Gravity and Linear Regression

Don’t show this to your flat-earther friend

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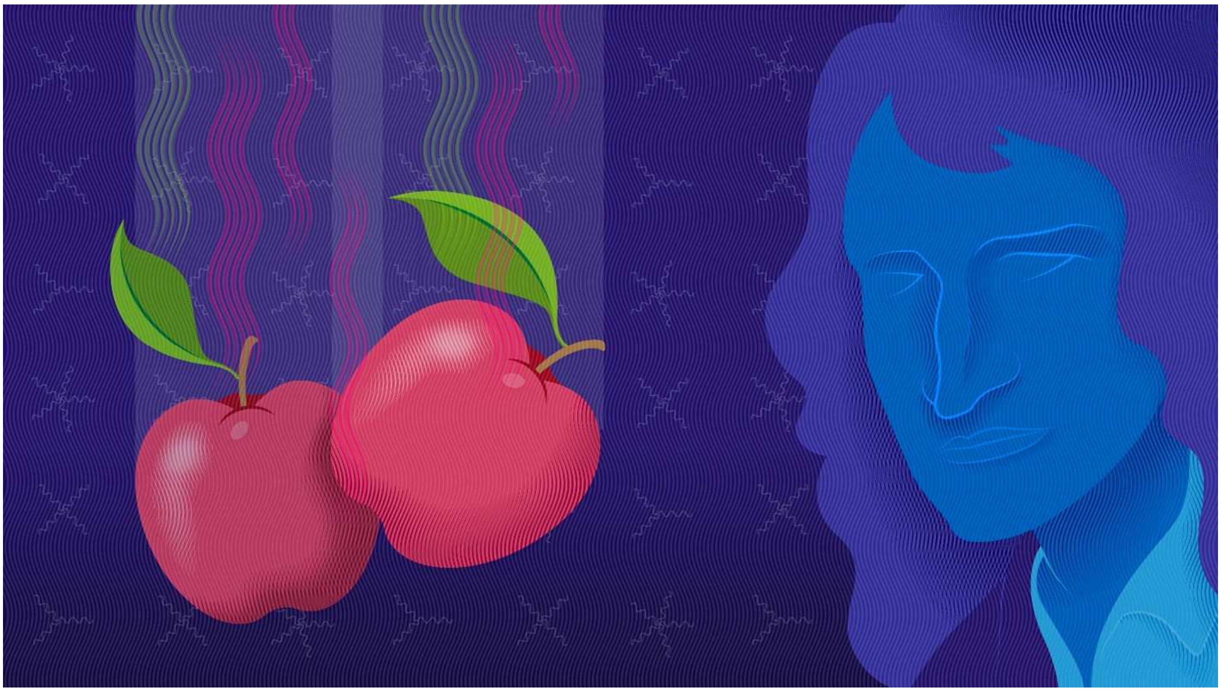
By: Juan Carlos Benavides B



Topics:

- Physics
- Statistics
- Data analysis
- Linear Regression Modelling

Tools:

- R

INTRODUCTION

The following is a simple but useful experiment for people interested in the use of statistics in science, physics, and data analysis or for others who want to wipe the smile off their flat-earther friend's face.

Despite the simplicity of this work, the physics methods could be useful for students and the statistical interpretation useful for people interested in data science applied to business.

The objectives are: to explore with math to prove there is a function that relates height and time (squared time) and a proportional constant (gravity), and finally to perform an experiment that is easily reproduced at home.

With a simple language but with the formal physics derivation, we will try to calculate the value of gravity and statistical inference in an experiment of one object moving throughout an inclined plane (Galilean freefall).

A joke about flat-earthers

I would say there is no general consensus among the flat-earthers – only they "know" what they believe and for many deny the existence of gravity. Yes, they are obligated to say gravity does not exist because gravity is responsible to shape planets as a sphere.

Instead, they say the flat Earth is moving throughout space with the same acceleration of gravity and that is why we humans think gravity exists.

Figure 1. Flat Earth traveling throughout space with ***g*** acceleration.

This model sounds possible at the beginning but has a little problem: it considers **all** the Earth moves with the same acceleration and, it is unable to explain why we can measure the different acceleration of gravity in world, for instance:

City	Acceleration in m/s^2	Acceleration in ft/s^2
Amsterdam	9.813	32.190
Athens	9.800	32.150
Brussels	9.811	32.190
Buenos Aires	9.797	32.140
Chicago	9.803	32.160
Copenhagen	9.815	32.200
Frankfurt	9.810	32.190
Havana	9.788	32.110
London	9.812	32.190
Los Angeles	9.796	32.140
Madrid	9.800	32.150
Mexico City	9.779	32.080
Montréal	9.789	32.120
New York City	9.802	32.160
Oslo	9.819	32.210
Ottawa	9.806	32.170
Paris	9.809	32.180
Rio de Janeiro	9.788	32.110
Rome	9.803	32.160
San Francisco	9.800	32.150
Singapore	9.781	32.090
Sydney	9.797	32.140
Tokyo	9.798	32.150
Vancouver	9.809	32.180
Washington, D.C.	9.801	32.160
Zurich	9.807	32.180

Table 1. Values of gravity acceleration in different cities.

METHODOLOGY

We will use the equation for freefall:

$$y = \frac{1}{2}gt^2$$

But following the steps of Galileo Galilei, we will modify this equation for the time of an object moving (“falling”) along an inclined plane.
Then we will vary the height of falling *y* and measure the time *t* of the object falling (a ball) for every different height, then all the data gathered in a table will be performed as a linear regression using the software R.
For measuring the time a chronometer able to measure centiseconds will be used. The units in this report correspond to International Units System, seconds for time and meters for height, thus acceleration will have the units m/s2.
If you are interested to reproduce this experiment at home you will need some practice and training in your reaction time for more accurately measuring the falling time.
This will suffice accurate enough results but if more scientific equipment such as laser triggered timers are used, results will only be all the more precise
For Costa Rica, where the experiment was performed, the theoretical value of gravity is: (9.77589 ± 0.00341) m/s2, this result or something similar is what we are comparing to respective to statistical confidence.

FOUNDATION THEORIES

Why does *g* have different values around the globe?

According to Newton´s Law of Gravitation, the value of gravity acceleration changes in function of the distance between the object and the center mass of the planet ***R***.

$$F_g = \frac{GM_p m_o}{R^2}$$

Remembering that ***Fg*** is:

$$F_g = gm_o$$

It is easy to realize that:

$$g = \frac{GM_p}{R^2}$$

From here we can see how ***g*** changes in function of ***R*** (distance between the object and center of mass in the planet), and knowing the Earth is not a perfect sphere (***R*** is different along the globe) it will be logical to have distinct values of ***g***, such as the values shown in table 1.

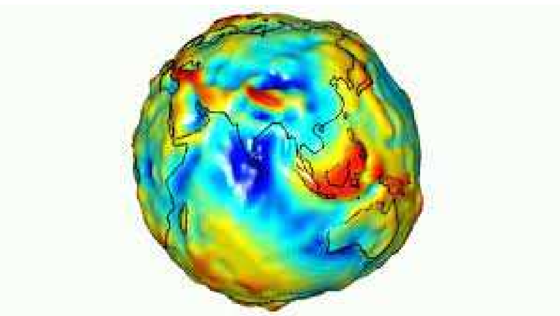


Figure 2. Radius is not the same in all the surface of Earth.

But it is very difficult to calculate the value of ***g*** in an experiment using this equation, instead, our approach will be simpler.

A mathematical explanation of gravity

Considering ***m*** as the mass of the object and ***y*** as the fall distance (until the object hits the ground).

An energetic approach for the case of a body in freefall, the kinetic energy, and the potential energy will be:

$$T = \frac{1}{2}m\ddot{y}$$

$$V = mgy$$

Then the Lagrangian function for a body in freefall will be:

$$L = \frac{1}{2}m\ddot{y} - mgy$$

The Euler-Lagrange equation for freefall case will be:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = 0$$

(You can find a demonstration of this equation in the appendix section).

Doing the operations over the Lagrangian function we will obtain:

$$m\ddot{y} + mg = 0$$

Where we can see that:

$$\ddot{y} = -g$$

Or

$$\frac{d^2y}{dt^2} = -g$$

The last expression requires that a constant acceleration must exist in the ***y*** direction vertical acceleration (**a**$_v$).

Another conclusion we can obtain from the last analysis is: the effect of gravity over an object does not depend on its mass.

This differential equation can be solved for initial conditions of speed and position when time is equal to zero. Calling $\dot{y}_0$the initial speed and y_0 the initial position.

The solution will be:

$$y = -\frac{1}{2}gt^2 + \dot{y}_0 t + y_0$$

Which is the equation we know for freefall calculation.

To simplify the experiment we will let the ball fall from the point where we will measure starting from zero speed, then the equation will be simpler too:

$$y = \frac{1}{2}gt^2$$

But to perform an experiment using this equation is difficult because the time of reaction is too short and with a simple chronometer you will have several errors in measurement unless you have an automatic laser chronometer at home.

To solve this problem we will use a trick Galileo applied almost 400 years ago. He had the same problem performing his experiments, but he realized over an inclined plane things fall more slowly. By using this approach we can be more precise with measuring the time.

Inclined plane (Galilean version of freefalling)

Now it is necessary to get an equation of motion for the ball rolling down an inclined plane.

For this calculation, we will use traditional Newtonian analysis and it means we will need some drawings:

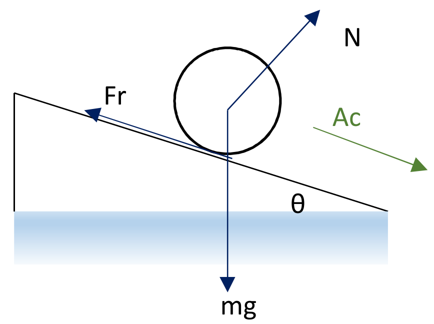


Figure 3. Forces acting over a ball rolling down an inclined plane.

For a sphere rolling down over a surface with friction (**F$_r$**) the sum of forces will be:

$$mg\ sen\theta - F_r = ma_c$$

And from rotational dynamic:

$$F_r R = \frac{I a_c}{R}$$

Remembering the inertia moment for a sphere of radius ***R***:

$$I = \frac{2}{5}mR^2$$

Combining these equations:

$$a_c = \frac{5}{7}g\ sen\theta$$

And we can convert the equation of free falling we got previously for vertical acceleration (**a$_v$**):

$$y = \frac{1}{2}a_v t^2$$

Using this geometrical relation:

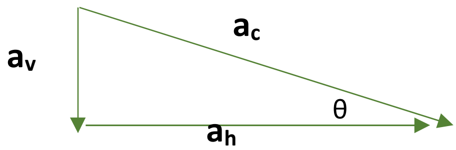


Figure 4. Acceleration components.

Finally:

$$y = \frac{5}{14} g\ sen^2(\theta)\ t^2$$

This is the equation for the motion of a sphere over an inclined plane, we can see height is still dependent on time squared, and ***g*** is still constant.

Converting into a line

The data we can collect from the experiment will not fit exactly into a line, because of limitations in the method of measurement but we can model as a line in the way:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

This can be re-expressed using a condensed formulation (matrix notation):

$$Y = X\beta + \varepsilon$$

Where the vector of coefficients can be calculated using matrix algebra, as follows:

$$\beta = (X^T X)^{-1} X^T Y$$

We can realize that the equation of motion we reached previously is a parabolic equation:

$$y = \frac{5}{14} g\ sen^2(\theta)\ t^2$$

However, for the statistical analysis we will use linear regression, which requires converting this last equation in the known form of a line.

$$y = \beta_1 x + \beta_0$$

By direct comparison (and a trick similar to variable change), we can isolate variables and constants

$$y = \ \beta_1\ x + \beta_0$$

$$y = \left(\frac{5}{14} g\ sen^2\ (\theta)\right)(t^2) + 0$$

Then in both equations ***y*** is playing the same roll measuring the falling distance and the other variables will have this line equivalence:

$$\beta_1 = \frac{5}{14} g\ sen^2(\theta) \qquad \text{(slope)}$$

$$x = t^2$$

$$\beta_0 = 0 \qquad \text{(Intercept)}$$

Finally, all of this means that in a linear regression we can get the value of the gravity acceleration, using the relation of slope and ***g*** as follows:

$$g = \frac{14\ \beta_1}{5\ sen^2(\theta)}$$

Now we are ready to start the analysis.

ANALYSIS

Calculating the model

The constant inclined plane used for this experiment is a slope of 3 meters long and 26 centimeters height (it is recommended to keep this relation height/length low, this will allow the ball to move slowly).

To collect the data we will release a ball, from different points keeping the slope constant, this will variate heights and we will measure time with a chronometer.

The height chosen to start to measure is 0.26 meters in this case three measures were taken at every point.

In this way varying distances and measuring times the data set obtained is the following (table 2):

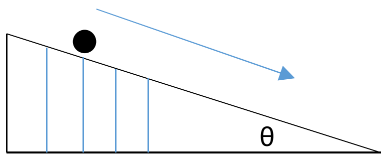


Figure 5. Different points of measures over the inclined plane.

Experiment´s measures	
height (m)	*time (s)*
0.26	3.15
0.26	3.14
0.26	3.16
0.24	3.00
0.24	3.05
0.24	3.03
0.23	2.91
0.23	2.95
0.23	2.94
0.21	2.85
0.21	2.82
0.21	2.83
0.19	2.70
0.19	2.70
0.19	2.72
0.17	2.55
0.17	2.57
0.17	2.56
0.16	2.45
0.16	2.45
0.16	2.46
0.14	2.29
0.14	2.31
0.14	2.30
0.12	2.15
0.12	2.16
0.12	2.17
0.10	1.97
0.10	1.98
0.10	1.99

Table 2. Data collected in the experiment.

According to theory, this data behaves following a parabolic curve that fits with the equation:

$$y = \frac{5}{14} g\ sen^2(\theta)\ t^2$$

We will convert this data into a line, then we will apply second power to the measures of time, and we can obtain the new data:

Variable change for Linear Model	
$y = h$	$x = t^2$
0.26	9.9225
0.26	9.8596
0.26	9.9856
0.24	9.0000
0.24	9.3025
0.24	9.1809
0.23	8.4681
0.23	8.7025
0.23	8.6436
0.21	8.1225
0.21	7.9524
0.21	8.0089
0.19	7.2900
0.19	7.2900
0.19	7.3984
0.17	6.5025
0.17	6.6049
0.17	6.5536
0.16	6.0025
0.16	6.0025
0.16	6.0516
0.14	5.2441
0.14	5.3361
0.14	5.2900
0.12	4.6225
0.12	4.6656
0.12	4.7089
0.10	3.8809
0.10	3.9204
0.10	3.9601

Table 3. Data transformed into linear behavior.

Over this new data, we will perform the statistical analysis to get the constant of gravity and evaluate its significance using the correlation coefficient and confidence intervals.

```
> summary(linear_model)

Call:
lm(formula = y ~ x, data = data_set)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.0047856 -0.0013841  0.0000098  0.0014421  0.0071917 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0003222  0.0016326  -0.197    0.845    
x            0.0262367  0.0002266 115.763   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.002355 on 28 degrees of freedom
Multiple R-squared:  0.9979,	Adjusted R-squared:  0.9978 
F-statistic: 1.34e+04 on 1 and 28 DF,  p-value: < 2.2e-16
```

Figure 6. Summary of statistical analysis.

Figure 6 shows the summary of the statistical analysis over the transformed data in table 3.

According to statistical theory in the calculation of coefficients we consider:

$$H_0 : \beta_i = 0$$

and

$$H_A : \beta_i \neq 0$$

We can see that the relation between the height and time squared is equal to 0.2624 (slope) and since its p-value tends to zero we can reject the null hypothesis and this association between the predictor and the response is true.

Then, similarly, the value in the intercept with ***Y*** axis is -0.00032, as close to zero as we were expecting when we derived the theoretical model, but we can see also (from figure 6) that its p-value is high and we can infer this coefficient is not represented in the model.

Finally our model will be:

$$y = 0.2624\, t^2$$

From this model we can recall the value we are looking for:

$$g = \frac{14\, \beta_1}{5\, sen^2(\theta)}$$

And finally we can get the value for gravity acceleration according to our experiment:

$$g = (9.78 \pm 0.08)\ m/s^2$$

Evaluating the model

To assess the accuracy of our model we can start plotting the line of the model together with the data in table 3.

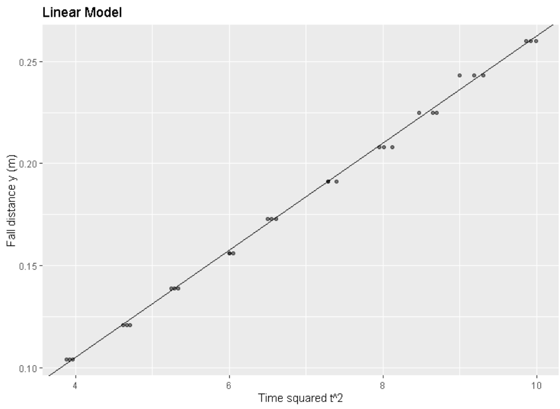


Figure 7. Graph of linear model.

Figure 7 shows how the data collected in the experiment (transformed data) create an almost perfect line, just as we were expecting from the derived model.

The correlation coefficient for our data (table 3) is r= 0.998 (calculated from figure 6). This means we have a high correlation between variables and they can be predictably captured along a line.

From figure 6 we can see that RSE = 0.23%, which indicates the model is highly accurate and its error is very low.

Finally, comparing the obtained value of gravity acceleration 9.78 m/s2 against the expected theoretical value 9.77 m/s2, we can see there is a difference that corresponds to 0.05%. A low difference in values of gravity considering the simplicity of the experiment performed.

CONCLUSIONS

The effect of gravity over an object does not depend on its mass.

There is a relationship $y = ct^2$ that explains how height variates in function of time.

The relationship $y = ct^2$ for an object rolling down over an inclined plane (the motion is the same effect as free fall).

In this relationship $y = ct^2$, ***c*** is a constant related directly with gravity.

The performed experiment has a high correlation coefficient, equal to 0.998.

The performed experiment has a low residual standard error, equal to 0.0023.

After the transformation over the collected data, the data can be represented as a line on a graph.

The slope in the line of this experiment is equal to 0.02624 ±0.002-

The slope obtained implies an acceleration gravity equal to $(9.78 \pm 0.08)\ m/s^2$

Despite the simplicity of this experiment (easy to perform at home), the value of gravity acceleration obtained is very accurate.

Finally but no less importantly, unfortunately for our flat earthers friends, their model does not obey any law of physics. But they have the opportunity to rewrite physics and to win a Nobel Prize.

APPENDIX

Euler–Lagrange equation demonstration

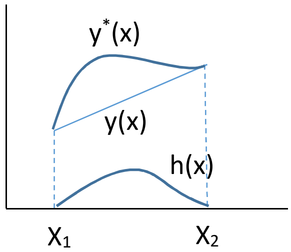
Least action principle, mathematical approach:

For an unknown function ***y(x)*** with fixed initial and final conditions on x_1 and x_2:

Must exist ***F(y,y′,x)***

And

$$I(y) = \int_{x_1}^{x_2} F(y, y', x)dx$$



Then:

$$y^*(x) = y(x) + \varepsilon h(x)$$

$$I^*(\varepsilon) = \int_{x_1}^{x_2} F(y^*, y^{*'}, x)dx$$

$$y^{*\prime}(x) = y'(x) + \varepsilon h'(x)$$

$$\frac{\partial y^*}{\partial \varepsilon} = 0 + \frac{\partial \varepsilon h(x)}{\partial \varepsilon} + \frac{\varepsilon \partial h(x)}{\partial \varepsilon} = h(x) \qquad (1)$$

$$\frac{\partial y^{*\prime}}{\partial \varepsilon} = 0 + \frac{\partial \varepsilon h\prime(x)}{\partial \varepsilon} + \frac{\varepsilon \partial h\prime(x)}{\partial \varepsilon} = h'(x) \qquad (2)$$

Minimizing the action:

$$\frac{dI^*}{d\varepsilon} = 0 \qquad \text{for } \varepsilon = 0$$

$$\varepsilon = 0 \rightarrow y^* = y \qquad (3)$$

$$\frac{dI^*}{d\varepsilon} = \frac{d}{d\varepsilon}\int_{x_1}^{x_2} F(y^*, y^{*\prime}, x)dx = 0$$

$$\int_{x_1}^{x_2} \left(\frac{\partial F}{\partial y^*}\frac{\partial y^*}{\partial \varepsilon} + \frac{\partial F}{\partial y^*}\frac{\partial y^{*\prime}}{\partial \varepsilon}\right) dx = 0$$

Using (1), (2) and (3) in last expression

$$\int_{x_1}^{x_2} \left(\frac{\partial F}{\partial y} h(x) + \frac{\partial F}{\partial y'} h'(x)\right) dx = 0$$

Integration by parts:

$$\int \left(\frac{\partial F}{\partial y} h(x)dx\right) + \frac{\partial F}{\partial y'} h(x) - \int \frac{d}{dx}\left(\frac{\partial F}{\partial y'} h(x)dx\right) = 0$$

$$\int_{x_1}^{x_2} \left(\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'}\right) h(x)dx = 0$$

Finally:

$$\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0$$

Or the physics version:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$