

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \cosh(x) = \frac{e^x + e^{-x}}{2} \quad \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\tanh'(x) = 1 - \tanh^2(x)$$

$$S_1 = \tanh(WX_1 + WS_0 + b_1) \quad O_1 = VS_1 + b_2$$

$$S_2 = \tanh(WX_2 + WS_1 + b_1) \quad O_2 = VS_2 + b_2$$

$$S_3 = \tanh(WX_3 + WS_2 + b_1) \quad O_3 = VS_3 + b_3$$

$$L_3 = \frac{1}{2} (Y_3 - O_3)^2$$

$$\frac{\partial L_3}{\partial U} = \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial U}$$

$$= (O_3 - Y_3) \cdot V \cdot \frac{\partial S_3}{\partial U}$$

$$\frac{\partial S_3}{\partial U} = \dots$$

$$\text{令 } \theta_3 = WX_3 + WS_2 + b_1 \quad S_3 = \tanh(\theta_3)$$

$$\textcircled{1} \frac{\partial S_3}{\partial U} = \frac{\partial S_3}{\partial \theta_3} \cdot \frac{\partial \theta_3}{\partial U}$$

$$= \frac{\partial S_3}{\partial \theta_3} \cdot \left(\frac{\partial \theta_3}{\partial U} + \frac{\partial \theta_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial U} \right)$$

$$= \frac{\partial \tanh(\theta_3)}{\partial \theta_3} x_3 + \frac{\partial \tanh(\theta_3)}{\partial \theta_3} \cdot \left[\frac{\partial S_2}{\partial U} \right] \cdot W$$

$$\textcircled{2} \frac{\partial S_2}{\partial U} = \frac{\partial \tanh(\theta_2)}{\partial \theta_2} x_2 + \frac{\partial \tanh(\theta_2)}{\partial \theta_2} \cdot \left[\frac{\partial S_1}{\partial U} \right] \cdot W$$

$$\textcircled{3} \frac{\partial S_1}{\partial U} = \frac{\partial \tanh(\theta_1)}{\partial \theta_1} x_1$$

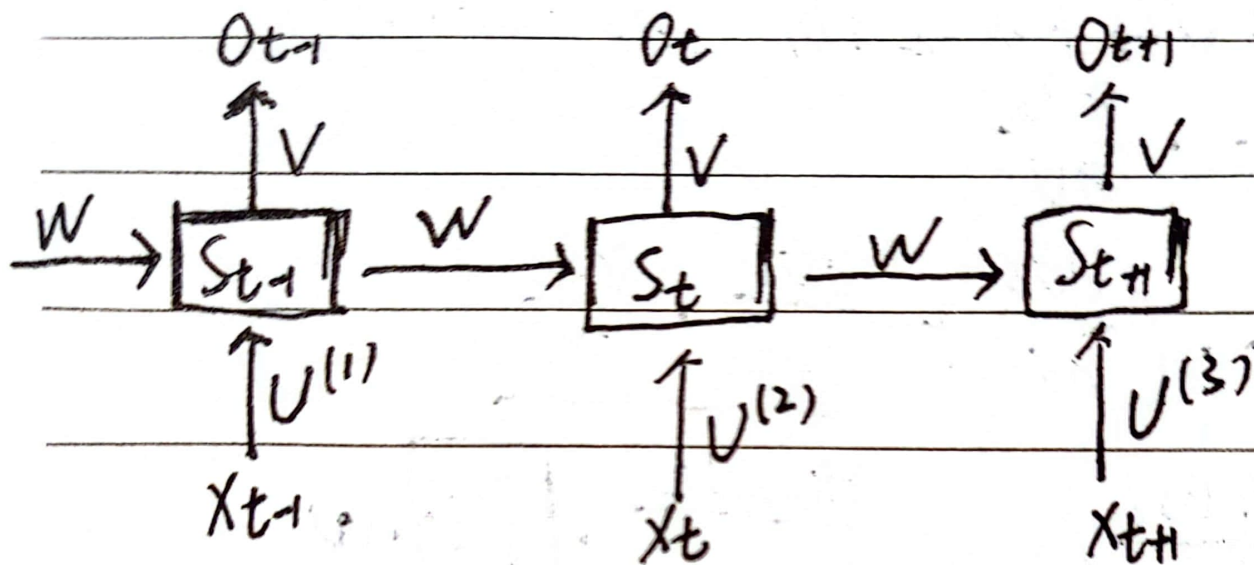
$$\begin{aligned}
 \textcircled{4} \quad \frac{\partial s_3}{\partial v} &= \frac{\partial \tanh(\theta_3)}{\partial \theta_3} x_3 + \frac{\partial \tanh(\theta_3)}{\partial \theta_3} \cdot w \left(\frac{\partial \tanh(\theta_2)}{\partial \theta_2} x_2 + \frac{\partial \tanh(\theta_2)}{\partial \theta_2} \cdot w \left(\frac{\partial \tanh(\theta_1)}{\partial \theta_1} x_1 \right) \right) \\
 &= \frac{\partial \tanh(\theta_3)}{\partial \theta_3} x_3 + \frac{\partial \tanh(\theta_3)}{\partial \theta_3} \cdot \frac{\partial \tanh(\theta_2)}{\partial \theta_2} \cdot x_2 w \\
 &\quad + \frac{\partial \tanh(\theta_3)}{\partial \theta_3} \cdot \frac{\partial \tanh(\theta_2)}{\partial \theta_2} \cdot \frac{\partial \tanh(\theta_1)}{\partial \theta_1} x_1 w^2 \\
 \xrightarrow{\text{---}} \sum_{k=1}^3 \frac{\partial L_3}{\partial v} &= \frac{\partial L_3}{\partial \theta_3} \cdot \frac{\partial \theta_3}{\partial s_3} \cdot \left(\prod_{j=k}^3 \frac{\partial \tanh(\theta_j)}{\partial \theta_j} \right) x_k w^{k-1}
 \end{aligned}$$

$$\tanh'(x) = 1 - \tanh^2(x)$$

$$\tanh(x) \in [-1, 1] \quad \tanh'(x) \in [0, 1]$$

假设 $t=20$

$$\begin{aligned}
 \frac{\partial L_{20}}{\partial v} &= \frac{\partial L_{20}}{\partial \theta_{20}} \cdot \frac{\partial \theta_{20}}{\partial s_{20}} \cdot \left(\frac{\partial \tanh(\theta_{20})}{\partial \theta_{20}} \cdot x_{20} \right. \\
 &\quad + \frac{\partial \tanh(\theta_{20})}{\partial \theta_{20}} \cdot \frac{\partial \tanh(\theta_{19})}{\partial \theta_{19}} x_{19} w \\
 &\quad + \frac{\partial \tanh(\theta_{20})}{\partial \theta_{20}} \cdot \frac{\partial \tanh(\theta_{19})}{\partial \theta_{19}} \cdot \frac{\partial \tanh(\theta_{18})}{\partial \theta_{18}} x_{18} w^2 \\
 &\quad + \dots + 0 + 0 + \dots \left. \right)
 \end{aligned}$$



$$t=3, \quad \frac{\partial L_3}{\partial U} = \frac{\partial L_3}{\partial U^{(3)}} + \frac{\partial L_3}{\partial U^{(2)}} + \frac{\partial L_3}{\partial U^{(1)}} \quad [\text{从前向传播}]$$

$$1> \frac{\partial L_3}{\partial U^{(3)}} = \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial U^{(3)}}$$

$$2> \frac{\partial L_3}{\partial U^{(2)}} = \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial U^{(2)}}$$

$$3> \frac{\partial L_3}{\partial U^{(1)}} = \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial S_1} \cdot \frac{\partial S_1}{\partial U^{(1)}}$$

$$\frac{\partial L_3}{\partial U} = \frac{\partial L_3}{\partial U^{(3)}} + \frac{\partial L_3}{\partial U^{(2)}} + \frac{\partial L_3}{\partial U^{(1)}}$$

$$= \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial U^{(3)}}$$

$$+ \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial U^{(2)}}$$

$$+ \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial S_1} \cdot \frac{\partial S_1}{\partial U^{(1)}}$$

$$= \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial U^{(3)}}$$

$$+ \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \left(\frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial U^{(2)}} + \frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial S_1} \cdot \frac{\partial S_1}{\partial U^{(1)}} \right)$$

$$= \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial U^{(3)}}$$

$$+ \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \left(\frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial S_1} \cdot \frac{\partial S_1}{\partial U^{(1)}} \right)_{K=1} + \frac{\partial S_3}{\partial S_2} \cdot \frac{\partial S_2}{\partial U^{(2)}} \Bigg)_{K=2}$$

$$= \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \frac{\partial S_3}{\partial U^{(3)}}$$

$$+ \sum_{K=1}^2 \cdot \frac{\partial L_3}{\partial O_3} \cdot \frac{\partial O_3}{\partial S_3} \cdot \left(\prod_{j=K+1}^3 \frac{\partial S_j}{\partial S_{j-1}} \right) \frac{\partial S_K}{\partial U^{(K)}}$$

$k \uparrow, j \uparrow$, 连乘项减少; $k \downarrow, j \downarrow$ 连乘项增多

在任意时刻下, 有.

$$\frac{\partial L_t}{\partial U} = \frac{\partial L_t}{\partial U^{(t)}} + \dots + \frac{\partial L_t}{\partial U^{(3)}} + \frac{\partial L_t}{\partial U^{(2)}} + \frac{\partial L_t}{\partial U^{(1)}}$$

$$= \frac{\partial L_t}{\partial O_t} \cdot \frac{\partial O_t}{\partial S_t} \cdot \frac{\partial S_t}{\partial U^{(t)}}$$

$$+ \sum_{k=1}^{t-1} \frac{\partial L_t}{\partial O_t} \cdot \frac{\partial O_t}{\partial S_t} \left(\prod_{j=k+1}^t \frac{\partial S_j}{\partial S_{j-1}} \right) \frac{\partial S_k}{\partial U^{(k)}}$$

$$\frac{\partial S_j}{\partial S_{j-1}} = \tanh'(\theta_j) W$$

$$= [1 - \tanh^2(\theta_j)] W \quad \left. \begin{array}{l} W \uparrow \text{ 梯度爆炸} \\ W \downarrow \text{ 梯度消失} \end{array} \right\}$$

$k \downarrow, j \downarrow$ 连乘的项越多.

前面的消息无法传到当前时刻, 失去列性.

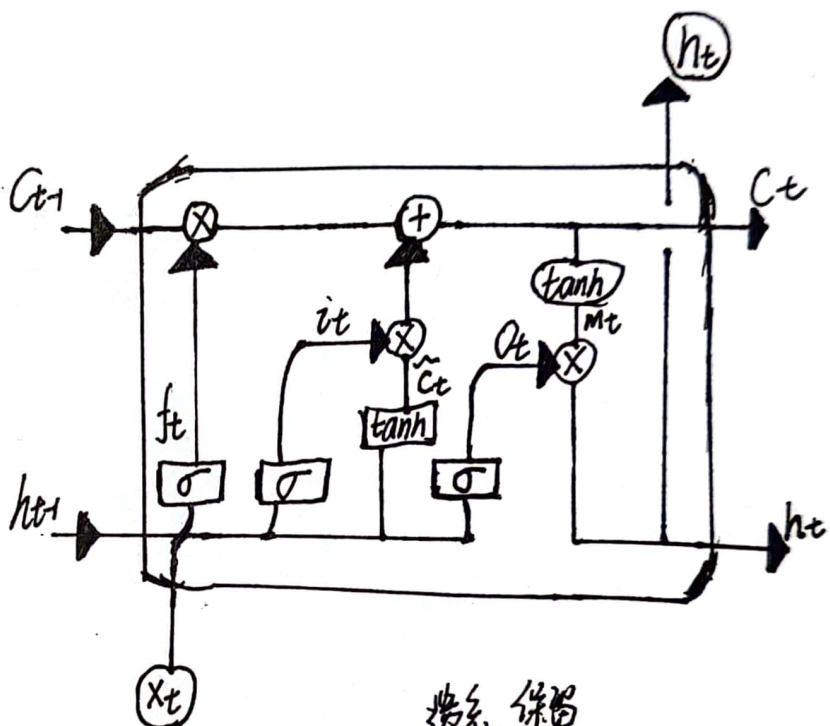
$t=20$

$$\frac{\partial L_{20}}{\partial U} = \frac{\partial L_{20}}{\partial O_{20}} \cdot \frac{\partial O_{20}}{\partial S_{20}} \cdot \frac{\partial S_{20}}{\partial U^{(20)}}$$

$$+ \frac{\partial L_{20}}{\partial O_{20}} \cdot \frac{\partial O_{20}}{\partial S_{20}} \cdot \left(\begin{array}{l} \frac{\partial S_{20}}{\partial S_{19}} \cdot \frac{\partial S_{19}}{\partial U^{(19)}} \\ + \frac{\partial S_{20}}{\partial S_{19}} \cdot \frac{\partial S_{19}}{\partial S_{18}} \cdot \frac{\partial S_{18}}{\partial U^{(18)}} \\ + \dots \end{array} \right)$$

$$+ \dots$$

$$\left(\begin{array}{l} + \dots \\ + \dots \end{array} \right)$$



$$f_t = \sigma(w_f)$$

$$i_t = \sigma(w_i)$$

$$o_t = \sigma(w_o)$$

eg: 场景: 期末考试周
A时间步: 考高等数学
下一时间步: 考线性代数

x_t : 复习线性代数

h_t : 考试得分

c_t : 新的记忆

c_{t-1} : 旧的记忆, 考完高数的记忆

h_{t-1} : 考完高数当下的状态

目标: ④ 尽可能好 [能复习]

$\sigma: [0, 1]$ 遗忘保留

f_t : 考完高数遗忘与遗忘相关的知识 [遗忘门]

~~遗忘~~

\tilde{c}_t : \tanh : 相当于复习现代生条新的记忆

i_t : 生成的新的记忆并保留有用, 通过 σ 进行更新 [更新门]

c_t : 旧的记忆 + 新的记忆 = 新记忆

$f_t * c_{t-1} + i_t * \tilde{c}_t$
数学思维能力 现代考点内容

$$t=3$$

$$\frac{\partial L_3}{\partial W_{sf}} = \frac{\partial L_3}{\partial W_{sf}^{(3)}} + \frac{\partial L_3}{\partial W_{sf}^{(2)}} + \frac{\partial L_3}{\partial W_{sf}^{(1)}}$$

$$\begin{aligned} \textcircled{1} \frac{\partial L_3}{\partial W_{sf}^{(3)}} &= \frac{\partial L_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3} \cdot \frac{\partial h_3}{\partial m_3} \cdot \frac{\partial m_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial f_3} \cdot \frac{\partial f_3}{\partial W_{sf}^{(3)}} \\ &= \frac{\partial L_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3} \cdot \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial f_3} \cdot \frac{\partial f_3}{\partial W_{sf}^{(3)}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \frac{\partial L_3}{\partial W_{sf}^{(2)}} &= \frac{\partial L_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3} \cdot \left(\begin{aligned} &\frac{\partial h_3}{\partial o_3} \cdot \frac{\partial o_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \cdot \frac{\partial c_2}{\partial f_2} \cdot \frac{\partial f_2}{\partial W_{sf}^{(2)}} \\ &+ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial c_2} \cdot \frac{\partial c_2}{\partial f_2} \cdot \frac{\partial f_2}{\partial W_{sf}^{(2)}} \\ &+ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial f_3} \cdot \frac{\partial f_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \cdot \frac{\partial c_2}{\partial f_2} \cdot \frac{\partial f_2}{\partial W_{sf}^{(2)}} \\ &+ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial i_3} \cdot \frac{\partial i_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \cdot \frac{\partial c_2}{\partial f_2} \cdot \frac{\partial f_2}{\partial W_{sf}^{(2)}} \\ &+ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial g_3} \cdot \frac{\partial g_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \cdot \frac{\partial c_2}{\partial f_2} \cdot \frac{\partial f_2}{\partial W_{sf}^{(2)}} \end{aligned} \right) \end{aligned}$$

$$\begin{aligned} &= \frac{\partial L_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3} \cdot \left(\begin{aligned} &\frac{\partial h_3}{\partial o_3} \cdot \frac{\partial o_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \\ &+ \frac{\partial h_3}{\partial c_3} \cdot \left(\begin{aligned} &\frac{\partial c_3}{\partial c_2} \cdot \frac{\partial f_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \\ &+ \frac{\partial c_3}{\partial i_3} \cdot \frac{\partial i_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \\ &+ \frac{\partial c_3}{\partial g_3} \cdot \frac{\partial g_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \end{aligned} \right) \cdot \frac{\partial c_2}{\partial f_2} \cdot \frac{\partial f_2}{\partial W_{sf}^{(2)}} \end{aligned} \right) \end{aligned}$$

(3) (1) $t_3 \rightarrow t_2$.

$$y_3 \rightarrow h_3 \rightarrow \begin{cases} o_3 \rightarrow h_2 \\ \cancel{c_3 \rightarrow f_3 \rightarrow h_2} \\ c_3 \rightarrow f_3 \rightarrow h_2 \\ c_3 \rightarrow i_3 \rightarrow h_2 \\ c_3 \rightarrow c_2 \\ c_3 \rightarrow g_3 \rightarrow h_2 \end{cases}$$

(2) $t_2 \rightarrow t_1$

$$\left\{ \begin{array}{l} h_2 \rightarrow \begin{cases} o_2 \rightarrow h_1 \\ \cancel{c_2} \\ \cancel{c_2 \rightarrow f_2 \rightarrow h_1} \\ c_2 \rightarrow f_2 \rightarrow h_1 \\ c_2 \rightarrow i_2 \rightarrow h_1 \\ c_2 \rightarrow g_2 \rightarrow h_1 \end{cases} \\ \\ c_2 \rightarrow \begin{cases} c_1 \\ \cancel{f_2 \rightarrow h_2 \rightarrow c_1} \\ f_2 \rightarrow h_2 \rightarrow c_1 \\ i_2 \rightarrow h_2 \rightarrow c_1 \\ g_2 \rightarrow h_2 \rightarrow c_1 \end{cases} \end{array} \right\} \rightarrow c_1 \rightarrow f_1 \rightarrow W_{sf}^{(1)}$$

$$=$$

$$\left\{ \begin{array}{l} h_2 \rightarrow \begin{cases} o_2 \rightarrow h_1 \\ c_2 \rightarrow c_1 \end{cases} \rightarrow f_1 \\ \\ c_2 \rightarrow c_1 \rightarrow f_1 \end{array} \right\}$$

$$= \frac{\partial l_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3} \cdot \left(\begin{array}{cc} \frac{\partial h_3}{\partial o_3} \cdot \frac{\partial o_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} & \frac{\partial l_2}{\partial f_2} \cdot \frac{\partial f_2}{\partial W_{sf}^{(2)}} \\ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial c_2} & \end{array} \right)$$

$$\begin{aligned}
 & (3) t_3 \rightarrow t_1 \\
 & y_3 \rightarrow h_3 \rightarrow \left(\begin{array}{l} O_3 \rightarrow h_2 \rightarrow \left(\begin{array}{l} O_2 \rightarrow h_1 \\ C_2 \rightarrow C_1 \end{array} \right) \rightarrow f_1 \rightarrow W_{sf}^{(1)} \\ C_3 \rightarrow f_3 \rightarrow h_2 \rightarrow \left(\begin{array}{l} O_2 \rightarrow h_1 \\ C_2 \rightarrow C_1 \end{array} \right) \rightarrow f_1 \rightarrow W_{sf}^{(1)} \\ C_3 \rightarrow i_3 \rightarrow h_2 \rightarrow \left(\begin{array}{l} O_2 \rightarrow h_1 \\ C_2 \rightarrow C_1 \end{array} \right) \rightarrow f_1 \rightarrow W_{sf}^{(1)} \\ C_3 \rightarrow g_3 \rightarrow h_2 \rightarrow \left(\begin{array}{l} O_2 \rightarrow h_1 \\ C_2 \rightarrow C_1 \end{array} \right) \rightarrow f_1 \rightarrow W_{sf}^{(1)} \\ C_3 \rightarrow C_2 \rightarrow C_1 \rightarrow f_1 \rightarrow W_{sf}^{(1)} \end{array} \right)
 \end{aligned}$$

$$= y_3 \rightarrow h_3 \rightarrow \left(\begin{array}{l} O_3 \rightarrow h_2 \\ C_3 \rightarrow f_3 \rightarrow h_2 \\ C_3 \rightarrow i_3 \rightarrow h_2 \\ C_3 \rightarrow g_3 \rightarrow h_2 \end{array} \right) \rightarrow O_2 \rightarrow h_1 \rightarrow C_1 \rightarrow f_1 \rightarrow W_{sf}^{(1)}$$

$$\left(\begin{array}{l} O_3 \rightarrow h_2 \\ C_3 \\ C_3 \rightarrow f_3 \rightarrow h_2 \\ C_3 \rightarrow i_3 \rightarrow h_2 \\ C_3 \rightarrow g_3 \rightarrow h_2 \end{array} \right) \rightarrow C_2 \rightarrow C_1 \rightarrow f_1 \rightarrow W_{sf}^{(1)}$$

$$\begin{aligned}
 & = y_3 \rightarrow h_3 \rightarrow \left(\begin{array}{l} O_3 \rightarrow h_2 \\ C_3 \rightarrow f_3 \rightarrow h_2 \\ C_3 \rightarrow i_3 \rightarrow h_2 \\ C_3 \rightarrow g_3 \rightarrow h_2 \end{array} \right) \rightarrow O_2 \rightarrow h_1 \rightarrow C_1 \\
 & \quad \downarrow \\
 & \quad \rightarrow f_1 \rightarrow W_{sf}^{(1)} \\
 & \quad \downarrow \\
 & \quad C_3 \rightarrow C_2 \rightarrow C_1 \rightarrow f_1 \rightarrow W_{sf}^{(1)}
 \end{aligned}$$

(24)

合起来。

$$\frac{\partial L_3}{\partial W_{hf}^{(1)}} = \frac{\partial L_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3}$$

~~$$\frac{\partial L_3}{\partial W_{hf}^{(1)}} = \frac{\partial L_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3}$$~~

~~$$\frac{\partial L_3}{\partial W_{hf}^{(1)}} = \frac{\partial L_3}{\partial y_3} \cdot \frac{\partial y_3}{\partial h_3}$$~~

知识总结

$$\left(\begin{array}{c} \frac{\partial h_3}{\partial o_3} \cdot \frac{\partial o_3}{\partial h_2} \\ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial f_3} \cdot \frac{\partial f_3}{\partial h_2} \\ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial i_3} \cdot \frac{\partial i_3}{\partial h_2} \\ + \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial g_3} \cdot \frac{\partial g_3}{\partial h_2} \end{array} \right) \cdot \left(\begin{array}{c} \frac{\partial h_2}{\partial o_2} \cdot \frac{\partial o_2}{\partial h_1} \\ \cdot \frac{\partial h_1}{\partial c_1} \end{array} \right) \cdot \left(\begin{array}{c} \frac{\partial c_1}{\partial f_1} \cdot \frac{\partial f_1}{\partial W_{hf}^{(1)}} \end{array} \right)$$

$$\left(\begin{array}{c} \frac{\partial h_3}{\partial o_3} \cdot \frac{\partial o_3}{\partial h_2} \cdot \frac{\partial h_2}{\partial c_2} \cdot \frac{\partial c_2}{\partial c_1} \\ \frac{\partial h_3}{\partial c_3} \cdot \frac{\partial c_3}{\partial c_2} \cdot \frac{\partial c_2}{\partial c_1} \end{array} \right)$$

$$\frac{\partial \mathcal{L}_t}{\partial \mathcal{L}_{t-1}} = \frac{\partial \mathcal{L}_t}{\partial \mathcal{L}_{t-1}} \quad ; \quad f_t$$

$$+ \frac{\partial \mathcal{L}_t}{\partial g_t} \cdot \frac{\partial g_t}{\partial h_{t-1}} \cdot \frac{\partial h_{t-1}}{\partial m_{t-1}} \cdot \frac{\partial m_{t-1}}{\partial \mathcal{L}_{t-1}}$$

$$+ \frac{\partial \mathcal{L}_t}{\partial \tilde{z}_t} \cdot \frac{\partial \tilde{z}_t}{\partial h_{t-1}} \cdot \frac{\partial h_{t-1}}{\partial m_{t-1}} \cdot \frac{\partial m_{t-1}}{\partial \mathcal{L}_{t-1}}$$

$$\neq \frac{\partial \mathcal{L}_t}{\partial f_t} \cdot \frac{\partial f_t}{\partial h_{t-1}} \cdot \frac{\partial h_{t-1}}{\partial m_{t-1}} \cdot \frac{\partial m_{t-1}}{\partial \mathcal{L}_{t-1}}$$

$$= f_t$$

$$+ \tilde{z}_t \cdot \tanh'(\theta) \cdot o_{t-1} \cdot \tanh'(\mathcal{L}_{t-1}) \cdot W_{hg}$$

$$+ g_t \cdot \sigma'(\theta) \cdot o_{t-1} \cdot \tanh'(\mathcal{L}_{t-1}) \cdot W_{hg}$$

$$+ \mathcal{L}_{t-1} \cdot \sigma'(\theta) \cdot o_{t-1} \cdot \tanh'(\mathcal{L}_{t-1}) \cdot W_{hf}$$

通过调节 W_{hf} , W_{ht} , W_{hg} 可以灵活控制 $\frac{\partial \mathcal{L}_t}{\partial \mathcal{L}_{t-1}}$ 的值
 当要从 n 时刻长期记忆某个东西直到 m 时刻时, 该路径上的 $\prod_{t=n}^m \frac{\partial \mathcal{L}_t}{\partial \mathcal{L}_{t-1}} \approx 1 \times 1 \times \dots \times 1$, 从而缓解梯度消失.

LSTM 可以通过调节参数 W_{hi} , W_{hf} , W_{hg} 灵活控制记忆细胞 $\frac{\partial C_t}{\partial C_{t-1}}$, 使得其值接近 1。为什么

RNN 中 $\frac{\partial S_t}{\partial S_{t-1}}$ 不可以通过调节参数 U 来控制其值近似为 1?

$\frac{\partial C_t}{\partial C_{t-1}}$ 是多个 W 的线性相加的综合结果, 其中某个参数 W 很大或者很小, 可以由其它的 W

进行协调, 不会影响最后的结果。RNN 中 $\frac{\partial S_t}{\partial S_{t-1}}$ 只由一个参数 U 控制, 就会导致梯度爆炸或梯度消失。[梯度正常+梯度消失=梯度正常]。