Investigação Operacional
Operational Research

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OPERATIONS RESEARCH
FOR WHEN "GOOD ENOUGH" ISN'T
GOOD ENOUGH
quickmeme.com

What is Operations Research?
"The science of the better"

Operations Research (OR) is the discipline of applying advanced analytical methods to help make better decisions.

- www.informs.org

Operations Research is the art of winning wars without actually fighting.

- Arthur Clarke

Operations Research is the art of giving bad answers to problems where otherwise worse answers are given.

- T.L. Satty.

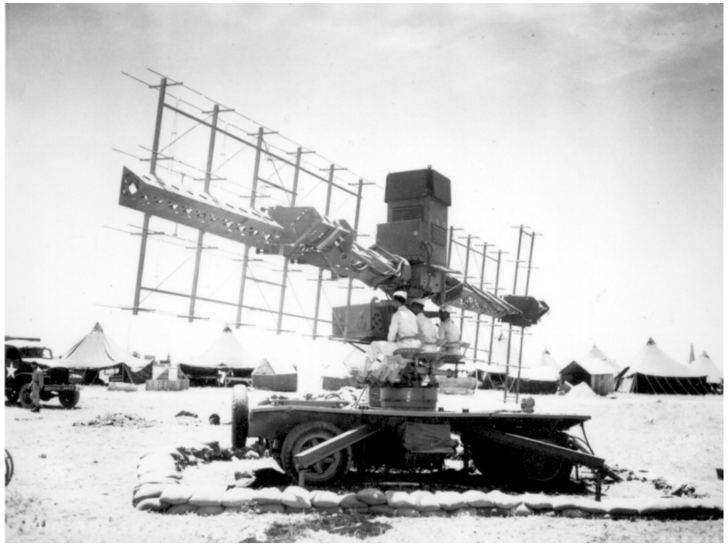
Operational Research was born during the II world war

The origins of Operational Research

(in the US it is called Operations Research)

During 2nd World War, some military leaders asked for help to reputed scientists and mathematicians in order to analyse and solve several **military operational** problems:

- radar installation
- railways and submarine operations
- planning guard teams
- mines and bombs placing.



This field of study was initially called *Military Operations Research (!)*

Later, since these methodologies and techniques could be applied to several different areas, the name was adapted to *Operations Research* in the USA.

Operational Research is a “baby war”

- Operational Research methods were developed during the second World War as analysts undertook a number of crucial projects that aided the war effort.
- Britain introduced the convoy system to reduce shipping losses, but while the principle of using warships to accompany merchant ships was generally accepted, it was unclear whether it was better for convoys to be small or large. But the number of escort ships was limited.
- Convoys travel at the speed of the slowest member, so small convoys can travel faster. It was also argued that small convoys would be harder for German U-boats to detect.
- On the other hand, large convoys could deploy more warships against an attacker.
- The O.R. teams showed that the losses suffered by convoys depended largely on the number of escort vessels present, rather than on the overall size of the convoy. Their conclusion, therefore, was that a few large convoys are more defensible than many small ones.
- Throughout the war, O.R. scientists looked for ways to make better decisions in such areas as logistics and training schedules. After the war it soon became evident that O.R. techniques could be applied to similar problems in industry.

OR backgorund

1947	Project Scoop (Scientific Computation of Optimum Programs) with George Dantzig and others. It was developed Simplex Method for linear problems.
1950's	Considerable activity, with several mathematical developments, particularly in queuing theory and mathematical programming.
1960's	More and more activity, developments and ideas.
1970's	Some disappointment and less activity. Discovery of NP-complete problems. More realistic expectations.
1980's	Personal computers arising. Easier access to huge volumes of information. Managers more willing to use mathematical models.
1990's	Increase in using and developing decision support systems based on OR models. New technologies for optimization simulation and modeling languages. Connection between OR and AI techniques.

OR in the XXI century

Lots of opportunities for work and research on OR

- Data, data, data...
- Machine Learning and AI methods apply several OR techniques
- Increasing need of support for decision making
- Increasing need of coordination for an efficient use of the available resources
- Transports, Mobility, Environment
- Manufacturing, Finance, Health,

Methodology of OR

One possible definition of OR:

"*OR is the scientific method applied to the decision problems context*"

A decision problem exists when:

- there is at least one decision agent (someone able to make decisions);
- There are at least two alternative lines of action to follow;
- There is at least one objective, once the decision agent chooses one line of action;
- The lines of action do not attain the same level of satisfaction for the objective(s).

Methodology of OR

The scientific method consists on the application of the following phases that overlap and interact with each other.

1. Problem definition and data gathering
2. Modeling the problem through a mathematical formulation
3. Model validation
4. Obtaining one (or more) solutions for the proposed model
5. Implementing the obtained solution or system

1. Problem definition and data gathering

- Need to:
  - study the organization and the system for which the problem appears
  - Identify the decision agents
  - Identify the main objectives of the organization (strategic, tactical, operational)
  - Select the objectives suited for the problem
  - Identify the minimal, reasonable, and ideal levels for objective satisfaction
- Need of multidisciplinary teams
- Gathering and selecting relevant information:
  - already available (databases, other systems)
  - to collect (e.g: a new database, surveys)

The outcome: a report containing a short and clear description of the problem, presenting guidelines and recommendations for its resolution. This document will evolve throughout the project being updated whenever new information is collected.

2. Modeling the problem through a mathematical formulation

A model: an idealized representation of reality

Mathematical model: a set of mathematical expressions representing the behavior of a complex system.

Choosing the most adequate model is a complex task:

- When the model is too simple, probably it will not consider some important aspects of the problem.
- When the model is too complex, it may no not be computationally tractable

A problem may be modeled in different ways, so choosing the appropriate model could be a success decision factor for the project.

It is very important to consider the availability and precision of the model input data.

3. Model validation

Model validation usually implies the implementation and execution (in a computer) of the chosen algorithm in order to guarantee that:

- ✓ input data and parameters do not contain errors
- ✓ the algorithm does not have logical errors
- ✓ The software does not have errors
- ✓ The algorithm represents correctly the model
- ✓ The results seem reasonable: sometimes, the algorithm is executed with historical data (if available) and the algorithm results are compared with the real past results.

4. Obtaining one (or more) solutions for the proposed model

We can use generic software (Excel, Lingo, CPLEX) or develop a particular algorithm for the specific problem.

In practice, the proposal of a solution involves the analysis of several solutions obtained under different conditions in order to acquire some sensibility to the data variability.

For example, if the input data was different would the solution be affected? Why or why not? This type of questions is called what-if analysis.

Attention: the optimal solutions are obtained for a particular model !!

– they should correspond to "satisfying" solutions for the problem.

– the ideal solution may not be attainable.

It’s better

to solve approximately the exact problem

than

to solve exactly the approximate problem

5. Implementing the obtained solution or system

This phase involves the implementation of the results of the study or the implementation of the algorithm as an operational tool or a software application (such as a Decision Support System).

Many OR projects successfully cross the previous phases and fail in the implementation …lots of work that will not have any effect in the organization…

In order to avoid this we have to:

- Timely plan the implementation phase;
- Involve the client since the beginning;
- Provide adequate formation to the users;
- Provide user manuals and project documentation;
- Keep on testing and validating the proposed solutions, correcting deviations that can still occur.

10 guidelines for a good problem formulation

1. Do not create a complex model when a simple one is enough.
2. Do not fit the problem to a particular resolution technique that we want to use.
3. Solve accurately the chosen model. Only then you are able to know if some inconsistencies of the solutions provided by the model are related to the model…
4. Validate the model before implementing it.
5. The model is not the reality.
6. The model is not forced to do and it cannot be criticized by not doing what it was not meant to do.
7. Do not overestimate the models.
8. One of the main advantages of modeling is the modeling process.
9. A model is not better than the information that we used to build it.
10. Models never replace decision agents.

Operations Research Transforms Scheduling of Chilean Soccer League

- For the past 12 years, the Chilean Professional Soccer Association (ANFP) has applied **operations research** (OR) techniques to schedule soccer leagues in Chile.
- Using integer programming-based methods, the ANFP decides which matches are played in each round, taking into account various objectives, such as holding down costs and ensuring engaging tournaments for the fans.
- It has scheduled more than 50 tournaments using this approach, resulting in an estimated direct economic impact of about **$59 million**, including reductions in television broadcaster operating costs, growth in soccer pay-television subscriptions, increased ticket revenue, and lower travel costs for the teams.

UPS ORION: Driving Performance by Optimizing Driver Behavior

- The number of possible routes a UPS driver can take on any day to make their deliveries is enormous.
- Optimizing the route for delivery, fuel, and time would save the company significant money. Unlike a traditional traveling salesman problem, finding the shortest route alone isn't the answer.
- Once ORION is fully deployed in the US, it's expected to save UPS up to **$400 million per year**, and reduce greenhouse emissions by 100,000 metric tons every year.

- UPS contributes to the broader Data Science community through the UPS George D. Smith Prize.
- UPS looks at learning institutions that are preparing students to practice operations research in the field and having an impact.
- The award is a $10,000 prize and a very prestigious trophy.

A very small example of

A Linear Programming Problem

Gold was found in Alentejo...

News (http://sic.sapo.pt/)

"Iberian Resources , an Australian company expects to start, by the end of 2007, the exploration of a gold mine in Alentejo. The investment will round 20 millions of Euros and will create 130 jobs. SIC knows that the company forecasts a potential of 56 tons of gold in this area."

The problem:

The company The2Mines wants to explore two gold mines in Alentejo, which extract 3 different brands of gold: high, medium and low quality. There is an agreement that obliges the company to provide weekly 12 tons of high quality gold, 8 tons of medium quality gold and 24 tons of low quality gold.

The two mines have different production capacities as we can see in the table below.

Mine	Daily cost	Production (ton/day)		
	(€ 1000)	High	Medium	Low
X	180	6	3	4
Y	160	1	1	6

How many days a week must each mine work in order to satisfy the demand?

Gold was found in Alentejo... (2)

Mine	Daily Cost	Production (ton/day)		
	(€ 1000)	High	Medium	Low
X	180	6	3	4
Y	160	1	1	6
Weekly demand		**12**	**8**	**24**

How many days a week must each mine work in order to satisfy the demand?

We can imagine several scenarios:

Solution 1 Each mine operates 1 day per week: in this case, 7 tons of high quality gold will be extracted per week, which does not satisfy the demand of 12 tons of high quality gold per week. The solutions is not feasible

Solution 2 Mine X operates 4 days a week and mine Y operates 3 days a week. In this case, the production satisfies the demand (the solution is feasible) but with a high production cost…

We could keep on guessing but now we’ll try a structured approach to the problem!

The real question is :

How many days a week must each mine work in order to

satisfy the demand

with the minimum operation costs?

Problem Formulation

Step 1: Identify the decision variables

Which are the decisions that we can make (unknown or decision variables)?
Which are the problem parameters (uncontrollable variables)?

Step 2: Identify the constraints of the problem

Which are the bounds imposed to our decisions?

Step 3: Identify the objective(s) for the problem

Which is the criterion that allow us to distinguish the good from the less good solutions?

Gold was found in Alentejo... (3)

Decision variables:

x = number of operating days per week for mine X
y = = number of operating days per week for mine Y
x >=0 and y >=0.

Constraints:

Production constraints: balance between demand and production limits

$$6x + y \geq 12$$
$$3x + y \geq 8$$
$$4x + 6y \geq 24$$

Note that these constraints are inequalities instead of equalities. Why?

Constraints on the number of operating days per week:

$$x \leq 5$$
$$y \leq 5$$

Objective:

We want to minimize the extraction costs, given by: $180x + 160y$

Gold was found in Alentejo… (4)

Complete formulation of the problem:

$$\text{minimize } 180x + 160y$$

Subject to

$$6x + y \geq 12$$
$$3x + y \geq 8$$
$$4x + 6y \geq 24$$
$$x \leq 5$$
$$y \leq 5$$
$$x, y \geq 0$$

This formulation has the following characteristics:

- All the variables are continuous (they can have fractional values)
- There is a single objective (minimization or maximization)
- The objective and the constraints are linear

Any formulation that satisfies all these three conditions corresponds to a Linear Programming problem (LP).

Implicitly, we assume that the two mines can operate in parts of a day, since they are continuous. The problems in which variables can only assume integer values are called Integer Programming problems.

In fact, these variables could assume integer values but, for the sake of simplicity we let them take fractional values. This is particularly relevant since Integer programming problems are more complex than Linear Programming problems. Frequently, when continuous variables take very large values, the fractional part can be ignored…

Main objectives of the course

To provide the students with competences to :

- identify decision problems;
- apply the different stages of the resolution of a decision problem, namely:
  - defining and structuring problems
  - building models
  - using quantitative methods to obtain a solution
- critically analyse a solution
- understand the relevance of the “agent of change” role in a organization

Course Syllabus

(i) Linear Programming
- ✓ Problems formulation
- ✓ Graphic resolution and sensitivity analysis
- ✓ Simplex Method
- ✓ Duality

(ii) Particular cases of Linear Programming
- ✓ Transportation Problems
- ✓ Assignment problems
- ✓ Network Problems

(iii) Integer Programming

(iv) Decision Theory

Theoretical lessons

nth	Week	M	T	W	T	F	S	S
		2020						
eb	1	10	11 T1-PL	12	13	14	15	16
	2	17	18 T2-PL	19	20	21	22	23
	3	24	25	26	27	28	29	1
lar	4	2	3 T3-PL	4	5	6	7	8
	5	9	10 T4-Transp	11	12	13	14	15
	6	16	17 T5-Transp	18	19	20	21	22
	7	23	24 T6-Afet	25	26	27	28	29
pr	8	30	31 T7-Redes	1	2	3	4	5
	9	6	7	8	9	10	11	12
	10	13	14 T8-ProgInt	15	16	17	18	19
	11	20	21 T9-TDec	22	23	24	25	26
ay	12	27	28	29	30	1	2	3
	13	4	5	6	7	8	9	10
	14	11	12 T10-TDec	13	14	15	16	17
	15	18	19 Exercício	20	21	22	23	24
	16	25	26	27	28	29	1	2

Month	Week	M	T	W	T	F	S	S
		2020						
Feb	1	10	11	12	13	14	15	16
	2	17 P1-Form	18 P1-Form	19 P1-Form	20	21 P1-Form	22	23
	3	24	25	26 P2-Simplex	27	28 P2-Simplex	29	1
Mar	4	2 P2-Simplex	3 P2-Simplex	4 P3-2Fases	5	6 P3-2Fases	7	8
	5	9 P3-2Fases	10 P3-2Fases	11 P4-Transp	12	13 P4-Transp	14	15
	6	16 P4-Transp	17 P4-Transp	18 P5-Transp	19	20 P5-Transp	21	22
	7	23 P5-Transp	24 P5-Transp	25 P6-Afet	26	27 P6-Afet	28	29
Apr	8	30 P6-Afet	31 P6-Afet	1 Duvidas	2	3 Duvidas	4	5
	9	6	7	8	9	10	11	12
	10	13	14 P8-Redes	15 P8-Redes	16	17 P8-Redes	18	19
	11	20 P8-Redes	21 P9-ProgInt	22 P9-ProgInt	23	24 P9-ProgInt	25	26
May	12	27 P9-ProgInt	28 P7-Excel	29 P7-Excel	30	1	2	3
	13	4	5	6	7	8	9	10
	14	11 P10-TDec	12 P10-TDec	13 P10-TDec	14	15 P10-TDec	16	17
	15	18 P11-TDec	19 P11-TDec	20 P11-TDec	21	22 P11-TDec	23	24
	16	25	26	27	28	29	1	2

Teóricas	T1-PL, T2-PL, T3-PL	Programação Linear
	T4-Transp, T5-Transp	Transportes
	T6-Afet	Afetação
	ExercicioAula	Exercicio aula teórica
	T7-Redes	Otimização em redes
	T8-ProgInt	Programação inteira
	T9-TDec, T10-TDec	Teoria da decisão
Práticas	P1-Form	Formulação
	P2-Simplex	Res. Gráfica+Simplex
	P3-2Fases	Simplex+2 fases
	P4-Transp	Método transportes
	P5-Transp	Formulação
	P6-Afet	Método Húngaro
	P8-Redes	Redes
	P9-ProgInt	Programação Inteira
	P10-TDec	Teoria da decisão
	P11-TDec	Árvores de decisão com inf. experimental

Course Evaluation

Distributed evaluation without final exam

2 tests

1st test (45% of the final grade)

Subjects: LP Formulation , Graphical resolution, Sensitivity Analysis, Simplex Method and Duality, Transportation Problems.

2nd test (45% of the final grade): during the exams' period

Subjects: Assignment Problem, Integer Programming, Network Problems, Decision Theory

Short exercises and Class participation: 10% of the final grade

Bibliography and software

- Hillier, Frederick S. e Lieberman, Gerald (2005). *Introduction to Operations Research, 8th edition*, Mc Graw-Hill.
- Camanho, Ana (2006). *Acetatos de Apoio à disciplina de Investigação Operacional*, GEIN-DEMEGI-FEUP.
- Oliveira, José Fernando e Carravilla, Maria Antónia. *Transparências de apoio à leccionação de aulas teóricas de Investigação Operacional* (2000) LEEC-FEUP (http://paginas.fe.up.pt/~mac/)
- Porta Nova, Acácio. *Apontamentos de Investigação Operacional* (2003). LEGI-IST (http://alfa.ist.utl.pt/~apnova/investigacao.operacional/)

web:

- http://www.informs.org/ INFORMS Online - American OR Society
- http://mat.gsia.cmu.edu/ Michael Trick's Operations Research Page
- http://www.apdio.pt Associação Portuguesa de Investigação Operacional

Software

- Excel
- Lingo (http://www.lindo.com/)
- QSB
- CPLEX

Some interesting videos about OR

- http://www.youtube.com/watch?v=QL07LfbQujM&feature=player_embedded
- http://www.youtube.com/watch?v=XdMoy48mBog
- http://www.youtube.com/watch?v=-dKKN-hnDzI
- http://www.youtube.com/watch?v=bS9Py1vDFUM
- http://www.youtube.com/watch?v=tEXUV8vsDo8
- http://www.youtube.com/watch?v=u2u-PRopD4M

INTRODUCTION TO LINEAR PROGRAMMING

Cereals, Ltd

Cereals, Ltd is a company specialized in preparing and packing wheat and corn for several retailing stores. There are no limits concerning the supply of these cereals and demand can be considered unlimited (in other words, the whole production is sold).
The production is processed in three phases: Pre-Processing (I), Processing (II) and Packing (III)

	I Pre-Processing	II Processing	III Packing	
Weekly production capacity (h)	120	100	150	
Time (h) needed to prepare 1 ton				Profit(€/ton)
Wheat	6 h	1 h	5 h	4
Corn	2 h	4 h	5 h	3

Currently, the company produces 18 tons of wheat and 6 tons of corn per week.
Is this the best option?

Cereals, Ltd - Formulation

Decision variables

x = tons of wheat to produce weekly

y = tons of corn to produce weekly

Constraints

$$6x + 2y \leq 120$$

$$x + 4y \leq 100$$

$$5x + 5y \leq 150$$

$$x \geq 0, y \geq 0$$

Objective function: to maximize the profit

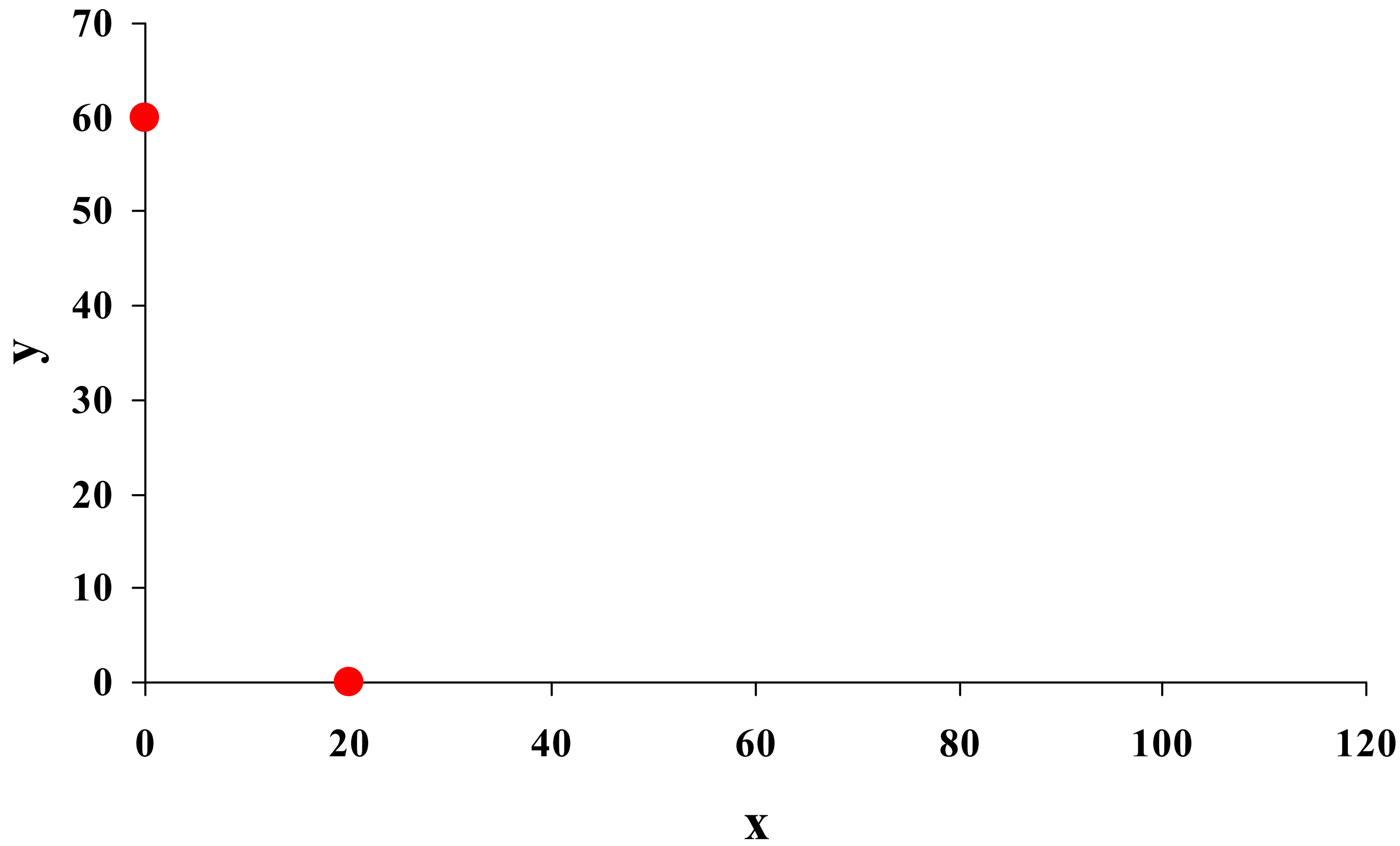
$$\max 4x + 3y$$

$$\max 4x + 3y$$

s.a

$$6x + 2y \le 120$$
$$x + 4y \le 100$$
$$5x + 5y \le 150$$
$$x \ge 0, y \ge 0$$

Graphical method



Each constraint can be represented graphically by a plane region.
For the inequality

$$6x + 2y \le 120$$

consider the straight line corresponding to this constraint:

$$6x + 2y = 120$$

And find any two points in the line:

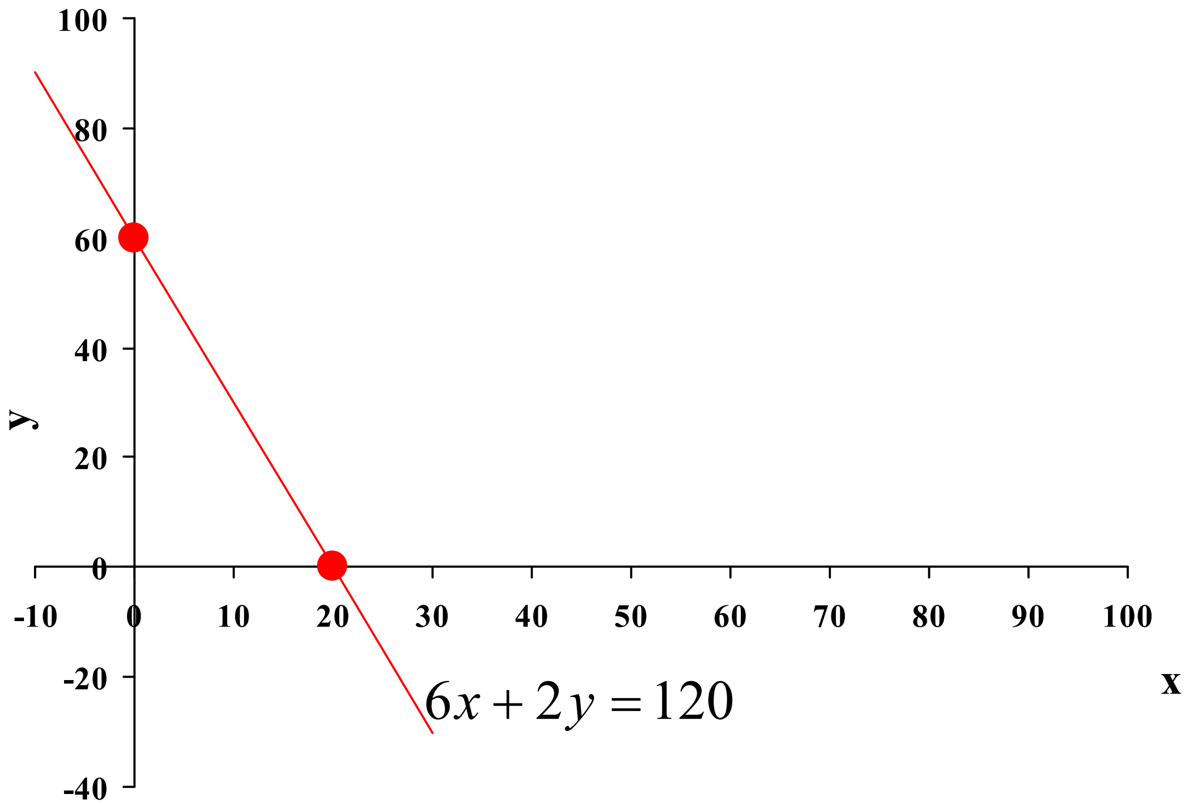
$$x = 0 \Rightarrow y = 60$$
$$y = 0 \Rightarrow x = 20$$

The points (0,60) and (20,0) belong to the line

$$\max\ 4x + 3y$$

s.a
$$6x + 2y \leq 120$$
$$x + 4y \leq 100$$
$$5x + 5y \leq 150$$
$$x \geq 0, y \geq 0$$

Graphical method



The straight line $6x + 2y = 120$ divides the plane in two half-planes

Which one of them satisfies the inequality $6x + 2y \leq 120$?

Consider, for example, the point (x,y)=(0,0)

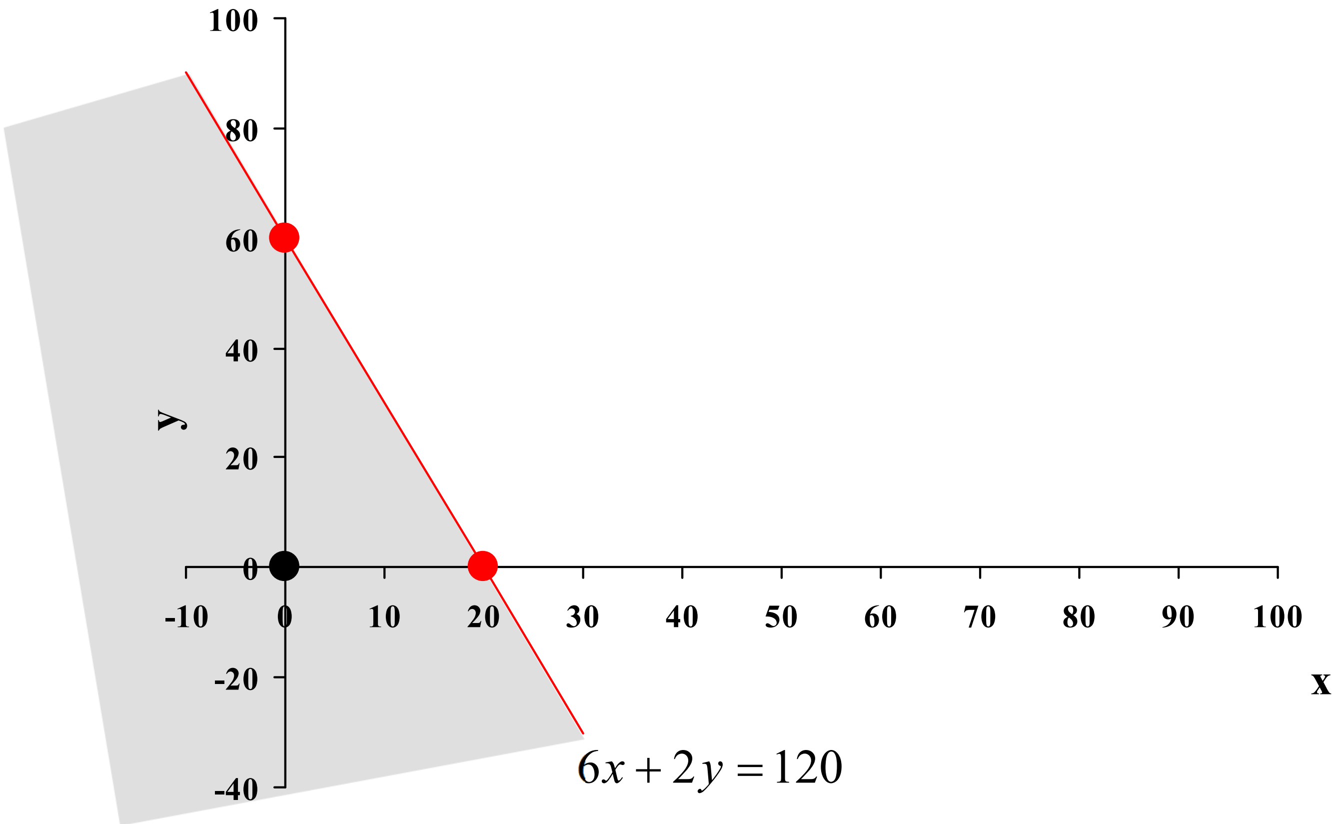
Replacing it in the inequality we have $6 \times 0 + 2 \times 0 = 0 \leq 120$

$$\max\ 4x + 3y$$

s.a

$$6x + 2y \le 120$$
$$x + 4y \le 100$$
$$5x + 5y \le 150$$
$$x \ge 0,\ y \ge 0$$

Graphical method

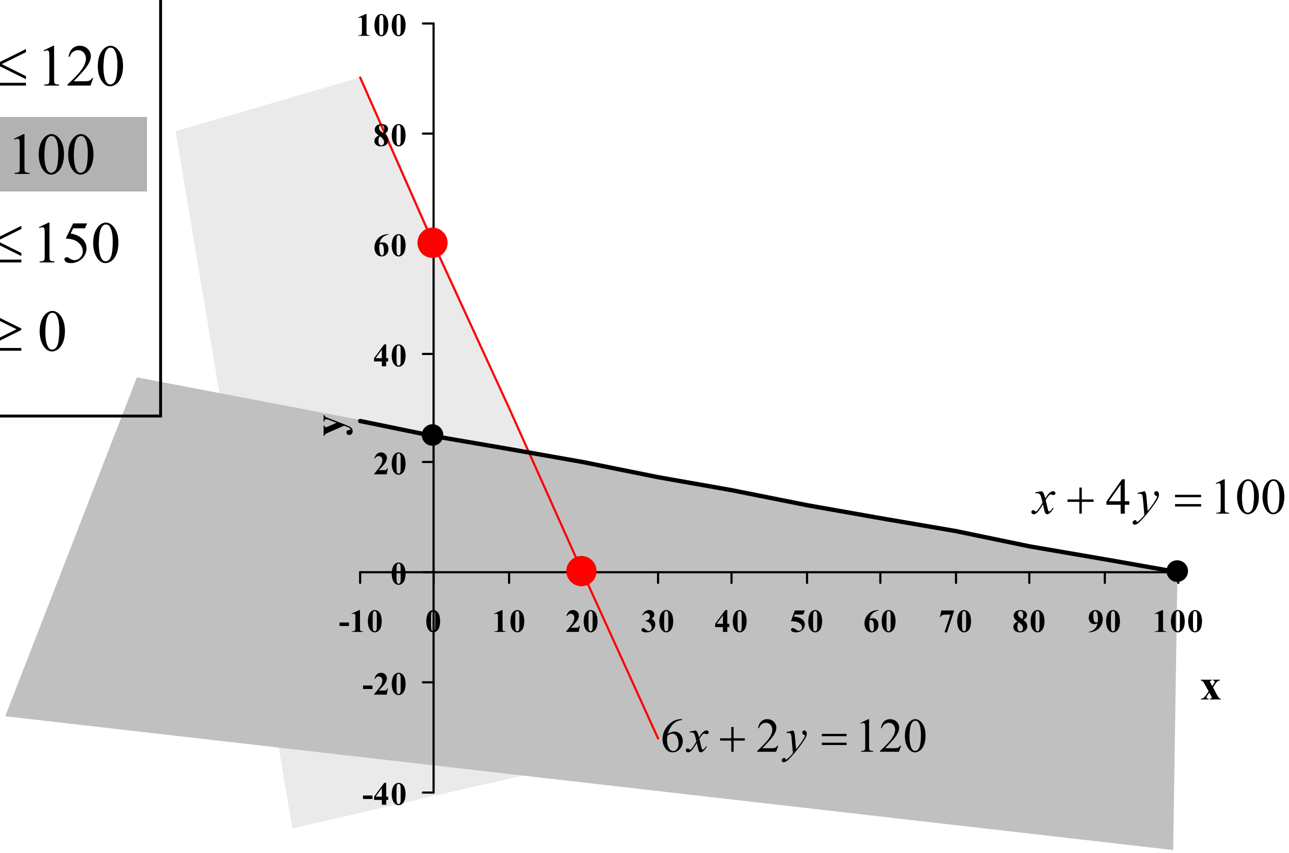


The inequality $6x + 2y \le 120$ is satisfied by all points in the shaded zone

$$\max 4x + 3y$$

s.a
$$6x + 2y \leq 120$$
$$x + 4y \leq 100$$
$$5x + 5y \leq 150$$
$$x \geq 0, y \geq 0$$

Graphical method



For the constraint
$$x + 4y \leq 100$$

Consider the straight line $x + 4y = 100$

$$x = 0 \Rightarrow y = 25$$
$$y = 0 \Rightarrow x = 100$$

Points (0,25) and (100,0) belong to the line

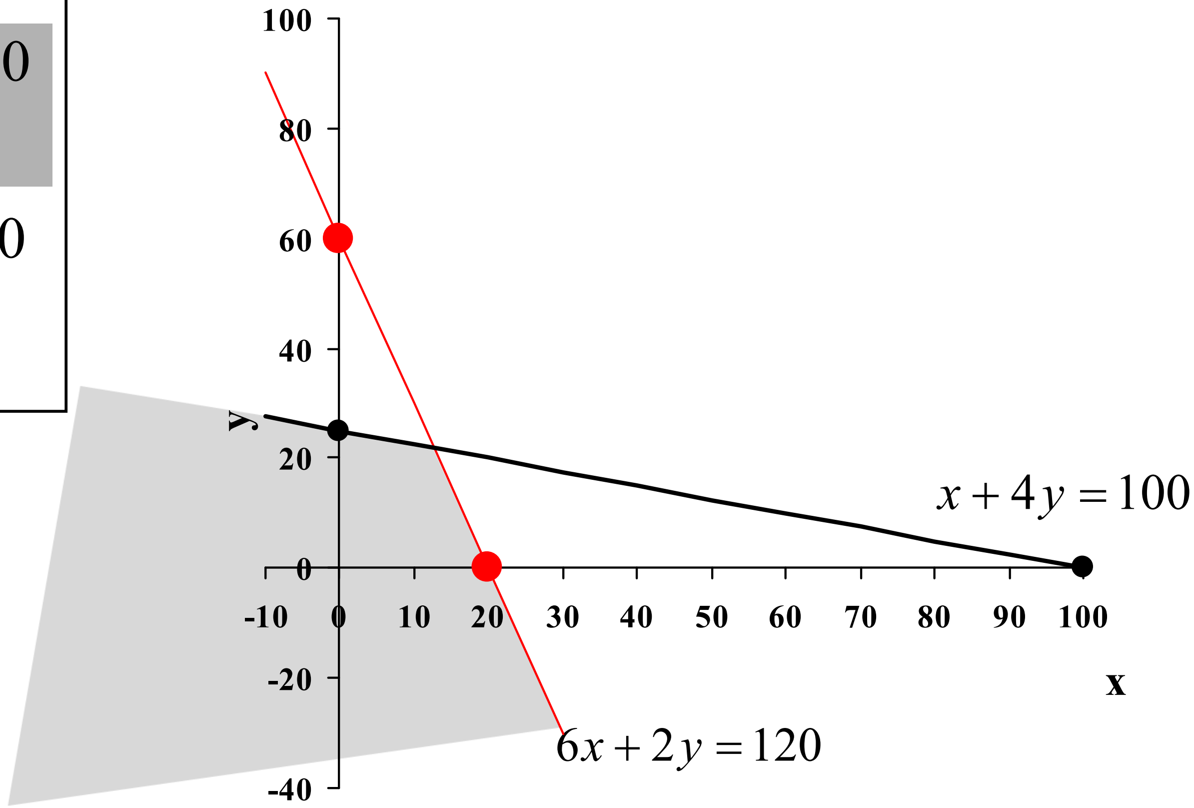
Point (0,0) satisfies the inequality $x + 4y \leq 100$

$$\max 4x + 3y$$

s.a

$$6x + 2y \le 120$$
$$x + 4y \le 100$$
$$5x + 5y \le 150$$
$$x \ge 0, y \ge 0$$

Graphical method

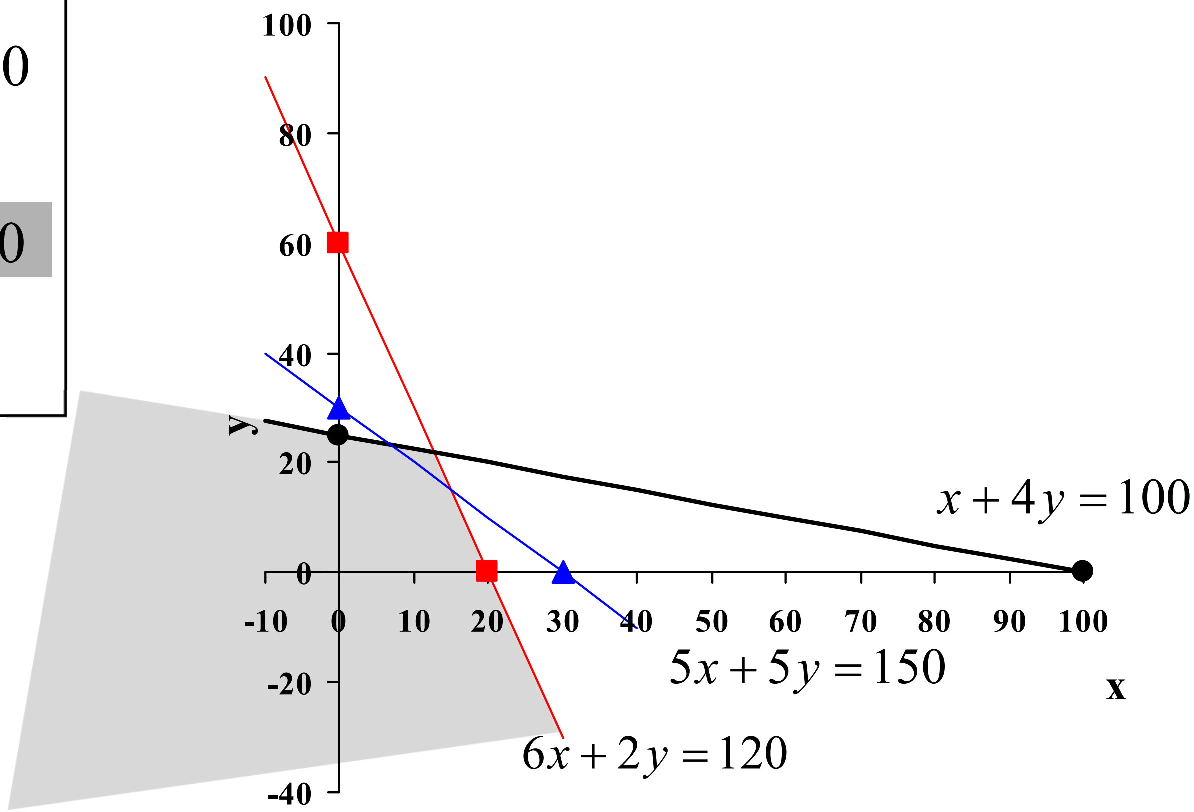


The two inequalities considered together are represented by the intersection of the half-planes, represented by the shaded area .

$$\max 4x + 3y$$

s.a
$$6x + 2y \leq 120$$
$$x + 4y \leq 100$$
$$5x + 5y \leq 150$$
$$x \geq 0, y \geq 0$$

Graphical method



For the constraint
$$5x + 5y \leq 150$$

Consider the straight line $5x + 5y = 150$

$$x = 0 \Rightarrow y = 30$$
$$y = 0 \Rightarrow x = 30$$

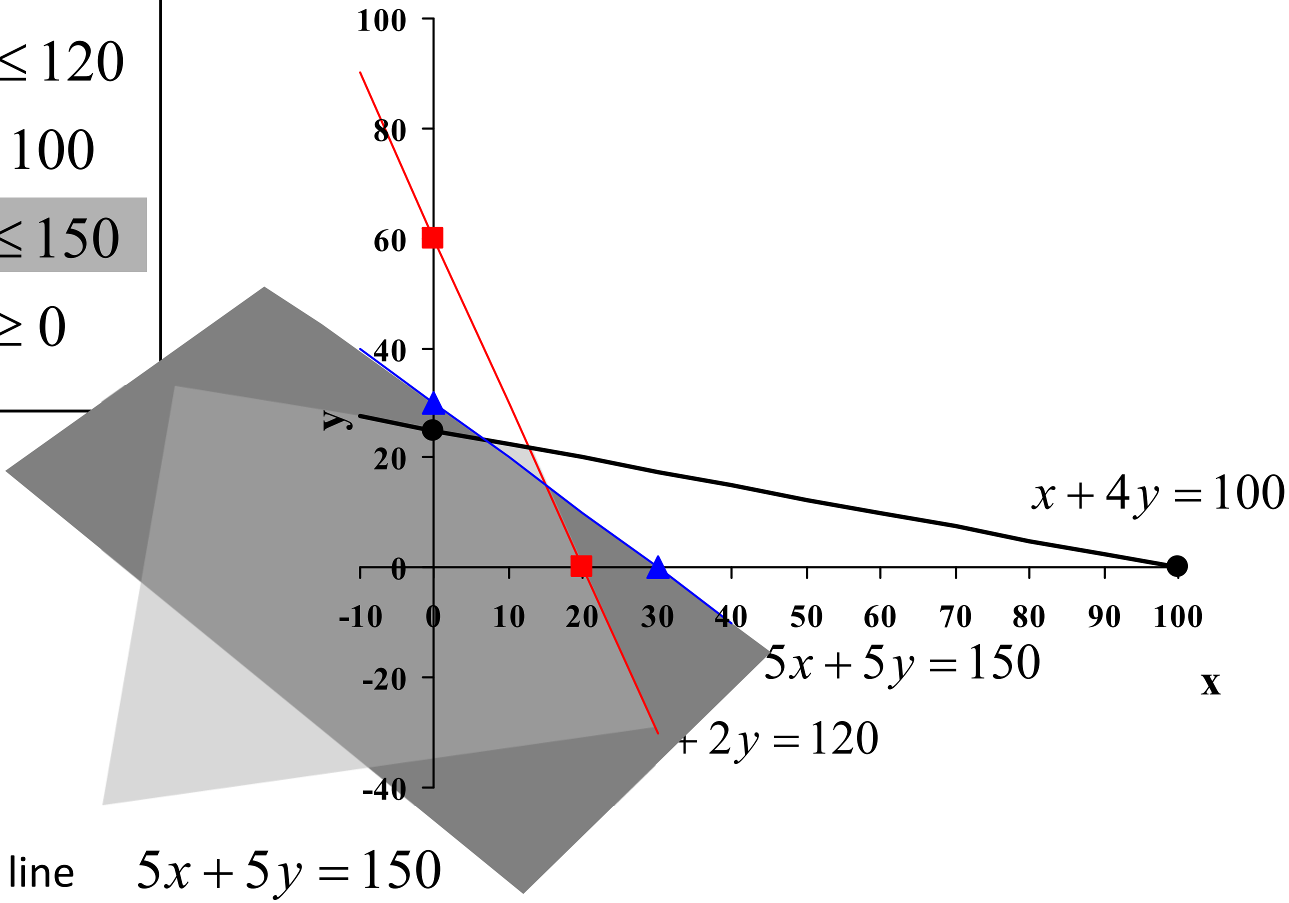
Points (0,30) and (30,0) belong to the straight line

Point (0,0) satisfies the inequality $5x + 5y \leq 150$

$$\max\ 4x + 3y$$

s.a
$$6x + 2y \le 120$$
$$x + 4y \le 100$$
$$5x + 5y \le 150$$
$$x \ge 0,\ y \ge 0$$

Graphical method



For the constraint
$$5x + 5y \le 150$$

Consider the straight line $5x + 5y = 150$

$$x = 0 \Rightarrow y = 30$$
$$y = 0 \Rightarrow x = 30$$

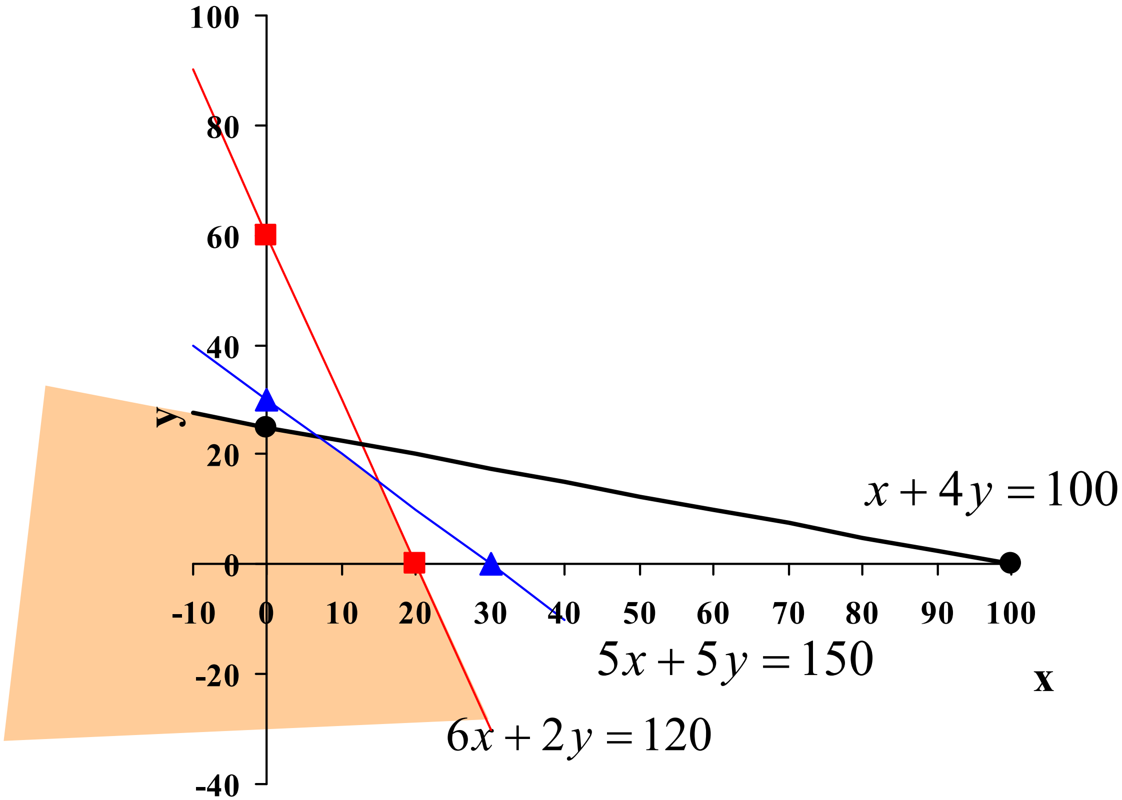
Points (0,30) and (30,0) belong to the straight line

Point (0,0) satisfies the inequality $5x + 5y \le 150$

$$\max\ 4x + 3y$$

s.a
$$6x + 2y \le 120$$
$$x + 4y \le 100$$
$$5x + 5y \le 150$$
$$x \ge 0, y \ge 0$$

Graphical method



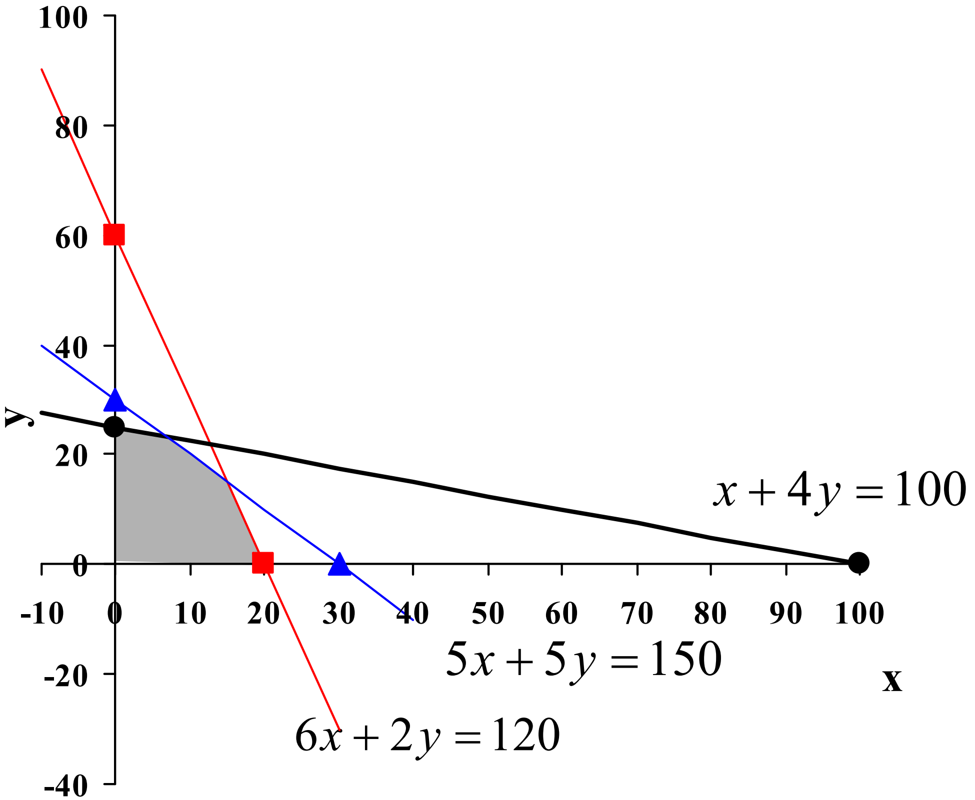
The three inequalities are represented by the shaded area.

$$\max 4x + 3y$$

s.a

$$6x + 2y \leq 120$$
$$x + 4y \leq 100$$
$$5x + 5y \leq 150$$
$$x \geq 0, y \geq 0$$

Graphical method



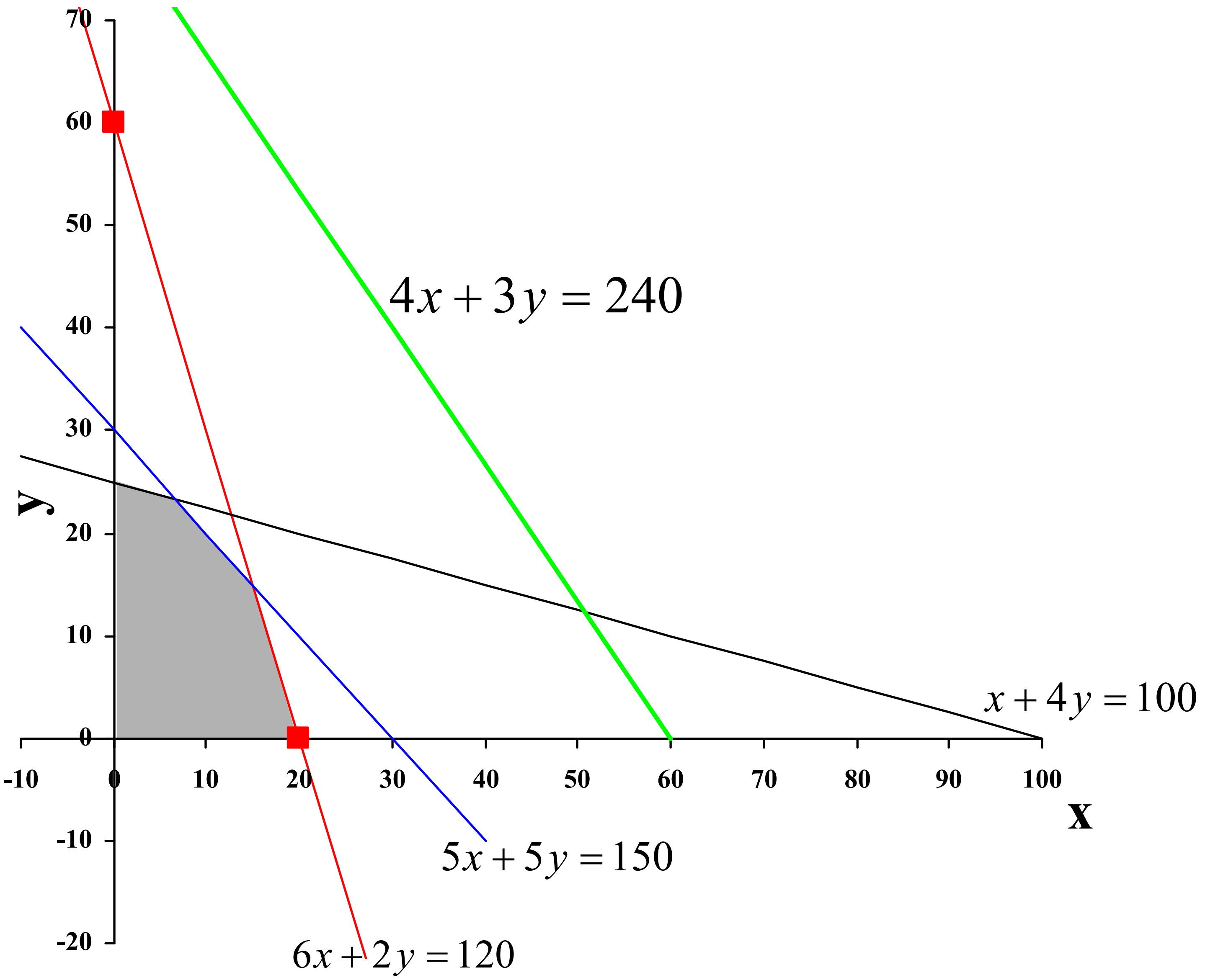
Considering also the non-negativity constraints $x \geq 0, y \geq 0$

The shaded area represents the feasible solutions region

$$\max\ 4x + 3y$$

s.a
$$6x + 2y \le 120$$
$$x + 4y \le 100$$
$$5x + 5y \le 150$$
$$x \ge 0, y \ge 0$$

Graphical method



How can we represent the objective function?

Assign an arbitrary value to the objective function.

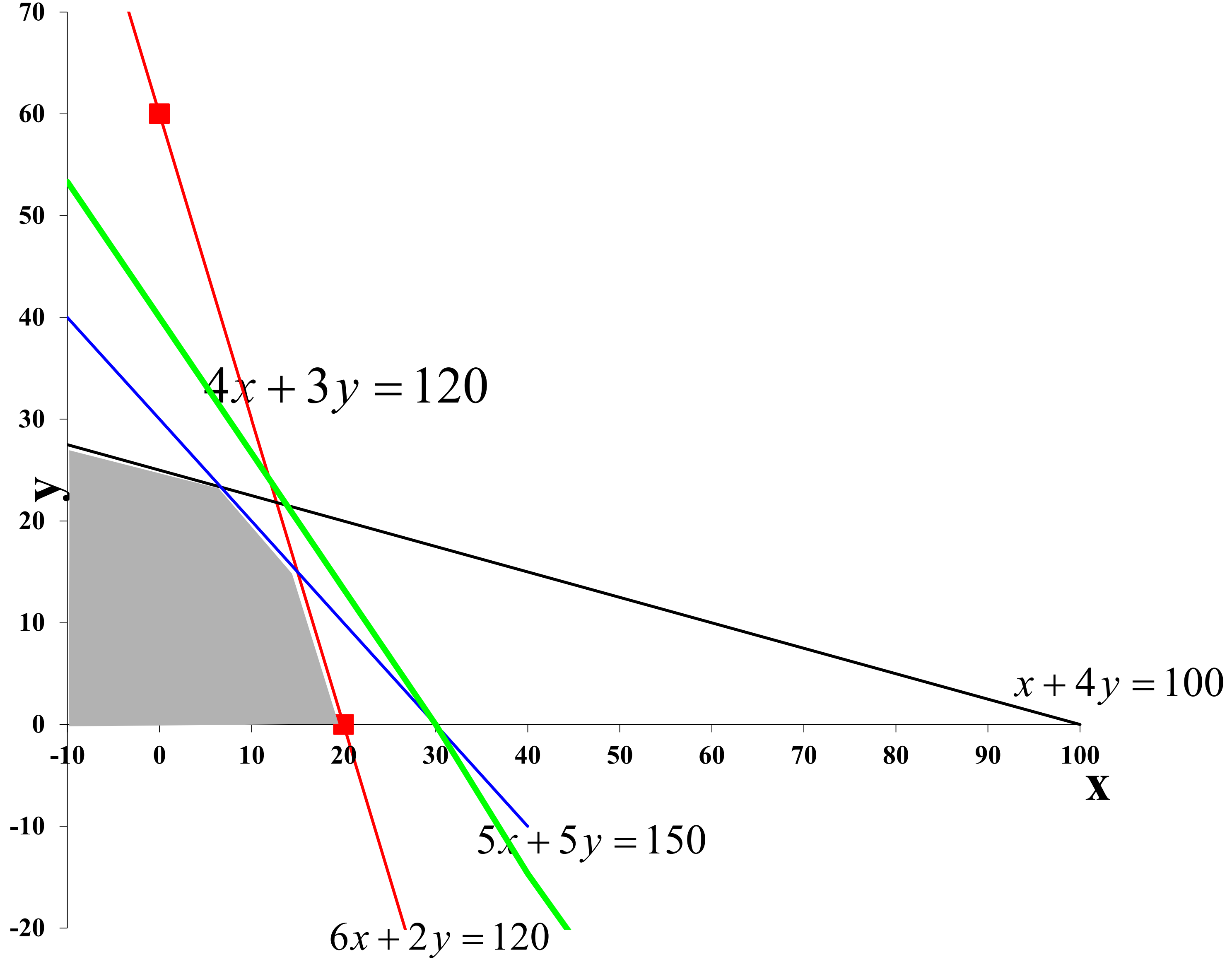
For example, to obtain a profit of 240 €:

$$4x + 3y = 240$$

$$\max\ 4x+3y$$

s.a
$$6x+2y\le 120$$
$$x+4y\le 100$$
$$5x+5y\le 150$$
$$x\ge 0,\ y\ge 0$$

Graphical method



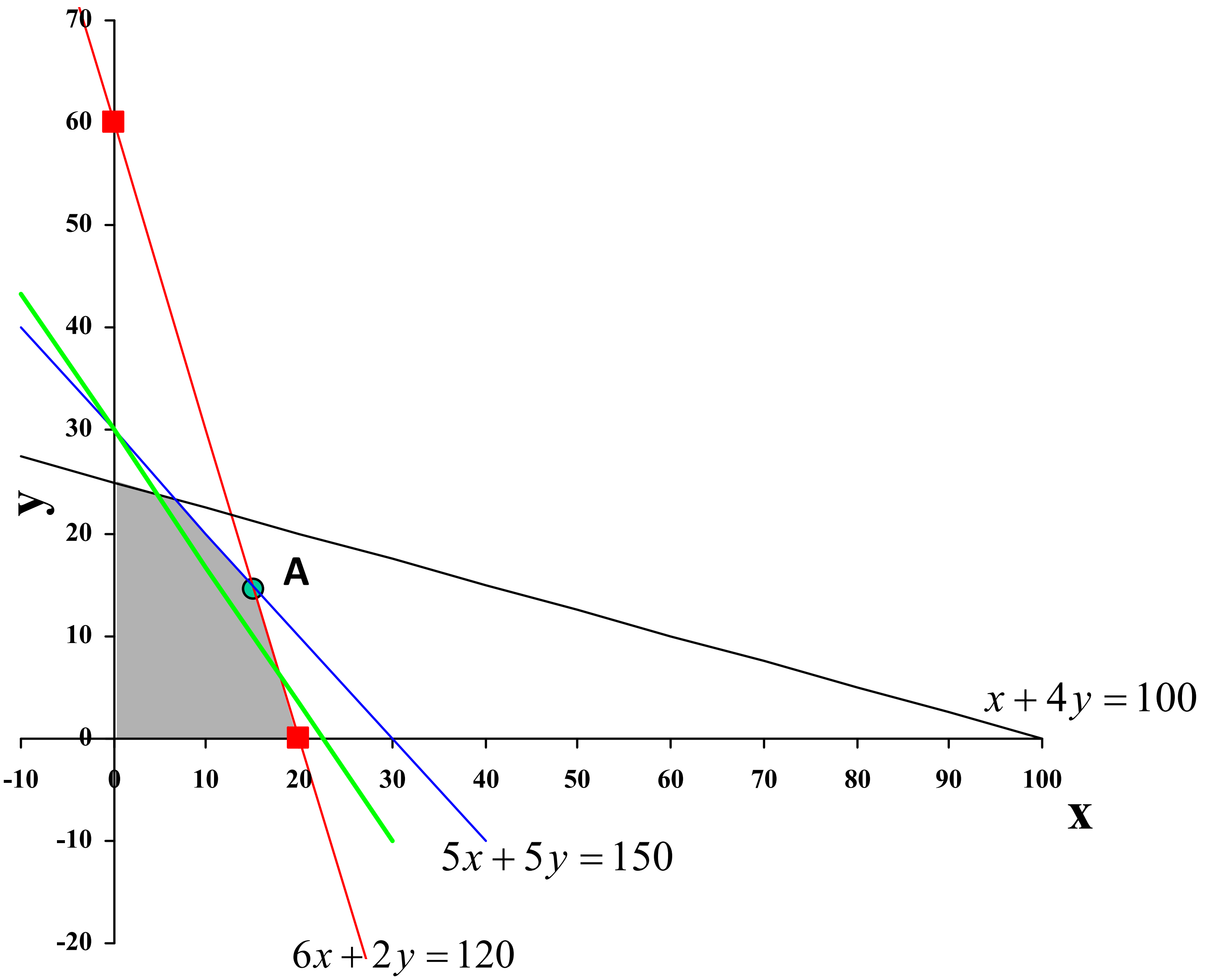
And for a profit of 120 €:

$$4x+3y=120$$

$$\max\ 4x+3y$$

s.a
$$6x+2y\le 120$$
$$x+4y\le 100$$
$$5x+5y\le 150$$
$$x\ge 0,\ y\ge 0$$

Graphical method



Currently, the company produces 18 tons of wheat and 6 tons of corn each week.
Hence the week profit is 90 €

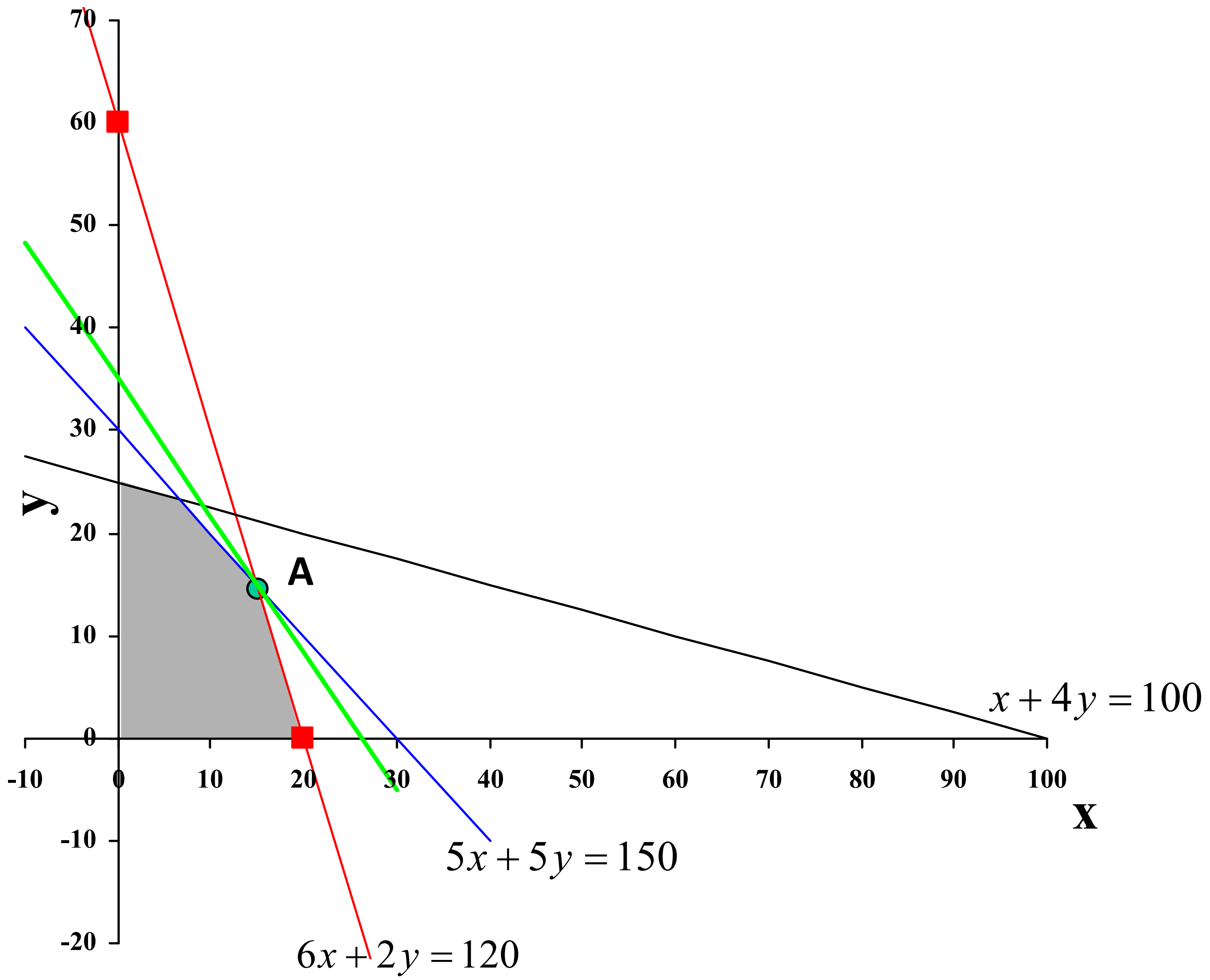
$$4x+3y=90$$

Is the current solution the optimal solution?

$$\max\ 4x + 3y$$

s.a
$$6x + 2y \le 120$$
$$x + 4y \le 100$$
$$5x + 5y \le 150$$
$$x \ge 0, y \ge 0$$

Graphical method



In order to obtain the optimal solution we have to calculate the intersection point (A) of the straight lines:

$$6x + 2y = 120$$
$$5x + 5y = 150$$

A = (15,15)

This production plan yields a profit of 105 €

$$4x + 3y = 105$$

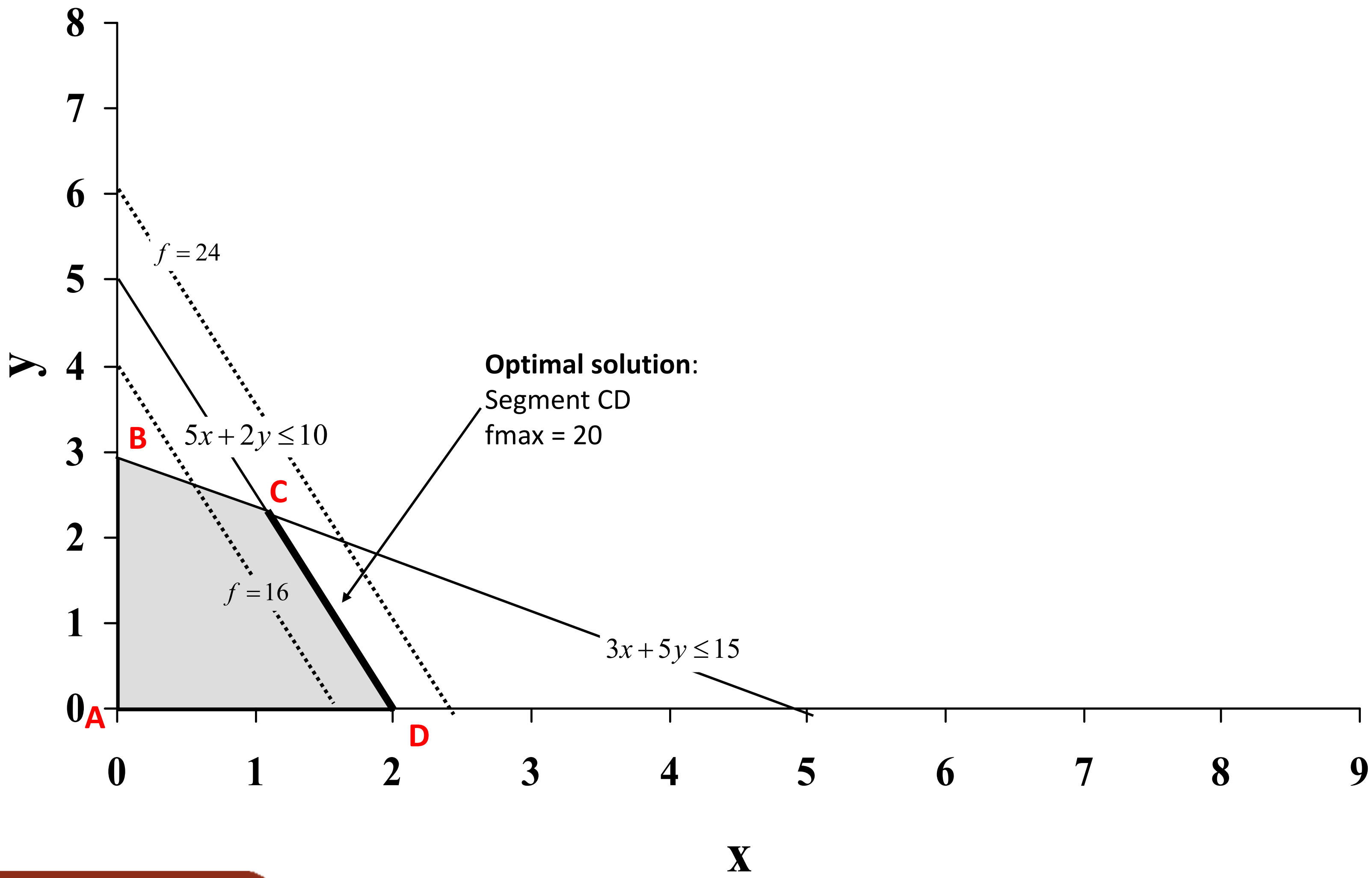
Particular cases of Linear Programming

$$\max 10x + 4y$$

s.a

$$3x + 5y \le 15$$
$$5x + 2y \le 10$$
$$x \ge 0, y \ge 0$$

Infinite optimal solutions

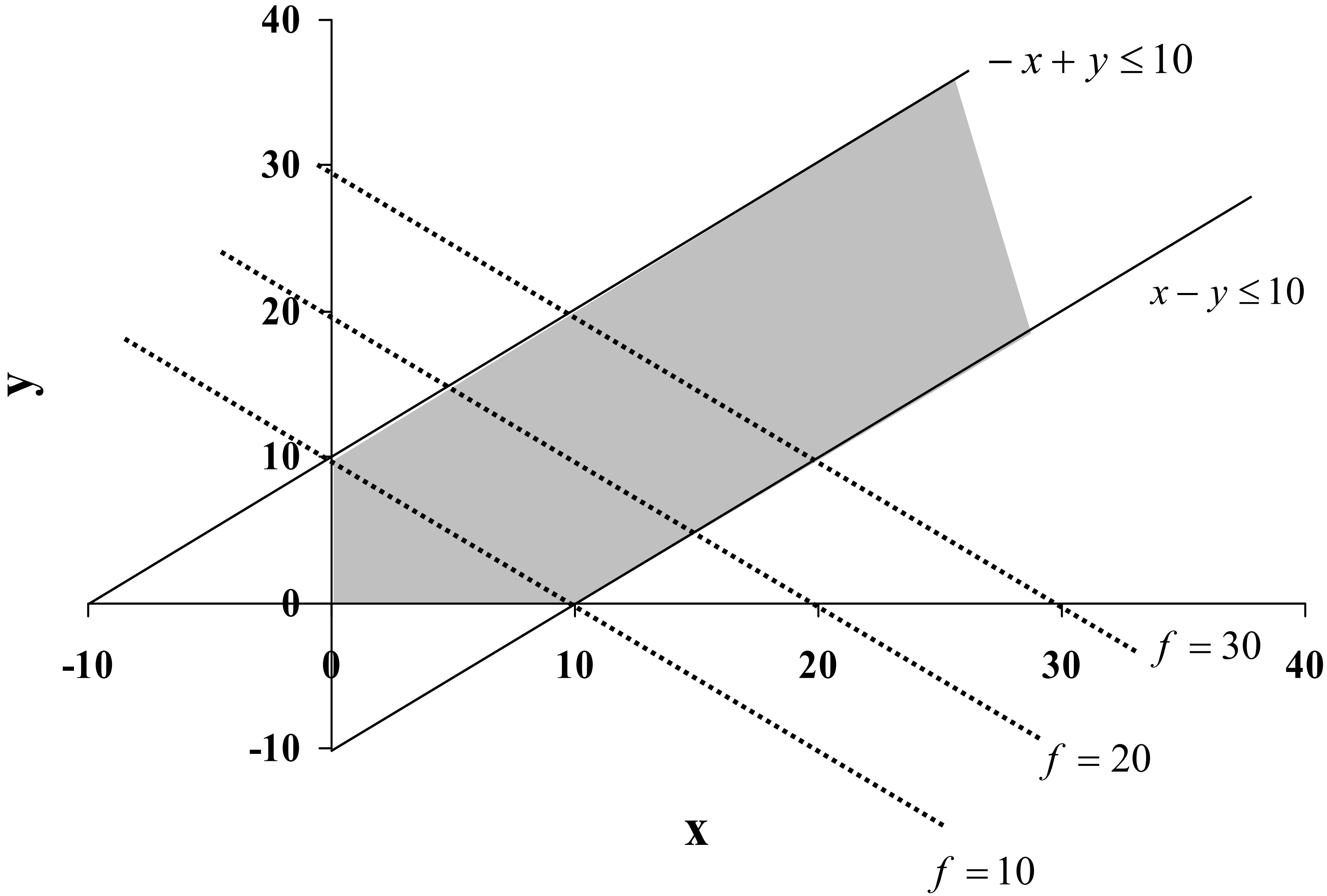


Unlimited optimal solution

$$\max x + y$$

s.a

$$x - y \leq 10$$
$$-x + y \leq 10$$
$$x \geq 0, y \geq 0$$

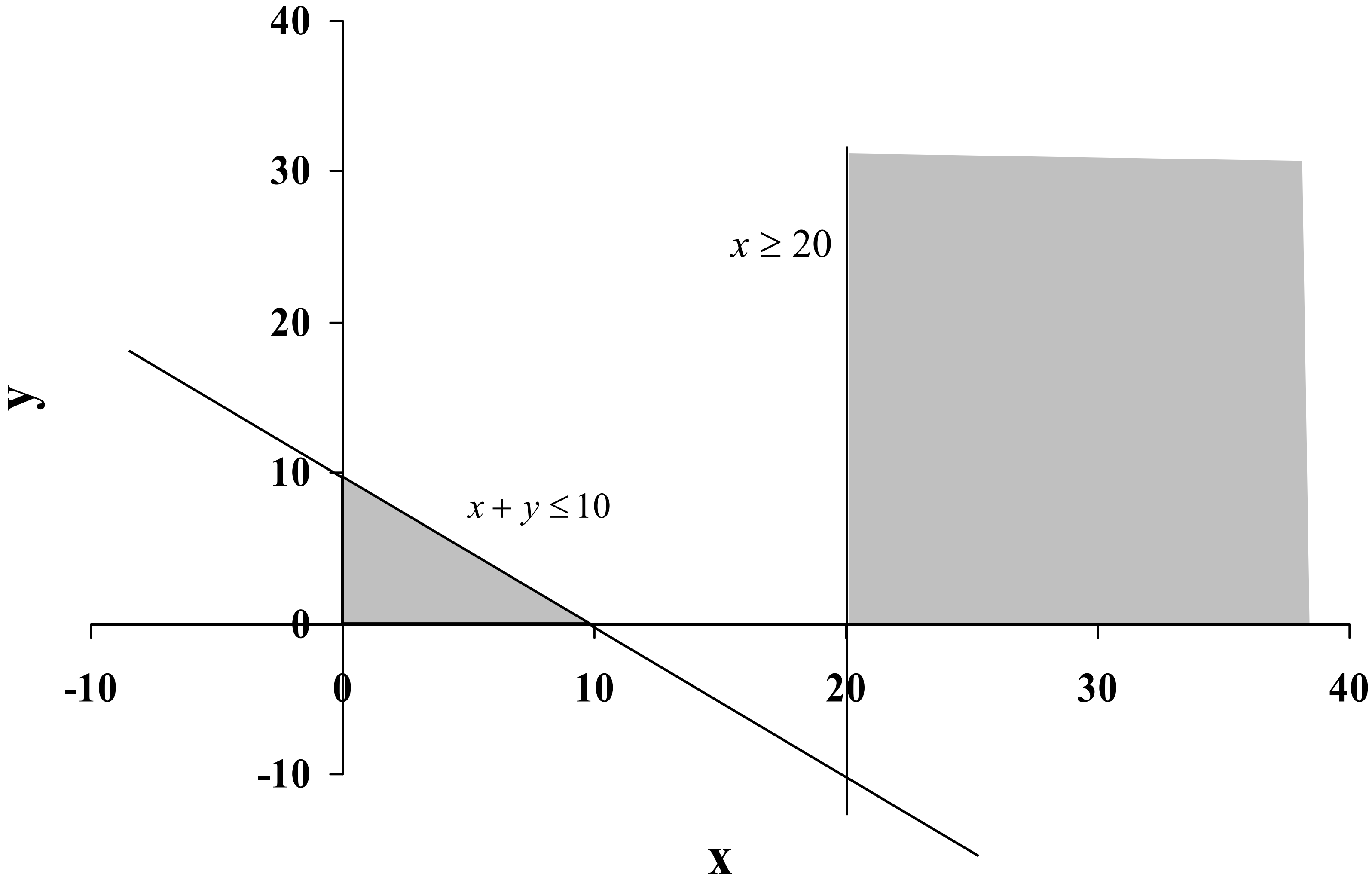


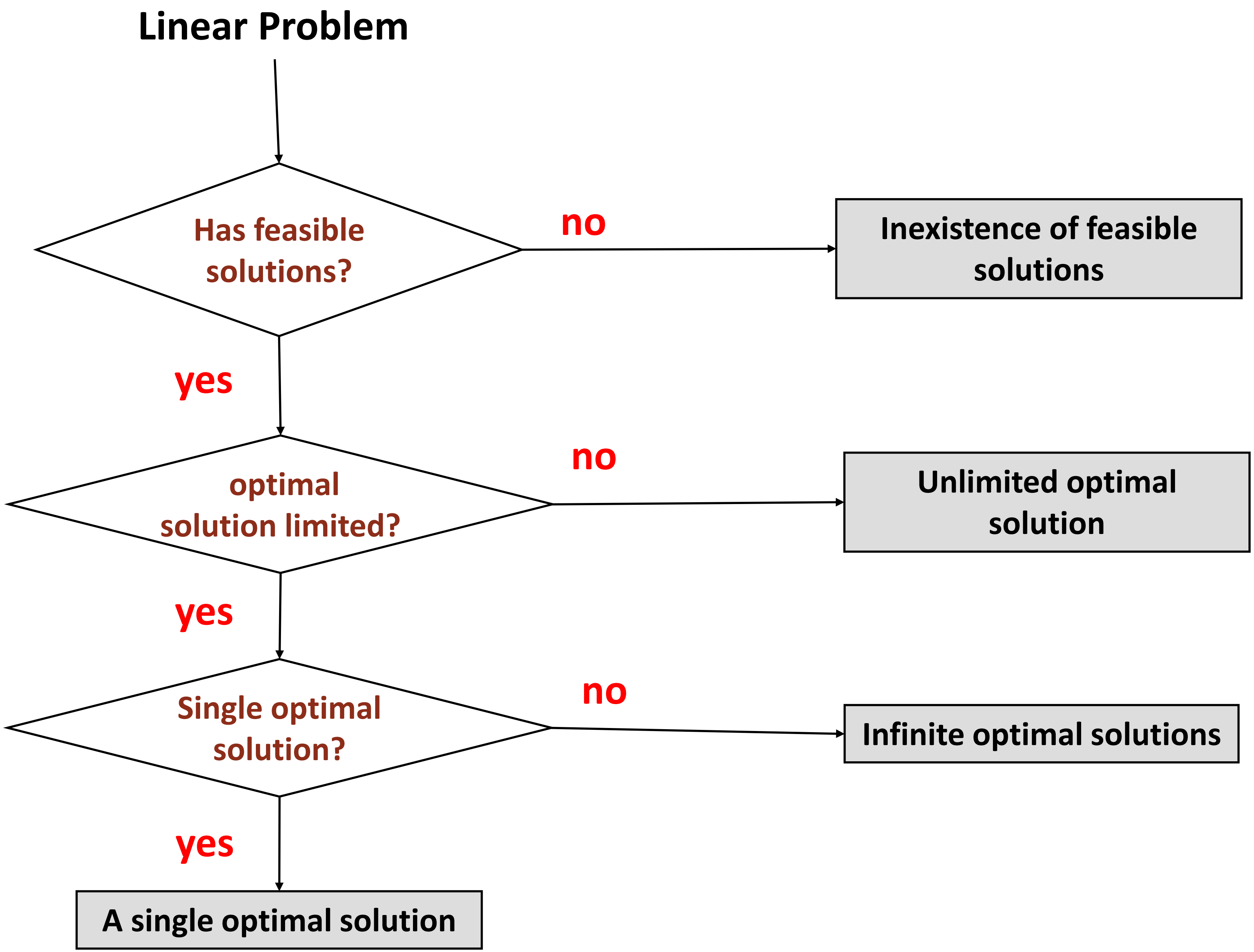
Inexistence of a feasible solution

$$\max x + 2y$$

s.a

$$x + y \leq 10$$
$$x \geq 20$$
$$x \geq 0, y \geq 0$$



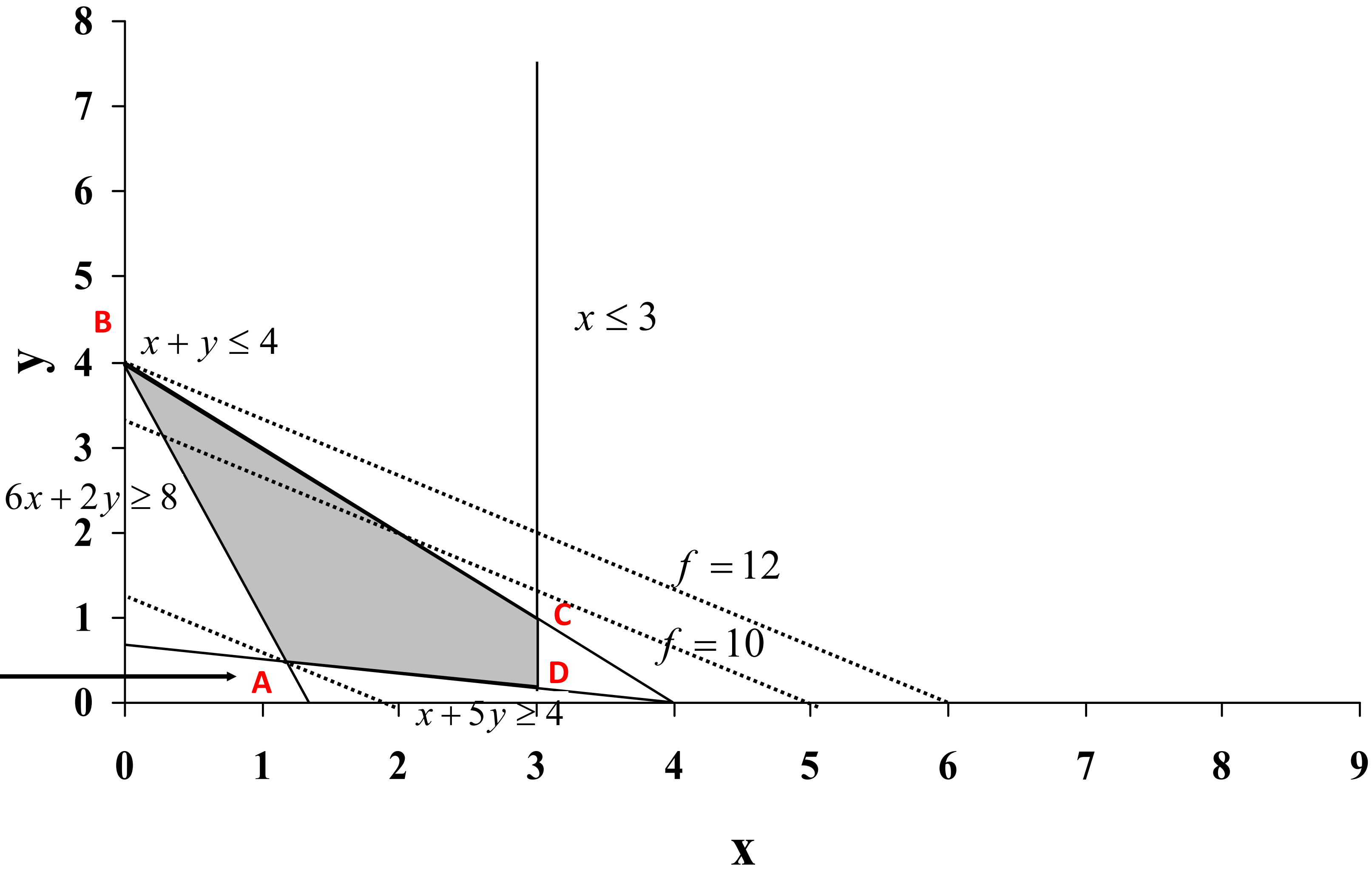

Linear Problem
Has feasible solutions?
no
Inexistence of feasible solutions
yes
optimal solution limited?
no
Unlimited optimal solution
yes
Single optimal solution?
no
Infinite optimal solutions
yes
A single optimal solution

$$\min 2x + 3y$$

s.a

$$x + y \leq 4$$
$$6x + 2y \geq 8$$
$$x + 5y \geq 4$$
$$x \leq 3$$
$$x \geq 0, y \geq 0$$

Exercise



Optimal solution:
Point A
fmin = 4
X = 1,14, y = 0,57

$$\max\ 4x + 3y$$

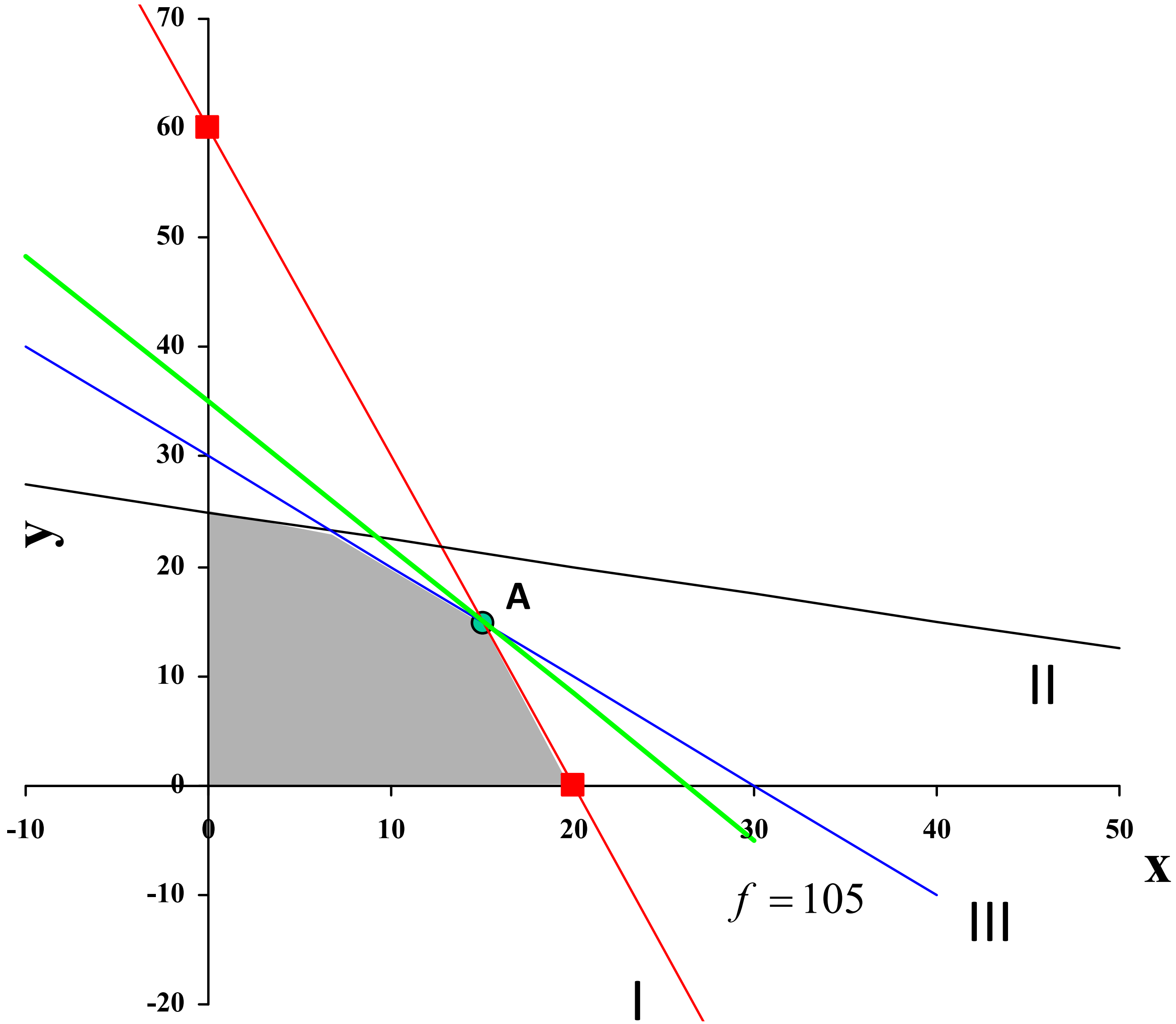
s.a $\quad 6x + 2y \leq 120$

$$x + 4y \leq 100$$

$$5x + 5y \leq 150$$

$$x \geq 0, y \geq 0$$

Consider again the example of Cereals, Ltd



Optimal solution A = (15,15)

Profit: $\quad 4x + 3y = 105$

What if the profit of each ton of wheat increases to 4,35 €?

What if the production capacity in section III (packing) is reduced to 125 h/week?