Mathematical Model for Decentralized P2P Energy Trading

The study considers a residential energy community composed of a set of nodes N, indexed by $\boldsymbol{i} = 0, 1, ..., N$. Although a node can represent a household or a building within the distribution network, this study focuses solely on households. All households are connected to the main grid and can inject/withdraw power through that connection. Part of their electricity demand is fixed and described by a load profile. The baseload demand of household $\boldsymbol{i}$ at timestep $t \in T = \{t_0, t_0 + \Delta t, t_0 + 2\Delta t, ..., T\}$ is denoted as $\boldsymbol{P^l_{i,t}}$, and is assumed to be deterministic. The power withdrawn/injected from/to the grid is denoted as $\boldsymbol{p^g_{i,t}}$ and is associated with a price λ^{buy} for buying energy from the grid and λ^{sell} for selling energy back to the grid. These price signals vary with $\boldsymbol{t}$ but are identical for all households in the community.

The cost function for each household $\boldsymbol{i}$ at timestep $\boldsymbol{t}$ can then be formulated as follows:

$$C^{\mathrm{g}}_{i,t}(p^{\mathrm{g}}_{i,t}) = \lambda^{\mathrm{buy}}_t \cdot [p^{\mathrm{g}}_{i,t}]^- - \lambda^{\mathrm{sell}}_t \cdot [p^{\mathrm{g}}_{i,t}]^+ \ \ \forall\, i \in \mathcal{N}, t \in \mathcal{T}$$

where

$$[p^{\mathrm{g}}_{i,t}]^+ = \max\{p^{\mathrm{g}}_{i,t}, 0\} \ \ \forall\, i \in \mathcal{N}, t \in \mathcal{T},$$
$$[p^{\mathrm{g}}_{i,t}]^- = \max\{-p^{\mathrm{g}}_{i,t}, 0\} \ \ \forall\, i \in \mathcal{N}, t \in \mathcal{T}.$$

In P2P trading, every household can exchange a certain amount of net power with other households within the network. The net power exchange for household $\boldsymbol{i}$ at time $\boldsymbol{t}$ is denoted as $\boldsymbol{p_{i,t}}$, where a positive value indicates surplus power (power available for sale) and a negative value indicates residual demand (power that needs to be bought). The net power $\boldsymbol{p_{i,t}}$ is determined using an energy balancing formula as follows:

$$p_{i,t} = p^{\mathrm{g}}_{i,t} + P^{\mathrm{pv}}_{i,t} + p^{\mathrm{b}}_{i,t} - P^{\mathrm{l}}_{i,t} - p^{\mathrm{ev}}_{i,t} - p^{\mathrm{hp}}_{i,t} \ \ \forall\, i \in \mathcal{N}, t \in \mathcal{T}.$$

For each household $\boldsymbol{i}$ in the set **N** (with |N|=10|) and each time step $\boldsymbol{t}$ in the set **T**:

- $p^g_{i,t}$: Power imported from (or exported to) the main grid.
- $P^{pv}_{i,t}$: Solar power generation.
- $P^b_{i,t}$: Wind power generation.
- $P^l_{i,t}$: Electrical load.
- $p^{ev}_{i,t}$: Power from EV.
- $p^{hp}_{i,t}$: Power from HP.

Market Design

This model facilitates the assignment of a bilateral trading coefficient to each trade. We assume that all households in the network are rational and nonstrategic market agents. The objective is to minimize the overall costs in the network for all households $\boldsymbol{N}$ over a time horizon $\boldsymbol{T}$. This includes the costs (revenues) associated with withdrawing (injecting) power to the grid, as well as the costs (revenues) of trading energy bilaterally with other households. The problem can be formulated as:

$$\text{minimize} \quad \sum_{t=0}^{T}\sum_{i=0}^{\mathcal{N}}\left[C^{\mathrm{g}}_{i,t}(p^{\mathrm{g}}_{i,t}) + \sum_{j=0}^{\mathcal{M}}\gamma_{ij,t}|d_{ij,t}|\right], \qquad (16a)$$

$$\text{subject to:} \quad (4)\text{–}(7), (9)\text{–}(15),$$

$$p_{i,t} = \sum_{j=0}^{\mathcal{M}} d_{ij,t}, \quad [\mu_{i,t}]\forall\, i \in \mathcal{N}, t \in \mathcal{T}, \qquad (16b)$$

$$\boldsymbol{D}_t = -\boldsymbol{D}_t^{\intercal} \quad [\Xi_t]\forall\, t \in \mathcal{T}. \qquad (16c)$$

In this formulation, $\boldsymbol{M}$ indexed by $\boldsymbol{j}$ represents the set of trading partners of household $\boldsymbol{i}$. Bilateral trading preferences are determined by the household owner using the parameter $\gamma_{(ij,t)}$, which is a bilateral trading coefficient imposed by i on the trade with household $\boldsymbol{j}$, and $d_{(ij,t)}$ is the quantity of energy traded between $\boldsymbol{i}$ and $\boldsymbol{j}$. The total amount of traded energy between $\boldsymbol{i}$ and all other households at each $\boldsymbol{t}$ must equal the net power of $\boldsymbol{i}$ at that $\boldsymbol{t}$ as determined. The dual variable $\mu_{(i,t)}$ associated with this constraint represents the perceived energy price by $\boldsymbol{i}$. The matrix $\boldsymbol{D}_t$ contains the quantities of all bilateral trades in the network at each $\boldsymbol{t}$, and the associated dual variable matrix Ξ_t contains the prices of all trades at that $\boldsymbol{t}$. The reciprocity of trading quantities, as well as of trading prices Ξ, is ensured by (16c) at the optimal solution of problem (16).

Decentralized Formulation

In this work, we employ the general-form consensus optimization of the Alternating Direction Method of Multipliers (ADMMs) to decompose the centralized formulation in (16) into several subproblems, which are solved independently by each household. The individual solutions are then coordinated to obtain the global solution.

After locally solving its subproblem, each household determines its optimal local trading quantities schedule, denoted as $\boldsymbol{D}$, which serves as a coupling variable corresponding to the global variable $\boldsymbol{C}$. Following, $(\boldsymbol{C} - \boldsymbol{C}^{\boldsymbol{T}})/2 = \boldsymbol{D}$ is defined as the average of the trading quantity proposed by household $\boldsymbol{i}$ to $\boldsymbol{j}$ and that proposed by $\boldsymbol{j}$ to **i**. Consensus among households is achieved when these trading values are equal with a condition guaranteed by ADMM at optimality since the optimization problem formulated in (16) is convex.

Notably, each household's local private energy information remains confidential and is not shared with the network.

Using this consensus constraint, the fully decentralized augmented Lagrangian for the bilateral trading model at each iteration ***k*** for each household ***i*** can be formulated as:

$$(p_i^g, D_i)^{k+1} = \underset{p_i^g, D_i}{\operatorname{argmin}} \sum_{t=0}^{\mathcal{T}} \Bigg[C_{i,t}^g(p_{i,t}^g) + \sum_{j=0}^{\mathcal{M}} \Bigg[\gamma_{ij,t} |d_{ij,t}^{k+1}| + (\rho/2) \left(\frac{d_{ij,t}^k - d_{ji,t}^k}{2} - d_{ij,t}^{k+1} + \xi_{ij,t}^k/\rho \right)^2 \Bigg] \Bigg]$$

Where ρ > 0 is the penalty parameter and ξ is the dual variable representing the price of each bilateral trade, updated at each ADMM iteration k, as follows:

$$\xi_{ij,t}^{k+1} = \xi_{ij,t}^k - \rho(d_{ij,t}^{k+1} + d_{ji,t}^{k+1})/2.$$

The consensus is achieved when the ADMM algorithm converges, which is determined using the following conditions:

$$\left\| r^k \right\|_2 \leq \epsilon_p, \quad \left\| s^k \right\|_2 \leq \epsilon_d$$

Where ***r*** and ***s*** are the primal and dual residuals, and ***ϵp*** and ***ϵd*** are the tolerances for the primal and dual residuals, which are typically assigned very low values.

Bilateral Trading Strategies

A matrix of bilateral trading coefficients, ***Γ***, is defined, containing all bilateral trading coefficient values between any household and its peers in the network. This matrix is used to indicate preferred trading partners and enable product differentiation. According to the formulation, the smaller the value of ***γij,t,*** the more favorable the associated trade with household ***j***. In this model, two strategies for defining the matrix ***Γ*** are proposed.

1. Supply-Demand Matching Strategy (ST1)

In this strategy, the bilateral trading coefficients are determined based on the matching between power demand and surplus PV power generation of households. The willingness of household ***i*** to trade at time ***t*** is proportional to the magnitude of its expected deficit demand or surplus PV generation. Following this approach, households with surplus power are more likely to trade with those experiencing a power deficit, and vice versa. This strategy prioritizes an economic mechanism to structure a bilateral trading market.

To implement these assumptions in the bilateral trading coefficients, several steps are followed:

1. The expected net budget matrix P^{net}, is determined, containing the net power of all households across all time steps.
2. From this matrix, two new matrices, P^{Buy} and P^{Sell}, are defined, representing the amount of net power each household wants to buy or sell at every timestep.
3. Each row in P^{Buy} and P^{Sell} is normalized according to the maximum deficit and surplus budget in that row.
4. Subsequently, the matrices $\Gamma^{b,rel}$ and $\Gamma^{s,rel}$ are generated.

These matrices represent the relative willingness of households to buy or sell energy. The parameter $\boldsymbol{\chi}$ defines the maximum baseline value for bilateral trading coefficients. Using $\Gamma^{b,rel}$ and $\Gamma^{s,rel}$, the final 3-D matrix of bilateral trading coefficients, Γ, is constructed.

When $\boldsymbol{\gamma_{ij,t}=\chi}$, households ***i*** and ***j*** are considered highly unlikely to engage in trading. The parameter $\boldsymbol{\chi}$ is set at a high value, ensuring that all bilateral trading coefficients fall within the range 0 to χ. The complete procedure for this strategy (ST1) is outlined in Algorithm 1[1].

Algorithm 1: ST1: Supply-Demand Matching Strategy.

foreach *household* $i \in \mathcal{N}$ **do**
- $P_i^l \leftarrow$ power demand profile
- $P_i^{pv} \leftarrow$ PV generation profile

Build the net budget matrix $\boldsymbol{P}^{net}$ (N x T):

$$\boldsymbol{P}^{net} = \boldsymbol{P}^{pv} - \boldsymbol{P}^{l}$$

Split $\boldsymbol{P}^{net}$ in two matrices $\boldsymbol{P}^{buy}$ and $\boldsymbol{P}^{sell}$, where each element in these matrices is defined as:

$$p_{i,t}^{buy} = \begin{cases} 0 & \text{if } p_{i,t}^{net} \geq 0 \\ p_{i,t}^{net} & \text{if } p_{i,t}^{net} < 0 \end{cases} \tag{20}$$

$$p_{i,t}^{sell} = \begin{cases} p_{i,t}^{net} & \text{if } p_{i,t}^{net} > 0 \\ 0 & \text{if } p_{i,t}^{net} \leq 0 \end{cases} \tag{21}$$

Set the maximum coefficients baseline value (χ)

Normalize each row in $\boldsymbol{P}^{buy}$ and $\boldsymbol{P}^{sell}$ using $\overline{P}_{max}^{buy}$ and $\overline{P}_{max}^{sell}$, as:

$$\boldsymbol{\Gamma}^{b,rel} = \frac{\chi}{2} \frac{\boldsymbol{P}^{buy}}{\overline{P}_{max}^{buy}} \tag{22}$$

$$\boldsymbol{\Gamma}^{s,rel} = \frac{\chi}{2} \frac{\boldsymbol{P}^{sell}}{\overline{P}_{max}^{sell}} \tag{23}$$

Build the 3D matrix of bilateral trading coefficients $\boldsymbol{\Gamma}$ based on $\boldsymbol{\Gamma}^{b,rel}$ and $\boldsymbol{\Gamma}^{s,rel}$, as:

$$\gamma_{ij,t} = \begin{cases} \chi & \text{if } \gamma_{i,t}^{s,rel}, \gamma_{j,t}^{s,rel} > 0 \\ \chi & \text{if } \gamma_{i,t}^{b,rel}, \gamma_{j,t}^{b,rel} > 0 \\ \chi - (\gamma_{i,t}^{b,rel} + \gamma_{j,t}^{b,rel} + \gamma_{i,t}^{s,rel} + \gamma_{j,t}^{s,rel}) & \text{otherwise} \end{cases} \tag{24}$$

Figure 1- Algorithm 1[1].

2. Distance-Based Matching Strategy (ST2)

The second proposed strategy for matching households is based on their distance within the network. Here, distance is defined as the number of connections between households. For example, directly connected households have a distance of 1. The distance between households depends on the network topology, which is assumed to be known in this strategy. Typically, households in distribution networks are connected in a radial topology, meaning the network can be represented as a multiway.

This strategy prioritizes reducing long-distance electricity flows, potentially improving energy efficiency by encouraging P2P trading among nearby households. The first step in this strategy is to construct an ***N***×***N*** child and parent matrix based on the network topology. The neighbors of a given household are identified as its parent and children. A household ***j*** that is closer to the root of the tree is considered the parent of ***i***, while ***i*** is regarded as the child of ***j***. Following this, two separate vectors, ***vp*** and ***vc*** , are created to store the list of all parents and children in the network, respectively. Finally, the distance between each pair of households is computed using a recursive function, ***"findDist(i,j)"***. The detailed descriptions of the ST2 algorithm and the ***"findDist(i,j)"*** function are provided in Algorithm 2[1].

Algorithm 2: ST2: Distance-Based Matching Strategy.

Receive the network topology
Build the $N \times N$ child and parent matrices based on the network topology
Identify the children and parents IDs of each household and store them in two separate vectors (**vp** and **vc**)
Find and store the distance d_{ij} between each two households ($i \in N$ and $j \in N$) in the network using the $findDist(i,j)$ function
Function $findDist\ (i,j)$ **:**
    $d_{ij} \leftarrow 0$ (initial value of the distance);
    if $i == j$ **then**
        return d_{ij} (a household's distance to itself is zero);
    else if $vp_i == j \,||\, vp_j == i$ **then**
        return d_{ij}+1 (one household is a parent of the other);
    else if $vp_i == vp_j$ **then**
        return d_{ij}+2 (households are children of the same parent);
    else
        $n \leftarrow max(vp_i, vp_j)$
        $m \leftarrow min(i,j)$
        if $vp_n == m$ **then**
            return $findDist(vp_n, m)$ (m is a grandparent of n);
        else if $vp_m == n$ **then**
            return $findDist(vp_m, n)$ (n is a grandparent of m);
        else
            return $findDist(n,m) + 1$ (other cases);
End Function
Set the trading coefficients based on distance:
foreach $i \in N$ **do**
    foreach $j \in N$ **do**
        $\gamma_{ij} \leftarrow d_{ij} = findDist(i,j)$
Return the final bilateral trading coefficients matrix $\mathbf{\Gamma}$

Figure 2- Algorithm 2 [1].

References

[1] T. AlSkaif, J. L. Crespo-Vazquez, M. Sekuloski, G. van Leeuwen, and J. P. S. Catalão, "Blockchain-Based Fully Peer-to-Peer Energy Trading Strategies for Residential Energy Systems," *IEEE Trans. Ind. Inform.*, vol. 18, no. 1, pp. 231–241, 2022, doi: 10.1109/TII.2021.3077008.