2-18-23

Homework Big O notation

1. Determine the big-O for each of the following functions.
   a. $f(n) = 100$ $O(1)$
   b. $f(n) = 7n - 13$ $O(n)$
   c. $f(n) = n^2 + n^2\log(n) + n!$ $O(n!)$
   d. $f(n) = n^4 + 3^n$ $O(n^3)$
   e. $f(n) = 5n^3 + 3n\log(n)$ $O(\log n)$
   f. $f(n) = n^5 + n^3 n! + 4n - 1$ $O(n!)$
   g. $f(n) = (n2^n + 3n^5)(2n^7 + 2\log(n))$ $O(\log n)$
2. Determine T(n) relationship between processing time and n. Then determine a big-O estimate for the number of operations (a Simple Statement takes one unit of time) used in this segment of an algorithm.

```
for(int i = 0; i<n; i++){
          for(int j = 0; j<n; j++){
                    Simple Statement
          }
}
for(int t = 0; t<n; t++){
          Simple Statement
          Simple Statement
          Simple Statement
          Simple Statement
}

Simple Statement
Simple Statement
Simple Statement
Simple Statement
```

$i=0 \ldots I=n-1$
$J=0 \ldots J=n-1$
$\underbrace{+1 \ldots = 1n\cdot n}_{n \text{ times}}$

$t=0 \ldots t=n-1$
$\underbrace{+4 \ldots = 4n}_{n \text{ times}}$

$+5$

$$T(n) = n^2 + 4n + 5 = O(n^2)$$

3. Determine T(n) relationship between processing time and n. Then determine a big-O estimate for the number of operations (a Simple Statement takes one unit of time) used in this segment of an algorithm.

```
for(int i = 0; i<n; i++){
          for(int j = i; j<n; j++){
                    Simple Statement
                    Simple Statement

          }
}
```

$i=0,\ i=1 \ldots i=n-1$
$J=0,\ J=1 \ldots J=n-1$
$\underbrace{+2 \quad +2 \quad = 2\cdot n\cdot n}_{n \text{ times}}$

$$T(n) = 2n^2 = O(\cancel{2}n^2) = O(n^2)$$
$$T(n) = O(n^2)$$

```
for(int i = 0; i<n; i=i + 2){
        Simple Statement
        Simple Statement
        Simple Statement

}
```

4. Determine T(n) relationship between processing time and n. Then determine a big-O estimate for the number of operations (a Simple Statement takes one unit of time) used in this segment of an algorithm.

```
for(int i = 0; i<n; i++){

    for(int j = 0; j<n; j++){

        for(int k = 0; k<n; k++){

            Simple Statement

            Simple Statement

        }

    }

}
```

$i=0 \ldots i=n-1$

$j=0 \ldots j=n-1$

$k=0 \ldots k=n-1$

$+2 \quad +2=2\cdot n$

$2n^3$

```
for(int i = 0; i<n; i++){
    for(int j = 0; j<n; j++){
        Simple Statement
        Simple Statement
        Simple Statement
        Simple Statement

    }
}
```

$i=0 \ldots i=n-1$

$j=0 \ldots j=n-1$

$+4 \quad +4=4\cdot n$

$4n^2$

```
for(int i = 0; i<n; i=i + 2){
        Simple Statement
        Simple Statement
        Simple Statement
        Simple Statement
        Simple Statement
}
Simple Statement
Simple Statement
Simple Statement
```

$i=0 \ldots i=n+2$

$+5 \quad +5=\frac{5\cdot n}{2}$

$+3$

$$T(n)=2n^3+4n^2+\frac{5n}{2}+3=O(n^3)$$

5. Find $C, n_0, f(n)$ such that $|T(n)| \leq C|f(n)|$ whenever $n > n_0$.
   a. $T(n) = n^3 - 5n^2 + 20n - 10$ $\quad C \geq 6$

6. Find $C, n_0, f(n)$ such that $|T(n)| \leq C|f(n)|$ whenever $n > n_0$.
   a. $T(n) = 2n^2 + 3n - 4$ $\quad C \geq 1$

7. Find $C, n_0, f(n)$ such that $|T(n)| \leq C|f(n)|$ whenever $n > n_0$.
   a. $T(n) = 2^n + 4n^2 + 3$ $\quad C \geq 9$

8. Find $C, n_0, f(n)$ such that $|T(n)| \leq C|f(n)|$ whenever $n > n_0$.
   a. $T(n) = n^2 + 10n + 10$ $\quad C \geq 21$

9. Find $C, n_0, f(n)$ such that $|T(n)| \leq C|f(n)|$ whenever $n > n_0$.
   a. $T(n) = n^3 + n^2 + n - 1$ $\quad C \geq 2$

10. Find $C, n_0, f(n)$ such that $|T(n)| \leq C|f(n)|$ whenever $n > n_0$.
   a. $T(n) = 20n - 10$ $\quad C \geq 10$

5. $n^3 - 5n^2 + 20n - 10 < cn^3$
$1^3 - 5\cdot 1^2 + 20\cdot 1 - 10 < c\cdot 1^3$
$1 - 5 + 20 - 10 < c$
$-4 + 20 - 10 < c$
$16 - 10 < c$
$6 < c$

6. $2n^2 + 3n - 4 < cn^2$
$2\cdot 1^2 + 3\cdot 1 - 4 < c\cdot 1^2$
$2 + 3 - 4 < c$
$1 < c$

7. $2n + 4n^2 + 3 < cn^2$
$2\cdot 1 + 4\cdot 1^2 + 3 < c\cdot 1^2$
$2 + 4 + 3 < c$
$9 < c$

8. $n^2 + 10n + 10 < cn^2$
$1^2 + 10\cdot 1 + 10 < c\cdot 1^2$
$1 + 10 + 10 < c$
$21 < c$

9. $n^3 + n^2 + n - 1 < cn^3$
$1^3 + 1^2 + 1 - 1 < c\cdot 1^3$
$1 + 1 < c$
$2 < c$

10. $20n - 10 < cn$
$20\cdot 1 - 10 < c\cdot 1$
$20 - 10 < c$
$10 < c$