Smoothed Particle Hidrodynamics for Impact Simulations

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Outline

1. Introduction
   - Problem statement
   - Project Description
2. SPH Formulation
   - Physical Model
   - Particle Aproximation
3. Results
   - Simmulations

Motivation

Objectives

Integrate bullet deformation & target fracture in SPH simulations

- Apply SPH Matlab routines from reference
- Develop new SPH formulations
- Implement Algorithm Modifications
- Evaluate Performance
- Manage code by version control software

General Description

Smoothed Particle Hydrodynamics (*SPH*)

- Numerical method for approximating PDEs solutions
- Meshless*
- A set of *particles* represent the total physical domain
- Langrangian description*
- Applications on Astrophysics, CFD, Solid Mechanics...

Integral Representation

A function and its spatial derivative can be represented in an integral form

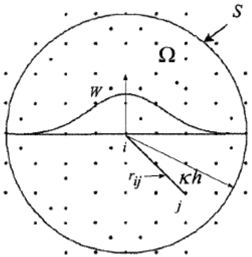
$$f(x) = \int_{\Omega} f(x')W(x - x', h)dx'$$

$$\nabla \cdot f(x) = -\int_{\Omega} f(x') \cdot \nabla W(x - x', h)dx'$$

- Ω: Defined domain for $f(x)$
- W: Smoothing function (*kernel**)
- h: Smoothing length

Smoothing Function (*kernel*)

Kernel properties $W(c - x', h)$



$$\lim_{h \to 0} W(x - x', h) = \delta(x - x')$$

$$\int_{\Omega} W(x - x', h) dx' = 1$$

$$W(x - x') = 0, \text{ for } |x - x'| > kh$$

Conservation Equations

Continuum domain

- Continuity

$$\frac{D\rho}{Dt} = -\rho \frac{\partial \nu^{\beta}}{\partial x^{\beta}}$$

- Momentum

$$\frac{D\nu^{\alpha}}{Dt} = \frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}}$$

- Energy

$$\frac{De}{Dt} = \frac{\sigma^{\alpha\beta}}{\rho} \frac{\partial \nu^{\alpha}}{\partial x^{\beta}}$$

Stress tensor* $\sigma^{\alpha\beta}$

Constitutive Model

Stress tensor

$$\sigma^{\alpha\beta} = -P\delta^{\alpha\beta} + \tau^{\alpha\beta}$$

Jaumann

$$\dot{\tau}^{\alpha\beta} = G\left(\epsilon^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\epsilon^{\gamma\gamma}\right) + \tau^{\alpha\gamma}R^{\beta\gamma} + \tau^{\alpha\beta}R^{\alpha\gamma}$$

Mie Gruniensen

$$P(e,\rho) = \left(1 - \frac{1}{2}\Gamma\eta\right)P_H\rho + \Gamma\rho e$$

- G is the shear modulus
- R si the rotation tensor
- P_H refers to Hugoniot curve
- Γ is the Gruneinsen Parameter
- η is the density change rate

Material Model

Grady & Kipp fragmentation model

- Existence of incipient flaws
- Number of flaws per unit volume $n(\epsilon) = k\epsilon^m$
- Local stress-release due to grow of cracks
  'Damage' D $0 \leq \ D \leq 1$

$$\sigma_D = \sigma(1 - D)$$

Particle Aproximation

Contiuum $\rightarrow$ Discrete

- Function

$$f(x_i) = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W_{ij}$$

$$W_{ij} = W(x_i - x_j, h)$$

- Function Spatial Derivative

$$\nabla \cdot f(x_i) = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) \cdot \nabla_i W_{ij}$$

$$\nabla_i W_{ij} = \frac{x_i - x_j}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} = \frac{x_{ij}}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}}$$

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Conservation Equations

Discrete Domain

- Conservation of mass

$$\rho_i = \sum_{j=1}^{N} m_j W_{ij}$$

- Momentum

$$\frac{D\nu_i^{\alpha}}{Dt} = \sum_{j=1}^{N} m_j \frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i \rho_j} \frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

- Energy

$$\frac{De_i}{Dt} = \sum_{j=1}^{N} m_j \frac{\rho_i + \rho_j}{\rho_i \rho_j} \nu_{ij}^{\beta} \frac{\partial W_{ij}}{\partial x_i^{\beta}} + \frac{\mu_i}{2\rho_i} \epsilon_i^{\alpha\beta} \epsilon_j^{\alpha\beta}$$

Material Model

Material model in discrete domain

Tensile Road

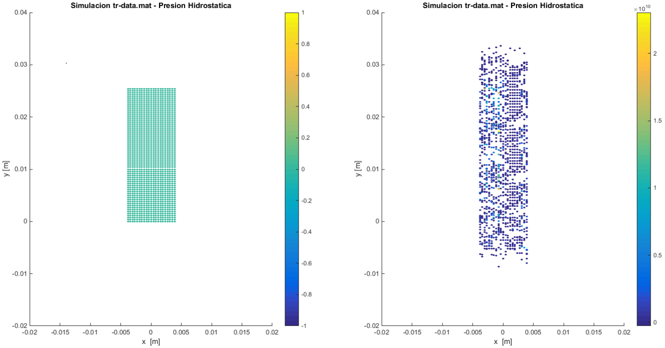


Figure: Fracture simulation of basalt road in tension

Bullet Impact

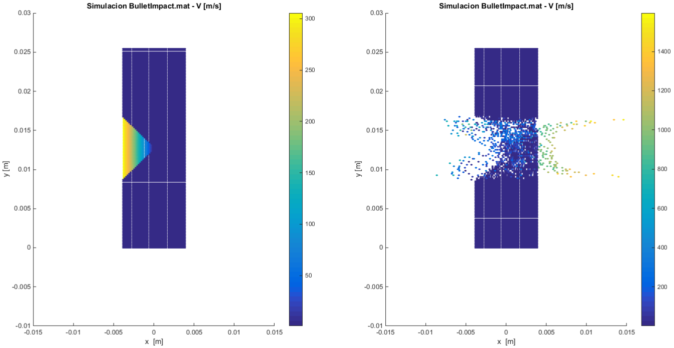
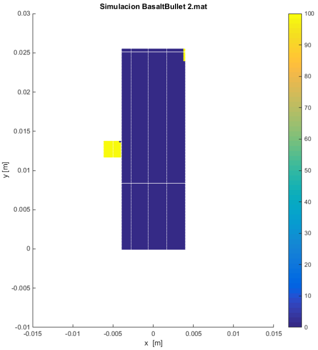


Figure: Simulation of Basalt tarjet under initial speed dsitribution

Basalt bullet impacting on basalt tarjet



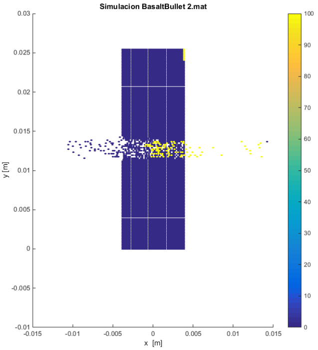


Figure: Simulation of Basalt bullet at initial velocity of $v_x = 300m/s$ impacting on basalt tarjet