

HOMEWORK ASSIGNMENT 1

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(1) Problem A

- (a) $P(\text{first arrival at 4-min}) = P(\text{no 3-min and 1 or more 4-min})$
 $= P(\text{no 3-min}) P(1 \text{ or more 4-min} \mid \text{no 3-min}) \quad (2.7)$
 $= 0.5^3 \times (1 - P(\text{no 4-min} \mid \text{no 3-min}))$
 $= 0.5^3 \times (1 - 0.5^3)$
 $= 7/64$
- (b) $P(C_1 + C_2 + C_3 = 10)$
 $= P(3, 3, 4) + P(3, 4, 3) + P(4, 3, 3)$
 $= P(C_1 = 3)P(C_2 = 3)P(C_3 = 4) \times 3$
 $= 0.5 \times 0.5 \times 0.25 \times 3$
 $= 1/16$
- (c) $P(\text{all 3-min} \mid \text{all same})$
 $= P(\text{all 3-min and all same}) / P(\text{all same}) \quad (2.7)$
 $= 0.5^3 / (0.5^3 + 0.25^3 + 0.25^3)$
 $= 0.8$

(2) Problem B

$$\begin{aligned}
 & P(A_2 \geq 1 \mid 2 \text{ attempts}) \\
 & P(2 \text{ attempts}) \\
 & = P(A_1 = 0 \text{ and } A_2 = 2 \text{ or } A_1 = 1 \text{ and } A_2 = 1 \text{ or } A_1 = 2 \text{ and } A_2 = 0) \\
 & = P(A_1 = 0 \text{ and } A_2 = 2) + (A_1 = 1 \text{ and } A_2 = 1) + (A_1 = 2 \text{ and } A_2 = 0) \\
 & (2.2) \\
 & = P(A_1 = 0)P(A_2 = 2 \mid A_1 = 0) + P(A_1 = 1)P(A_2 = 1 \mid A_1 = 1) + P(A_2 = 0 \mid A_1 = 2) \quad (2.7) \\
 & = (1-p)^2 \times p^2 + (p \times (1-p))(q \times p(1-p) + (q \times (1-p)(p)) + ((1-p) \times p) \\
 & \quad p) + (1-p)^2 \times p^2 \\
 & P(A_2 \geq 1 \text{ and } 2 \text{ attempts}) / P(2 \text{ attempts}) \\
 & = P(A_1 = 0)P(A_2 = 2 \mid A_1 = 0) + P(A_1 = 1)P(A_2 = 1 \mid A_1 = 1) / P(A_1 = 0)P(A_2 = 2 \mid A_1 = 0) \\
 & \quad + P(A_1 = 1)P(A_2 = 1 \mid A_1 = 1) + P(A_2 = 0 \mid A_1 = 2) \\
 & = (1-p)^2 \times p^2 + (p \times (1-p))(q \times p(1-p) + (q \times (1-p)(p)) + ((1-p) \times p) / ((1-p)^2 \times p^2 \\
 & \quad + (p \times (1-p))(q \times p(1-p) + (q \times (1-p)(p)) + ((1-p) \times p) + (1-p)^2 \times p^2
 \end{aligned}$$

(3) See code.