Jérémy Chavarot

CS-328 : HW 2

2.4.4. Deriving the linear system for the color space transformation

At the beginning the problem is constructed as follows:

$$\underset{24\times 3}{C_{camera}} \cdot \underset{3\times 3}{F} = \underset{24\times 3}{C_{target}}$$

unknown

The $C_{camera} \cdot F$ product gives:

$$C_{camera} \cdot F = \begin{pmatrix} r_1 & g_1 & b_1 \\ \vdots & \vdots & \vdots \\ r_{24} & g_{24} & b_{24} \end{pmatrix} \begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix}$$

$$= \begin{pmatrix} r_1 x_1 + g_1 x_4 + b_1 x_7 & \cdots & r_1 x_3 + g_1 x_6 + b_1 x_9 \\ \vdots & \cdots & \\ r_{24} x_1 + g_{24} x_4 + b_{24} x_7 & \cdots & r_{24} x_3 + g_{24} x_6 + b_{24} x_9 \end{pmatrix}$$

Now we know what the composants

of Ctarget correspond to. For a least squares system of the form $A\vec{x} = \vec{b}$, we reinterpret $F_{3\times3}$ as a 3×1 column vector $\vec{x}$.
That is: $A_{?\times 9} \cdot x_{9\times1} = b_{?\times1}$

Say we want to reinterpret the 24×3 Ctarget matrix as a 72×1 column vector $\vec{b}$ so not to lose the 72 observations.
Hence the system becomes:

$$A_{72\times9} \cdot x_{9\times1} = b_{72\times1}$$

This implies:

$$A = \begin{pmatrix} r_1 & 0 & 0 & g_1 & 0 & 0 & b_1 & 0 & 0 \\ 0 & r_1 & 0 & 0 & g_1 & 0 & 0 & b_1 & 0 \\ 0 & 0 & r_1 & 0 & 0 & g_1 & 0 & 0 & b_1 \\ \vdots & & & & \ddots & & & & \vdots \\ r_{24} & 0 & 0 & g_{24} & 0 & 0 & b_{24} & 0 & 0 \\ 0 & r_{24} & 0 & 0 & g_{24} & 0 & 0 & b_{24} & 0 \\ 0 & 0 & r_{24} & 0 & 0 & g_{24} & 0 & 0 & b_{24} \end{pmatrix}$$

and

$$b = \begin{pmatrix} r_1 x_1 + g_1 x_4 + b_1 x_7 \\ r_1 x_2 + g_1 x_5 + b_1 x_8 \\ \vdots \\ r_{24} x_3 + g_{24} x_6 + b_{24} x_9 \end{pmatrix}$$