HOMEWORK ASSIGNMENT 3

SIYUAN YAO, ZHENGFEI CHEN, KAI YAN

(1) Problem 1
See code.

(2) Problem 2
$r = 3, p = 0.4$
$EN = r \cdot \frac{1}{p} = 7.5$
$Var(N) = r \cdot \frac{1-p}{p^2} = 11.25$
Standard Deviation of B $= \sqrt{Var(N)} = 3.35$
$pN(k) = \binom{k-1}{r-1} \cdot (1-p)^{k-r} \cdot p^r$
$skewness = E[(N-7.5)^3/3.35^3] = E[(N-7.5)^3/37.73]$
$= \sum_{N=3}^{\infty} [[((N-7.5)^3)/37.73] \cdot \binom{N-1}{2} \cdot (0.4^3) \cdot (0.6^{N-3})] = 1.1925$

(3) Problem 3

(a) long run average number of rolls between wins
Let's call the number of rolls between wins M. We are to find E(M); if there are M rolls between consecutive wins, then in long run, every $(M+1)$ rolls contain a win;
let $Avg(R)$ denotes the average of all the rolls we got between one win and another;
$E(M+1) = E\left(\frac{8}{Avg(R)} + 1\right) = \frac{E(8)}{E(Avg(R))} + 1$
the expected value of $Avg(R)$ is just ER, which is 3.5 according to textbook;
so $E(M) = E(M+1) - 1 = \frac{E(8)}{ER} - 1 = 8/3.5 - 1 = 2.2857 - 1 = 1.2857$

(b) long run value of total winnings per turn.
this should be equal to $\sum_{i=0}^{5} P(\text{land on i and win})(i+1)$
P(land on i and win) = P(land on i)·P(win | land on i)
P(land on i) for all i should be equal, which will be 1/8.
P(land on 0 and win) = (1/8)·(1)
P(land on 1 and win) = (1/8)·(5/6)
P(land on 2 and win) = (1/8)·(4/6)
P(land on 3 and win) = (1/8)·(3/6)

P(land on 4 and win) = (1/8)·(2/6)
P(land on 5 and win) = (1/8)·(1/6)

$$\sum_{i=0}^{5} P(\text{land on i and win})(i+1)$$

$= 1 \cdot (1/8) \cdot 1 + 2 \cdot (1/8) \cdot (5/6) + 3 \cdot (1/8) \cdot (4/6) + 4 \cdot (1/8) \cdot (3/6) + 5 \cdot (1/8) \cdot (2/6) + 6 \cdot (1/8) \cdot (1/6)$
$= 1.666667$
so the value should be 1.666667

(c) ET_j:
if we start at square j, we need to get a total number of (8-j) to win; let S_i denotes the total squares we've advanced given i rolls; for example, if we rolled a 1, then a 2, then S2 = 1+2 = 3;
$P(T_j = N) = P[(S_n - 1 < (8-j))$ and $(S_n >= (8-j))]$
for ET_7: we definitely will win in 1 roll, so $ET_7 = 1$;
for ET_6: we have 5/6 chance to win in 1 roll, and 1/6 chance that we roll a 1 first, then roll anything to win
so $ET_6 = 1 \cdot 5/6 + 2 \cdot 1/6 = 1.16666$
for ET_5: we have 4/6 chance to win in 1 roll, and 1/6 chance that we enter a situation which is exactly the same as ET_6, and 1/6 chance we enter a situation that's exactly like ET_7;
so $ET_5 = 1 \cdot 4/6 + (1 + ET_6) \cdot 1/6 + (1 + ET_7) \cdot 1/6 = 1.36111$

so we see the pattern here: if we don't make it to win from our original state, we are entering one of the other states: for example, if we start out at position 5 and didn't win, we are either entering position 6 or 7, and thus we can use the ET of those state, except that we take one more steps to arrive, so we can do $(1 + ET_i) \cdot P(j \to i)$ where j is our original state and i is the expected new state.
consequenctly:
$ET_4 = 1 \cdot 3/6 + (1+ET_5) \cdot 1/6 + (1+ET_6) \cdot 1/6 + (1+ET_7) \cdot 1/6 = 1.58796$
$ET_3 = 1 \cdot 2/6 + (1 + ET_4) \cdot 1/6 + (1 + ET_5) \cdot 1/6 + (1 + ET_6) \cdot 1/6 + (1 + ET_7) \cdot 1/6 = 1.85262$
$ET_2 = 1 \cdot 1/6 + (1 + ET_3) \cdot 1/6 + (1 + ET_4) \cdot 1/6 + (1 + ET_5) \cdot 1/6 + (1 + ET_6) \cdot r1/6 + (1 + ET_7) \cdot 1/6 = 2.16139$
$ET_1 = (1 + ET_2) \cdot 1/6 + (1 + ET_3) \cdot 1/6 + (1 + ET_4) \cdot 1/6 + (1 + ET_5) \cdot 1/6 + (1 + ET_6) \cdot 1/6 + (1 + ET_7) \cdot 1/6 = 2.52162$
$ET_0 = (1 + ET_1) \cdot 1/6 + (1 + ET_2) \cdot 1/6 + (1 + ET_3) \cdot 1/6 + (1 + ET_4) \cdot 1/6 + (1 + ET_5) \cdot 1/6 + (1 + ET_6) \cdot 1/6 = 2.77523$