MODELING CUSTOMER RELATIONSHIPS
AS MARKOV CHAINS

P h i l l i p   E .   P f e i f e r
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INTRODUCTION

The lifetime value of a customer is an important and useful concept in interactive marketing. Courtheaux (1986) illustrates its usefulness for a number of managerial problems—the most obvious if not the most important being the budgeting of marketing expenditures for customer acquisition. It can also be used to help allocate spending across media (mail vs. telephone vs. television), vehicles (list A vs. list B), and programs (free gift vs. special price), as well as to inform decisions with respect to retaining existing customers (see, e.g., Hughes, 1997). Jackson (1996) even argues that its use helps firms to achieve a strategic competitive advantage.

Dwyer (1989) helped to popularize the lifetime value (LTV) concept by illustrating the calculation of LTV for both a customer retention and a customer migration situation. Customer retention refers to situations in which customers who are not retained are considered lost for good. In a customer retention situation, nonresponse signals the end of the firm's relationship with the customer. In contrast, customer migration situations are those in which nonresponse does not necessarily signal the end of the relationship. Besides articulating this distinction between customer retention and migration, Dwyer listed several impediments to the use of LTV.

More recently, Berger and Nasr (1998) argue for the impor-

PHILLIP E. PFEIFER AND ROBERT L. CARRAWAY are with the Darden School of Business, Charlottesville, Virginia.

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tance of moving beyond numerical illustrations of the calculation of LTV to consider mathematical models for LTV. They offer five such mathematical models, couched in the Dwyer customer retention/migration classification scheme. While Dwyer illustrates the calculation of LTV using two extensive numerical examples, Berger and Nasr provide mathematical equations for LTV for five situations—four involving customer retention and one involving customer migration.

Blattberg and Deighton (1996) also formulate a mathematical model for LTV, for the purpose of helping managers decide the optimal balance between customer acquisition spending (investments to convince prospects to become customers) and customer retention spending (investments to convince current customers to continue purchasing from the firm). Whereas the Berger and Nasr models apply only to customers (people or organizations who have already purchased from the firm), the Blattberg and Deighton model applies specifically to prospects (people or organizations who have yet to purchase from the firm). Because the Blattberg and Deighton model uses an infinite horizon, their resulting equation for LTV is quite simple and easy to evaluate, and involves none of the summation operators on which the Berger and Nasr finite-horizon models rely.

This paper builds directly on the work of Berger and Nasr (1998) and Blattberg and Deighton (1996) in that it introduces a general class of mathematical models, Markov Chain Models, which are appropriate for modeling customer relationships and calculating LTV. The major advantage of the Markov chain model (MCM) is its flexibility. Almost all of the situations previously modeled are amenable to Markov chain modeling. The MCM can handle both customer migration and customer retention situations. It can apply either to a customer or to a prospect. In addition, the flexibility inherent in the MCM means that it can be used in many other situations not covered by previous models. MCMs have been used successfully in several areas (White, 1993) including marketing (Bronnenberg, 1998, is a recent example). A major purpose of this paper will be to explore their usefulness in modeling the relationship between an individual customer and a marketing firm.

In addition to its flexibility, the MCM offers other advantages. Because it is a probabilistic model, it explicitly accounts for the uncertainty surrounding customer relationships. It uses the language of probability and expected value—language that allows one to talk about the firm's future relationship with an individual customer. As direct marketers move toward true one-to-one marketing, we suggest that their language will also change. Rather than talking about groups or cohorts of customers, direct marketers will talk about Jane Doe. Rather than talking about retention *rates,* they will talk about the *probability* Jane Doe will be retained. Rather than talking about *average profits* from a segment of customers, they will talk about the *expected profit* from their relationship with Jane Doe. Because the MCM incorporates this language of probability and expected value, it is ideally suited for facilitating true one-to-one marketing.

Another advantage of the MCM is that it is supported by a well-developed theory about how these models can be used for decision making (see, e.g., Puterman, 1994). We will introduce the theory surrounding Markov decision processes, and illustrate how this theory can help direct marketers manage and optimize their relationships with individual customers.

The MCM also works quite nicely with the popular Recency, Frequency, Monetary value (RFM) framework (see, e.g., David Shepard Associates, Inc., 1995), which direct marketers use to categorize customers and manage customer relationships. We will illustrate the relationship between the MCM and the RFM framework.

This paper is organized as follows. After this introduction, we present the fundamentals of the MCM. The section following that will illustrate the use of the MCM for a variety of customer relationship assumptions. For these illustrations, we will choose customer situations similar to those addressed previously in the literature. A final section will illustrate the use of MCM and Markov decision processes to help a firm optimize a customer relationship. For this illustration, we look at a catalog company decid-

ing when to curtail its relationship with the customer. This last example incorporates both recency and frequency as determinants of the status of the firm's relationship with the customer. The paper ends with a summary and conclusions.

The Markov Chain Model

First we illustrate the fundamentals of Markov chain modeling. Consider the following situation: the ABC direct marketing company is trying to acquire Jane Doe as a customer. If successful, ABC expects to receive *NC* in net contribution to company profits on Jane's initial purchase and on each succeeding purchase. Purchases are made at most once a period, at the end of the period. Consistent with the customer migration models of Dwyer and of Berger and Nasr, it is possible that Jane will go several periods between purchases. Periods are of equal length, and the firm uses a per-period discount rate *d* to account for the time value of money.

For each period that Jane is considered an active customer, ABC will spend throughout the period in efforts to remarket to Jane. Let *M* represent the present value of those remarketing expenditures. Furthermore, the firm believes the probability Jane will purchase at the end of any period is a function only of Jane's recency, the number of periods since Jane last purchased. (If Jane purchased at the end of last period, Jane is at recency 1 for the current period.) Let the probability Jane will purchase at the end of any period be p_r, where *r* is Jane Doe's recency. For the purposes of illustration, assume that if and when Jane reaches $r = 5$, the firm will curtail all future efforts to remarket to Jane.

Thus, there are five possible states of the firm's relationship with Jane Doe at the end of any period, corresponding to recencies 1 though 4 and a fifth state, "noncustomer" or "former customer," which we will label $r = 5$. A key feature of the firm's relationship with Jane Doe is that the future prospects for that relationship are a function only of the current state of the relationship (defined by Jane Doe's recency) and not of the particular path Jane Doe took to reach her current state. This property is
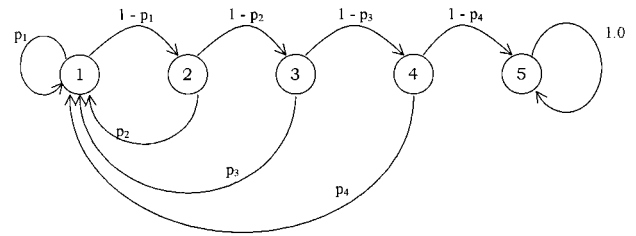
Graphical Representation of the Markov Chain Model of the Firm's Relationship with Jane Doe

called the Markov property. The Markov property is a necessary condition for a stochastic system to be a Markov chain. It is a property of the Berger and Nasr models, the Dwyer models, and the Blattberg and Deighton model. Because all these models exhibit the Markov property with constant probabilities, they can all be represented as Markov chains. Figure 1 is a graphical representation of the MCM for the firm's relationship with Jane Doe.

The probabilities of moving from one state to another in a single period are called transition probabilities. For Jane Doe, the following 5×5 transition probability matrix summarizes the transition probabilities shown graphically in Figure 1. The last row of this matrix reflects the assumption that if Jane is at recency 5 in any future period, she will remain at recency 5 for the next and all future periods. In the language of Markov chains, $r = 5$ or "former customer" is an absorbing state. Once Jane enters that state she remains in that state.

$$\mathbf{P} = \begin{bmatrix} p_1 & 1 - p_1 & 0 & 0 & 0 \\ p_2 & 0 & 1 - p_2 & 0 & 0 \\ p_3 & 0 & 0 & 1 - p_3 & 0 \\ p_4 & 0 & 0 & 0 & 1 - p_4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix **P** is the one-step transition matrix. The *t*-step transition matrix is defined to be the matrix of probabilities of moving from one state to another in exactly *t* periods. It is a well-known property of Markov chain modeling that the *t*-step transition matrix is simply the matrix product of *t* one-step transition matrices. Thus

the (i, j) element of matrix $\mathbf{P}^t$ (found by multiplying $\mathbf{P}$ by itself t times) is the probability Jane Doe will be at recency j at the end of period t given that she started at recency i. In essence, $\mathbf{P}^t$ is a tidy way both to summarize and to calculate the probability forecast of Jane Doe's recency at any future time point t.

Now that we have a probability forecast for ABC's future relationship with Jane Doe, we turn to the economic evaluation of that relationship. The cash flows received by the firm in any future period will be a function of Jane Doe's recency. $\mathbf{R}$, a 5×1 column vector of rewards, summarizes these cash flows:

$$\mathbf{R} = \begin{bmatrix} NC - M \\ -M \\ -M \\ -M \\ 0 \end{bmatrix}$$

If at the end of any future period t Jane Doe makes a purchase and thereby transitions to $r = 1$ for the next period, the firm receives NC at time t and is committed to marketing to Jane throughout the next period. Recalling that M represents the present value of these marketing expenditures, the total reward to the firm at time t if Jane transitions to $r = 1$ is $NC - M$. If Jane does not make a purchase and transitions to recencies 2, 3, or 4, the firm's reward is $-M$, reflecting its decision to continue to remarket to Jane during the coming period. However, if Jane transitions to $r = 5$, the firm has decided to curtail the relationship. The corresponding reward is zero.

If the ABC company decides to evaluate its relationship with Jane Doe using a horizon of length T, the remaining challenge is to combine the probability forecasts reflected in $\mathbf{P}^t$ with the reward structure reflected in $\mathbf{R}$. If the firm is risk neutral, it will be willing to make decisions based on *expected* net present value. If so, the only remaining challenge is to take $\mathbf{P}^t$, $\mathbf{R}$, T and d and develop an expression for the expected present value of the firm's relationship with Jane Doe. The theory of the Markov decision process provides the mechanism for doing so:

$$\mathbf{V}^T = \sum_{t=0}^{T} [(1 + d)^{-1}\mathbf{P}]^t\mathbf{R} \qquad (1)$$

where $\mathbf{V}^T$ is the 5×1 column vector of expected present value over T periods. The elements of $\mathbf{V}^T$ correspond to the five possible initial states of the Jane Doe relationship. The top element of $\mathbf{V}^T$ corresponding to $r = 1$ (a Jane Doe relationship that starts with a purchase at $t = 0$) will be of particular interest. This is the expected present value of the cash flows from the firm's proposed relationship with Jane Doe. It is, in other words, the expected LTV for Jane Doe. Notice that this value accounts for NC from the initial purchase at $t = 0$ as well as a trailing expenditure of M at time T if Jane is an active customer at T.

Rather than selecting some finite horizon T over which to evaluate its relationship with Jane Doe, the firm might decide to select an infinite horizon. (Because expected cash flows from a customer relationship usually become quite small at large values of t, the numerical differences between selecting a long but finite horizon and an infinite horizon are usually negligible. The infinite horizon approach has the advantage of being computationally simpler.) For an infinite horizon, it can be shown that

$$\mathbf{V} \equiv \lim_{T \to \infty} \mathbf{V}^T$$

$$= \{\mathbf{I} - (1 + d)^{-1}\mathbf{P}\}^{-1}\mathbf{R} \qquad (2)$$

where $\mathbf{I}$ is the identity matrix.

Let us illustrate the use of (1) and (2) using a numerical example. Suppose that for Jane Doe, $d = 0.2$, $NC = \$40$, and $M = \$4$. The numerical values for $\mathbf{R}$ turn out to be

$$\mathbf{R} = \begin{bmatrix} \$36 \\ \$(4) \\ \$(4) \\ \$(4) \\ \$0 \end{bmatrix}$$

Suppose further that $p_1 = 0.3$, $p_2 = 0.2$, $p_3 = 0.15$, and $p_4 = 0.05$. The one-, two-, three-, and four-step transition matrices are then

$$\mathbf{P} = \begin{bmatrix} 0.3 & 0.7 & 0 & 0 & 0 \\ 0.2 & 0 & 0.8 & 0 & 0 \\ 0.15 & 0 & 0 & 0.85 & 0 \\ 0.05 & 0 & 0 & 0 & 0.95 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}^2 = \begin{bmatrix} 0.2300 & 0.2100 & 0.5600 & 0 & 0 \\ 0.1800 & 0.1400 & 0 & 0.6800 & 0 \\ 0.0875 & 0.1050 & 0 & 0 & 0.8075 \\ 0.0150 & 0.0350 & 0 & 0 & 0.9500 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}^3 = \begin{bmatrix} 0.1950 & 0.1610 & 0.1680 & 0.4760 & 0 \\ 0.1160 & 0.1260 & 0.1120 & 0 & 0.6460 \\ 0.0473 & 0.0613 & 0.0840 & 0 & 0.8075 \\ 0.0115 & 0.0105 & 0.0280 & 0 & 0.9500 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{P}^4 = \begin{bmatrix} 0.1397 & 0.1365 & 0.1288 & 0.1428 & 0.4522 \\ 0.0768 & 0.0812 & 0.1008 & 0.0952 & 0.6460 \\ 0.0390 & 0.0331 & 0.0490 & 0.0714 & 0.8075 \\ 0.0098 & 0.0081 & 0.0084 & 0.0238 & 0.9500 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The upper row of $\mathbf{P}^4$ tells us that if the firm successfully initiates a relationship with Jane Doe at $t = 0$, there is a 0.1397 probability Jane will make a purchase at $t = 4$, a 0.1365 probability Jane will reach recency 2 at $t = 4, \ldots$, and a 0.4522 probability the firm will curtail its relationship with Jane at the end of period 4 because she did not make a purchase in any of the four preceding periods.

Substituting these transition matrices, $\mathbf{R}$ as given above, and $d = 0.2$ into (1) gives

$$\mathbf{V}^4 = \begin{bmatrix} \$50.115 \\ \$4.220 \\ \$0.592 \\ \$(1.980) \\ \$0 \end{bmatrix}$$

which represents the expected present value of the Jane Doe relationship over a four period horizon. The expected LTV of the proposed

relationship with Jane Doe is $50.11. This value consists of $40 from the initial purchase and $10.11 of expected present value from future cash flows.

If and when Jane reaches recency 2, her expected customer lifetime value decreases to $4.22. Notice that $4.22 is the present value now of a Jane Doe customer relationship that begins with Jane having just transitioned to recency 2. (This is a Jane Doe who purchased one period ago but failed to purchase in this period.) Similarly, if Jane transitions to recency 3, the firm's relationship with Jane has an expected present value of $0.59, while if Jane transitions to recency 4, the expected present value is a *negative* $1.98.

This negative expected value for a recency 4 Jane Doe means that the ABC firm loses money in its efforts to remarket to a recency 4 Jane Doe. Perhaps this is because our analysis used the relatively short time horizon of $T = 4$?

A simple inversion of the 5×5 matrix required of (2) allows us to calculate $\mathbf{V}$, the expected net present value of the Jane Doe relationship for an infinite horizon:

$$\mathbf{V} = \begin{bmatrix} \$52.320 \\ \$5.554 \\ \$1.251 \\ \$(1.820) \\ \$0 \end{bmatrix}$$

The differences in the results reflect the expected present value of cash flows after $t = 4$. Using an infinite horizon, the expected LTV for Jane Doe increases to $52.32.

While all the expected present values have improved under the longer horizon, the expected present value is still negative at recency 4. Given the economic and probabilistic assumptions of the model, the firm would do better if it curtailed its relationship with Jane Doe at recency 4 rather than recency 5.

It is a simple matter to modify the Markov decision model to evaluate this proposed change in policy. One way would be to reformulate the model using four rather than five states. A quicker short cut is to modify the existing five-state model

by setting p_4 equal to zero and the fourth element of $\mathbf{R}$ equal to zero. These two changes reflect the fact that under the new policy, no money will be spent on a recency 4 Jane Doe and she will automatically transition to recency 5. Making these changes and recalculating $\mathbf{V}$ gives

$$\mathbf{V} = \begin{bmatrix} \$53.149 \\ \$6.621 \\ \$2.644 \\ \$0 \\ \$0 \end{bmatrix}$$

As expected, under the new policy the expected present value of a recency 4 Jane Doe is zero. It is also apparent that because the new policy more effectively deals with Jane Doe if and when she reaches recency 4, the change improves the value of the entire relationship. The new expected LTV for the Jane Doe relationship is $53.15. The improved policy has added $0.83 to the expected LTV.

This simple example illustrates the notion that firms can use MCMs not only to evaluate proposed customer relationships, but also to help manage and improve those relationships. Markov chain modeling assists not only with prospecting decisions (should we try to initiate a relationship with Jane Doe?) but also with retention and termination decisions. Finally, notice that any improvements to a customer relationship are immediately incorporated into the LTV calculation through the MCM.

MARKOV CHAIN MODELS OF CUSTOMER RELATIONSHIPS

The ABC firm's potential relationship with Jane Doe was our first example of a customer relationship amenable to Markov chain modeling. The Jane Doe relationship might be characterized as a customer migration situation with purchase probabilities dependent on recency. This is a case 5 situation, as defined by Berger and Nasr.

Now suppose that, in addition to purchase probabilities, remarketing expenditures and net contribution also depend on recency. In other words, suppose the amount the firm spends on remarketing is adjusted based on recency. In addition, suppose the expected net contribution from Jane Doe purchases is thought to depend on the recency state from which the purchase is made. Purchase probabilities, net contributions, and remarketing expenses varying with recency are characteristics of the situation considered by Dwyer in his customer migration example.

To model this situation using an MCM will require an expanded state space. In order to account for net contributions that depend on Jane Doe's recency at time of purchase, we will need to separate the recency 1 state into four new states. These four new states will be labeled 1_1, 1_2, 1_3, and 1_4 corresponding to Jane Doe purchasing at the end of a period in which she was at recency 1, 2, 3, and 4 respectively. The transition matrix and reward vector for this more complicated customer migration situation are given below:

State					P					R
1_1	p_{11}	0	0	0	$1 - p_{11}$	0	0	0		$NC_{11} - M_{11}$
1_2	p_{12}	0	0	0	$1 - p_{12}$	0	0	0		$NC_{12} - M_{12}$
1_3	p_{13}	0	0	0	$1 - p_{13}$	0	0	0		$NC_{13} - M_{13}$
1_4	p_{14}	0	0	0	$1 - p_{14}$	0	0	0		$NC_{14} - M_{14}$
2	0	p_2	0	0	0	$1 - p_2$	0	0		$-M_2$
3	0	0	p_3	0	0	0	$1 - p_3$	0		$-M_3$
4	0	0	0	p_4	0	0	0	$1 - p_4$		$-M_4$
5	0	0	0	0	0	0	0	1		0

For example, notice that if Jane Doe purchases at recency 2, she transitions to state 1_2, where the firm receives NC_{12} in net contribution and spends M_{12} for remarketing. Purchases at recency 3, however, send Jane to state 1_3, where the firm receives NC_{13} and spends M_{13}. Just as net contribution and remarketing expenditures depend on the recency at which Jane purchased, so too do her subsequent purchase probabilities p_{11}, p_{12}, p_{13}, and p_{14}. Breaking out the original recency 1 state into four substates has allowed us to apply the MCM to a situation where purchase amounts, marketing expenditures, and repurchase probabilities depend on customer recency at the time of purchase.

So far we have considered only customer migration situations. The MCM can also be used for customer retention situations. The simplest of such situations is one in which a customer has a constant probability p of responding in each period, and any nonresponse signals the end of the customer relationship. The MCM for this situation requires only two states: customer and former customer. The **P** and **R** matrices are given below:

State	**P**		**R**	
Customer	p	$1 - p$	$NC - M$	
Former Customer	0	1	0	

This is a very simple model—so simple that the advantages of Markov chain modeling are negligible. Berger and Nasr consider this simplest customer-migration situation as case 1. Notice that because the MCM applies to a period of arbitrary length (discount rate d is defined as the per-period discount rate), this MCM also applies to the situation considered by Berger and Nasr as case 2a.

It is a relatively simple matter to expand the MCM to handle the firm's relationship with a prospect. Suppose an expenditure A gives the firm probability p_a of acquiring the prospect.

The MCM model for this prospecting/customer retention situation requires three states: prospect, customer, and former customer. The transition matrix and reward vector are as follows:

State	**P**			**R**	
Prospect	0	p_a	$1 - p_a$	$-A$	
Customer	0	p	$1 - p$	$NC - M$	
Former Customer	0	0	1	0	

Once again, this model is simple enough that the benefits of Markov Chain modeling are limited. In fact, this model is simple enough that equation (2) can be solved algebraically to give the following expected values:

$$V_{\text{Prospect}} = -A + (1 + d)^{-1} p_a V_{\text{Customer}}$$

$$V_{\text{Customer}} = (NC - M)/[1 - p/(1 + d)]$$

$$V_{\text{Former Customer}} = 0$$

This example illustrates that while it is possible to extend the MCM to include prospecting, there is usually little reason to do so. The transition from prospect to customer is usually simple enough that it can be treated algebraically, without recourse to the MCM.

Next, consider a customer retention situation where purchase amounts, repurchase probabilities, and remarketing expenditures all depend on frequency—the number of times the customer has purchased from the firm. For the purposes of this illustration, suppose these variables change for frequencies 1, 2, and 3, but remain constant for frequencies of 4 or more. The MCM of this situation requires five states: frequencies 1, 2, 3, and 4 together with "former customer." The frequency 4 state might be better labeled "frequency 4 or greater." The transition matrix and reward vector are as follows:

State	**P**					**R**
frequency 1	0	p_1	0	0	$1 - p_1$	$NC_1 - M_1$
frequency 2	0	0	p_2	0	$1 - p_2$	$NC_2 - M_2$
frequency 3	0	0	0	p_3	$1 - p_3$	$NC_3 - M_3$
frequency 4	0	0	0	p_4	$1 - p_4$	$NC_4 - M_4$
Former Customer	0	0	0	0	1	0

Purchase probabilities, net contributions, and remarketing expenses varying with frequency in a customer retention situation are characteristics of the situation considered by Dwyer in his customer retention example. Here we have shown the MCM for this same situation.

As our final example of MCM of a customer relationship, consider a customer migration situation in which the firm believes that purchase probabilities, net contribution, and remarketing expenditures all depend on the customer's recency, frequency, and monetary value of past purchases. This is the familiar RFM method for categorizing customers—applied here to characterize the status of the firm's relationship with the customer. Let r be the customer's recency, let f be the customer's frequency, and let m be an index corresponding to categories associated with the monetary value of the customer's past purchases.

The MCM model for this situation will use states defined by (r, f, m) where each of these elements are integers with some known upper bound. The upper bound for r might be the recency at which the firm ends the relationship. The highest frequency value might be a "frequency 4 or greater" type of category. Finally, the upper bound for m will simply be the number of monetary value categories the firm has created. The end result is some finite number of states defined using (r, f, m) that characterize the status of the firm's relationship with the customer.

Because the number of states in this general RFM-based MCM model can be quite large, we will not attempt to portray the transition probability matrix and reward vector. Instead we offer Figure 2. Figure 2 focuses on state (r, f, m) and shows all possible transitions to and from that state. The only path to state (r, f, m) is a nonresponse from state $(r - 1, f, m)$. Similarly, a nonresponse from state (r, f, m) transitions the customer to state $(r + 1, f, m)$. A response or purchase from state (r, f, m) transitions the customer to a recency 1 state, to frequency $f + 1$ (assuming the next higher frequency is not above the upper bound for frequency), and to one of several possible monetary value states. Figure 2 uses $m - 1$, m, and $m + 1$ as three
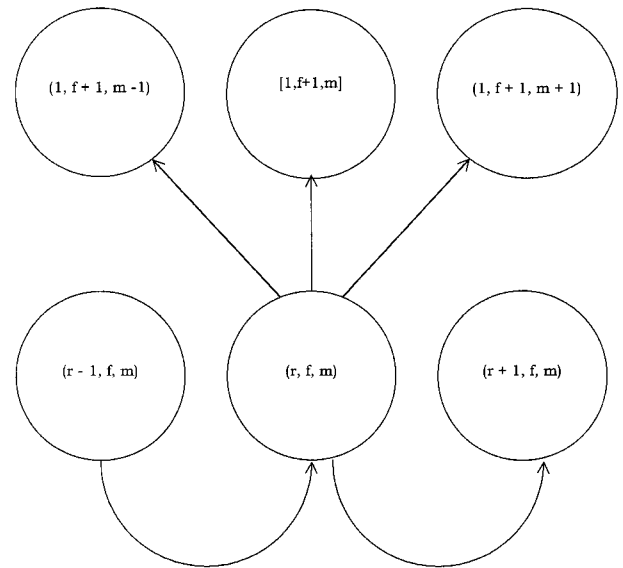

FIGURE 2
Graphical Representation of Transitions into and out of State (r, f, m) for an RFM-based MCM

possibilities. The firm receives a reward which depends on the monetary value category to which the customer transitions—higher net contribution, for example, if the customer moves to a higher monetary value category.

A challenge in constructing an RFM-based MCM will be to define monetary value categories in such a way that the Markov property will hold. Recall that the Markov property requires that future prospects for the customer relationship depend only on the current state of the relationship. By their very nature, monetary value categories based on moving averages will be non-Markovian. Instead of moving averages, monetary value categories based on the single last purchase amount or the cumulative total of all previous purchase amounts will be better suited for Markov chain modeling.

EXAMPLE USE OF MARKOV DECISION PROCESSES TO MANAGE A CUSTOMER RELATIONSHIP

Up to this point we have attempted to illustrate how the MCM can be used to model a firm's relationship with a customer in a variety of situations. Presumably, the most important use of

these models is in calculating LTV as a function of some small set of meaningful input parameters. LTV is perhaps most commonly used to help firms decide how much to spend in their attempts to initiate relationships.

In this section, we offer an extensive numerical example that will concentrate on the opposite end of the firm's relationship with the customer: the firm's decision to terminate the relationship. This example will demonstrate the use of the theory of Markov decision processes to find the best decision strategy in a reasonably complicated situation.

A large catalog company knows that a customer's response probabilities to its thrice-yearly catalog depend on both the customer's recency (how many periods/catalogs since the customer's last purchase) and frequency (how many times the customer has purchased from the firm). These purchase probabilities for a typical customer are given in Table 1.

While the MCM can accommodate purchase amounts and marketing expenses that vary with recency and frequency, for the purposes of this example we will keep things simple. Assume the customer's purchases bring the firm $60 in expected net contribution regardless of the customer's recency or frequency at the time of purchase. For each period the customer is active, the firm spends $1 mid-period in marketing—again regardless of the customer's recency or frequency. The firm uses a discount rate of 0.03 per period to account for the time value of money.

Heretofore the firm has curtailed its relationship with the customer after recency 24. This is why purchase probabilities for recencies greater than 24 are not available. In light of projected increases in paper and postage costs, the firm is intent on reexamining this policy. Perhaps the firm should be more aggressive in paring its mailing list of dormant customers? Or perhaps the firm's policy should depend on both recency and frequency—cutting off low frequency customers sooner than higher frequency customers? It seems fairly obvious that such a policy will do better—but how much better? Will the increase in profitability justify

TABLE I
Catalog Customer Repurchase Probabilities

Recency	Frequency				
	1	2	3	4	≥5
1	0.103	0.121	0.143	0.151	0.163
2	0.076	0.090	0.106	0.112	0.121
3	0.059	0.069	0.081	0.086	0.093
4	0.045	0.053	0.062	0.066	0.071
5	0.038	0.045	0.053	0.056	0.061
6	0.035	0.041	0.049	0.051	0.056
7	0.030	0.035	0.041	0.043	0.047
8	0.027	0.032	0.038	0.040	0.043
9	0.025	0.029	0.035	0.037	0.040
10	0.021	0.025	0.030	0.031	0.034
11	0.021	0.024	0.028	0.030	0.033
12	0.020	0.024	0.028	0.030	0.032
13	0.017	0.020	0.024	0.025	0.027
14	0.017	0.020	0.024	0.025	0.027
15	0.016	0.019	0.022	0.024	0.026
16	0.015	0.018	0.021	0.022	0.024
17	0.014	0.017	0.020	0.021	0.022
18	0.013	0.016	0.018	0.019	0.021
19	0.013	0.015	0.018	0.019	0.020
20	0.012	0.014	0.017	0.018	0.019
21	0.012	0.014	0.016	0.017	0.018
22	0.011	0.013	0.015	0.016	0.017
23	0.011	0.012	0.015	0.015	0.017
24	0.010	0.012	0.014	0.015	0.016

the costs associated with implementing the change?

The Markov chain model of this catalog firm's relationship with a customer will require 121 states. The first 120 states will be the 24×5 combination of 24 possible recencies ($r = 1$ to 24) with 5 possible frequencies ($f = 1$ to 5). Notice that frequencies greater than or equal to 5 are all lumped together as frequency 5. The final state is the familiar "former customer" state that we will label recency 25 and frequency 1. The probabilities in Table 1 will be labeled $p_{r,f}$, where (r, f) refers to the recency and frequency of the customer.

The construction of the 121×121 transition matrix follows directly from the problem de-

scription. The customer transitions to state (25, 1) from states (24, 1) through (24, 5) with probabilities $1 - p_{24,1}$ through $1 - p_{24,5}$ respectively. The customer transitions to state (1, 5) from states (1, 4) through (24, 4) with probabilities $p_{1,4}$ through $p_{24,4}$ respectively, and from states (1, 5) through (24, 5) with probabilities $p_{1,5}$ through $p_{24,5}$ respectively. For $f = 2, 3,$ or 4 the customer transitions to state (1, f) from states (1, $f - 1$) through (24, $f - 1$) with probabilities $p_{1,f-1}$ through $p_{24,f-1}$ respectively. Finally, for recencies 2 through 24 inclusive, the customer transitions to state (r, f) from state ($r - 1, f$) with probability $1 - p_{r-1,f}$.

The 121-component reward vector is defined as follows:

$$\mathbf{R}_{rf} = \begin{bmatrix} NC - M \\ -M \\ 0 \end{bmatrix} \quad \begin{array}{l} r = 1 \\ r = 2, 3, \ldots, 24 \\ r = 25 \end{array}$$

where M is equal to the present value of remarketing expenditures or $\$1/(1.03)^{1/2}$. Notice the very simple structure of these cash flows. As mentioned earlier, useful extensions to this model would be to consider net contributions that vary with frequency and marketing expenditures that vary with both recency and frequency. These extensions are easy to implement because they involve changes to the reward vector only.

We are now in a position to evaluate the firm's current policy of curtailing the customer relationship after recency 24. Using equation (2) and the inversion of a 121×121 matrix allows us to calculate the 121×1 column vector $\mathbf{V}$. The first element of $\mathbf{V}$ is calculated to be $\$89.264$. The expected present value of the catalog firm's relationship with the customer is $\$89.264$. Included in this number is the $\$60$ net contribution from the initial purchase—so that $\$29.264$ of the present value of the relationship comes from cash flows after the initial purchase.

Examining the calculated expected values for the remaining possible states of the firm's relationship tells us that the policy of curtailing the relationship after recency 24 is a fairly good one. Only one state, state (24, 1) has a negative

expected value. The firm could do a little bit better if it dropped frequency 1 customers after recency 23, rather than 24.

To illustrate in detail how the Markov decision model can be used to help the firm manage its relationship with the customer, suppose that paper and postage increases raise the cost of remarketing from $\$1$ to $\$2$ per period, with no other changes. Suppose further that the firm is free to change its mailing policy as it sees fit with no effect on the customer's repurchase probabilities. (While this might be true in the short run, it would not be true in the long run. If customers learn to expect a certain number of contacts and plan or allocate their purchases accordingly, cutting back on the number of contacts might increase the purchase probabilities at lower recencies.)

The firm's challenge is to find the optimal contact policy. We saw earlier that the optimal policy at $M = \$1$ was to curtail mailings after recency 23 for a frequency 1 customer and after recency 24 for frequencies 2, 3, 4, and 5. Let us refer to this policy as (23, 24, 24, 24, 24), reflecting the recency cutoffs for each of the five possible frequencies. Our challenge is to find the best contact policy (the best string of five recency cutoff values) now that mid-period marketing expenses are $\$2$.

The problem described is an example of a Markov decision problem, and there is a large body of knowledge about how to solve these kinds of problems. We will illustrate one popular approach called the *policy improvement algorithm* (see Hillier & Lieberman, 1986, p. 715).

The policy improvement algorithm begins by evaluating (calculating $\mathbf{V}$) for some arbitrary policy. As our initial arbitrary policy we use the firm's current policy of curtailing contact after recency 24: (24, 24, 24, 24, 24). Using (2) to evaluate $\mathbf{V}$ for this policy gives the results in Table 2. The expected customer lifetime value using this policy is $\$69.470$. Subtracting the initial $\$60$ contribution, we see that the higher remarketing cost has cut the value of the firm's future relationship with the customer rather drastically—from $\$29.264$ down to $\$9.470$ if the firm sticks to its initial (24, 24, 24, 24, 24) policy. We can see from Table 2, however, that there

TABLE 2
Calculated **V** for Catalog-Firm Policy (24, 24, 24, 24, 24) $NC = \$60$, $M = \$2$, and $d = 0.03$

Recency	Frequency				
	1	2	3	4	5
1	$ 69.470	$ 79.194	$ 88.052	$ 92.381	$ 95.821
2	$ 4.069	$ 12.697	$ 20.710	$ 24.632	$ 27.837
3	$ 0.184	$ 7.903	$ 15.169	$ 18.737	$ 21.704
4	$ (2.563)	$ 4.415	$ 11.050	$ 14.320	$ 17.069
5	$ (4.371)	$ 2.027	$ 8.155	$ 11.185	$ 13.751
6	$ (5.715)	$ 0.172	$ 5.843	$ 8.660	$ 11.057
7	$ (6.861)	$ (1.473)	$ 3.750	$ 6.351	$ 8.576
8	$ (7.597)	$ (2.634)	$ 2.204	$ 4.613	$ 6.692
9	$ (8.153)	$ (3.598)	$ 0.868	$ 3.090	$ 5.027
10	$ (8.553)	$ (4.384)	$ (0.282)	$ 1.771	$ 3.554
11	$ (8.651)	$ (4.808)	$ (1.016)	$ 0.882	$ 2.538
12	$ (8.682)	$ (5.169)	$ (1.689)	$ 0.056	$ 1.581
13	$ (8.679)	$ (5.512)	$ (2.362)	$ (0.772)	$ 0.611
14	$ (8.426)	$ (5.547)	$ (2.685)	$ (1.231)	$ 0.035
15	$ (8.142)	$ (5.565)	$ (2.996)	$ (1.685)	$ (0.546)
16	$ (7.765)	$ (5.480)	$ (3.197)	$ (2.035)	$ (1.021)
17	$ (7.257)	$ (5.247)	$ (3.243)	$ (2.213)	$ (1.315)
18	$ (6.646)	$ (4.909)	$ (3.174)	$ (2.270)	$ (1.494)
19	$ (5.940)	$ (4.460)	$ (2.984)	$ (2.210)	$ (1.544)
20	$ (5.168)	$ (3.953)	$ (2.737)	$ (2.098)	$ (1.547)
21	$ (4.295)	$ (3.331)	$ (2.362)	$ (1.850)	$ (1.411)
22	$ (3.343)	$ (2.625)	$ (1.902)	$ (1.521)	$ (1.189)
23	$ (2.310)	$ (1.833)	$ (1.354)	$ (1.097)	$ (0.878)
24	$ (1.194)	$ (0.953)	$ (0.715)	$ (0.585)	$ (0.473)
25	$ 0				

are many states in which the value of the firm's relationship is negative. Improvement will be possible.

The next step in the policy improvement algorithm is to revisit the firm's decision at each and every state and replace the current decision with a best decision based on the calculated values. In our example, this means contacting only those customers at states with positive values and not contacting the rest. The improved policy is therefore (3, 6, 9, 12, 14). Thus ends the first iteration in the policy improvement algorithm. Policy (3, 6, 9, 12, 14) becomes the new candidate policy.

To evaluate policy (3, 6, 9, 12, 14), we mod-ified the Markov decision model by changing the purchase probabilities to zero for any state outside the firm's contact policy. In addition, we set the reward for transitioning to any of these states at zero. In effect, if the customer transitions to a state outside the firm's contact policy she or he will transition with probability 1 and with reward zero to state (25, 1). The results of evaluating policy (3, 6, 9, 12, 14) are given in Table 3. The expected customer lifetime value for this strategy has improved to $71.487.

Turning to the policy improvement step, we notice that every state contacted in the current policy has positive expected value. So there is

TABLE 3
Calculated **V** for Catalog-Firm Policy (3, 6, 9, 12, 14) NC = $60, M = $2, and d = 0.03

Recency	Frequency				
	1	2	3	4	5
1	$ 71.487	$ 80.085	$ 88.337	$ 92.781	$ 96.131
2	$ 6.281	$ 13.702	$ 20.987	$ 25.063	$ 28.159
3	$ 2.578	$ 9.011	$ 15.440	$ 19.197	$ 22.038
4	$ 0	$ 5.620	$ 11.318	$ 14.809	$ 17.417
5	$ 0	$ 3.321	$ 8.424	$ 11.702	$ 14.113
6	$ 0	$ 1.554	$ 6.113	$ 9.207	$ 11.433
7	$ 0	$ 0	$ 4.021	$ 6.927	$ 8.969
8	$ 0	$ 0	$ 2.478	$ 5.219	$ 7.101
9	$ 0	$ 0	$ 1.146	$ 3.728	$ 5.454
10	$ 0	$ 0	$ 0	$ 2.441	$ 3.999
11	$ 0	$ 0	$ 0	$ 1.584	$ 3.000
12	$ 0	$ 0	$ 0	$ 0.792	$ 2.063
13	$ 0	$ 0	$ 0	$ 0	$ 1.114
14	$ 0	$ 0	$ 0	$ 0	$ 0.559
15	$ 0	$ 0	$ 0	$ 0	$ 0
16	$ 0	$ 0	$ 0	$ 0	$ 0
17	$ 0	$ 0	$ 0	$ 0	$ 0
18	$ 0	$ 0	$ 0	$ 0	$ 0
19	$ 0	$ 0	$ 0	$ 0	$ 0
20	$ 0	$ 0	$ 0	$ 0	$ 0
21	$ 0	$ 0	$ 0	$ 0	$ 0
22	$ 0	$ 0	$ 0	$ 0	$ 0
23	$ 0	$ 0	$ 0	$ 0	$ 0
24	$ 0	$ 0	$ 0	$ 0	$ 0
25	$ 0				

no way to improve the current policy by making further cutbacks. What we need to do now is consider contacting some of the states we no longer contact. For example, suppose we contacted state (4, 1). Such a contact would cost us $2 in the middle of the period but would bring us a 0.0450 probability of transitioning to state (1, 2) and a 0.9550 probability of transitioning to state (5, 1) at the end of the period. The expected present value of doing so is

$$(1.03)^{-1/2}(-\$2) + (1.03)^{-1}[0.045(\$80.085)$$
$$+ 0.9550(\$0)] = \$1.528.$$

You will notice that we have evaluated the proposed change in policy using the values calculated for the current policy. While this evaluation will not hold if we make more than one change to the current policy, it is as prescribed by the policy improvement algorithm. After performing similar calculations for all states not contacted under the current policy, we find several that project a profitable contact. The improved policy that results is (8, 12, 15, 16, 17).

The expected customer lifetime value for the (8, 12, 15, 16, 17) policy is calculated to be $74.519. All contacted states have positive ex-

pected values and one of these, state (9, 1), projects a small positive profit if contacted.

Our final stage of the policy improvement algorithm evaluates policy (9, 12, 15, 16, 17) to find an expected lifetime value of $74.523, all contacted states with positive values, and no other states projecting a positive profit if contacted. The policy improvement algorithm is complete. We have found the optimal policy.

The net result of the increase in marketing costs from $1 to $2 has been a change in optimal policy from (23, 24, 24, 24, 24) to (9, 12, 15, 16, 17) and a decrease in expected customer lifetime value from $89.267 to $74.523. Subtracting the initial $60 contribution shows us that the expected future value of the firm's relationship with the customer decreases from $29.267 to $14.523.

SUMMARY AND CONCLUSIONS

This paper introduced a general class of mathematical models, Markov Chain Models, which are appropriate for modeling customer relationships and calculating LTVs. A major advantage of this class of models is its flexibility. This flexibility was demonstrated by showing how the MCM could handle the wide variety of the customer relationship situations previously modeled in the literature. The MCM is particularly useful in modeling complicated customer-relationship situations for which algebraic solutions will not be possible.

A second advantage of the MCM is that it is a probabilistic model. It incorporates the language of probability and expected value—language that will help marketers talk about relationships with individual customers.

The MCM is also supported by a well-developed theory about how these models can be used for decision making. The use of this theory was demonstrated by a comprehensive numerical example in which a catalog company adjusted its contact policy in response to increased mailing costs.

REFERENCES

Berger, P.D., & Nasr, N.I. (1998). Customer Lifetime Value: Marketing Models and Applications. Journal of Interactive Marketing, 12 (1), 17–29.

Blattberg, R.C. (1998). Managing the Firm Using Lifetime-Customer Value. Chain Store Age, January, 46–49.

Blattberg, R.C. (1987). Research Opportunities in Direct Marketing. Journal of Direct Marketing, 1 (1), 7–14.

Blattberg, R., & Deighton, J. (1996). Manage Marketing by the Customer Equity Test. Harvard Business Review, July–August, 136–144.

Bronnenberg, B.J. (1998). Advertising Frequency Decisions in a Discrete Markov Process Under a Budget Constraint. Journal of Marketing Research, 35 (3), 399–406.

Courtheaux, R.J. (1986). Database Marketing: Developing a Profitable Mailing Plan. Catalog Age, June–July.

David Shepard Associates, Inc. (1995). The New Direct Marketing (2nd ed.). Burr Ridge, IL: Irwin.

Dwyer, F.R. (1989). Customer Lifetime Valuation to Support Marketing Decision Making. Journal of Direct Marketing, 3 (4), 8–15 (reprinted in 11 (4), 6–13).

Hillier, F.S., & Lieberman, G.J. (1986). Introduction to Operations Research (4th ed.). Oakland, CA: Holden–Day.

Hughes, A.M. (1997). Customer Retention: Integrating Lifetime Value into Marketing Strategies. Journal of Database Marketing, 5 (2), 171–178.

Jackson, B. (1985). Winning and Keeping Industrial Customers. Lexington, MA: Lexington Books.

Jackson, D.R. (1996). Achieving Strategic Competitive Advantage Through the Application of the Long-Term Customer Value Concept. Journal of Database Marketing, 4 (2), 174–186.

Puterman, M.J. (1994). Markov Decision Processes: Discrete Stochastic Dynamic Programming. New York: John Wiley & Sons.

White, D.J. (1993). A Survey of Applications of Markov Decision Processes. The Journal of the Operational Research Society, 44 (11), 1073–1096.