HOMEWORK ASSIGNMENT 4

SIYUAN YAO, ZHENGFEI CHEN, KAI YAN

(1) Problem A

(a) Since ducd wants to find the density at the value x, it with solved original function $f_x(x)$ with given value:
$ducd(x,c) = \frac{3}{2 \cdot (c^{1.5} - 1)} \cdot x^{0.5}$

(b) pucd wants to find the cdf at value in q, which means it want to find the Fx(x), which is the integral of the fx(x).
$pucd(q,c) = \int_1^q \frac{3}{2 \cdot (c^{1.5} - 1)} \cdot x^{0.5} dx = \frac{q^{1.5}}{c^{1.5} - 1} - \frac{1^{1.5}}{c^{1.5} - 1} = \frac{q^{1.5} - 1}{c^{1.5} - 1}$

(c) qucd wants to find the quantiles at the values of q. This means it will get want to find the inverse of cdf function.
$qucd(q,c) = (q \cdot (c^{1.5} - 1) + 1)^{\frac{2}{3}}$

(d) In rucd, we want to generate n random values. So, we use runif to generate n random probabilities and use qucd to get the value of t's that have those probabilities.

```
rucd <- function(n,c) {
        tmp <- runif(n);
        qucd(tmp,c);
};
```

(2) Extra Credit

(a) Expected value

According to the given formula:
$EV = E[g(U)]$
where
$g(t) = E(V|U = t)$
we plug in
$V = D$
and
$U = (N, p)$
and get:

$ED = E(E(D|(N, p) = t))$

since " Given N and p, D has a binomial distribution with N trials and success probability p",we have:

$ED = E(N * p)$

since N and p are independent as stated in the prompt

$ED = E(N * p) = E(N) * E(p)$

The expected value for Poisson distribution is λ , while the expected value for Beta distribution is $\alpha/(\alpha + \beta)$
so we get:

$ED = E(N * p) = E(N) * E(p) = \lambda * \alpha/(\alpha + \beta)$

(b) Variance

again,according to the given formula:
$Var(V) = E[v(U)] + Var[e(U)]$
where $v(t) = Var(V|U = t)$ and $e(t) = E(V|U = t)$
from that we can derive:
$Var(V) = E[v(U)] + E[(e(U))^2] + (E[e(U)])^2$
again,we plug in
$V = D$
and
$U = (N, p)$
so:
$Var(D) = E[v(D)] + E[(e(D))^2] + (E[e(D)])^2$
$= \int N * fu(N) * p * fu(p) * (1 - p * fu(p))dNdp$
$+ \int (N * fu(N) * p * fu(p))^2)dNdp$
$- \int (N * fu(N) * p * fu(p)))dNdp$
we plug in:
$\int((p * fu(p))^2)dp = E(p^2) = Var(p) + (E(p))^2 = (\alpha + (\alpha)^2 * (\alpha + \beta + 1)/((\alpha + \beta)^2) * (\alpha + \beta + 1))$

and:
$\int(N * fu(N))^2 dN = E(N^2) = Var(N) + (E(N))^2 = \lambda + (\lambda)^2$
then we can integral everything and get: $Var(D) =$

$((\lambda)^2 * (\alpha * \lambda + (\alpha)^2 * (\alpha + \lambda + 1))/((\alpha + \lambda)^2 * (\alpha + \lambda + 1))) + (\lambda * a/(\alpha + \beta)) - (\lambda * \alpha/(\alpha + \beta))$