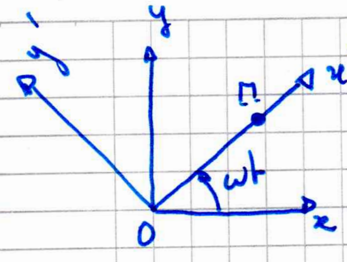
Mvt circulaire



$$\vec{OM}_R \begin{vmatrix} \cos\omega t \\ \sin\omega t \end{vmatrix} \qquad \left.\frac{d\vec{OM}}{dt}\right|_R = \begin{vmatrix} -\omega \sin\omega t \\ \omega\cos\omega t \end{vmatrix}$$

$$\boxed{\left.\frac{d\vec{OM}}{dt}\right|_R = \omega\, \vec{e}_z \wedge \vec{OM}_R} \qquad \text{Hyp} \quad \omega = c^{te}$$

$$= \omega\, \vec{e}_z \wedge (x\,\vec{e}_x + y\,\vec{e}_y)$$

$$= +\omega x\, \vec{e}_y - \omega y\, \vec{e}_x$$

Spirale $\quad \vec{OM}_R \begin{vmatrix} t\cos\omega t \\ t\sin\omega t \end{vmatrix} \qquad \left.\dfrac{d\vec{OM}}{dt}\right|_R = \begin{vmatrix} \cos\omega t - \omega t\sin\omega t \\ \sin\omega t + \omega t\cos\omega t \end{vmatrix} = \vec{e}_x{}'(t) + \omega\,\vec{e}_z \wedge \vec{OM}$

$$\vec{OM}_{R'} = t\,\vec{e}_x{}'(t) \quad \text{avec} \quad \vec{e}_x{}'(t) = \cos\omega t\,\vec{e}_x + \sin\omega t\,\vec{e}_y$$

$$\left.\frac{d\vec{OM}}{dt}\right|_R = \underbrace{\left.\frac{d\vec{OM}}{dt}\right|_{R'}} + \omega_{R'/R}\,\vec{e}_z \wedge \vec{OM}(t)$$

Ds R', M se déplace le long de x', mvt rectiligne uniforme

$$\left.\frac{d\vec{OM}}{dt}\right|_{R'} = 1 \cdot \vec{e}_x{}'$$

Spirale accélérée $\quad \vec{OM}_R = \begin{vmatrix} t^2\cos\omega t \\ t^2\sin\omega t \end{vmatrix} \qquad \left.\dfrac{d\vec{OM}}{dt}\right|_R = \begin{vmatrix} 2t\cos\omega t - \omega t^2\sin\omega t \\ 2t\sin\omega t + \omega t^2\cos\omega t \end{vmatrix}$

$$\left.\frac{d\vec{OM}}{dt}\right|_R = 2t\,\vec{e}_x{}'(t) + \omega_{R'/R}\,\vec{e}_z \wedge \vec{OM}(t)$$

$$\left.\frac{d^2\vec{OM}}{dt^2}\right|_R = 2\,\vec{e}_x{}'(t) + 2t\left.\frac{d}{dt}\vec{e}_x{}'\right|_R + \omega_{R'/R}\,\vec{e}_z \wedge \left.\frac{d\vec{OM}}{dt}\right|_R$$

$$\vec{e}_x{}' \begin{vmatrix} \cos\omega t \\ \sin\omega t \end{vmatrix} \qquad \left.\frac{d\vec{e}_x{}'}{dt}\right|_R = \begin{vmatrix} -\omega\sin\omega t \\ \omega\cos\omega t \end{vmatrix} = \omega\,\vec{e}_y{}' = \omega\,\vec{e}_z \wedge \vec{e}_x{}'$$

$$\vec{e}_z \wedge \left.\frac{d\vec{OM}}{dt}\right|_R = 2t\,\vec{e}_y{}' + \omega_{R'/R}\,\vec{e}_z \wedge (\vec{e}_z \wedge \vec{OM}) \qquad \text{or} \quad \vec{a}\wedge(\vec{b}\wedge\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$

$$= 2t\,\vec{e}_y{}' + \omega_{R'/R}\left[(\vec{e}_z\cdot\vec{OM})\,\vec{e}_z - \vec{OM}\right]$$

$$\left.\frac{d^2\vec{OM}}{dt^2}\right|_R = 2\,\vec{e}_x{}'(t) + 4\omega t\,\vec{e}_y{}' - \omega^2\,\vec{OM} = \left.\frac{d^2\vec{OM}}{dt^2}\right|_{R'} + 2\omega_{R'/R}\,\vec{e}_z \wedge \left.\frac{d\vec{OM}}{dt}\right|_{R'} + \omega_{R'/R}^2\,\vec{e}_z \wedge (\vec{e}_z \wedge \vec{OM})$$

$$\left.\frac{d^2\vec{OM}}{dt^2}\right|_R = \left(\left.\frac{d}{dt}\right|_{R'} + \omega_{R'/R}\,\vec{q}\wedge\right)\left(\left.\frac{d}{dt}\right|_{R'} + \omega_{R'/R}\,\vec{q}\wedge\right)\vec{OM}$$

$$= \left.\frac{d^2\vec{OM}}{dt^2}\right|_{R'} + 2\omega_{R'/R}\,\vec{q}\wedge\left.\frac{d\vec{OM}}{dt}\right|_{R'} + \omega^2_{R'/R}\,\vec{q}\wedge(\vec{q}\wedge\vec{OM})$$

Soit une masse test m soumise à une force est $\vec{F}$

$$m\left.\frac{d^2\vec{OM}}{dt^2}\right|_R = m\left.\frac{d^2\vec{OM}}{dt^2}\right|_{R'} + 2\omega_{R'/R}\,m\vec{q}\wedge\left.\frac{d\vec{OM}}{dt}\right|_{R'} + m\omega^2_{R'/R}\,\vec{q}\wedge(\vec{q}\wedge\vec{OM}) = \vec{F}$$

si $\left.\frac{d\vec{OM}}{dt}\right|_{R'} \equiv \vec{0}$ alors $\vec{OM}$ vérifie $\vec{F} - m\omega^2_{R'/R}\underbrace{\vec{q}\wedge(\vec{q}\wedge\vec{OM})}_{(\vec{q}\cdot\vec{OM})\vec{q}-\vec{OM}} = \vec{0}$

si $\vec{F}$ est dans le plan Oxy alors $\vec{OM}$ aussi, la condition devient :

$$\vec{F} + m\omega^2_{R'/R}\,\vec{OM} = \vec{0} \qquad \text{Hyp } \omega_{R'/R} = c^{te}$$

Cas du couple Terre - Soleil.

La période de révolution de la Terre autour du Soleil est donnée par :

$$T^2 = 4\pi^2\frac{a^3}{G M_s} \quad \text{où } a \text{ est le demi-grand axe}$$

$$\omega^2_{R'/R} = \frac{4\pi^2}{T^2} = \frac{G M_s}{a^3}$$

Système à 3 corps Terre - Soleil - satellite. repérés par $\vec{r}_1$, $\vec{r}_2$ et $\vec{r}$

$$\vec{F} = -G\,m\,M_s\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3} - G\,m\,\underbrace{m_2}_{\text{masse de la Terre}}\frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3} \qquad \text{où } \vec{r}_1 \equiv \vec{0}$$

Point de Lagrange.

Il est défini tel que :

$$-Gm M_s \frac{\vec{r}}{r^3} - G m m_2 \frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3} + \omega^2_{R'/R}\,\vec{r} = \vec{0}$$

Extrémum de l'énergie potentielle

$$V = -\frac{Gm M_s}{r} - \frac{G m m_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2} m\, \omega^2_{R'/R}\, r^2$$

ou encore

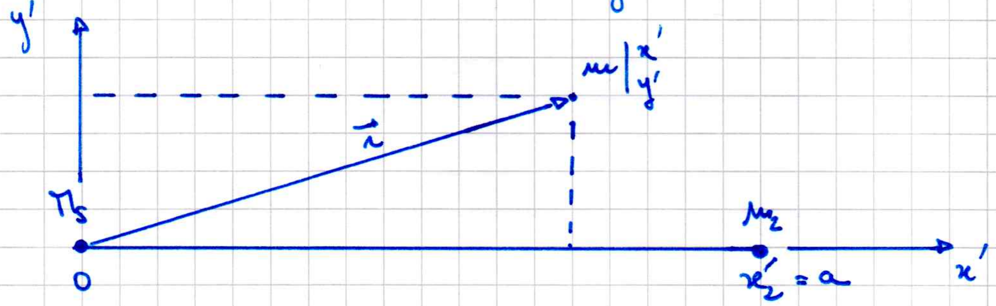
$$\frac{V}{m} = -\frac{G M_s}{r} - \frac{G m_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}\omega^2_{R'/R}\, r^2 \quad \text{avec} \quad \omega^2_{R'/R} = \frac{G M_s}{a^3}$$

A.N. $M_s = 1.9884\ 10^{30}$ kg

$m_2 = 5.9722\ 10^{24}$ kg

$a = 149\ 597\ 870\ 700$ m

$G = 6.67430\ 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$



$$\frac{V}{mG} = -\frac{M_s}{r} - \frac{m_2}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}\frac{M_s}{a^3} r^2$$

$$U = \frac{V}{Gm M_s} = -\frac{1}{r} - \frac{\alpha}{|\vec{r}-\vec{r}_2|} - \frac{1}{2}\frac{r^2}{a^3} \quad \text{où} \quad \alpha = \frac{m_2}{M_s} = 3.003\ 10^{-6}$$

Lois de Kepler

$$\frac{d^2(\vec r_2-\vec r_1)}{dt^2} = -\left[\frac{1}{m_1}+\frac{1}{m_2}\right] G \frac{m_1 m_2}{r^3}\vec r \quad \text{où} \quad \vec r = \vec r_2 - \vec r_1$$

$$\mu \frac{d^2\vec r}{dt^2} = -G\frac{m_1 m_2}{r^3}\vec r \quad \text{où} \quad \mu = \frac{m_1 m_2}{m_1+m_2}$$

$$\Rightarrow \quad m_2 = \frac{\mu}{m_1}(m_1+m_2) = \mu(1+\alpha)$$

$$\mu \frac{d^2\vec r}{dt^2} = -\underbrace{G(1+\alpha)}_{"G'"}\frac{m_1 \mu}{r^3}\vec r = -G' m_1 \frac{\mu}{r^2}\hat r \quad \text{force centrale}$$

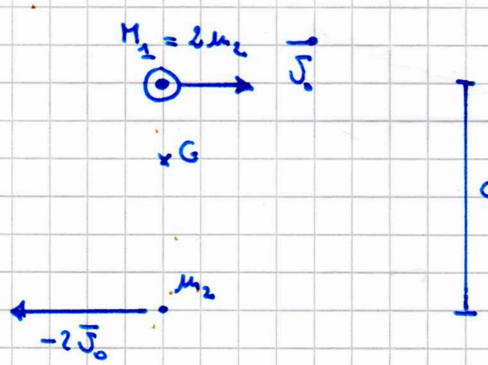
$$T^2 = 4\pi^2\frac{a^3}{G' m_1} = \frac{4\pi^2}{1+\alpha}\frac{a^3}{G m_1}$$

A OK enfin $\omega^2 = \frac{4\pi^2}{T^2} = (1+\alpha)\frac{G m_1}{a^3}$

$$\mu\, \vec r \wedge \frac{d^2\vec r}{dt^2} = \vec 0 \qquad \vec\sigma_1 = \vec r\wedge \mu\vec v = \vec{c^{te}} \quad \text{où} \quad \vec v = \frac{d\vec r}{dt}$$

si mvt circulaire uniforme $\vec v = \vec\omega\wedge\vec r$ avec $\vec\omega = \vec{c^{te}}$

$$\vec\sigma_1 = \mu\,\vec r\wedge(\vec\omega\wedge\vec r) = \mu\left(\vec\omega\, r^2 - \underbrace{(\vec\omega\cdot\vec r)}_{0}\,\vec r\right) = \mu r^2\vec\omega$$



$$m_1\frac{d^2\vec r_1}{dt^2} = G\frac{m_1 m_2}{r^3}\vec r \quad \text{où} \quad \vec r = \vec r_2 - \vec r_1$$

$$m_2\frac{d^2\vec r_2}{dt^2} = -G\frac{m_1 m_2}{r^3}\vec r$$

D'après wikipedia $\omega^2 = \frac{G(m_1+m_2)}{r^3}$ où $\vec r = \vec r_2 - \vec r_1$

A.N. $a = 1$, $G = 1$ $v_0 = 1/3$ pour obtenir une orbite circulaire

```matlab
clc
clear all
close all

% prelimaries
G=1;
M1=2; m2=1;
r=3;
omega2 = G*(M1+m2)/r^3;
disp('HYP orbite circulaire r=3')
T=2*pi/sqrt(omega2)

t_range=[0 T];
v0=0.333333; % adapter pour orbite circulaire

x1 =0;  y1 =1; x2 =0;     y2 =-2;
vx1=v0; vy1=0; vx2=-2*v0; vy2= 0;
u0 = [x1 y1 x2 y2 vx1 vy1 vx2 vy2]';

options = odeset('RelTol',1e-8,'AbsTol',1e-10);
[t, u] = ode45(@binar, t_range, u0, options);

x1 =u(:, 1); y1 =u(:, 2);  x2 =u(:, 3); y2 =u(:, 4);
vx1=u(:, 5); vy1=u(:, 6);  vx2=u(:, 7); vy2=u(:, 8);

r = hypot(x2-x1, y2-y1);
e = min(r)/max(r);
disp(['min(r)= ' num2str(min(r)) ' max(r)= '
num2str(max(r)) ' e= ' num2str(e)])

plot(x2-x1, y2-y1, 'r')
daspect([1 1 1])
title('Trajectory r2-r1');
grid on

figure
plot(x1, y1, 'r')
hold on
plot(x2, y2, 'b')
grid on
daspect([1 1 1])

figure
teta1=atan2(y1, x1);
teta2=atan2(y2, x2);
plot(t, teta1, 'r')
hold on
plot(t, teta2, 'b')
grid on
```

```
function du_dt = binar(t, u)

M1=2; m2=1;
G =1;

x1 =u(1); y1 =u(2);  x2 =u(3); y2 =u(4);
vx1=u(5); vy1=u(6);  vx2=u(7); vy2=u(8);

dx1_dt = vx1; dy1_dt = vy1;
dx2_dt = vx2; dy2_dt = vy2;
r=hypot(x2-x1, y2-y1);

dvx1_dt = +G*m2/r^3*(x2-x1);
dvy1_dt = +G*m2/r^3*(y2-y1);
dvx2_dt = -G*M1/r^3*(x2-x1);
dvy2_dt = -G*M1/r^3*(y2-y1);

du_dt = [dx1_dt;  dy1_dt;  dx2_dt;  dy2_dt; ...
         dvx1_dt; dvy1_dt; dvx2_dt; dvy2_dt;];
end
```