Homework III, Theory of Computation 2022

Submission: The deadline for Homework 3 is 23:59 on Friday 27 May. Please submit your solutions on Moodle. Typing your solutions using a typesetting system such as LaTeX is strongly encouraged! If you must handwrite your solutions, write cleanly and with a pen. Messy and unreadable homeworks will not be graded. No late homeworks will be accepted.

Writing: Please be precise, concise and (reasonably) formal. Keep in mind that many of the problems ask you to provide a proof of a statement (as opposed to, say, just to provide an example). Therefore, make sure that your reasoning is correct and there are no holes in it. A solution that is hard/impossible to decipher/follow might not get full credit (even if it is in principle correct). You do not need to reprove anything that was shown in the class—just state clearly what was proved and where.

Collaboration: These problem sets are meant to be worked on in groups of 3–5 students. Please submit only one writeup per team—it should contain the names of all the students. You are strongly encouraged to solve these problems by yourself. If you must, you may use books or online resources to help solve homework problems, but you must credit all such sources in your writeup and you must never copy material verbatim. Even though only one writeup is submitted, it is expected that each one of the team members is able to fully explain the solutions if requested to do so.

Grading: Each of the two problems will be graded on a scale from 0 to 5.

Warning: Your attention is drawn to the EPFL policy on academic dishonesty. In particular, you should be aware that copying solutions, in whole or in part, from other students in the class or any other source without acknowledgement constitutes cheating. Any student found to be cheating risks automatically failing the class and being referred to the appropriate office.

Homework 3

1 Let $G = (V, E)$ be an undirected graph. A set of vertices $D \subseteq V$ is *dense* if every vertex $v \in V \setminus D$ has a neighbour in D (that is, $\{v, u\} \in E$ for some $u \in D$). Define

$$\textsc{DenseSet} \;=\; \{\langle G, k\rangle : G \text{ has a dense set of size at most } k\}.$$

Prove that DenseSet is **NP**-complete. In your proof, you may assume the **NP**-completeness of any of the problems discussed in class (SAT, Independent Set, Clique, Subset Sum, Vertex Cover, Set Cover, etc.).

2 Denote by $\mathbb{Z}$ the set of integers and by $\mathbb{N} = \{1, 2, 3, \ldots\}$ the set of positive integers. Consider the following two variants of SubsetSum (the first one of which is the version defined in class):

$$\begin{aligned}\textsc{SubsetSum}^{+} &= \{\langle X, s\rangle : X \subseteq \mathbb{N} \text{ is a multiset and some subset of } X \text{ sums to } s\},\\ \textsc{SubsetSum}^{\pm} &= \{\langle X, s\rangle : X \subseteq \mathbb{Z} \text{ is a multiset and some subset of } X \text{ sums to } s\}.\end{aligned}$$

Describe a *direct* reduction $\textsc{SubsetSum}^{\pm} \leq_p \textsc{SubsetSum}^{+}$. That is, your reduction must directly transform an input of $\textsc{SubsetSum}^{\pm}$ to an input of $\textsc{SubsetSum}^{+}$. You are not allowed to use the Cook–Levin theorem or the knowledge that $\textsc{SubsetSum}^{+}$ is **NP**-complete.