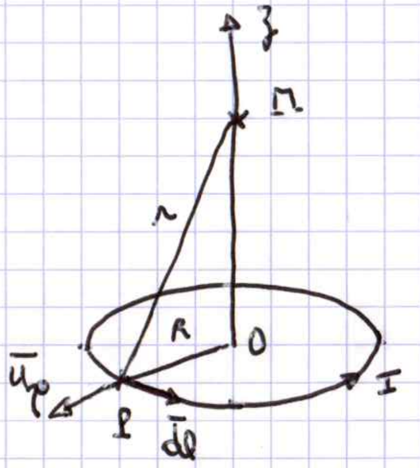
Application 2 = spire circulaire . champ sur l'axe (3)

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\vec{dl} \wedge \vec{r}}{r^3} \quad \text{où } \vec{r} = \vec{PM} \quad \text{ici } r = c^{te} \text{ à } z \text{ fixé}$$



$$\vec{dl} = R\, d\varphi\, \vec{u_\varphi}$$

$$\vec{r} = \vec{PO} + \vec{OM}$$

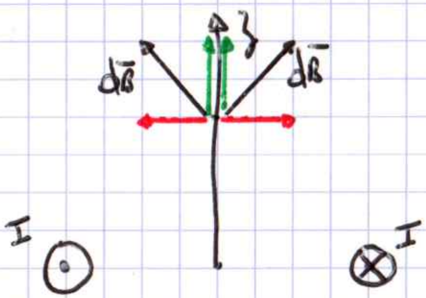
$$\vec{r} = -R\,\vec{u_\rho} + z\,\vec{u_z}$$

Conclusion :

$$\vec{dl} \wedge \vec{r} = R\, d\varphi\, \vec{u_\varphi} \wedge (-R\vec{u_\rho} + z\vec{u_z})$$

$$\vec{dl} \wedge \vec{r} = +R^2\, d\varphi\, \vec{u_z} + R\, z\, d\varphi\, \vec{u_\rho}$$

Noter que $\vec{u_\rho} = \vec{f}(\varphi)$



Les composantes radiales se compensent 2 à 2.

Seule reste la composante longitudinale de $d\vec{B}$

(4)

Donc

$$dB_z = \vec{u_z} \cdot d\vec{B} = \frac{\mu_0 I}{4\pi r^3} R^2\, d\varphi \quad \text{où } \varphi \in [0, 2\pi]$$

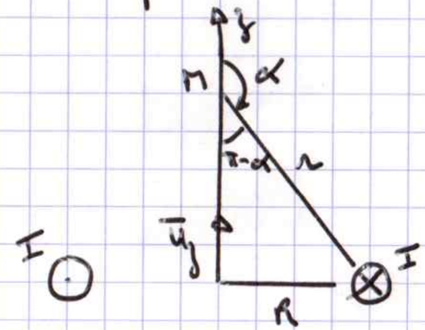
On en déduit :

$$B_z(M) = \frac{\mu_0 I}{4\pi r^3} R^2 \int_0^{2\pi} d\varphi = \frac{\mu_0 I}{2 r^3} R^2$$

Le long de l'axe : $\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{r^3} \vec{u_z}$

où $r^2 = R^2 + z^2$

Autre expression :



$$\sin\alpha = \sin(\pi - \alpha) = \frac{R}{r}$$

$$\vec{B}(M) = \frac{\mu_0 I}{2R} \sin^3\alpha\, \vec{u_z}$$