Homework 6 Solutions

Questions

The next 7 questions refer to the following problem:

Clifton Distributing needs to ship goods from three plants (P1, P2, P3) to three distribution centers (C1, C2, C3). Clifton can use a cross-docking facility (F) in which shipments from multiple plants can be consolidated. The benefit of shipment consolidation is that larger, and fuller, trucks can be employed for transporting the goods from the facility to their final destination, which results in a lower per-unit transportation cost.

The distance in miles between each pair of locations between which a shipment is possible is given in the following table:

Distance	C1	C2	C3	F
P1	400	1300	1000	350
P2	300	1500	1200	450
P3	500	1600	1150	800
F	260	1000	900	-

The supply at the plants and the demand at the centers is given in the next two tables:

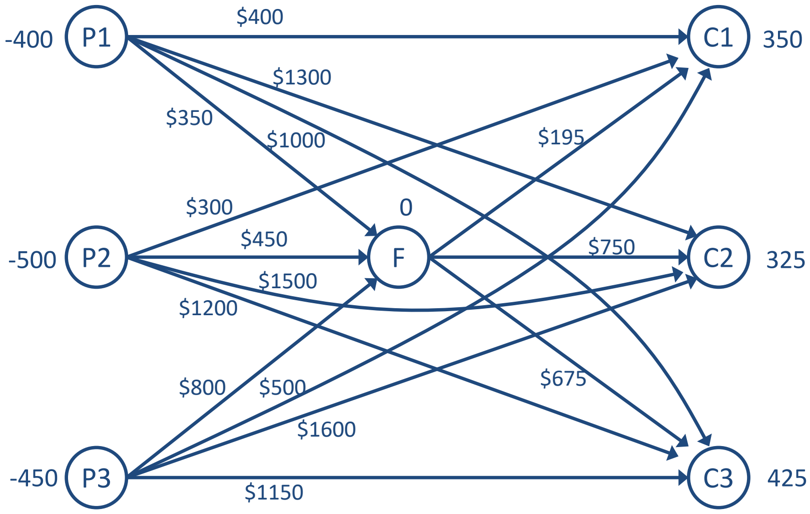
	P1	P2	P3
Supply	400	500	450

	C1	C2	C3
Demand	350	325	425

The unit cost of transportation from the plants to all other locations is \$1 per mile. However, the unit cost of transportation from the cross-docking facility to the centers is \$0.75 per mile. Clifton wants to determine a production plan that minimizes total transportation cost, while meeting all demand.

We will represent the problem as a network flow problem, using variables x_{ij} to represent the flow from location i to location j.

1. Draw the network representation for this problem.



2. What is the unit cost of the arc from P1 to C2?
   $1300
3. What is the unit cost of the arc from P3 to F?
   $800
4. What is the unit cost of the arc from F to C1?
   $195
5. Write down the flow conservation constraint for plant P2.
   $-(x_{P2,C1} + x_{P2,C2} + x_{P2,C3} + x_{P2,F}) \geq -500$
   OR $x_{P2,C1} + x_{P2,C2} + x_{P2,C3} + x_{P2,F} \leq 500$
6. Write down the flow conservation constraint for facility F.
   $(x_{P1,F} + x_{P2,F} + x_{P3,F}) - (x_{F,C1} + x_{F,C2} + x_{F,C3}) = 0$
7. Write down the flow conservation constraint for center C1.
   $x_{P1,C1} + x_{P2,C1} + x_{P3,C1} + x_{F,C1} = 350$
   ($x_{P1,C1} + x_{P2,C1} + x_{P3,C1} + x_{F,C1} \geq 350$ would also be correct)

The next 8 questions refer to the following problem:

This question refers to the 'maximum flow problem' in the slides of this week (slides 35–38), and the associated worksheets in the Excel Spreadsheet `week8_empty.xlsx`.

First, we solve the problem by implementing it in Excel using the 2-dimensional matrix representation in worksheet '`Max flow`'.

8. Write down the Excel formula that goes in cell `B18` of worksheet '`Max flow`'.
   `=C11`
9. Write down the Solver constraint formula that restricts each arc flow to be at most its capacity.
   `=$C$6:$H$11 <= $C$24:$H$29`
10. Is the optimal solution unique?
    No, the sensitivity report contains zeroes in the allowable increase/decrease for the objective coefficients of the variable cells.

Next we solve the problem by implementing it in Excel using the arc-list representation in worksheet `Max flow (2)`.

11. Write down the Excel formula that goes in cell `J6` of worksheet '`Max flow (2)`'.
    `=SUMIF($E$4:$E$13,H6,$A$4:$A$13)`
12. Write down the Excel formula that goes in cell `K9` of worksheet '`Max flow (2)`'.
    `=SUMIF($C$4:$C$13,H9,$A$4:$A$13)`
13. The optimal objective value is 1,625,000 calls. What is the lowest capacity (in 1,000s) for the segment between Washington DC and Kansas City that still allows to attain this maximum solution?
    200. There are two ways to find this number.
    - Trial and error; try out lower values until the objective starts to change. This may require many steps though.
    - Modify the optimization problem by changing the objective into minimizing the flow from DC to Kansas, and adding a constraint that fixes the flow from San Francisco to DC to 1,625.