HOMEWORK ASSIGNMENT 1

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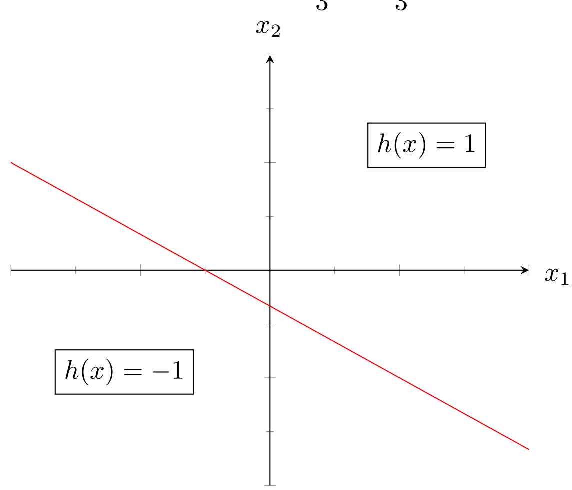
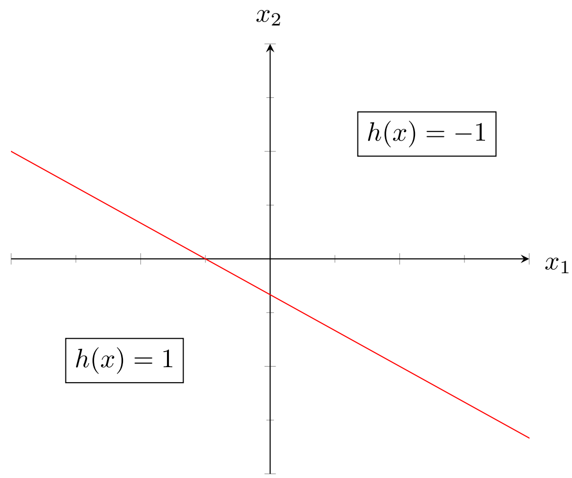
(1) Problem 1

(a) We know $w^T x = w_0 \cdot 1 + w_1 x_1 + w_2 x_2$.
So suppose $w^T x > 0$ and $w_2 > 0$ then $w_0 + w_1 x_1 + w_2 x_2 > 0$
$w_2 x_2 > -w_1 x_1 - w_0$
$x_2 > -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$
If $w_2 < 0$, we have $x_2 < -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$
Suppose $w^T x < 0$ and $w_2 > 0$ then $w_0 + w_1 x_1 + w_2 x_2 < 0$
Then we have $x_2 < -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$
If $w_2 < 0$, we have $x_2 > -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$
So the line is $x_2 = -\frac{w_1}{w_2} x_1 - \frac{w_0}{w_2}$,
and slope $a = -\frac{w_1}{w_2}$, intercept $b = -\frac{w_0}{w_2}$

(b) $w = [1, 2, 3]^T$, then $x_2 = -\frac{2}{3} x_1 - \frac{1}{3}$



$w = -[1, 2, 3]^T$, then $x_2 = -\frac{2}{3}x_1 - \frac{1}{3}$



(2) Problem 2

(a) Since $f(x) = +1$, for $h \neq f$, we only consider $h(x) = -1$.
$h(x) = -1$ has condition: $x = x_k$ and k is even and $1 \leq k \leq M + N$

And $E_{off}(h, f) = \frac{1}{M}\sum_{m=1}^{M}[h(x_{N+m}) \neq f(x_{N+m})]$

$E_{off}(h, f) = \frac{M+1}{2M}$ if N odd, M odd

$E_{off}(h, f) = \frac{1}{2}$ if N odd, M even

$E_{off}(h, f) = \frac{1}{2}$ if N even, M even

$E_{off}(h, f) = \frac{M-1}{2M}$ if N even, M odd

(b) 2^M possible f can generate $\mathcal{D}$

(c) Since if $f \neq h$ can have $E_{off}(h, f) = \frac{k}{M}$, there are $\binom{M}{k}$ possible f

(d) $E_f[E_{off}(h, f)]$

$$= \sum_{i=1}^{2^M} P(f_i)[E_{off}(h, f_i)]$$

$$= \frac{1}{2^M}\sum_{i=1}^{2^M}[E_{off}(h, f_i)]$$

$$= \frac{1}{2^M}\left[\binom{M}{0}\cdot\frac{0}{M} + \binom{M}{1}\cdot\frac{1}{M} + ... + \binom{M}{M-1}\cdot\frac{M-1}{M} + \binom{M}{M}\cdot\frac{M}{M}\right]$$

$$= \frac{1}{2^M}\cdot\frac{1}{M}\sum_{i=1}^{M} i\binom{M}{i}$$

$$= \frac{1}{2^M} \cdot \frac{1}{M} \sum_{i=1}^{M} i \cdot \frac{M!}{i!(M-i)!}$$
$$= \frac{1}{2^M} \cdot (M-1)! \sum_{i=1}^{M} i \cdot \frac{1}{(i-1)!(M-i)!}$$
$$= \frac{(M-1)!}{2^M} \left[\frac{1}{(M-1)!} + \frac{1}{2(M-2)!} + ... + \frac{1}{(M-1)!} \right]$$
$$= \frac{(M-1)!}{2^M} \cdot \frac{2^M}{2(M-1)!}$$
$$= \frac{1}{2}$$

(e) We know from (d) that $E_f[E_{off}(h, f)] = \frac{1}{M \cdot 2^M} \sum_{i=1}^{M} i \binom{M}{i}$

Since h does not affect the off-training-set error:

$E_f[E_{off}(A_1(\mathcal{D}), f)] = \frac{1}{M \cdot 2^M} \sum_{i=1}^{M} i \binom{M}{i} = E_f[E_{off}(A_2(\mathcal{D}), f)]$

(3) Problem 3

(a) Step size: 5.2e-05
f(w)=0.03149046
This step size will generate a weight list that has almost lowest f(w)

(b) Prediction accuracy: 0.986504723347 The lower the f(w) I got, the higher accuracy the model has.

(c) Step size: 5.2e-05
Prediction accuracy: 0.987404408457

Step size: 0.00052
Prediction accuracy: 0.986954565902

Step size: 5.2e-06
Prediction accuracy: 0.986504723347
Because it is a randomized model, the accuracy could vary a litte. Same step-size could have higher or lower accuracy.

(d) Step size: 8.2e-05 Decay rate: 100 Decay value: 0.1
Prediction accuracy: 0.986504723347

Step size: 8.2e-05 Decay rate: 100 Decay value: 0.01
Prediction accuracy: 0.968061178587

Step size: 8.2e-05 Decay rate: 10 Decay value: 0.1
Prediction accuracy: 0.985605038237

(4) Problem 4

(a) In case $sign(w^T x_n) \neq y_n$,
$y_i w^T x_n < 0$ and $\max(0, 1 - y_i w^T x_n) > 1$.

In case $sign(w^T x_n) = y_n$,
$y_i w^T x_n > 0$ and $0 \leq \max(0, 1 - y_i w^T x_n) \leq 1$.
So $e_n(w)$ is an upper bound for $[sign(w^T x_n) \neq y_n]$

(b) Step size: 6.5e-6
f(w)=0.08732331
Prediction accuracy: 0.985605038237