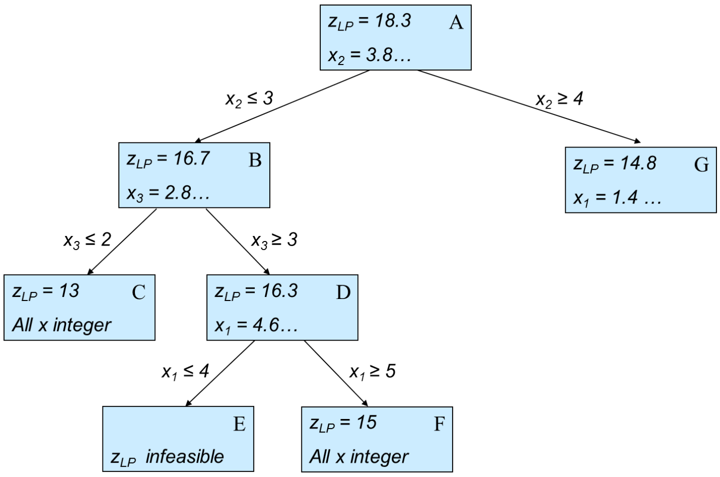
HW 8 Solutions

Questions

The next three questions refer to the following problem:

Consider the partial branch-and-bound tree for a maximization problem depicted in the figure below:



Each subproblem is represented by a box containing the objective value of the associated linear program, as well as information regarding the solution being integer, having a fractional variable, or being infeasible. To distinguish the subproblems, they are assigned letters A up to G, where subproblem A represents the original LP model. So far, the branch-and-bound method has considered seven subproblems, in the order A, B, C, D, E, F, G.

1. What are the branching constraints that are added to the original linear programming model for subproblem E?

   $x_2 \leq 3$, $x_3 \geq 3$, $x_1 \leq 4$

2. What is the objective value of the incumbent solution so far (i.e., after considering subproblems A up to G)?

   $z_{\text{inc}} = 15$

3. Does subproblem G need to be branched upon further? If so, which are the constraints we need to add to each subproblem below G?

   We don't need to branch further: subproblem G is bounded because the objective value of the LP solution is worse than the incumbent.

The next six questions refer to the following problem:

This question refers to the 'Minimum Order' problem in the slides of the class on Integer Programming:

A portfolio manager has \$100,000 to invest. She will choose her investments from the following list of funds:

Fund	1	2	3	4	5	6
Return	15%	13%	11%	18%	12%	17%

If she invests in a fund, she must invest at least \$10,000 in that fund. She can invest no more than \$46,000 in any fund. The objective is to maximize total return.

We will formulate the problem as an integer linear programming model. For this, we introduce two types of variables for fund $i = 1, \ldots, 6$:

- Binary variable x_i represents whether we use fund i ($x_i = 1$) or not ($x_i = 0$),
- Nonnegative variable y_i represents the amount invested in fund i.

4. Write down the objective function.

   maximize $15y_1 + 13y_2 + 11y_3 + 18y_4 + 12y_5 + 17y_6$.

5. Write down the constraint for the total investment amount of \$100,000.

   $$\sum_{i=1}^{6} y_i = 100,000$$

6. Write down the constraints that ensure that we invest between \$10,000 and \$46,000 for a selected fund.

   We can do this with two separate constraints, one for each bound
   $y_i \geq 10,000\ x_i$ for each fund $i = 1, \ldots, 6$,
   $y_i \leq 46,000\ x_i$ for each fund $i = 1, \ldots, 6$.

The fund manager needs to respect a number of additional requirements. Formulate each of the following requirements as a new constraint to our integer linear programming model:

7. We can select at most one fund out of funds 1, 2, and 4.

   $x_1 + x_2 + x_4 \leq 1$

8. If both funds 2 and 3 are selected, then fund 5 cannot be selected.

   $x_5 \leq 2 - x_2 - x_3$

9. If fund 3 is selected, then we must invest at least \$20,000 in fund 6.

   $y_6 \geq 20,000\ x_3$

The next seven questions refer to the following problem:

This question refers to 'Facility Location / Distribution: example' in the slides.

The purpose of this question is to find a solution to the integer linear programming model using Excel Solver. The template worksheet can be found in `HW8.xlsx`.

Complete the Excel implementation in worksheet '`HW 8 Facility Location`', and solve the problem using Solver.

10. Write down the Excel formula that goes in cell `J21`.
11. Write down the Excel formula that goes in cell `L21`.
12. Write down the Excel formula that goes in cell `H22`.
13. Write down the Excel formula that goes in cell `H24`.

    For questions 10–13, see `HW8Sol.xlsx`.

14. In the optimal solution, how many plants are built in total?

    3

15. What is the optimal objective value?

    $7,425,000