Smoothed Particle Hydrodynamics for Simulations of High Speed Impacts

J. C. Alfonso D. Luna A. Gonzalez

Department of Mechanical Engineering
Universidad de los Andes

May 28, 2015/*IMEC Special Problem*

Outline

Universidad de los Andes

General Description

Smoothed Particle Hydrodynamics (*SPH*)

- Numerical method for approximating PDEs solutions
- Mesh-less*
- A set of *particles* represent the total physical domain
- Lagrangian description*
- Applications on Astrophysics, CFD, Solid Mechanics...

Objectives

Simulate interaction between bullet deformation & target fracture

- Using Matlab routines from reference:
  - Implement boundary treatment
  - Simulate ductile deformation
  - Simulate bullet deformation & target fracture
- Evaluate Performance
- Manage code by version control software

Integral Representation

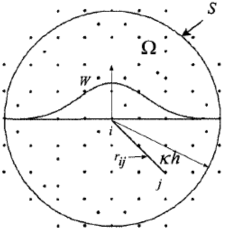
A function and its spatial derivative (Divergence) can be represented in an integral form

$$f(x) = \int_{\Omega} f(x')W(x - x', h)dx'$$

$$\nabla \cdot f(x) = -\int_{\Omega} f(x') \cdot \nabla W(x - x', h)dx'$$

- Ω: Defined domain for $f(x)$
- W: Smoothing function (*kernel**)
- h: Smoothing length

Kernel - Particle Approximation



$$\rightarrow f(x_i) = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W_{ij}$$

$$\rightarrow \nabla \cdot f(x_i) = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) \cdot \nabla_i W_{ij}$$

Conservation Equations

Continuum domain

- Continuity

$$\frac{D\rho}{Dt} = -\rho \frac{\partial v^{\beta}}{\partial x^{\beta}}$$

- Momentum

$$\frac{Dv^{\alpha}}{Dt} = \frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}}$$

- Energy

$$\frac{De}{Dt} = \frac{\sigma^{\alpha\beta}}{\rho} \frac{\partial v^{\alpha}}{\partial x^{\beta}}$$

Stress tensor* $\sigma^{\alpha\beta}$

Constitutive Model

Stress tensor

$$\sigma^{\alpha\beta} = -P\delta^{\alpha\beta} + \tau^{\alpha\beta}$$

Jaumann

$$\dot{\tau}^{\alpha\beta} = G\left(\epsilon^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\epsilon^{\gamma\gamma}\right) + \tau^{\alpha\gamma}R^{\beta\gamma} + \tau^{\alpha\beta}R^{\alpha\gamma}$$

Mie Gruniensen

$$P(e,\rho) = \left(1 - \frac{1}{2}\Gamma\eta\right)P_H\rho + \Gamma\rho e$$

- G is the shear modulus
- R si the rotation tensor
- P_H refers to Hugoniot curve
- Γ is the Gruneinsen Parameter
- η is the density change rate

Material Model

Grady & Kipp fragmentation model

- Incipient Flaws
- Number of flaws, per unit volume, having failure strain lower than ϵ (Probability distribution)

$$n(\epsilon) = k\epsilon^m$$

- Flaw activation $\rightarrow$ Crack growing

$$c_g = 0.4c_{longitudinal\ elastic\ wave}$$

$$a = c_g(t - t')$$

Material Model

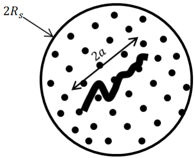
Grady & Kipp fragmentation model - Damage

Particles near a crack lose the ability to support stress

- State variable 'damage' $D \epsilon [0, 1]$

$$\sigma_D = \sigma(1 - D)$$

- D is the fractional volume that is relieved of stress by local growing crakcs



$$D = a^3/R_s^3$$

$$\frac{dD^{1/3}}{dt} = \frac{c_g}{R_s}$$

Boundary treatment

The boundary is composed of virtual particles that exert a repulsive boundary force:

$$PB_{ij} = D\left[\left(\frac{r_0}{r_{ij}}\right)^{n_1} - \left(\frac{r_0}{r_{ij}}\right)^{n_2}\right]\frac{x_{ij}}{r_{ij}^2}$$

- Reference parameters n_1 and n_2
- Scale factor $D \propto max(V^2)$
- Cutoff distance r_0

Constant properties
Physical interaction with real particles

State variables and time integration

The system is defined by the state variables:

- $\rho(t+dt) \leftarrow$ *Continuity Equation*
- $\sigma(t+dt)$
  - $\tau(t+dt) \leftarrow$ *Jauman*
  - $P(t+dt) \leftarrow Mie - Gruniensen$
- $v(t+dt) \leftarrow$ *Momentum Equation*
- $e(t+dt) \leftarrow$ *Energy Conservation*
- $x(t+dt) = vdt$
- $\epsilon(t+dt) = d\epsilon(\sigma)dt$
- $D(t+dt) = dD(\epsilon)dt$
- $m \leftarrow$ *Constant*

$$\Delta t = min\left(\frac{h_i}{c}\right)$$

Initial State of software

- SPH physical model implemented on matlab routines
- Target fragmentation for initial velocity distribution
- No boundary treatment
- No interaction between bullet and target
- No control version
- No documentation of functions

Target Fragmentation

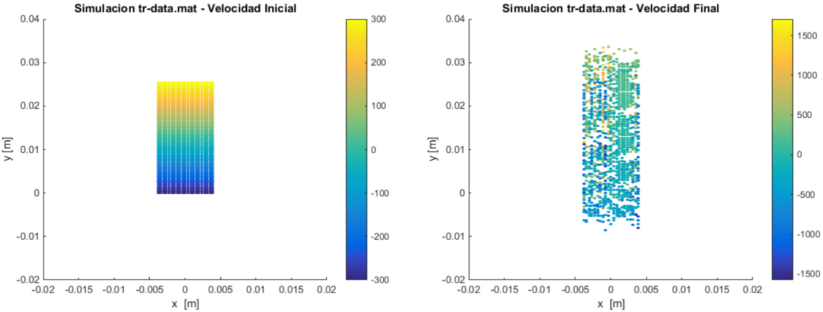


Figure: Fracture simulation of basalt target in tension

Target Fragmentation

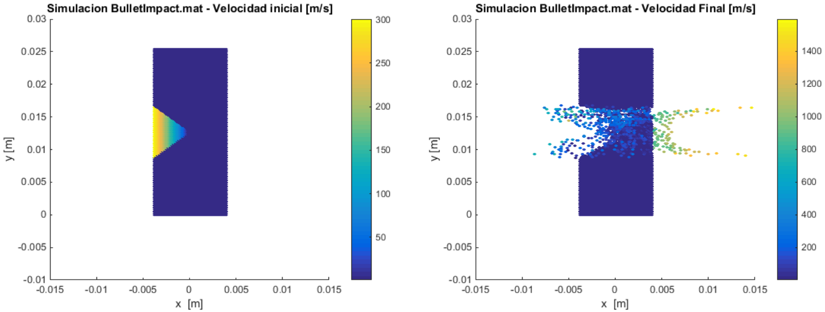


Figure: Simulation of Basalt target under initial speed distribution

Additions to the software

- Complete documentation of functions
- Control Version
  https://github.com/JC-AlfonsoR/SPH

Basalt bullet impacting on basalt target

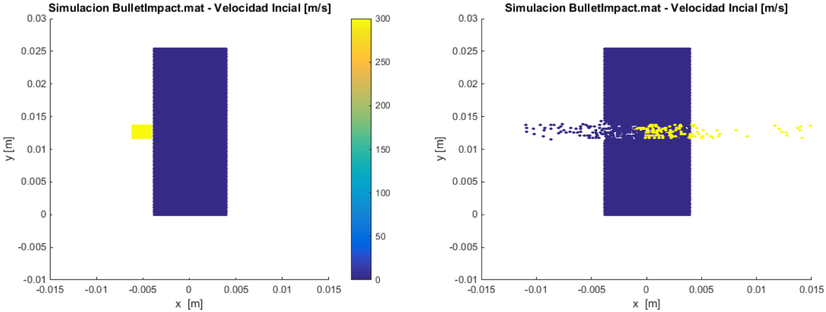
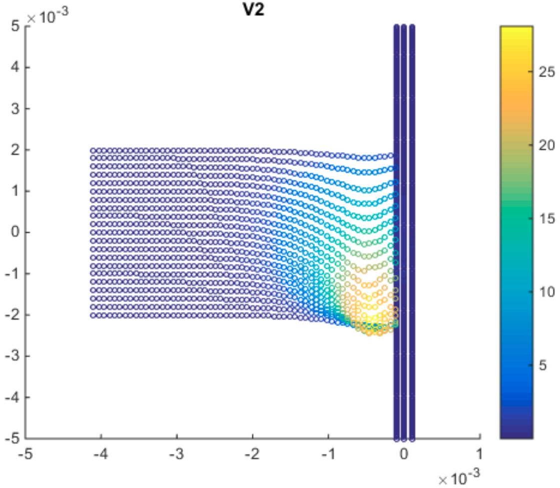
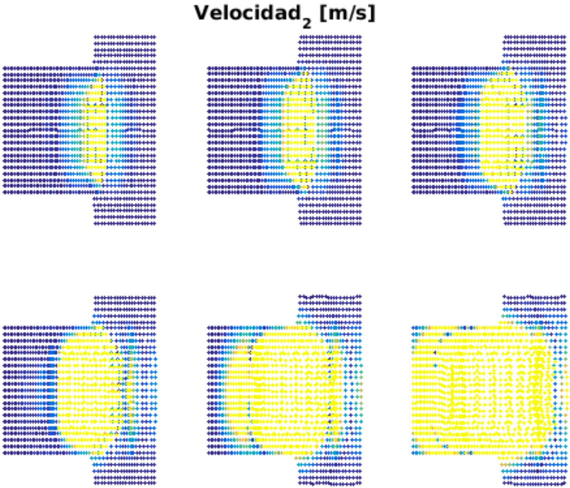


Figure: Simulation of Basalt bullet impacting on basalt target. The two bodies suffer fragile fracture. The bullet particles pass through the target body

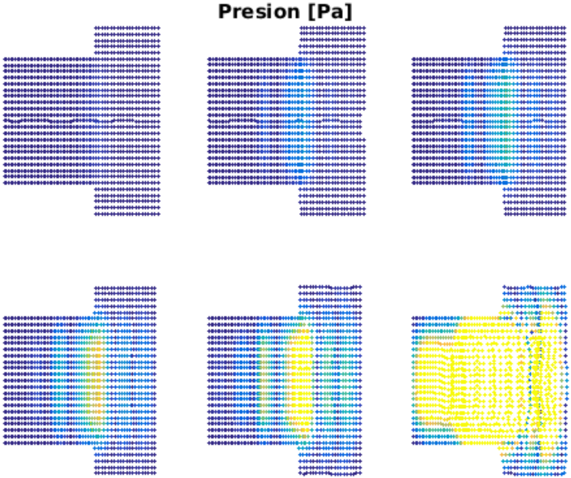
Boundary treatment



Impact



Impact



Conlcusions

Work Done

- Complete documentation of functions
- Control versions
- Boundary treatment
- Propagation of physical magnitudes
- $>$300 hours of computational time (using *Compufisica* resources) looking for optimal parameters.

Problems

- Magnitudes exceed the expected ones
- The excess on magnitudes generate wrong results

Solutions

- Fix *Kernel*
- Try new *kernels*

Universidad de los Andes