data:

$$\epsilon = y_{data} - y_{model} \tag{7}$$

The residual plot shows valuable information. For example, a systematic pattern in the residuals shows that our model cannot capture the full data. In principle, we should modify our, thereby correcting for the systematic error.

[ ]:
```
import matplotlib.gridspec as gridspec

fig = plt.figure(figsize=(8,6))
gs = gridspec.GridSpec(5, 4) #Creates a grid of 6 rows and 4 columns.
axes_main = plt.subplot(gs[1:5, :4]) #Main axis goes from row 1 to 6.
axes_residuals = plt.subplot(gs[0, :4],sharex=axes_main) #Residual axis is row␣
↪0.

axes_main.tick_params(axis='both',direction ='in',labelsize=12)
axes_main.scatter(t[::15],x[::15],color='black', marker =␣
↪'p',ec='black',fc='white',s=50,zorder=3)
axes_main.plot(t,sinus_model(t,*popt),color='darkred',ls='--', lw=2)
axes_main.set_xlabel('t (s)',fontsize=16)
axes_main.set_ylabel('x-position (m)',fontsize=16)
axes_main.set_xlim(np.min(t),5)

axes_residuals.tick_params(axis='both',direction ='in',labelsize=12)
axes_residuals.set_title('Residual plot of Pendelum',fontsize=16)
axes_residuals.errorbar(t,x-sinus_model(t,*popt),xerr=0,yerr=0,fmt='none',
                        color='black',ecolor='black',capsize=3)
axes_residuals.hlines(0,np.min(t),np.max(t),color='black',ls='--')
axes_residuals.set_ylabel(r'$\epsilon$ (m)',fontsize=16)
axes_residuals.set_xlim(np.min(t),5)

plt.tight_layout()
# plt.show()
plt.close()
```

2.6 Calculate derivative (nummerical & analytical)

In the previous section we plotted the data of the simple pendulum. The data looks very smooth and we can described the data (very well) by a simple pendulum model. However, in most situations, we are not interested in the position of the object, but we are interested in the velocity or (even) the acceleration of an object. If one has a (analytical) function that describes the position as a function of time, we can simply derive the velocity (or acceleration) by taking the (second) derivative with respect to time. Therefore

$$x(t) = A\sin(\omega_n t + \phi) + x_0 \tag{8}$$

$$v(t) = \frac{dx(t)}{dt} = \omega_n A \cos(\omega_n + \phi) \tag{9}$$

$$a(t) = \frac{d^2x(t)}{dt^2} = -\omega_n^2 A \sin(\omega_n + \phi) \tag{10}$$

The velocity (v) and acceleration (a) can be approximated using the estimated parameters (previous section).

However, in most (realistic) (bio)mechanical systems, we do not have an exact model that describes our measurements. We are forced to calculate a derivative using nummerical methods. A straigthforward method to calculate the derivative of a dataset is to use the Euler forward, backward or central method. The Euler central method works as following:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \tag{11}$$

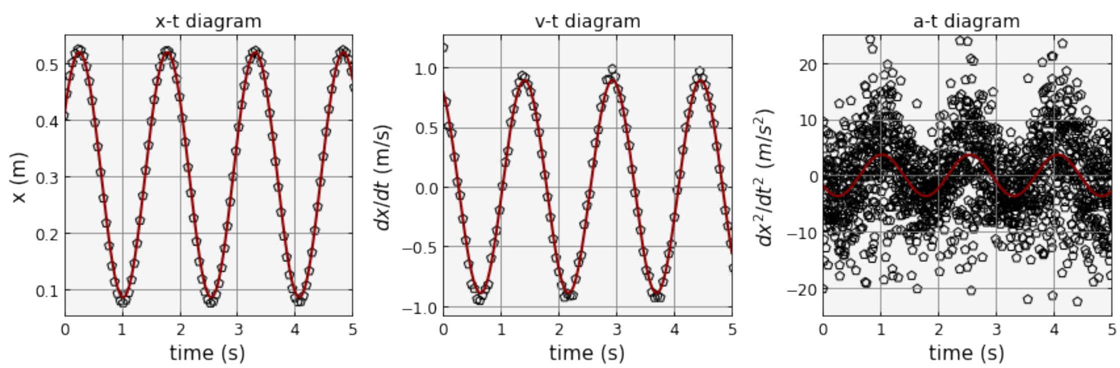
An intuitive explanation is that we approximate the derivative by evaluating the function $f(x)$ at $f(x+h)$ and $f(x-h)$, with h the stepsize. Next, we take the difference of these expressions and divide by twice the stepsize. The smaller the stepsize we use, the better the approximation will be! Other (more advanced) methods to approximate a derivative of a function f(x) are based on higher-order derivatives. These methods are ofter more precise (or faster), but are cumbersome to implement.

In Python, we can take the gradient function to calculate the derivative. It uses the Euler central method. Note that at the boundaries $t = t_0$ or $t = t_{final}$, the central method fails to work. However, the numpy gradient function switches to Euler forward/backward method to compensate for the loss of data at the edges.

The following links can provide some more information about the used methods. * Numpy Gradient function * Nummerical Differentiation using Python * Line 1-2: I take the derivative of the position data with respect to time (line 1). In line 2 I take the second derivative, to obtain the acceleration. * Line 4-12: I make a figure of 1 row and 3 columns. I plot the x-t data (black pentagons) (panel 1) and model (solid line, darkred). In panel 2 and 3 I plot the velocity and acceleration versus time. * Line 14-33: Some aesthetic aspects of the plot.

[ ]:
```
der1 = np.gradient(x,t)
der2 = np.gradient(der1,t)

fig,axes=plt.subplots(1,3,figsize=(12,4))
axes[0].scatter(t[::15],x[::15],color = 'black', s = 50, marker='p',ec='black',fc='none') #plot every tenth datapoint [::10]
axes[0].plot(t,sinus_model(t,*popt),color='darkred',lw=2)

axes[1].scatter(t[::15], der1[::15], color='black', s = 50, marker='p',ec='black',fc='none')
axes[1].plot(t,popt[2]*popt[1]*np.cos(popt[2]*t+popt[3]),color='darkred',lw=2)
```

```
axes[2].scatter(t[::], der2[::], color='black', s = 50, marker='p',ec='black',fc='none')
axes[2].plot(t,-popt[2]**2*popt[1]*np.sin(popt[2]*t+popt[3]),color='darkred',lw=2)

axes[2].set_ylim(-25,25)
axes[0].set_title('x-t diagram',fontsize=14)
axes[1].set_title('v-t diagram',fontsize=14)
axes[2].set_title('a-t diagram',fontsize=14)

for i in range(3):
    axes[i].set_xlim(0,5)
    axes[i].set_xlabel('time (s)',fontsize=15)
    axes[i].tick_params(direction="in",labelsize=12,bottom = True,top = True,left= True,right=True) #inward direction of tick-lines
    axes[i].set_facecolor('whitesmoke')
    axes[i].grid(True,color='gray')

axes[0].set_ylabel('x (m)',fontsize=15)
axes[1].set_ylabel(r'$dx/dt$ (m/s)',fontsize=15)
axes[2].set_ylabel(r'$dx^2/dt^2$ $(m/s^2)$',fontsize=15)
plt.tight_layout()
plt.show()

# location='user_defined_location'
# plt.savefig('location'+'simple_pendulum.svg')
```



2.7 Results

In the previous figure, we observe that the x-t and v-t diagram still look reasonable. However, the a-t diagram is very chaotic. It is difficult to observe the expected (analytical) acceleration. Before we try to resolve this issue, we first have to understand it a bit better.

The noisy data is a result of the method to calculate a derivative:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \tag{12}$$

In our experiment, we use a high-speed camera, that produces (around) 500 fps. This means that we have a timestep of h = 1/fps = 0.002 seconds. Any observed difference in x(t) is magnified in the calculation of the velocity by ~h^-1 and even by ~h^-2 for the acceleration. This is in principle not a problem of the method, but of the finite resolution of the camera. If we would have an even faster acquistion rate (fps) we could in principle obscure single pixel differences between frames. The downside is that, between these frames the acceleration is $a = 1\ /h^2\ [pixel/s^2]$. The acceleration that we obtain therefore depends on the resolution of the camera.

In practice, we also have to deal with noise, this makes the method to calculate a derivative even more problematic. To resolve these issues we can do the following methods: * Decrease resolution of temporal data; * Apply a smoothing technique to reduce the effect of noise.

Both methods decrease the resolution of the temporal data. In most (bio)mechanical experiments, this is not an issue.

Note that these are not the only two options, more advanced filtering techniques excist e.g. spectral filtering, kernel smoothing, etc., but these are tedious to implement and beyond the scope of the practical course.

For further reading I recommend the following papers: * Numerical differentiation of experimental data * Numerical differentiation of noisy data

2.8 Method 1: Change temporal resolution (easy)

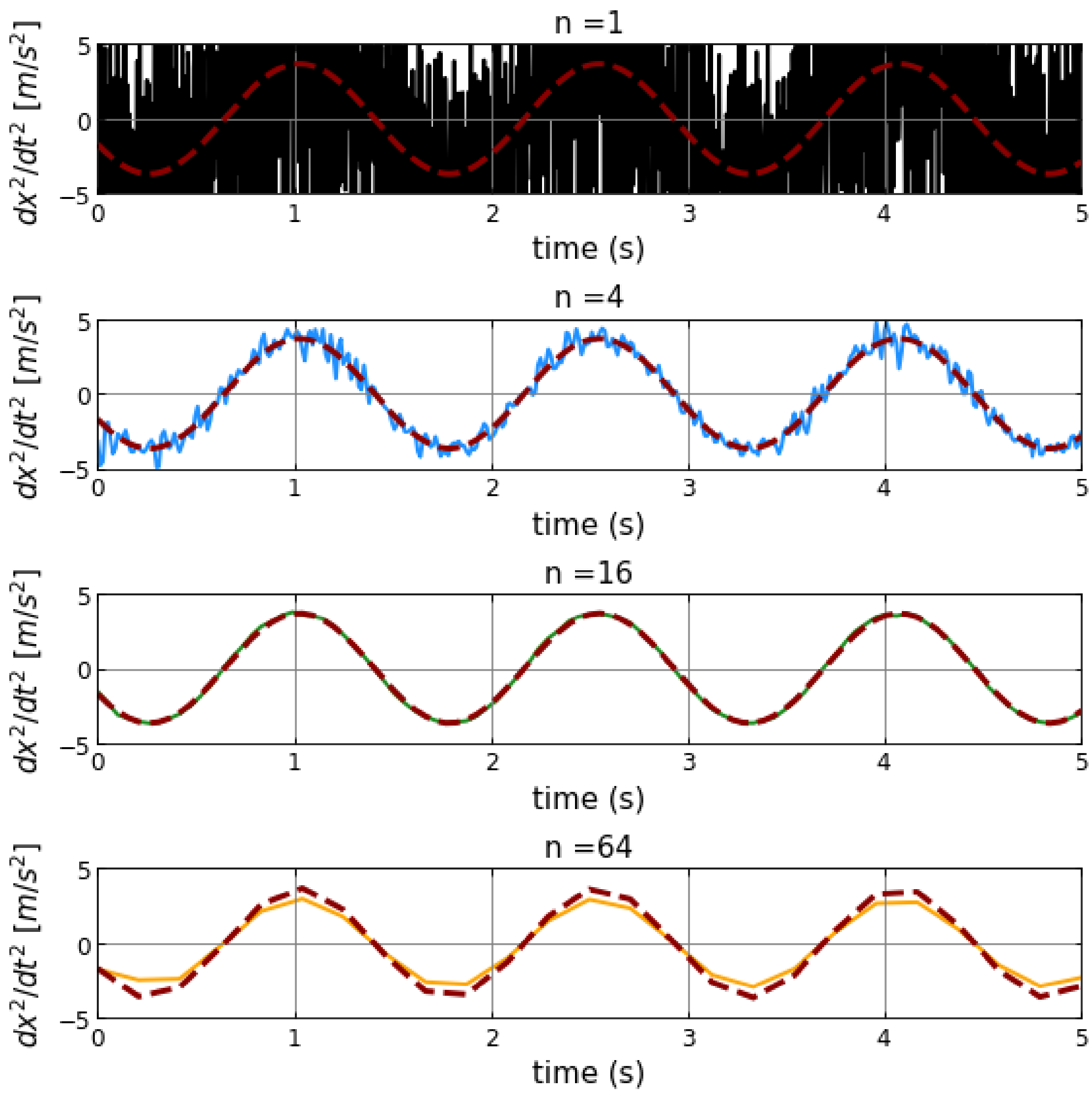
In this method we decrease the temporal resolution of the data. Instead of taking the derivative between each consequtive point, we calculate the derivative of each n-th consequtive point n ∈ {1,2,3,4}. We therefore (effectively) smooth the data. This also implies that we could've decreased our acquisition rate by a factor n. Keep this in mind during your own experimental work!

The downside of this method, is that we throw away a lot of data that could be potentially useful.

- line 2: define the n-th consequtive point.
- line 6-10: decrease the resolution of the time and position data. Calculate the new velocities and acceleration.
- line 11-12: plot nummerical and analytical derivative of the data.

[ ]:
```
fig,axes=plt.subplots(4,1,figsize=(8,8))
slicelist= [1,4,16,64]
lwlist=[2,2,2,2,2]
colorlist=['black','dodgerblue','forestgreen','orange','purple']
for i in range(len(slicelist)):
    x2 = x[::slicelist[i]]
    t2 = t[::slicelist[i]]
    der_slice1 = np.gradient(x2,t2)
    der_slice2 = np.gradient(der_slice1,t2)

    axes[i].plot(t2,der_slice2,color = colorlist[i],lw=lwlist[i],zorder=1)
```

```
    axes[i].plot(t2,-popt[2]**2*popt[1]*np.
 ↪sin(popt[2]*t2+popt[3]),color='darkred',lw=3,ls='--',zorder=2)

    axes[i].set_title("n =" + str(slicelist[i]),fontsize=15)
    axes[i].set_ylim(-5,5)
    axes[i].set_xlim(0,5)
    axes[i].set_xlabel('time (s)',fontsize=15)

    axes[i].tick_params(direction="in",labelsize=12,bottom = True,top =␣
 ↪True,left= True,right=True) #inward direction of tick-lines
    axes[i].grid(True,color='gray')
    axes[i].set_ylabel(r'$dx^2/dt^2$ $[m/s^2]$',fontsize=15)
plt.tight_layout()
```



2.9 Method 2: Smoothing data (harder)

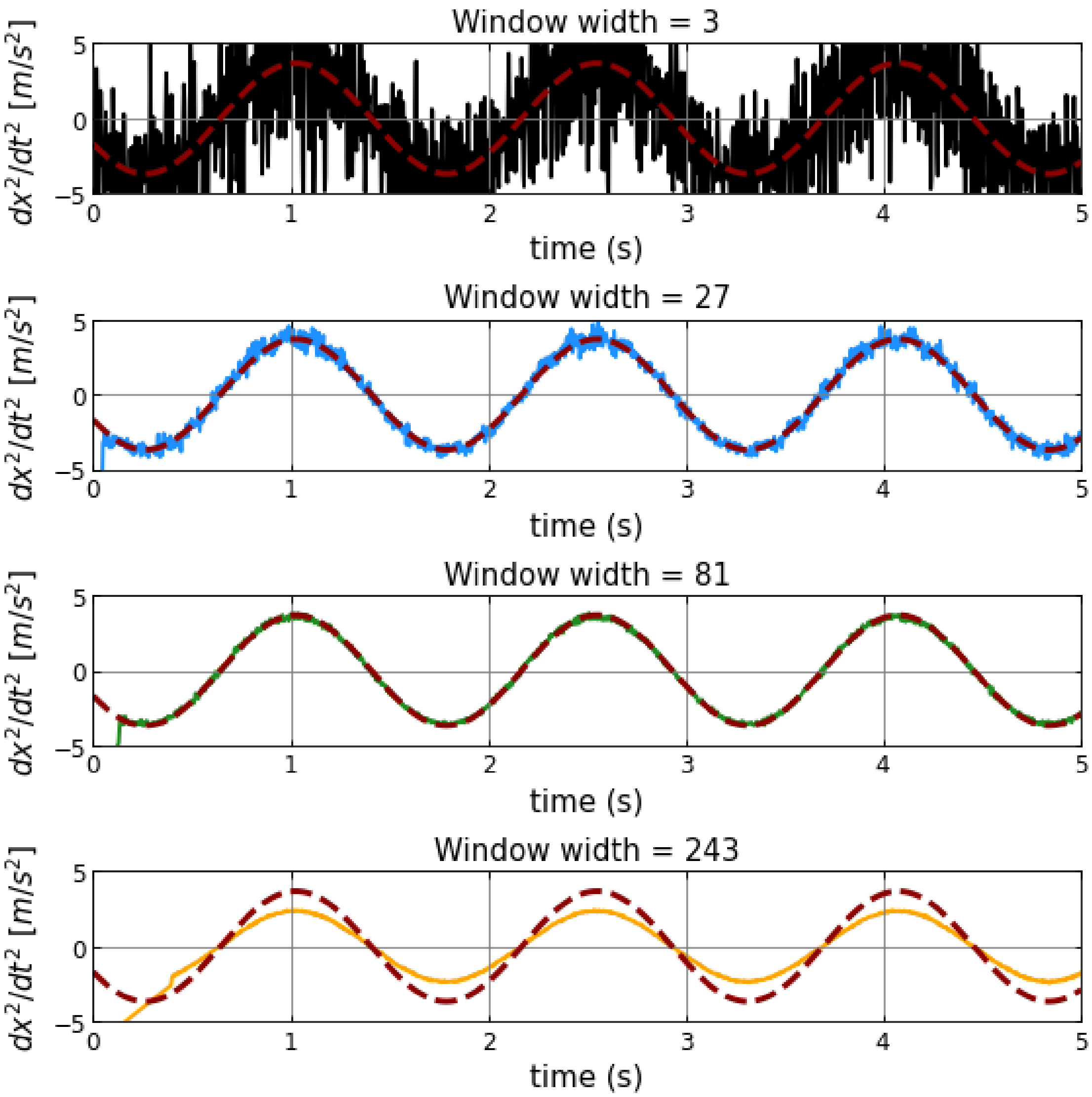
In this method we apply a smoothing technique to our data. Our starting point is the very noisy acceleration data. Instead of cropping the data, we can apply a smoothing technique: Savitzky-Golay filtering. The "savgol" filter is often used as a preprocessing in spectroscopy and signal processing. The filter can be used to reduce high frequency noise in a signal due to its smoothing properties and reduce low frequency signal (e.g., due to offsets and slopes) using differentiation [1].

The Savitzky-Golay (savgol) Filter: For a given signal measured at N points and a filter of window width, w, savgol calculates a polynomial fit of order o in each filter window as the filter is moved across the signal [1]. The result of this operation is a smoothened curve of the data. The downside is, just like the previous method, that the high-frequency components of the data are removed, i.e. a decrease in the resolution of the data [2].

- [1] Savitzky-Golay Smoothing and Differentiation Filter
- [2] Savitzky-Golay Smoothing Filters
- line 1: import savgol_filter
- line 4: define the number of neighbours (width) of the filter.
- line 9: apply savgol_filter, with window width defined in line 4 and a polynomial fit of order 1 (linear).

[ ]:

```
from scipy.signal import savgol_filter

fig,axes=plt.subplots(4,1,figsize=(8,8))
neighbourlist= [3,27,81,243]
lwlist=[2,2,2,2,2]
colorlist=['black','dodgerblue','forestgreen','orange','purple']

for i in range(len(slicelist)):
    axes[i].plot(t, savgol_filter(der2,neighbourlist[i],1),color =␣
 ↪colorlist[i],lw=lwlist[i],zorder=1)
    axes[i].plot(t,-popt[2]**2*popt[1]*np.
 ↪sin(popt[2]*t+popt[3]),color='darkred',lw=3,ls='--',zorder=2)

    axes[i].set_title("Window width = " + str(neighbourlist[i]),fontsize=15)
    axes[i].set_ylim(-5,5)
    axes[i].set_xlim(0,5)
    axes[i].set_xlabel('time (s)',fontsize=15)
    axes[i].tick_params(direction="in",labelsize=12,bottom = True,top =␣
 ↪True,left= True,right=True) #inward direction of tick-lines
    axes[i].grid(True,color='gray')
    axes[i].set_ylabel(r'$dx^2/dt^2$ $[m/s^2]$',fontsize=15)
plt.tight_layout()
```



2.10 Discussion

In the previous two figures, we observe the same trend, as we increase the strength of the filtering, the nummerical acceleration a_{obs} converges towards the analytical acceleration a_{fit} (based on a simple pendulum). The nummerical acceleration a_{obs} depends on the window width, i.e. the savgol_filter width or the concatenated data method. To find the optimal filtering window is challenging. In the last figure (of both methods) the filtering was too strong. We start to lose data.

Always (!) check if your filter solely removes the noise and not the signal your are looking for!

In principle we could quanitfy the deviation between the observed and expected data using the χ^2 analysis:

$$\chi^2 = \sum_1^N \frac{(y_i - f(x_i))^2}{\sigma_i^2} = \sum_1^N \frac{r_i^2}{\sigma_i^2}. \tag{13}$$

σ_i is the standard deviation 'error' on the observed datapoint y_i. Lastly, we should correct for the number of degrees of freedom. We have N data points and c parameters, so we have d = N - c degrees of freedom. We define the following:

$$\tilde{\chi}^2 = \frac{\chi^2}{d}, \tag{14}$$

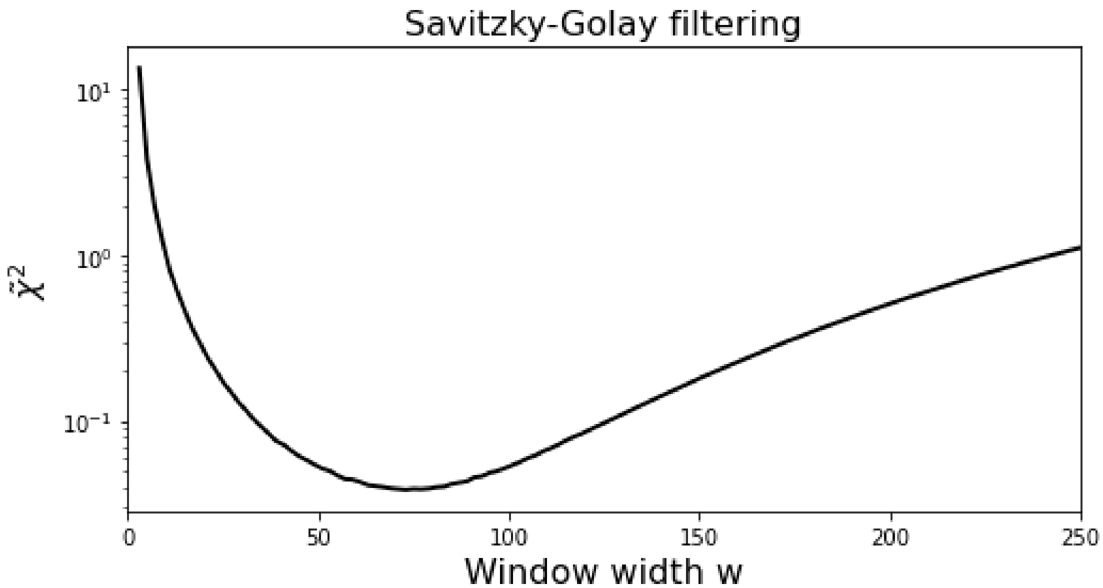
$\tilde{\chi}^2$ is the reduced χ^2, where we included the effect of the degrees of freedom. The value of $\tilde{\chi}^2$ is a measure of the deviation between the data and the model. This deviation is, in priniciple, a function of the window-width, and is therefore problem specific.

For a more general discussion about the significance and applicability of $\tilde{\chi}^2$ I refer to chapter 12 in An Introduction to Error Analysis by John R. Taylor.

2.10.1 Optional: Optimal Window width

In the previous cell we defined the $\tilde{\chi}^2$. In the next block I calculate the $\tilde{\chi}^2$ as a function of the window width. We observe that the optimal filtering window w = 79 neighbours. If our window width is larger than 79 neighbours we start losing data. Note that this procedure can only be applied if we know the model!

[ ]:
```python
rchi_2_list,width_filter = [],[]
for i in range(3,281,2):
    deviation = (savgol_filter(der2[i:],i,1) + popt[2]**2*popt[1]*np.
 ↪sin(popt[2]*t[i:]+popt[3]))
    rchi_2 = np.sum(deviation**2)/(len(t)-1)
    rchi_2_list.append(rchi_2)
    width_filter.append(i)


fig,axes=plt.subplots(1,1,figsize=(8,4))
axes.plot(width_filter,rchi_2_list, color = 'black', lw=2)
axes.set_title("Savitzky-Golay filtering",fontsize=16)
axes.set_ylabel(r"$\tilde{\chi}^2$" ,fontsize=16)
axes.set_xlabel(r"Window width w" ,fontsize=16)
axes.set_xlim(0,250)

axes.set_yscale('log')
plt.show()
```



3 Your Experiment starts here

- Author: Student name.
- Date: Date.
- About: Your experiment.
- TA:

Please ask your TA if you need to hand in your code at the end of the practical course.