HOMEWORK ASSIGNMENT 1

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(1) Problem A

We know that

$$\frac{P(S=j|T=i)=P(S=j \text{ and } T=i)}{P(T=i)}$$

and also, if S and T are independet, this becomes

$$P(S=j|T=i)=\frac{P(S=j)\cdot P(T=i)}{P(T=i)}=P(S=j)$$

However, if we consider the case where j = 2 and T = 0, we can see that they are not independent.

P(S=2) means that both dices roll to 1,

which has the possibility of $\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$.

P(T=0) means that both dices roll to odd number, which has the possibility of $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$.

However, $P(S=2|T=0)=\dfrac{P(S=2 \text{ and } T=0)}{P(T=0)}$: because for all S=2, only rolling two dices that both have value 1 will hold,

so P(S=2 and T=0)=P(S=2);

So $P(S=2|T=0)=\dfrac{P(S=2)}{P(T=0)}=\dfrac{\frac{1}{36}}{\frac{1}{4}}$, which is certainly not the same as P(S=2), which is $\frac{1}{36}$.

So S and T are not independent.

(2) Problem B

(a) Expected Value and Variance of the degree of v_1

Let D_n denotes the degree of node N at the time immediately after v_4 is added

$P(D_1=3)=P(N_3=1 \text{ and } N_4=1)=P(N_4=1|N_3=1)\cdot P(N_3=1)=2/4\cdot 1/2=1/4$

$P(D_1=2)=P(N_3=1 \text{ and } N_4=2)+P(N_3=1 \text{ and } N_4=3)+P(N_3=2 \text{ and } N_4=1)=1/2\cdot 1/4\cdot 3=3/8$

$P(D_1=1)=P(N_3=2 \text{ and } N_4=2)+P(N_3=2 \text{ and } N_4=3)=1/4+1/8=3/8$

$E(D_1) = 3 \cdot P(D_1 = 3) + 2 \cdot P(D_1 = 2) + 1 \cdot (D_1 = 1) = 3/4 + 6/8 + 3/8 = 15/8 = 1.875$

$Var(D_1) = E(D_1^2) - (ED_1)^2$
$= 3^2 \cdot P(D_1 = 3) + 2^2 \cdot P(D_1 = 2) + 1^2 \cdot (D_1 = 1) - (15/8)^2$
$= 9/4 + 12/8 + 3/8 - 225/16 = 144/64 + 96/64 + 24/64 - 225/64 = 39/64$
$Var(D_1) = 39/64$

(b) Covariance between the degrees of v_1 and v_2.
$E(D_2) = E(D_1) = 1.875$ because $(P(N_3 = 1) = P(N_3 = 2))$
$Cov(D_1, D_2) = E[(D_1 - ED_1)(D_2 - ED_2)] = (3 - 15/8)(1 - 15/8) \cdot 1/4 + (2 - 15/8)(1 - 15/8) \cdot 1/8 + (2 - 15/8)(2 - 15/8) \cdot 2/8 + (1 - 15/8)(2 - 15/8) \cdot 1/8 + (1 - 15/8)(3 - 15/8) \cdot 2/8 = -33/64$

(3) Problem C

$\mu = EB = \sum_{c=0}^{3} c \cdot P(X = c) = 0 \times 0.5 + 1 \times 0.4 + 2 \times 0.1 = 0.6$
$Var(B) = (0 - 0.6)^2 \cdot 0.5 + (1 - 0.6)^2 \cdot 0.4 + (2 - 0.6)^2 \cdot 0.1 = 0.44$
$\sigma =$ Standard Deviation of B $= \sqrt{Var(B)} = 0.66$
$E[(B - \mu)^3/\sigma^3] = \dfrac{(0 - 0.6)^3 \cdot 0.5 + (1 - 0.6)^3 \cdot 0.4 + (2 - 0.6)^3 \cdot 0.1}{0.66^3} = 0.658$

(4) Problem D

$$EB = \sum_{c=1}^{6} c \cdot \frac{1}{6} \cdot \frac{1}{6}$$
$$= \frac{1}{36} * \sum_{c=1}^{6} c$$
$$= \frac{1}{36} * 21 = 0.583$$
$$E(B)^2 = \sum_{c=1}^{6} c^2 \cdot \frac{1}{6} \cdot \frac{1}{6}$$
$$= \frac{1}{36} \cdot \sum_{c=1}^{6} c^2$$
$$= \frac{1}{36} \cdot 91 = 2.528$$