HW 9 Solutions

Questions

The next two questions refer to the following problem:

This question refers to the problem 'Minimum order - discounts' discussed in class and the associated **Excel worksheet** or see `Discount` sheet in `HW9.xlsx`. We refer to the variables on Lecture 19 slide 19 ($x_{\text{low},i}$, $x_{\text{high},i}$, $y_{\text{low},i}$, $y_{\text{high},i}$).

1. Complete the Excel implementation, and report the optimal solution.
   See `HW9Sol.xlsx`.

2. Suppose we can obtain the high-production discount for factory 3 only if we also have the high-production discount for factory 1. Represent this requirement as a constraint to be added our integer programming model. Does this requirement change the optimal objective value found under 1.?

   We can model this requirement as the following linear constraint:

   $$y_{\text{high},3} \leq y_{\text{high},1}$$

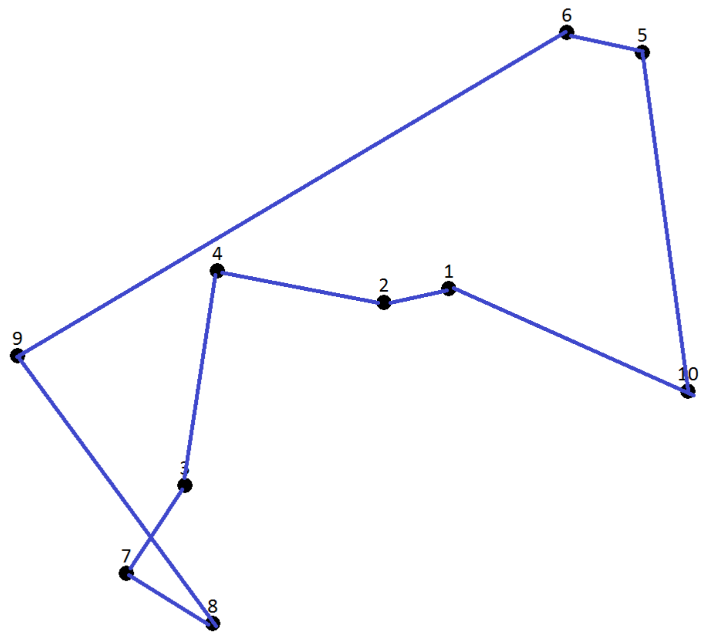
   Adding this constraint will not change the optimal objective value.

Remaining questions on next page

The next four questions refer to the following problem:

A parcel delivery service needs to visit 9 customers, depicted in the figure below. Here, the node labeled '1' represents the depot from which we need to start and finish our truck tour. The customers are labeled 2 up to 10. The distance matrix is given in the `Distance` sheet in `HW9.xlsx`.

Our goal is to find a tour visiting each location exactly once, with minimum total length (even though we may not be able to prove optimality).



We first apply the greedy 'nearest neighbor' heuristic to find a solution, starting from location 1 (see slides of the first TSP lecture). The resulting solution (the order in which the locations are visited) is:

1 - 2 - 4 - 3 - 7 - 8 - 9 - 6 - 5 - 10 - 1.

3. During the greedy heuristic, we encountered one 'tie': the current location had two unvisited neighbors that were equally close. At which location did this happen, and which were the two nearest neighbors? Which neighbor did we choose to arrive at the solution above?

   The tie occurs for location 4, at which point the nearest neighbors are 3 and 9 (both with distance 31). We chose neighbor 3 in the solution above.

4. What is the total length of this greedy tour?

   328

We next try to improve our greedy solution by applying the 2-exchange heuristic.

5. Consider the edges (2,4) and (6,9). What is the net impact on the tour length if we were to apply the 2-exchange to these edges?

   We remove edges (2,4) and (6,9) with distance $24 + 90 = 114$

   We add edges (2,9) and (4,6) with distance $52 + 60 = 112$

   Therefore, this 2-exchange will result in decreasing the total length by 2 units.

6. Consider the edges (3,7) and (8,9). What is the net impact on the tour length if we were to apply the 2-exchange to these edges?

   We remove edges (3,7) and (8,9) with distance $15 + 47 = 62$

   We add edges (3,8) and (7,9) with distance $20 + 34 = 54$

   Therefore, this 2-exchange will result in decreasing the total length by 8 units.