Verticals: Finance

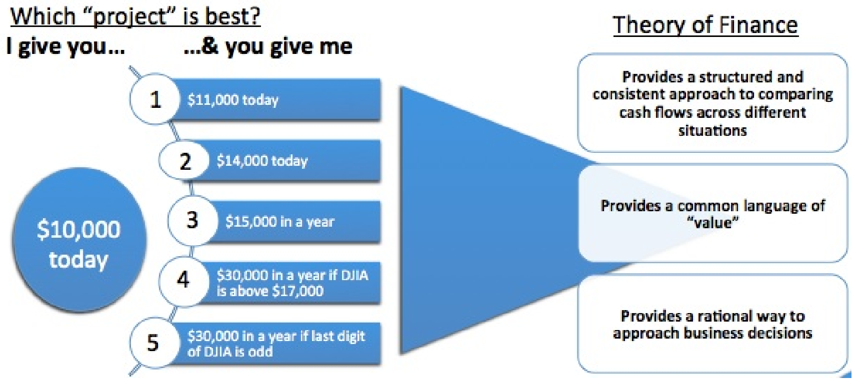
Outline

- Motivation
- Fundamental Challenges of Finance
- Role of Financial Markets and "price discovery" process
- Present Value and The Law of One Price or No Arbitrage
- NPV and alternative decision methods
- Mini Case

Motivation

Business decisions require valuation!

Why is Finance Important?



Fundamental Challenges

Financial Managers face two fundamental challenges:

- **Valuation of Assets**
- **Management of Assets**

- Any business decision is concerned with these two aspects.
- Valuation is the starting point for management.
- You can't manage what you cannot measure. Once we have a "value", we can manage!

What is *value*?

- Role of Financial Markets and “price discovery” process
- How does the market come up with the “value”? Let’s see financial markets at work!

How are decisions made?

Objectives+Valuations=Decisions

Framework for Financial Analysis

Accounting

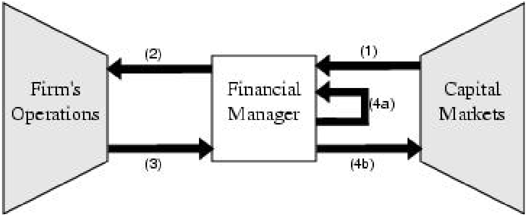
- The language of finance
- Difference between "Stock" vs. "Flow" variables

Balance Sheet and Income Statement

- Balance Sheet: Picture of the financial status quo (stock)
- Income Statement: rate of change of the status quo (flow)

Framework: Corporate Financial Decisions

Broadly stated, finance is concerned with the flow of funds (capital) across both time and agents.

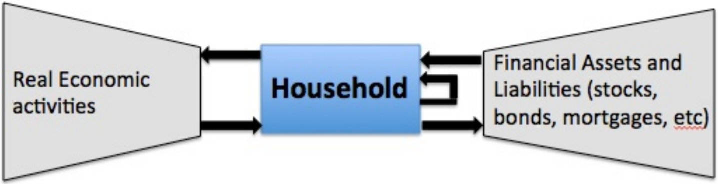


- For firms, these flows include:
  1. issues of securities to raise cash
  2. purchases of real assets used in the firm's operations
  3. cash inflows generated by the real assets
  4. cash inflows reinvested in the firm (4a)
  5. cash inflows returned to the firm's investors (4b)

How does this apply to you?

Personal Financial Decisions

1. cash raised from financial institutions
2. cash invested in real assets (tangible and intangible)
3. cash inflows generated by labor supply
4. cash consumed and reinvested in real assets
5. cash invested in financial assets



Time and Risk

Challenging factors that make finance challenging

1. Time
   - One dollar today is worth more than one dollar tomorrow
   - Time flows only in one direction (as far as we know)
   - How do we model temporal differences?

2. Risk
   - Under certainty, finance theory is complete (no more analysis needed)
   - Risk is difficult to measure. How should we model the "unknown"?

How to address these two issues?

- Historical Data
- Use Mathematics

I.1 Evaluating Investment Decisions with Present Value

Critical Concepts

- Cash flows and definition of "assets"
- The Present Value and Future Value
- Time Value of Money
- Shortcuts for special Cash flows: Perpetuity and Annuity

Assets and Cashflows

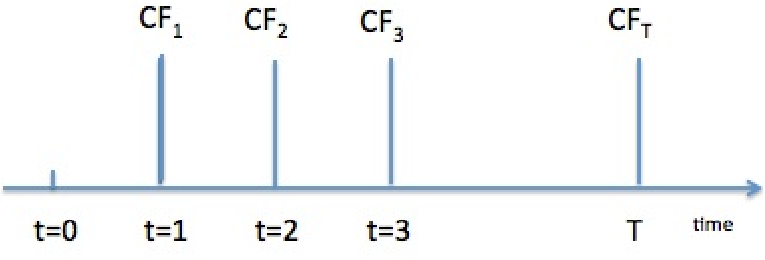
- A cash flow is money coming to you or going away from you. A flow of cash!
- At the most general level, an *asset* is a claim to a stream of cash flows.
- This definition encompasses both real and financial investments.
- Examples:
  - stock and bond
  - oil field, an electric car
  - business entity (start up)
  - patents and R&D
  - knowledge and reputation.

From a business prespective, an asset is simply a sequence of cash flows

$$Asset_t = [CF_t, CF_{t+1}, CF_{t+2}, ...]$$

Valuation Principle

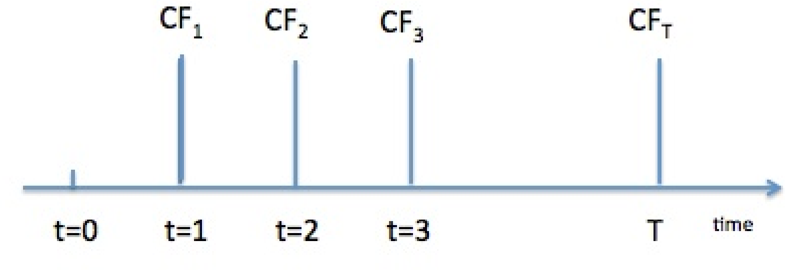
Valuing an asset requires valuing a sequence of cashflows.



- Always draw a timeline to visualize the timing of the cashflows!
- Cashflows at different points in time are different "currencies". I'll need to convert all the cash flows in the single currency that is today's dollars.

Valuation Principle

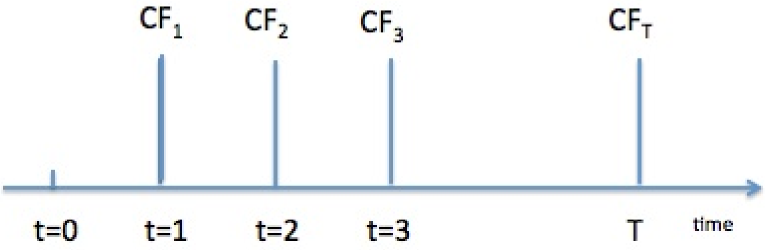
Valuing an asset requires valuing a sequence of cashflows.



- **Always draw a timeline to visualize the timing of the cashflows!**
- Cashflows at different points in time are different "currencies". I'll need to convert all the cash flows in the single currency that is today's dollars.

Valuation Principle

Valuing an asset requires valuing a sequence of cashflows.



- **Always draw a timeline to visualize the timing of the cashflows!**
- **Cashflows at different points in time are different "currencies". I'll need to convert all the cash flows in the single currency that is today's dollars.**

Using Market Prices to Obtain Value: Apples to Apples

To evaluate any decision:

- We want to “sum up” all the different elements of the asset.
- To do so, we need to convert all the elements into a single common denominator, a numeraire, (e.g., \$USD today).

Example (Using Market Prices)

A farmer is going to a local farmers market with an inventory of 1,000 apples. Another farmer offers to give him 700 oranges in exchange for his apples. Should the farmer accept this trade?

- Suppose $P_{orange} = \$0.80$, $P_{apple} = \$0.50$

$$\begin{aligned} \text{Value of the Transaction} &= \text{Value of Oranges - Value of Apples} \\ &= 700 \times P_{orange} - 1,000 \times P_{apple} \\ &= \$60 \end{aligned}$$

Using Market Prices to Obtain Value: Apples to Apples

To evaluate any decision:

- We want to "sum up" all the different elements of the asset.
- To do so, we need to convert all the elements into a single common denominator, a numeraire, (e.g., $USD today).

Example (Using Market Prices)

A farmer is going to a local farmers market with an inventory of 1,000 apples. Another farmer offers to give him 700 oranges in exchange for his apples. Should the farmer accept this trade?

- Suppose $P_{orange} = \$0.80$, $P_{apple} = \$0.50$

$$\begin{aligned} \text{Value of the Transaction} &= \text{Value of Oranges - Value of Apples} \\ &= 700 \times P_{orange} - 1{,}000 \times P_{apple} \\ &= \$60 \end{aligned}$$

Using Market Prices to Obtain Value: Apples to Apples

To evaluate any decision:

- We want to "sum up" all the different elements of the asset.
- To do so, we need to convert all the elements into a single common denominator, a numeraire, (e.g., \$USD today).

Example (Using Market Prices)

A farmer is going to a local farmers market with an inventory of 1,000 apples. Another farmer offers to give him 700 oranges in exchange for his apples. Should the farmer accept this trade?

- Suppose $P_{orange} = \$0.80$, $P_{apple} = \$0.50$

$$\begin{aligned}\text{Value of the Transaction} &= \text{Value of Oranges - Value of Apples}\\ &= 700 \times P_{orange} - 1{,}000 \times P_{apple}\\ &= \$60\end{aligned}$$

Using Market Prices to Obtain Value: Apples to Apples

To evaluate any decision:

- We want to "sum up" all the different elements of the asset.
- To do so, we need to convert all the elements into a single common denominator, a numeraire, (e.g., \$USD today).

Example (Using Market Prices)

A farmer is going to a local farmers market with an inventory of 1,000 apples. Another farmer offers to give him 700 oranges in exchange for his apples. Should the farmer accept this trade?

- Suppose $P_{orange} = \$0.80$, $P_{apple} = \$0.50$

$$\begin{aligned}
\text{Value of the Transaction} &= \text{Value of Oranges - Value of Apples} \\
&= 700 \times P_{orange} - 1{,}000 \times P_{apple} \\
&= \$60
\end{aligned}$$

Using Market Prices to Obtain Value: Apples to Apples

To evaluate any decision:

- We want to "sum up" all the different elements of the asset.
- To do so, we need to convert all the elements into a single common denominator, a numeraire, (e.g., \$USD today).

Example (Using Market Prices)

A farmer is going to a local farmers market with an inventory of 1,000 apples. Another farmer offers to give him 700 oranges in exchange for his apples. Should the farmer accept this trade?

- Suppose $P_{orange} = \$0.80$, $P_{apple} = \$0.50$

$$\begin{aligned}
\text{Value of the Transaction} &= \text{Value of Oranges - Value of Apples} \\
&= 700 \times P_{orange} - 1,000 \times P_{apple} \\
&= \$60
\end{aligned}$$

Using Market Prices to Obtain Value (cont'd)

The key here was that we used market prices to convert each element of the transaction (both oranges and apples) into dollars.

- What assumptions have we made in assessing the value of this trade opportunity?
  - Homogeneiety: about the quality of the fruit
  - Liquidity: about the ability to sell the fruit

Using Market Prices to Obtain Value (cont'd)

The key here was that we used market prices to convert each element of the transaction (both oranges and apples) into dollars.

- What assumptions have we made in assessing the value of this trade opportunity?
  - Homogeneiety: about the quality of the fruit
  - Liquidity: about the ability to sell the fruit

Using Market Prices to Obtain Value (cont'd)

The key here was that we used market prices to convert each element of the transaction (both oranges and apples) into dollars.

- What assumptions have we made in assessing the value of this trade opportunity?
  - Homogeneiety: about the quality of the fruit
  - Liquidity: about the ability to sell the fruit

Using Market Prices to Obtain Value (cont'd)

- Markets are *adaptive and competetive*:
  - There is a market price, everyone can buy and sell at the price.
- There is no such thing as a free lunch.
- All agents act to further their own self-interest.
- Other things equal, individuals:
  - Prefer more money to less (no-satiation)
  - Prefer money now to money later (impatience)
  - Prefer to avoid risk (risk aversion)
- How reasonable are these assumptions?
  - Reasonable, to a first approximation (especially in financial markets).
  - Plus, we got to start somewhere!

When decisions involve trade-offs over time

Example

A drug company is considering developing a vaccine.

Strategy A:

Date:	0	1	2	3
Cash Flows:	-400	200	200	200

Strategy B:

Date:	0	1	2	3
Cash Flows:	-400	-100	350	350

Which strategy creates the most value?

- We cannot just add up the cash flows – why?

Key: We need to convert the cash flows into a single common denominator so we can add them up.

- What common denominator can we use?
- What is the "exchange rate"?

When decisions involve trade-offs over time

Example

A drug company is considering developing a vaccine.

Strategy A:

Date:	0	1	2	3
Cash Flows:	-400	200	200	200

Strategy B:

Date:	0	1	2	3
Cash Flows:	-400	-100	350	350

Which strategy creates the most value?

- We cannot just add up the cash flows – why?

Key: We need to convert the cash flows into a single common denominator so we can add them up.

- What common denominator can we use?
- What is the "exchange rate"?

Interest Rates

To convert to a common denominator, we will need to move money through time (i.e., borrow or lend).

- The cost or benefit of doing so is represented by the **interest rate**.
- The interest rate is just a price!

Example (The Price of Consumption)

I lend you \$1 today. The annual interest rate is 7%. You repay \$1.07 in a year.

- Lender's perspective: I get to consume \$0.07 extra by being willing to delay consumption a year.
- Borrower's perspective: You get to consume \$1 today, but have to give up \$1.07 worth of consumption in a year. You give up \$0.07 by not being willing to delay.
- The price (for consuming today versus in the future) is just \$0.07.

Interest Rates

To convert to a common denominator, we will need to move money through time (i.e., borrow or lend).

- The cost or benefit of doing so is represented by the **interest rate**.
- The interest rate is just a price!

Example (The Price of Consumption)

I lend you \$1 today. The annual interest rate is 7%. You repay \$1.07 in a year.

- Lender's perspective: I get to consume \$0.07 extra by being willing to delay consumption a year.
- Borrower's perspective: You get to consume \$1 today, but have to give up \$1.07 worth of consumption in a year. You give up \$0.07 by not being willing to delay.
- The price (for consuming today versus in the future) is just \$0.07.

Present Value

Definition

The present value of a stream of cash flows is given by

$$\mathrm{PV} = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_T}{(1+r)^T}$$

1. Moving money backward in time (also called "discounting").
2. The units of PV are dollars today.
3. If you have cash flows in the future, PV is the amount you could **borrow** against them today.
4. If you wanted cash flows in the future, PV is the amount you need to **invest** in order to get them.
5. In a competitive market, PV is the **market value** of the cash flows.

Capital markets $\Rightarrow$ Market Value = Present Value

Key Idea: In a competitive market, the PV will determine (is equal to) what you can sell Asset F for today. That is,

present value = **market value** = **current price!**

- Therefore, we can value investments using present value.

Question: Why? What happens if market value does not equal present value?

Capital markets $\Rightarrow$ Market Value = Present Value

Key Idea: In a competitive market, the PV will determine (is equal to) what you can sell Asset F for today. That is,

$$\textbf{present value} = \textbf{market value} = \textbf{current price!}$$

- Therefore, we can value investments using present value.

Question: Why? What happens if market value does not equal present value?

Capital markets $\Rightarrow$ Market Value = Present Value

Key Idea: In a competitive market, the PV will determine (is equal to) what you can sell Asset F for today. That is,

present value = market value = current price!

- Therefore, we can value investments using present value.

Question: Why? What happens if market value does not equal present value?

- Traders can make riskless profits simply by buying and selling securities.
- That is, an arbitrage opportunity exists.

Capital markets $\Rightarrow$ Market Value = Present Value

Key Idea: In a competitive market, the PV will determine (is equal to) what you can sell Asset F for today. That is,

$$\textbf{present value} = \textbf{market value} = \textbf{current price!}$$

- Therefore, we can value investments using present value.

Question: Why? What happens if market value does not equal present value?

- Traders can make riskless profits simply by buying and selling securities.
- That is, an arbitrage opportunity exists.

No Arbitrage and the Law of One Price

Example

Suppose that you have a security that will pay \$11M a year from now. How much can you sell it for today? Suppose the interest rate is 10%.

$$\text{PV} = \frac{\$11\text{M}}{1 + 0.1} = \$10\text{M}$$

No Arbitrage and the Law of One Price

Example

Suppose that you have a security that will pay \$11M a year from now. How much can you sell it for today? Suppose the interest rate is 10%.

$$\text{PV} = \frac{\$11\text{M}}{1+0.1} = \$10\text{M}$$

No Arbitrage and the Law of One Price

Example

Suppose that you have a security that will pay \$11M a year from now. How much can you sell it for today? Suppose the interest rate is 10%.

$$\text{PV} = \frac{\$11\text{M}}{1 + 0.1} = \$10\text{M}$$

- Suppose you can buy or sell the security for \$9M. How can you profit from this? What will happen?

- Suppose that you can buy or sell the security for \$10.5M. How can you profit from this? What will happen?

No Arbitrage and the Law of One Price

Example

Suppose that you have a security that will pay \$11M a year from now. How much can you sell it for today? Suppose the interest rate is 10%.

$$\text{PV} = \frac{\$11\text{M}}{1 + 0.1} = \$10\text{M}$$

- Suppose you can buy or sell the security for \$9M. How can you profit from this? What will happen?
- Suppose that you can buy or sell the security for \$10.5M. How can you profit from this? What will happen?

No Arbitrage and the Law of One Price (cont'd)

This reasoning is called **no arbitrage pricing** or the **law of one price**.
It says that *in equilibrium*:

- If two securities generate the same cashflows at t=1,2,3,..., then they must sell for the same price at t=0.
- You cannot make riskless profits simply by trading assets in financial markets.
- The theory underlying this conclusion relies on competitive capital markets.

Big Takeaways: We should evaluate investments using present value!

- A manager should choose investments with the highest (net) present value in order to maximize the value of the firm.

No Arbitrage and the Law of One Price (cont'd)

This reasoning is called **no arbitrage pricing** or the **law of one price**.
It says that *in equilibrium*:

- If two securities generate the same cashflows at t=1,2,3,..., then they must sell for the same price at t=0.
- You cannot make riskless profits simply by trading assets in financial markets.
- The theory underlying this conclusion relies on competitive capital markets.

Big Takeaways: We should evaluate investments using present value!

- A manager should choose investments with the highest (net) present value in order to maximize the value of the firm.

Example (adding up PV's: Vaccine cont'd.)

Assume that the firm can borrow and lend at $r = 5\%$.

Strategy A:

Date:	0	1	2	3
Cash Flows:	-400	200	200	200
Present Value:	-400	190.5	181.4	172.8
			NPV	**144.6**

"NPV" means **net present value**. It's often used instead of PV to indicate that some of the cash flows are negative.

Strategy B:

Date:	0	1	2	3
Cash Flows:	-400	-100	350	350
Present Value:	-400	-95.2	317.5	302.3
			NPV	**124.6**

The firm should choose strategy A because it increases firm value the most.

Example (adding up PV's: Vaccine cont'd.)

Assume that the firm can borrow and lend at $r = 5\%$.

Strategy A:

Date:	0	1	2	3
Cash Flows:	-400	200	200	200
Present Value:	-400	190.5	181.4	172.8
			NPV	**144.6**

"NPV" means **net present value**. It's often used instead of PV to indicate that some of the cash flows are negative.

Strategy B:

Date:	0	1	2	3
Cash Flows:	-400	-100	350	350
Present Value:	-400	-95.2	317.5	302.3
			NPV	**124.6**

The firm should choose strategy A because it increases firm value the most.

Example (adding up PV's: Vaccine cont'd.)

Assume that the firm can borrow and lend at $r = 5\%$.

Strategy A:

Date:	0	1	2	3
Cash Flows:	-400	200	200	200
Present Value:	-400	190.5	181.4	172.8
			NPV	**144.6**

"NPV" means **net present value**. It's often used instead of PV to indicate that some of the cash flows are negative.

Strategy B:

Date:	0	1	2	3
Cash Flows:	-400	-100	350	350
Present Value:	-400	-95.2	317.5	302.3
			NPV	**124.6**

The firm should choose strategy A because it increases firm value the most.

Future Value

Definition

You have an amount P today. The **Future Value (FV)** of P is the amount you will have at some point in the future if you invest P today.

$$\text{FV}_n = P \times (1 + r)^n$$

1. Moving money forward in time (also called "compounding").
2. You do this by investing, i.e. lending.
3. Once we converted to future value at date n, we can simply add the elements up again.

Example

A bank pays 4.75% per year on a 10-year CD and you deposit $10,000. What is the FV of your investment? That is, how much money will you have in 10 years?

$$\text{FV}_{10} = 10,000 \times (1 + 0.0475)^{10} = \$15,905.24$$

Note: Both PV and FV allow us to convert cash flows into a common denominator.

Future Value

Definition

You have an amount P today. The **Future Value (FV)** of P is the amount you will have at some point in the future if you invest P today.

$$\mathrm{FV}_n = P \times (1 + r)^n$$

1. Moving money forward in time (also called "compounding").
2. You do this by investing, i.e. lending.
3. Once we converted to future value at date n, we can simply add the elements up again.

Example

A bank pays 4.75% per year on a 10-year CD and you deposit $10,000. What is the FV of your investment? That is, how much money will you have in 10 years?

$$\mathrm{FV}_{10} = 10,000 \times (1 + 0.0475)^{10} = \$15,905.24$$

Note: Both PV and FV allow us to convert cash flows into a common denominator.

Key Takeaways: Present Value and Future Value

- How to aggregate and measure cash flows arriving over time:
  - Present Value: $PV_T = \frac{C}{(1+r)^T}$
  - Future Value: $FV_T = C \times (1+r)^T$
- With multiple cash flows, to get PV (FV), compute PV (FV) of each and then add them up.
- In perfect capital markets, no arbitrage implies that:

$$\text{Market Price} = \text{Present Value}$$

I.2 Time Saving Formulas: Shortcuts to Present Value and Future Value

Shortcuts to calculating PVs

There are four types of cash flows that we will encounter frequently and have formulas which make them easy to compute:

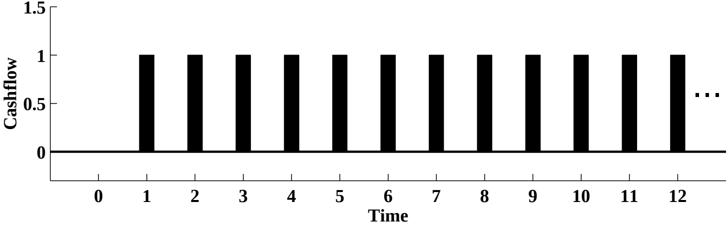
1. Perpetuity
   - Examples: UK consols, stock dividends of a large firm.
2. Growing perpetuity
   - Examples: Dividends of small (growing) firm
3. Annuity
   - Examples: Mortgage/loan payments, insurance premiums, pension payments
4. Growing Annuity
   - Examples: Venture capital investment, retirement benefits

Perpetuities

Definition

A **perpetuity** is a cash flow stream that provides an identical cash flow of \$C at the end of each period, forever.

$$\text{Present Value of a Perpetuity} = \frac{C}{r}$$



Important: The formula gives the PV as of 1 period before the first cash flow:

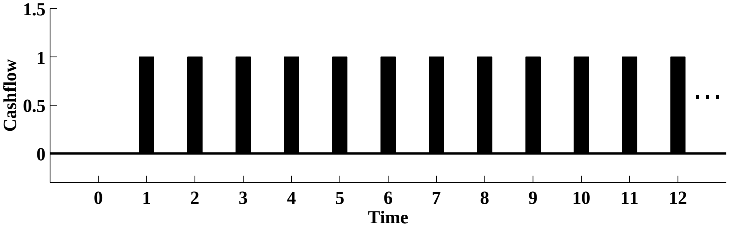
- If first cash flow is at t=1, the formula gives the PV is as of t=0.
- Same disclaimer applies to next three formulas.

Perpetuities

Definition

A **perpetuity** is a cash flow stream that provides an identical cash flow of \$C at the end of each period, forever.

$$\text{Present Value of a Perpetuity} = \frac{C}{r}$$



Important: The formula gives the PV as of 1 period before the first cash flow:

- If first cash flow is at t=1, the formula gives the PV is as of t=0.
- Same disclaimer applies to next three formulas.

Perpetuities: UK Consol Bonds

Example

- In the 1800's, the British government consolidated the huge debt accumulated during the Napoleonic wars and replaced it with a single issue of bonds with no termination date and a coupon rate of 2.5% (i.e., makes payments of 2.5% of the face value).
- These bonds, called *consols*, are still traded today.
- Suppose that the current interest rate in the U.K. is 9%. What is the PV of a consol with a £1,000 face value?

This is just a perpetuity promising to pay £25 each year. Its PV is

$$PV_0 = \frac{25}{0.09} = £277.78$$

Perpetuities: UK Consol Bonds

Example

- In the 1800's, the British government consolidated the huge debt accumulated during the Napoleonic wars and replaced it with a single issue of bonds with no termination date and a coupon rate of 2.5% (i.e., makes payments of 2.5% of the face value).
- These bonds, called *consols*, are still traded today.
- Suppose that the current interest rate in the U.K. is 9%. What is the PV of a consol with a £1,000 face value?

This is just a perpetuity promising to pay £25 each year. Its PV is

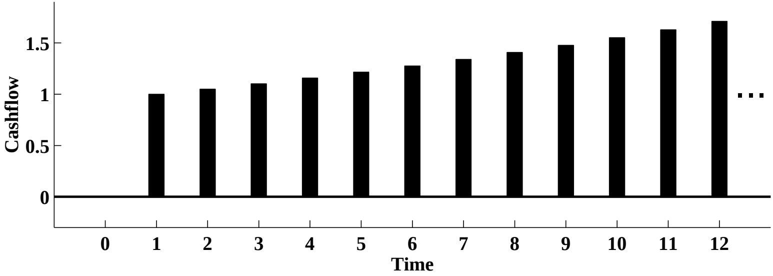
$$PV_0 = \frac{25}{0.09} = £277.78$$

Growing Perpetuity

Definition

A **growing perpetuity** provides a cash flow of \$C at the end of this period, with subsequent cash flows growing at a rate of g each period.

Cash flows from a \$1 growing perpetuity look like this for $g = 5\%$:



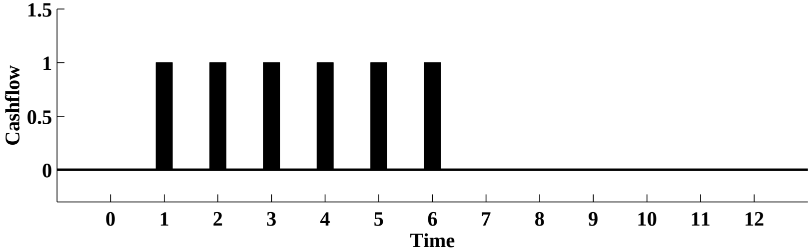
- Present value: $\boxed{PV = \frac{C}{r-g}}$, when $r - g > 0$.
- What if the first payment occurs today (at time 0) rather than in a year?

Annuities

Definition

An **annuity** provides an identical cash flow of \$C each period, starting at the end of this period and lasting for *n* periods.

$$\text{Present Value of an Annuity} = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

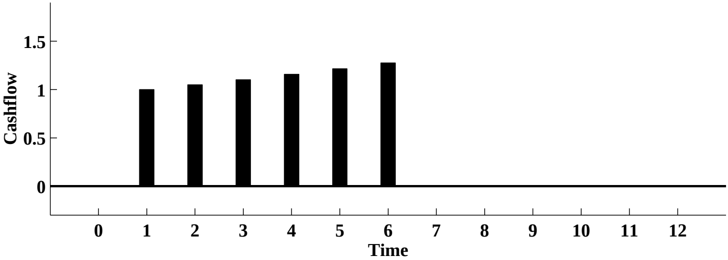


Growing Annuities

Definition

A **growing annuity** provides a cash flow of \$C at the end of this period, with subsequent cash flows growing at a rate of g each period, and lasting for n periods.

$$PV = \frac{C}{r-g} \times \left(1 - \left(\frac{1+g}{1+r}\right)^n\right)$$



Question: What if $r = g$?

- Take the limit as $g \to r$ (using L'Hopital's rule): $C \times \frac{n}{1+r}$

I.3 The NPV Decision Rule and Alternatives

Outline

We know how to value existing cash flows, but what is a good rule for selecting **new projects**?Will all shareholders agree on the rule?

- **The NPV decision rule**
- **Other tools for making investment decision? Are they useful?**
  - IRR and MIRR
  - Profitability Index
  - Payback Period and Discounted Payback Period
  - Best-case analysis

Outline

We know how to value existing cash flows, but what is a good rule for selecting **new projects**?Will all shareholders agree on the rule?

- **The NPV decision rule**
- **Other tools for making investment decision? Are they useful?**
  - IRR and MIRR
  - Profitability Index
  - Payback Period and Discounted Payback Period
  - Best-case analysis

The NPV rule

Recall that:

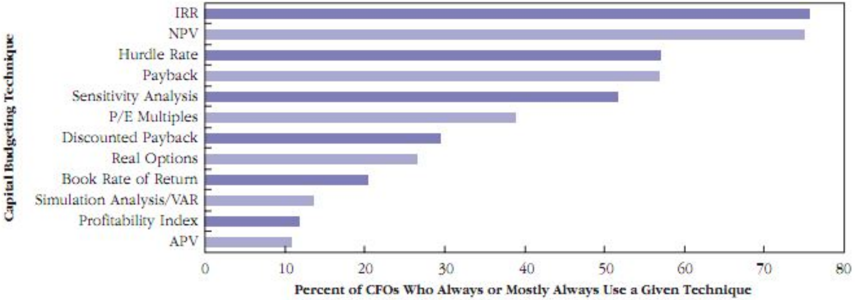
$$\text{NPV} = E(C_0) + \frac{E(C_1)}{1+r_1} + \cdots + \frac{E(C_n)}{(1+r_n)^n}$$

Where $E(C_t)$ is the **expected cash flow** and r is the firm's **cost of capital**. This represents the added market value of the project or firm.

The NPV decision rule:

1. In the case of a single project, or several independent projects, accept the project if and only if NPV > 0.
2. In the case of mutually exclusive projects, accept the project with the highest NPV, if that NPV > 0.

A Survey of CFOs



- John Graham and Campbell Harvey from Duke University surveyed 392 CFOs in 1999. **NPV and IRR are the most popular decision rules**, each used "Always or Almost Always" by about 75% of respondents.
- A similar study by Gitman and Forrester in 1977 found that only 9.8% of firms used the NPV rule – MBAs have been paying attention in class!

Summary of Alternative Evaluation Techniques

Some are best thought of as **as supplements to the basic NPV rule:**

- **IRR and MIRR**
  - Provide how much estimation error in the cost of capital can exist without altering the original decision. Also a good communication tool!
- **Profitability index**
  - A useful tool to maximize overall NPV from a set of projects when you face a constraint. Better than just picking projects in order of decreasing NPV.
- **Adjusted PV**, **APV**
  - This is just NPV where you account for the tax benefits of debt.
- **Real options theory**
  - Allows one to maximize NPV when the project has embedded options, e.g., the option to shut down early if bad new arrives. It is a way to value flexibility.

Internal Rate of Return (IRR)

Definition

A project's **IRR** is the interest rate that sets the net present value of the project cash flows equal to zero:

$$0 = C_0 + \frac{C_1}{(1+\mathrm{IRR})} + \frac{C_2}{(1+\mathrm{IRR})^2} + \cdots + \frac{C_n}{(1+\mathrm{IRR})^n}$$

Decision rule when using IRR:

- For **independent projects**: Accept a project if its IRR is greater than the opportunity cost of capital.
- For **mutually exclusive projects**: Among the projects, accept the one with the highest IRR provided it is greater than the opportunity cost of capital.

Basic idea: If IRR>Cost of capital, you can borrow at the cost of capital, gradually pay off investors and have money left over!

Internal Rate of Return (IRR)

Definition

A project's **IRR** is the interest rate that sets the net present value of the project cash flows equal to zero:

$$0 = C_0 + \frac{C_1}{(1+\mathrm{IRR})} + \frac{C_2}{(1+\mathrm{IRR})^2} + \cdots + \frac{C_n}{(1+\mathrm{IRR})^n}$$

Decision rule when using IRR:

- For **independent projects**: Accept a project if its IRR is greater than the opportunity cost of capital.
- For **mutually exclusive projects**: Among the projects, accept the one with the highest IRR provided it is greater than the opportunity cost of capital.

Basic idea: If IRR>Cost of capital, you can borrow at the cost of capital, gradually pay off investors and have money left over!

Internal Rate of Return (IRR)

Definition

A project's **IRR** is the interest rate that sets the net present value of the project cash flows equal to zero:

$$0 = C_0 + \frac{C_1}{(1+\text{IRR})} + \frac{C_2}{(1+\text{IRR})^2} + \cdots + \frac{C_n}{(1+\text{IRR})^n}$$

Decision rule when using IRR:

- For **independent projects**: Accept a project if its IRR is greater than the opportunity cost of capital.
- For **mutually exclusive projects**: Among the projects, accept the one with the highest IRR provided it is greater than the opportunity cost of capital.

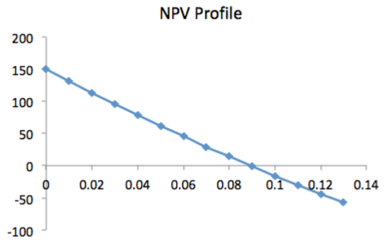
Basic idea: If IRR>Cost of capital, you can borrow at the cost of capital, gradually pay off investors and have money left over!

Question: When IRR and NPV decision rules agree

Date	Cash flow
0	-1000
1	500
2	550
3	100

=IRR(B2:B5)
8.92%

Potential discount rate, r	NPV for this discount rate
0	150.00
0.01	131.27
0.02	113.07
0.03	95.38
0.04	78.17
0.05	61.44
0.06	45.16
0.07	29.31
0.08	13.88
0.09	-1.14
0.1	-15.78
0.11	-30.04
0.12	-43.94
0.13	-57.49



- An NPV Profile plots the NPV as a function of the cost of capital.
- Suppose that the cost of capital is 6%
  - Is this a good project according to the IRR rule?
  - Is this a good project according to the NPV rule?

Why is IRR useful?

- **Estimation Error** IRR tells you how high the cost of capital can go before the project becomes unprofitable.
  - Even if you use NPV, you may still want to use IRR as a supplement.
- Useful **communication tool** for people who understand **returns**, but have trouble with the concept of present value.
- **IRR can sometimes give misleading or incorrect answers**
  - Important to know when issues arise.
  - Some of the problems with the IRR rule can be fixed with incremental IRR or modified IRR

Issue 1: IRR Ignores Scale

Suppose you have two projects that are mutually exclusive:

	C_0	C_1	IRR	NPV at 10%
Project 1 (small)	-10	20	100%	8.18
Project 2 (large)	-20	36	80%	12.73

- **Question:** Which project adds the **most value for shareholders?**
- **Question:** Which project does the IRR rule select?

A Fix for the Scale Issue

A way around this problem is to use **incremental cash flows**.

- Is the incremental investment (project 2-1) a good idea?

	C_0	C_1	IRR	NPV at 10%
Project 1 (small)	-10	20	100%	8.18
Project 2 (large)	-20	36	80%	12.73
Project 2-1	-10	16	60%	4.55

- The smallest project has IRR=100%>10%.
- The incremental project 2-1 has IRR=60%>10%.

Conclusion: Pick project 2.

Remark: Similar issue can arise when comparing projects with different horizons.

- The fix is the same, use incremental IRR on the longer-shorter project.

A Fix for the Scale Issue

A way around this problem is to use **incremental cash flows**.

- Is the incremental investment (project 2-1) a good idea?

	C_0	C_1	IRR	NPV at 10%
Project 1 (small)	-10	20	100%	8.18
Project 2 (large)	-20	36	80%	12.73
Project 2-1	-10	16	60%	4.55

- The smallest project has IRR=100%>10%.
- The incremental project 2-1 has IRR=60%>10%.

Conclusion: Pick project 2.

Remark: Similar issue can arise when comparing projects with different horizons.

- The fix is the same, use incremental IRR on the longer-shorter project.

A Fix for the Scale Issue

A way around this problem is to use **incremental cash flows**.

- Is the incremental investment (project 2-1) a good idea?

	C_0	C_1	IRR	NPV at 10%
Project 1 (small)	-10	20	100%	8.18
Project 2 (large)	-20	36	80%	12.73
Project 2-1	-10	16	60%	4.55

- The smallest project has IRR=100%>10%.
- The incremental project 2-1 has IRR=60%>10%.

Conclusion: Pick project 2.

Remark: Similar issue can arise when comparing projects with different horizons.

- The fix is the same, use incremental IRR on the longer-shorter project.

A Fix for the Scale Issue

A way around this problem is to use **incremental cash flows**.

- Is the incremental investment (project 2-1) a good idea?

	C_0	C_1	IRR	NPV at 10%
Project 1 (small)	-10	20	100%	8.18
Project 2 (large)	-20	36	80%	12.73
Project 2-1	-10	16	60%	4.55

- The smallest project has IRR$=$100%$>$10%.
- The incremental project 2-1 has IRR$=$60%$>$10%.

Conclusion: Pick project 2.

Remark: Similar issue can arise when comparing projects with different horizons.

- The fix is the same, use incremental IRR on the longer-shorter project.

A Fix for the Scale Issue

A way around this problem is to use **incremental cash flows**.

- Is the incremental investment (project 2-1) a good idea?

	C_0	C_1	IRR	NPV at 10%
Project 1 (small)	-10	20	100%	8.18
Project 2 (large)	-20	36	80%	12.73
Project 2-1	-10	16	60%	4.55

- The smallest project has IRR$=$100%$>$10%.
- The incremental project 2-1 has IRR$=$60%$>$10%.

Conclusion: Pick project 2.

Remark: Similar issue can arise when comparing projects with different horizons.

- The fix is the same, use incremental IRR on the longer-shorter project.

Issue 2: IRR is not the actual return!

You have a project with a 41% IRR that requires a $10 million initial investment.

- Your investor **thinks** that means that after 5 years, they will have
$$\$10M \times 1.41^5 = \$56M$$
- This is **only true if** you can reinvest any intermediate cash flows at a 41% return. See McKinsey article: "IRR: A Cautionary Tale."
- It is more realistic to assume intermediate cash flows can be invested at the opportunity cost of capital. To compute the actual return, you can use theCompound Annual Growth Rate (CAGR) or modified IRR given **an assumption** about reinvestment rate.

$$\text{Modified IRR} = \left(\frac{\text{FV of cash flows at date n given reinvestment rate}}{\text{PV of cash outflows discounted at cost of capital}}\right)^{1/n} - 1$$

Identical IRRs, but very different annual returns

Internal-rate-of-return (IRR) values are identical for 2 projects . . .

Project A							IRR
Year	0	1	2	3	4	5	
Cash flows, $ million	–10	5	5	5	5	5	41%

Project B							IRR
Year	0	1	2	3	4	5	
Cash flows, $ million	–10	5	5	5	5	5	41%

. . . however, interim cash flows are reinvested at different rates

Key assumption: reinvestment rate = IRR

Project A							CAGR[1]
Year	0	1	2	3	4	5	
Value of cash flows at year 5 if reinvested at 41%		5 →				20	41%
			5 →			14	41%
				5 →		10	41%
					5 →	7	41%
						5	
Year 5 value of $10 million investment =						**$56 million**	**41% CAGR[1]**

Key assumption: reinvestment rate = cost of capital

Project B							CAGR[1]
Year	0	1	2	3	4	5	
Value of cash flows at year 5 if reinvested at 8%		5 →				7	8%
			5 →			6	8%
				5 →		6	8%
					5 →	5	8%
						5	
Year 5 value of $10 million investment =						**$29 million**	**24% CAGR[1]**

True return is nearly 50% less because of lower reinvestment rate

[1] Compound annual growth rate.

More Potential Issues with IRR

- IRR may **not exist** or there can be **multiple IRRs**.
  - This can occur if cash flows change sign more than once
  - Comes up a lot in real estate when doing large renovations
  - To fix: use MIRR (or just NPV)
- IRR rule doesn't work if cash flows are positive and then negative.

Example (Cash Flows positive then negative)

C_0	C_1 %
100	-120

- IRR=20%. Suppose cost of capital is 10%.
- IRR rule likes the project, but that is **wrong** since

$$NPV = 100 - \frac{120}{1.10} = -\$9,091$$

- Project is like taking out a loan: if you want to use IRR rule, look for a low IRR, not a high one.

Summary on how to use IRR

- **The IRR is useful to know even if you use the NPV decision rule.**
  - It tells you how high the cost of capital can be before the NPV goes negative.
- The **IRR decision rule** is less useful.
  - At best it agrees with NPV.
- But, sometimes you need to **communicate** with people who do not understand present value. In that case, remember:
  - For mutually exclusive projects with different scale or length, use the incremental IRR approach.
  - Use modified IRR to calculate the actual return on the project or if cash flows flip sign more than once.
  - If first cash in, then cash out, look for a low IRR, not a high one.

Resource Constraints and Profitability Index

- **In theory**: Firm should take on **all positive NPV projects**.
- **in practice**: Firms faces resource constraints.
  - Capital, workers, engineers, programmers, factories, etc.

Definition (Profitability Index)

The **profitability index** of a project is the ratio of the present value to the resource consumed

$$PI = \frac{PV}{\text{Resource Consumed}}$$

You **still want to maximize the NPV** of the set of projects you choose.

- Need to consider both NPV and resource constraints requirements.
- You do not necessarily want to pick projects in oder of decreasing NPV.
- Intuitively, PI captures the 'bang for your buck".

Resource Constraints and Profitability Index

- **In theory**: Firm should take on **all positive NPV projects**.
- **in practice**: Firms faces resource constraints.
  - Capital, workers, engineers, programmers, factories, etc.

Definition (Profitability Index)

The **profitability index** of a project is the ratio of the present value to the resource consumed

$$PI = \frac{PV}{\text{Resource Consumed}}$$

You **still want to maximize the NPV** of the set of projects you choose.

- Need to consider both NPV and resource constraints requirements.
- You do not necessarily want to pick projects in oder of decreasing NPV.
- Intuitively, PI captures the 'bang for your buck".

Example (Profitability Index under Capital Constraints)

As an entrepreneur you have \$1,000,000 in available venture capital. You cannot raise more capital. You can choose any combination of the following projects:

Project	Cost at $t = 0$	PV of $C_1, C_2, \ldots$	NPV as of $t = 0$	PI
A	200,000	300,000	100,000	1.50
B	500,000	620,000	120,000	1.24
C	400,000	700,000	300,000	1.75
D	200,000	275,000	75,000	1.38
E	100,000	130,000	30,000	1.30
F	100,000	140,000	40,000	1.40

If you *incorrectly* pick projects in order of decreasing NPV, what's your total NPV?

$$C + B + F = \$460,000$$

If you instead pick projects in order of decreasing PI, what's your total NPV?

$$C + A + F + D + E = \$545,000$$

Could apply a similar methodology if **something else was constrained**.

- Suppose you have plenty of capital, but cannot hire any more programmers.

Example (Profitability Index under Capital Constraints)

As an entrepreneur you have \$1,000,000 in available venture capital. You cannot raise more capital. You can choose any combination of the following projects:

Project	Cost at $t = 0$	PV of $C_1, C_2, \ldots$	NPV as of $t = 0$	PI
A	200,000	300,000	100,000	1.50
B	500,000	620,000	120,000	1.24
C	400,000	700,000	300,000	1.75
D	200,000	275,000	75,000	1.38
E	100,000	130,000	30,000	1.30
F	100,000	140,000	40,000	1.40

If you *incorrectly* pick projects in order of decreasing NPV, what's your total NPV?

$$C + B + F = \$460,000$$

If you instead pick projects in order of decreasing PI, what's your total NPV?

$$C + A + F + D + E = \$545,000$$

Could apply a similar methodology if **something else was constrained.**

- Suppose you have plenty of capital, but cannot hire any more programmers.

Example (Profitability Index under Capital Constraints)

As an entrepreneur you have \$1,000,000 in available venture capital. You cannot raise more capital. You can choose any combination of the following projects:

Project	Cost at $t = 0$	PV of $C_1, C_2, \ldots$	NPV as of $t = 0$	PI
A	200,000	300,000	100,000	1.50
B	500,000	620,000	120,000	1.24
C	400,000	700,000	300,000	1.75
D	200,000	275,000	75,000	1.38
E	100,000	130,000	30,000	1.30
F	100,000	140,000	40,000	1.40

If you *incorrectly* pick projects in order of decreasing NPV, what's your total NPV?

$$C + B + F = \$460,000$$

If you instead pick projects in order of decreasing PI, what's your total NPV?

$$C + A + F + D + E = \$545,000$$

Could apply a similar methodology if **something else was constrained.**

- Suppose you have plenty of capital, but cannot hire any more programmers.

Example (Profitability Index under Capital Constraints)

As an entrepreneur you have \$1,000,000 in available venture capital. You cannot raise more capital. You can choose any combination of the following projects:

Project	Cost at $t = 0$	PV of $C_1, C_2, \ldots$	NPV as of $t = 0$	PI
A	200,000	300,000	100,000	1.50
B	500,000	620,000	120,000	1.24
C	400,000	700,000	300,000	1.75
D	200,000	275,000	75,000	1.38
E	100,000	130,000	30,000	1.30
F	100,000	140,000	40,000	1.40

If you *incorrectly* pick projects in order of decreasing NPV, what's your total NPV?

$$C + B + F = \$460,000$$

If you instead pick projects in order of decreasing PI, what's your total NPV?

$$C + A + F + D + E = \$545,000$$

Could apply a similar methodology if something else was constrained.

- Suppose you have plenty of capital, but cannot hire any more programmers.

Example (Profitability Index under Capital Constraints)

As an entrepreneur you have \$1,000,000 in available venture capital. You cannot raise more capital. You can choose any combination of the following projects:

Project	Cost at $t = 0$	PV of $C_1, C_2, \ldots$	NPV as of $t = 0$	PI
A	200,000	300,000	100,000	1.50
B	500,000	620,000	120,000	1.24
C	400,000	700,000	300,000	1.75
D	200,000	275,000	75,000	1.38
E	100,000	130,000	30,000	1.30
F	100,000	140,000	40,000	1.40

If you *incorrectly* pick projects in order of decreasing NPV, what's your total NPV?

$$C + B + F = \$460,000$$

If you instead pick projects in order of decreasing PI, what's your total NPV?

$$C + A + F + D + E = \$545,000$$

Could apply a similar methodology if something else was constrained.

- Suppose you have plenty of capital, but cannot hire any more programmers.

Example (Profitability Index under Capital Constraints)

As an entrepreneur you have \$1,000,000 in available venture capital. You cannot raise more capital. You can choose any combination of the following projects:

Project	Cost at $t = 0$	PV of $C_1, C_2, \ldots$	NPV as of $t = 0$	PI
A	200,000	300,000	100,000	1.50
B	500,000	620,000	120,000	1.24
C	400,000	700,000	300,000	1.75
D	200,000	275,000	75,000	1.38
E	100,000	130,000	30,000	1.30
F	100,000	140,000	40,000	1.40

If you *incorrectly* pick projects in order of decreasing NPV, what's your total NPV?

$$C + B + F = \$460,000$$

If you instead pick projects in order of decreasing PI, what's your total NPV?

$$C + A + F + D + E = \$545,000$$

Could apply a similar methodology if **something else was constrained**.

- Suppose you have plenty of capital, but cannot hire any more programmers.

Other (less useful) metrics

1. Payback Period: Number of years for **undiscounted** cash flows to pay back initial investment: the minimum T such that

$$C_1 + C_2 + \cdots + C_J \geq -C_0 = I_0$$

Example (Using Payback Period)

Let $T^* = 3$. Consider the independent projects:

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	J
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	20	80	5	10	10	3

Decision: Accept Project 2, not Project 1.

- Ignores time of value of money and cash flows in later years.

2. Discounted Payback Period: Number of years for **discounted cash flow to payback the initial investment.**
   - Still ignores cash flows in later years.

Other (less useful) metrics

1. Payback Period: Number of years for **undiscounted** cash flows to pay back initial investment: the minimum T such that

$$C_1 + C_2 + \cdots + C_J \geq -C_0 = I_0$$

Example (Using Payback Period)

Let $T^* = 3$. Consider the independent projects:

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	J
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	20	80	5	10	10	3

Decision: Accept Project 2, not Project 1.

- Ignores time of value of money and cash flows in later years.

2. Discounted Payback Period: Number of years for **discounted cash flow to payback the initial investment.**
   - Still ignores cash flows in later years.

Other (less useful) metrics

1. Payback Period: Number of years for **undiscounted** cash flows to pay back initial investment: the minimum T such that

$$C_1 + C_2 + \cdots + C_J \geq -C_0 = I_0$$

Example (Using Payback Period)

Let $T^* = 3$. Consider the independent projects:

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	J
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	20	80	5	10	10	3

Decision: Accept Project 2, not Project 1.

- Ignores time of value of money and cash flows in later years.

2. Discounted Payback Period: Number of years for **discounted cash flow to payback the initial investment.**
   - Still ignores cash flows in later years.

Other (less useful) metrics

1. Payback Period: Number of years for **undiscounted** cash flows to pay back initial investment: the minimum T such that

$$C_1 + C_2 + \cdots + C_J \geq -C_0 = I_0$$

Example (Using Payback Period)

Let $T^* = 3$. Consider the independent projects:

	C_0	C_1	C_2	C_3	C_4	C_5	C_6	J
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	20	80	5	10	10	3

Decision: Accept Project 2, not Project 1.

- Ignores time of value of money and cash flows in later years.

2. Discounted Payback Period: Number of years for **discounted cash flow to payback the initial investment.**

- Still ignores cash flows in later years.

Perhaps useful if cash flows are **sufficiently risky** beyond some date?

- Not really. A better way is to use **expected cash flows**, accounting for both good and bad outcomes by weighting by their probabilities.

Bottom Line: These tell you some information about project cash flows but should not be used as decision criterion.

Perhaps useful if cash flows are **sufficiently risky** beyond some date?

- Not really. A better way is to use **expected cash flows**, accounting for both good and bad outcomes by weighting by their probabilities.

Bottom Line: These tell you some information about project cash flows but should not be used as decision criterion.

Perhaps useful if cash flows are **sufficiently risky** beyond some date?

- Not really. A better way is to use **expected cash flows**, accounting for both good and bad outcomes by weighting by their probabilities.

Bottom Line: These tell you some information about project cash flows but should not be used as decision criterion.

Best-case cash flows and an extra risk premium

From McKinsey article: 'Avoiding risk premium...". You have a project with a cost of capital of 8% and two possible scenarios.

- **Best case** (50%): Cash flow at t=1: \$100M, g=2% in perpetuity.
- **Downside case** (50%): Cash Flow at t=1:\$60M, g=2% in perpetuity.

Two Approaches to Computing Project Value

1. Weigh each of the two scenarios by their probability and compute the expected net present value.
2. Focus on the best case, but add an extra risk premium to the cost of capital in oder to account for downside risk.

Importance of Choosing the Appropriate Risk Premium

Approach 1: using a probability-weighted scenario, $ million

Expected net present value (NPV)	Probability of scenario[1]	NPV at 8% WACC[2]	Cash flows: Year 1	Year 2	Year 3	. . .
1,333	Base case: 50%	1,667	100	102	104	
	Downside case: 50%	1,000	60	61	62	

Approach 2: adding a premium to the discount rate, $ million

Risk premium	NPV at 8% WACC plus risk premium
1.5 points	1,333
3.0 points	1,001
5.0 points	909

Question: Why does McKinsey argue that Approach 1 is preferable?

- Obviously, there are some "fudge factor" (extra premium) in Approach 2 that will lead to the same value. But that factor is project specific!

Importance of Choosing the Appropriate Risk Premium

Approach 1: using a probability-weighted scenario, $ million

Expected net present value (NPV)	Probability of scenario[1]	NPV at 8% WACC[2]	Cash flows Year 1	Year 2	Year 3	...
1,333	Base case: 50%	1,667	100	102	104	
	Downside case: 50%	1,000	60	61	62	

Approach 2: adding a premium to the discount rate, $ million

Risk premium	NPV at 8% WACC plus risk premium
1.5 points	1,333
3.0 points	1,001
5.0 points	909

Question: Why does McKinsey argue that Approach 1 is preferable?

- Obviously, there are some "fudge factor" (extra premium) in Approach 2 that will lead to the same value. But that factor is project specific!

Key Takeaways

- The **NPV decision rule**
  - Captures how much value the project adds
  - Does not require manager to account for shareholder's time preferences.
- Useful supplements:
  - **Internal Rate of Return**
    - Useful communication device
    - As a decision criterion, it is less useful.
    - May need to be adjusted using incremental IRR and MIRR.
  - **Profitability Index**
    - Good for dealing with resource constraints.
- Less useful metrics/alternatives you should know but not rely upon
  - **Payback Period** and **Discounted Payback Period**
  - **Best-case** cash flows with **extra risk premium**.