EPFL

Homework I, Theory of Computation 2022

Submission: **The deadline for Homework 1 is 23:59 on 24 March.** Please submit your solutions on Moodle. Typing your solutions using a typesetting system such as LaTeX is strongly encouraged! If you must handwrite your solutions, write cleanly and with a pen. Messy and unreadable homeworks will not be graded. No late homeworks will be accepted.

Writing: Please be precise, concise and (reasonably) formal. Keep in mind that many of the problems ask you to provide a proof of a statement (as opposed to, say, just to provide an example). Therefore, make sure that your reasoning is correct and there are no holes in it. A solution that is hard/impossible to decipher/follow might not get full credit (even if it is in principle correct). You do not need to reprove anything that was shown in the class—just state clearly what was proved and where.

Collaboration: These problem sets are meant to be worked on in groups of 3–5 students. Please submit only one writeup per team—it should contain the names of all the students. You are strongly encouraged to solve these problems by yourself. If you must, you may use books or online resources to help solve homework problems, but you must credit all such sources in your writeup and you must never copy material verbatim. Even though only one writeup is submitted, it is expected that each one of the team members is able to fully explain the solutions if requested to do so.

Grading: Each of the two problems will be graded on a scale from 0 to 5.

Warning: Your attention is drawn to the EPFL policy on academic dishonesty. In particular, you should be aware that copying solutions, in whole or in part, from other students in the class or any other source without acknowledgement constitutes cheating. Any student found to be cheating risks automatically failing the class and being referred to the appropriate office.

Homework 1

1 Pick either one of the automata below (Figure 1 or 2) and find a description of the language it recognizes. Prove by induction (or otherwise) that your description is correct. (*Note: You will be awarded full points for analysing just one of the below automata—you choose which one!*)
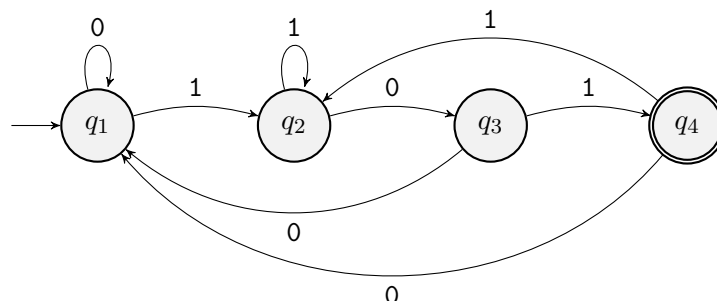
Figure 1. Original harder variant
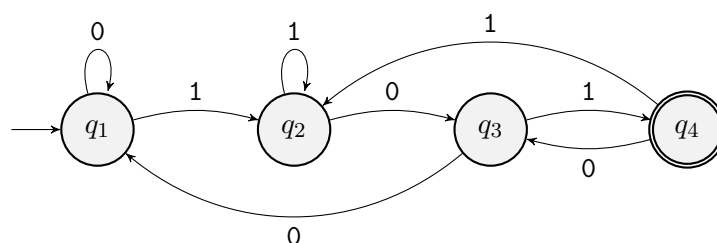
Figure 2. New easier variant

2 Closure properties:

2a For a language $L \subseteq \Sigma^*$ define its *triple* by

$$L^3 \ := \ \{www : w \in L\}.$$

Show that regular languages are *not* closed under tripling. That is, find some regular language L such that L^3 is not regular (and prove that it is not).

2b Suppose that $L \subseteq \Sigma^*$ is a regular language over a *unary alphabet*, $|\Sigma| = 1$. Show that in this case L^3 is regular.

*(It suffices that you describe a finite automaton for L^3. You do **not** need to prove correctness via induction.)*