

Smoothed Particle Hidrodynamics for Impact Simulations

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Outline

1 Introduction

- Problem statement
- Project Description

2 Implementation

- SPH Formulation

3 Results

- Simulations

Motivation

A. González-Mancera et al.

Objectives

Bullet Deformation & Target Fracture

- Apply SPH Matlab routines from reference
- Develop new SPH formulations
- Implement Algorithm Modification
- Evaluate Performance
- Manage Version Control Repository

General Description

- SPH is a numerical method for approximating PDEs solutions

Integral Representation

A function and its spatial derivative can be represented in an integral form

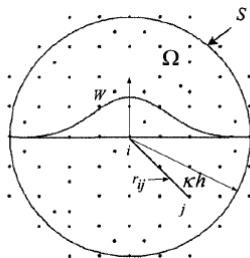
$$f(x) = \int_{\Omega} f(x') W(x - x', h) dx'$$

$$\nabla \cdot f(x) = - \int_{\Omega} f(x') \cdot \nabla W(x - x', h) dx'$$

- x, x' : spatial vectors
- $f(x)$: Function of x
- Ω : Defined domain for $f(x)$
- W : Smoothing function (*kernel**)
- h : Smoothing length

Smoothing Function (*kernel*)

Kernel properties



$$\lim_{h \rightarrow 0} W(x - x', h) = \delta(x - x')$$

$$\int_{\Omega} W(x - x', h) dx' = 1$$

$$W(x - x') = 0, \text{ for } |x - x'| > kh$$

Particle Aproximation

Contiuum \rightarrow Discrete

- Function

$$f(x_i) = \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) W_{ij}$$

$$W_{ij} = W(x_i - x_j, h)$$

- Function Spatial Derivative

$$\nabla \cdot f(x_i) = \sum_{j=1}^N \frac{m_j}{\rho_j} f(x_j) \cdot \nabla_i W_{ij}$$

$$\nabla_i W_{ij} = \frac{x_i - x_j}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} = \frac{x_{ij}}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}}$$

Conservation Equations

Continuoum domain

- Continuity

$$\frac{D\rho}{Dt} = -\rho \frac{\partial \nu^\beta}{\partial x^\beta}$$

- Momentum

$$\frac{D\nu^\alpha}{Dt} = \frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta}$$

- Energy

$$\frac{De}{Dt} = \frac{\sigma^{\alpha\beta}}{\rho} \frac{\partial \nu^\alpha}{\partial x^\beta}$$

Conservation Equations

Discrete Domain

- Conservation of mass

$$\rho_i = \sum_{j=1}^N m_j W_{ij}$$

- Momentum

$$\frac{D\nu_i^\alpha}{Dt} = \sum_{j=1}^N m_j \frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i \rho_j} \frac{\partial W_{ij}}{\partial x_i^\beta}$$

- Energy

$$\frac{De_i}{Dt} = \sum_{j=1}^N m_j \frac{\rho_i + \rho_j}{\rho_i \rho_j} \nu_{ij}^\beta \frac{\partial W_{ij}}{\partial x_i^\beta} + \frac{\mu_i}{2\rho_i} \epsilon_i^{\alpha\beta} \epsilon_j^{\alpha\beta}$$

Tensile Road

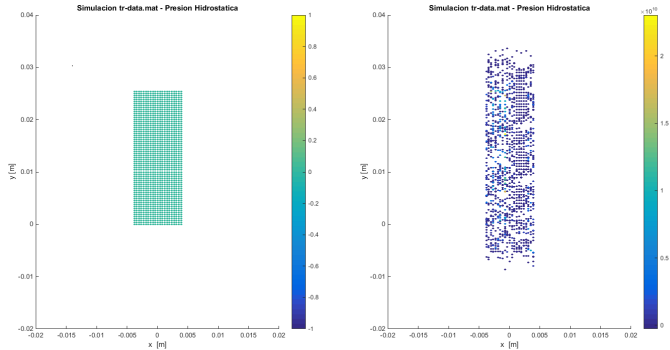


Figure: Fracture simulation of basalt road in tension

Bullet Impact

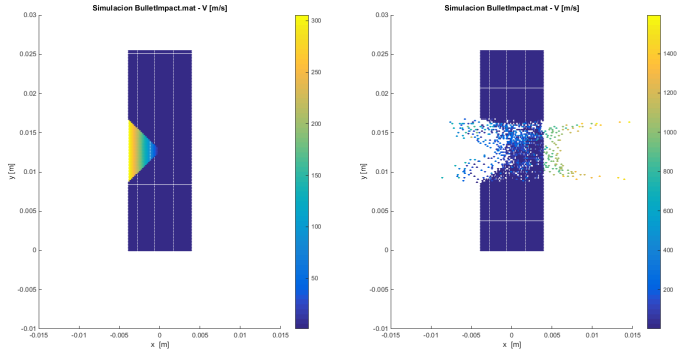


Figure: Simulation of Basalt target under initial speed dsitribution

Basalt bullet impacting on basalt target

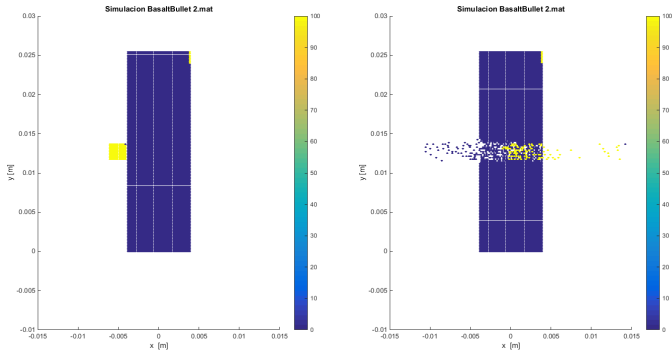


Figure: Simulation of Basalt bullet at initial velocity of $v_x = 300\text{m/s}$ impacting on basalt target