

## Question 9.2

The file *ex9\_2.dat* contains 105 weekly observations on sales revenue (SALES) and advertising expenditure (ADV) in millions of dollars for a large midwest department store in 2008 and 2009. The following relationship was estimated:

$$\widehat{\text{SALES}}_t = 25.34 + 1.842 \cdot \text{ADV}_t + 3.802 \cdot \text{ADV}_{t-1} + 2.265 \cdot \text{ADV}_{t-2}$$

(a) Describe the relationship between sales and advertising expenditure. Include an explanation of the lagged relationship. When does advertising have its greatest impact? What is the total effect of a sustained \$1 million increase in advertising expenditure?

[1.842, 3.802, 2.265] are all positive. Thus, advertising expenditure had a positive impact on sales revenue and there is a positive effect in the current week, next week and next next week. if we sustained \$1 million increase in advertising expenditure, It will increase sales revenue by:  $1.842 + 3.802 + 2.265 = 7.909$

(b) The estimated covariance matrix of the coefficients is

	<i>C</i>	<i>ADV</i>	<i>ADV</i> <sub><i>t</i>-1</sub>	<i>ADV</i> <sub><i>t</i>-2</sub>
<i>C</i>	2.5598	-0.7099	-0.1317	-0.7661
<i>ADV</i>	-0.7099	1.3946	-1.0406	0.0984
<i>ADV</i> <sub><i>t</i>-1</sub>	-0.1317	-1.0406	2.1606	-1.0367
<i>ADV</i> <sub><i>t</i>-2</sub>	-0.7661	0.0984	-1.0367	1.4214

Using a **one-tail test** and a 5% significance level, which lag coefficients are significantly different from zero? Do your conclusions change if you use a one-tail test? Do they change if you use a 10% significance level? [ $t = b_i / \text{se}(b_i)$ ,  $i = 0, 1, 2$ ]

One-tail test ADV:  $H_0: \beta_0 = 0$  against  $H_1: \beta_0 \neq 0$

$$t_{\text{adv}} = 1.842 / (1.3946)^{0.5} = 1.5598$$

$$\alpha = 0.05, t_{(0.95, 99)} = 1.660$$

Fail to reject the  $H_0$ , because  $1.5598 < 1.660$

$$\alpha = 0.1, t_{(0.90, 99)} = 1.290$$

Reject  $H_0$ , because  $1.5598 > 1.290$

One-tail test  $\text{ADV}_{t-1}$ :  $H_0: \beta_1 = 0$  against  $H_1: \beta_1 \neq 0$

$$t_{\text{adv1}} = 3.802 / (2.1606)^{0.5} = 2.5866$$

$$\alpha = 0.05, t_{(0.95, 99)} = 1.660$$

Reject  $H_0$ , because  $2.5866 > 1.660$

$$\alpha = 0.1, t_{(0.90, 99)} = 1.290$$

Reject  $H_0$ , because  $2.5866 > 1.290$

One-tail test  $\text{ADV}_{t-2}$ :  $H_0: \beta_2 = 0$  against  $H_1: \beta_2 \neq 0$

$$t_{\text{adv2}} = 2.265 / (1.4214)^{0.5} = 1.8998$$

$$\alpha = 0.05, t_{(0.95, 99)} = 1.660$$

Reject  $H_0$ , because  $1.8998 > 1.660$

$$\alpha = 0.1, t_{(0.90, 99)} = 1.290$$

Reject  $H_0$ , because  $1.8998 > 1.290$

According to the result, ADV lag coefficients are *not* significantly different from zero at 5% significance level. There are all significantly different from zero at 10% significance level. Only ADV change significantly different when change significance level.

**(c) Find 95% confidence intervals for the impact multiplier, the one-period interim multiplier, and the total multiplier.**

$$t_{(0.975,99)} = 1.984$$

Impact multiplier:

$$b_0 \pm t_c * se(b_0) = 1.842 \pm 1.984 * 1.1181 = [-0.501, 4.185]$$

One-period interim multiplier:

$$(b_0 + b_1) = 1.842 + 3.802 = 5.644$$

$$se(b_0 + b_1) = (1.3946 + 2.1606 + 2 * (-1.0406))^{0.5} = 1.2141$$

$$(b_0 + b_1) \pm t_c * se(b_0 + b_1) = [3.235, 8.053]$$

The total multiplier:

$$(b_0 + b_1 + b_3) = 1.842 + 3.802 + 2.265 = 7.909$$

$$se(b_0 + b_1 + b_3) = (1.3946 + 2.1606 + 1.4214 + 2 * (-1.0406) + 2 * 0.0984 + 2 * (-1.0367))^{0.5} = 1.0094$$

$$(b_0 + b_1 + b_3) \pm t_c * se(b_0 + b_1 + b_3) = [5.907, 9.991]$$

### Question 9.4

The following least squares residuals come from a sample of size  $T = 10$ :

$t$	1	2	3	4	5	6	7	8	9	10
$\hat{e}_t$	0.28	-0.31	-0.09	0.03	-0.37	-0.17	-0.39	-0.03	0.03	1.02

(a) Use a hand calculator to compute the sample autocorrelations:

$$r_1 = \frac{\sum_{t=2}^T \hat{e}_t \hat{e}_{t-1}}{\sum_{t=1}^T \hat{e}_t^2} \quad r_2 = \frac{\sum_{t=3}^T \hat{e}_t \hat{e}_{t-2}}{\sum_{t=1}^T \hat{e}_t^2}$$

	Numerator	Denominator		Numerator	Denominator
	-0.0868	0.0784		-0.0252	0.0784
	0.0279	0.0961		-0.0093	0.0961
	-0.0027	0.0081		0.0333	0.0081
	-0.0111	0.0009		-0.0051	0.0009
	0.0629	0.1369		0.1443	0.1369
	0.0663	0.0289		0.0051	0.0289
	0.0117	0.1521		-0.0117	0.1521
	-0.0009	0.0009		-0.0306	0.0009
	0.0306	0.0009			0.0009
		1.0404			1.0404
Sum	0.0979	1.5436	Sum	0.1008	1.5436
	$r_1 =$	0.0634		$r_2 =$	0.0653

(b) Test whether (i)  $r_1$  is significantly different from zero and (ii)  $r_2$  is significantly different from zero. Sketch the first two bars of the correlogram. Include the significance bounds.

(i)  $H_0: \rho_1 = 0$  against  $H_1: \rho_1 \neq 0$

$$Z = \sqrt{T} * r_1 = 10^{0.5} * 0.0634 = 0.201$$

Two-tail test,  $\alpha = 0.05$ ,  $Z_{(0.025)} = -1.96$ ,  $Z_{(0.975)} = 1.96$

So,  $-1.96 < 0.201 < 1.96$ , we do not reject  $H_0$  and conclude that  $r_1$  is not significantly different from zero.

(ii)  $H_0: \rho_2 = 0$  against  $H_1: \rho_2 \neq 0$

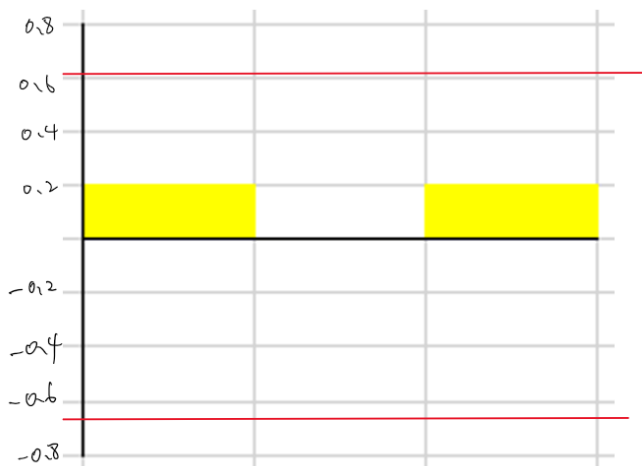
$$Z = \sqrt{T} * r_2 = 10^{0.5} * 0.0653 = 0.207$$

Two-tail test,  $\alpha = 0.05$ ,  $Z_{(0.025)} = -1.96$ ,  $Z_{(0.975)} = 1.96$

So,  $-1.96 < 0.207 < 1.96$ , we do not reject  $H_0$  and conclude that  $r_2$  is not significantly different from zero.

$$B = Z * \sigma / T^{0.5} = \pm 1.96 * 1 / 10^{0.5} = \pm 0.6198$$

The plot of first two bars of the correlogram Including the significance bounds:



**Question 9.6** (Note: in part (b) use only the correlogram)

Increases in the mortgage interest rate increase the cost of owning a house and lower the demand for houses. In this question we consider an equation where the monthly change in the number of new one-family houses sold in the U.S. depends on last month's change in the 30-year conventional mortgage rate. Let HOMES be the number of new houses sold (in thousands) and IRATE be the mortgage rate. Their monthly changes are denoted by  $DHOMES_t = HOMES_t - HOMES_{t-1}$  and  $DIRATE_t = IRATE_t - IRATE_{t-1}$ . Using data from January 1992 to March 2010 (stored in the file *homes.dat*), we obtain the following least squares regression estimates:

$$\widehat{DHOMES}_t = -2.077 - 53.51DIRATE_{t-1} \quad \text{obs} = 218$$

(se)            (3.498) (16.98)

**(a) Interpret the estimate -53.51. Construct and interpret a 95% confidence interval for the coefficient of  $DIRATE_{t-1}$**

If t-1 to t period increase 1% in the mortgage interest rate, it will lead to decrease the number of new houses sold between t-1 to t period by 53,510 units.

```
## {r}
t<-qt(c(0.025),df=216)
t
##
[1] -1.971007
```

$$b_2 \pm t_c * se(b_2) = -53.51 \pm 1.971 * 16.98 = [-86.98, -20.04]$$

**(b) Let  $\hat{e}_t$  denote the residuals from the above equation. Use the following estimated equation to conduct two separate tests for first-order autoregressive errors.**

$$\hat{e}_t = -0.1835 - 3.210DIRATE_{t-1} - 0.3306\hat{e}_{t-1} \quad R^2 = 0.1077$$

(se)            (16.087)            (0.0649)            obs = 218

The autocorrelation LM test:

$$LM\text{-statistic} = T * R_2 = 218 * 0.1077 = 23.4786$$

the critical value at 5% significance level is:  $\chi^2_c = \chi^2_{(0.95,1)} = 3.841$

Since the LM-statistic = 23.4786 >  $\chi^2_{(0.95,1)} = 3.841$ , we reject the null hypothesis. Therefore, we conclude that there has the autocorrelation.

The t-test for autocorrelation on the significance of the coefficient of  $e_{t-1}$ :

$$t\text{-statistic} = (-0.3306 - 0) / 0.0649 = -5.09$$

$$\text{The critical } t \text{ value } t_{c,s} = t_{(0.025,215)} = -1.971$$

Since the t-statistic = -5.09 <  $t_c = -1.971$ , we reject the null hypothesis. Therefore, we conclude that there has the autocorrelation.

```
## {r}
# 9.6(b)
t<-qt(c(0.025),df=215)
t
##
[1] -1.971059
```

$t_{(0.025,215)}$

```
## {r}
# 9.6(b)
chisqb<-qchisq(0.95, df=1)
chisqb
##
[1] 3.841459
```

$\chi^2_{(0.95,1)}$

(c) The model with AR (1) errors was estimated as

$$\widehat{DHOMES_t} = -2.124 - 58.61DIRATE_{t-1} \quad e_t = -0.3314e_{t-1} + \hat{v}_t$$

(se)            (2.497) (14.10)                            (0.0649)

obs = 217

Construct a 95% confidence interval for the coefficient of  $DIRATE_{t-1}$ , and comment on the effect of ignoring autocorrelation on inferences about this coefficient.

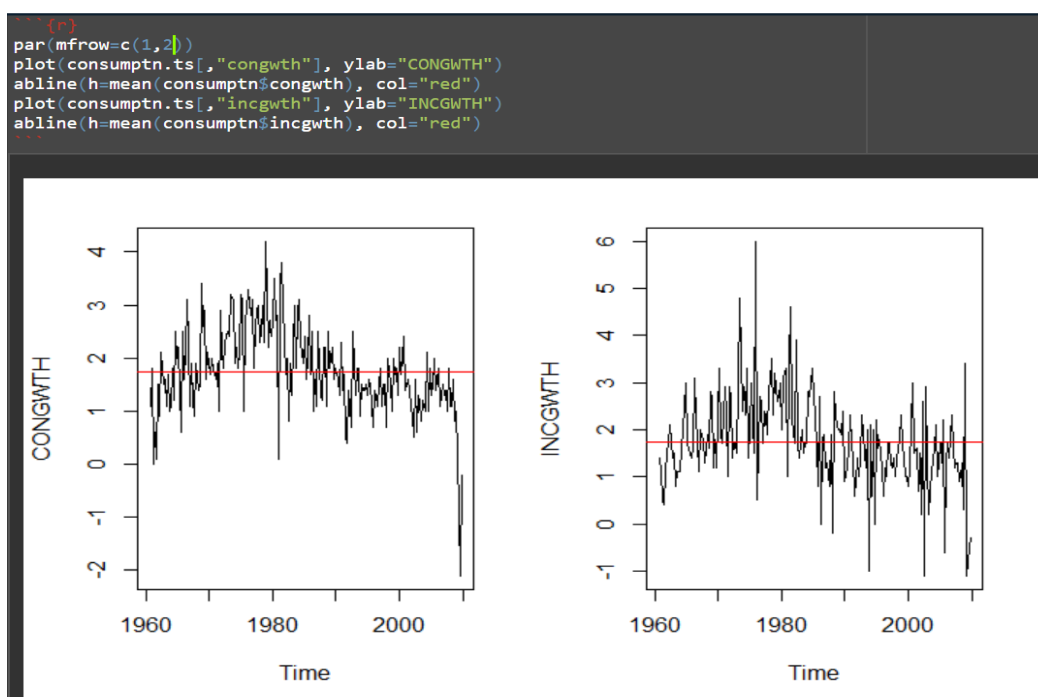
$$\beta_2 \pm t_c * se(\beta_2) = -58.61 \pm 1.971 * 14.10 = [-86.40, -30.82]$$

The estimated equation in the part a ignores autocorrelation. Ignoring autocorrelation on inferences about this coefficient gave a lower absolute value for the coefficient of interest and higher standard error. The confidence interval has a lower bound and higher upper bound. When autocorrelation is ignored, our inferences about the coefficient could be misleading because the wrong standard error is used.

### Question 9.22

An important relationship in macroeconomics is the consumption function. The file *consumptn.dat* contains quarterly data from 1960Q1 to 2009Q4 on the percentage changes in disposable personal income and personal consumption expenditures. We describe these variables as income growth (INCGWTH) and consumption growth (CONGWTH). To ensure that the same number of observations (197) are used for estimation in each of the models that we consider, use as your sample period 1960Q4 to 2009Q4. Where relevant, lagged variables on the right-hand side of equations can use values prior to 1960Q4.

(a) Graph the time series for CONGWTH and INCGWTH. Include a horizontal line at the mean of each series. Do the series appear to fluctuate around a constant mean?



The times series graphs for CONGWTH and INCGWTH above, their times series graphs appear to fluctuate around their respective constant means.

(b) Estimate the model  $\text{CONGWTH}_t = \delta + \delta_0 \text{INCGWTH}_{t-1} + v_t$ . Interpret the estimate for  $\delta_0$ . Check for serially correlated errors using the residual correlogram, and an LM test with two lagged errors. What do you conclude?

```
## (b)
consumptn.ar2 <- dynlm(congwth ~ L(incgwth,0), data=consumptn.ts)
summary(consumptn.ar2)
ehat <- resid(consumptn.ar2)
corrgm <- acf(ehat)
plot(corrgm)
round(corrgm$acf,4)
```

Time series regression with "ts" data:  
Start = 1960(4), End = 2009(4)

Call:  
dynlm(formula = congwth ~ L(incgwth, 0), data = consumptn.ts)

Residuals:

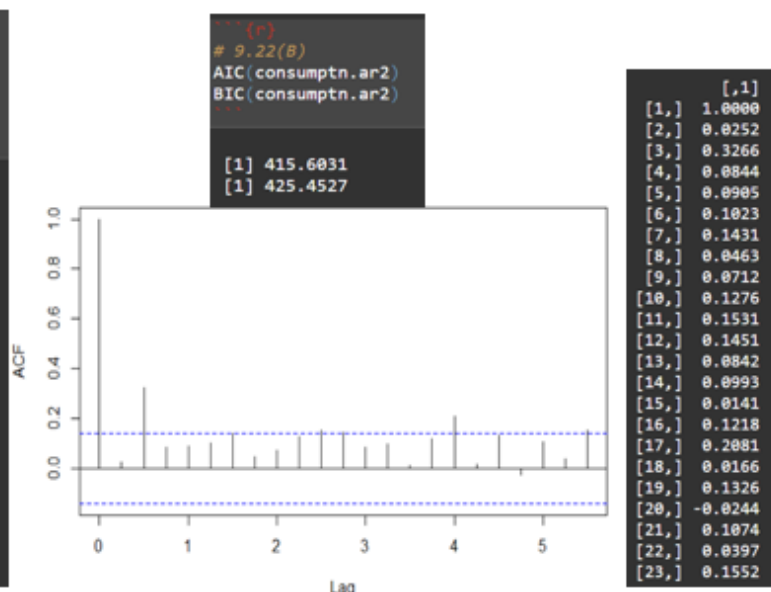
	Min	1Q	Median	3Q	Max
	-2.84905	-0.41333	-0.01333	0.37659	2.10138

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.97384	0.09961	9.776	<2e-16 ***
L(incgwth, 0)	0.44958	0.04967	9.052	<2e-16 ***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6878 on 195 degrees of freedom  
Multiple R-squared: 0.2959, Adjusted R-squared: 0.2923  
F-statistic: 81.94 on 1 and 195 DF, p-value: < 2.2e-16



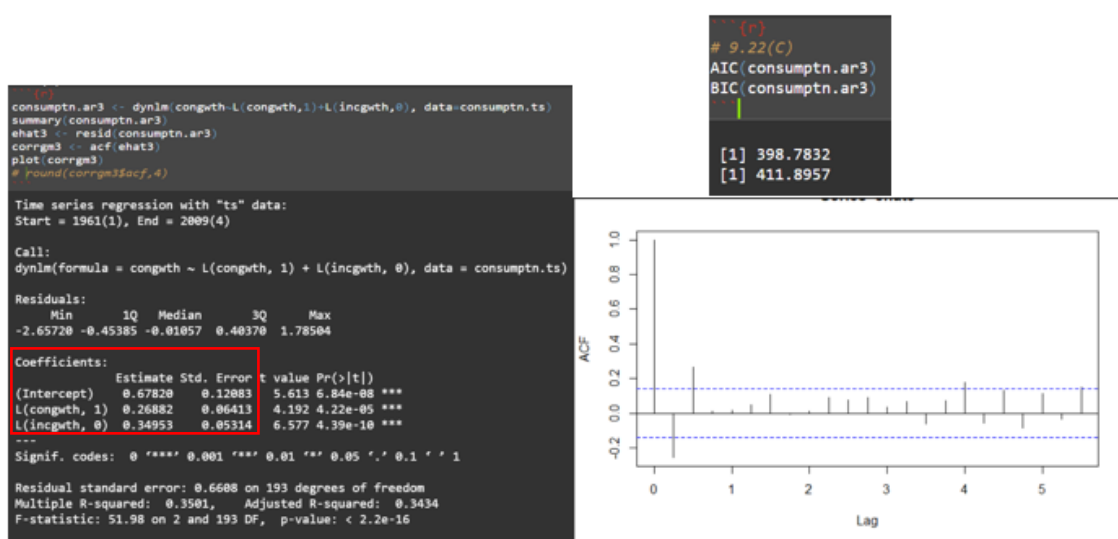
Estimated model:  $\widehat{\text{CONGWTH}}_t = 0.9738 + 0.4496 \cdot \text{INCGWTH}_t$

The estimate  $\delta_0 = 0.4496$  suggests that a 1% increase in the income growth rate increases the consumption growth rate by 0.45%. We also examine the residuals by seeing its ACF plot and an LM test with two lagged errors. The correlogram below shows significant serial correlation in the errors at lag 2. There is also some slight evidence of serially correlated errors at some longer lags (7, 11, 12, 16, 19).

For the LM test:

LM =  $N \cdot R^2 = 197 \cdot 0.2959 = 58.2923$  is  $> \chi^2_{(2)} = 21.93$ , with a  $p$ -value less than 0.00005, a strong indication of serially correlated errors.

(c) Estimate the model  $\text{CONGWTH}_t = \delta + \theta_1 \text{CONGWTH}_{t-1} + \delta_0 \text{INCGWTH}_t + v_t$ . Is this model an improvement over that in part (b)? Is the estimate for  $\theta_1$  significantly different from zero? Have the values for the AIC and the SC gone down? Has serial correlation in the errors been eliminated?

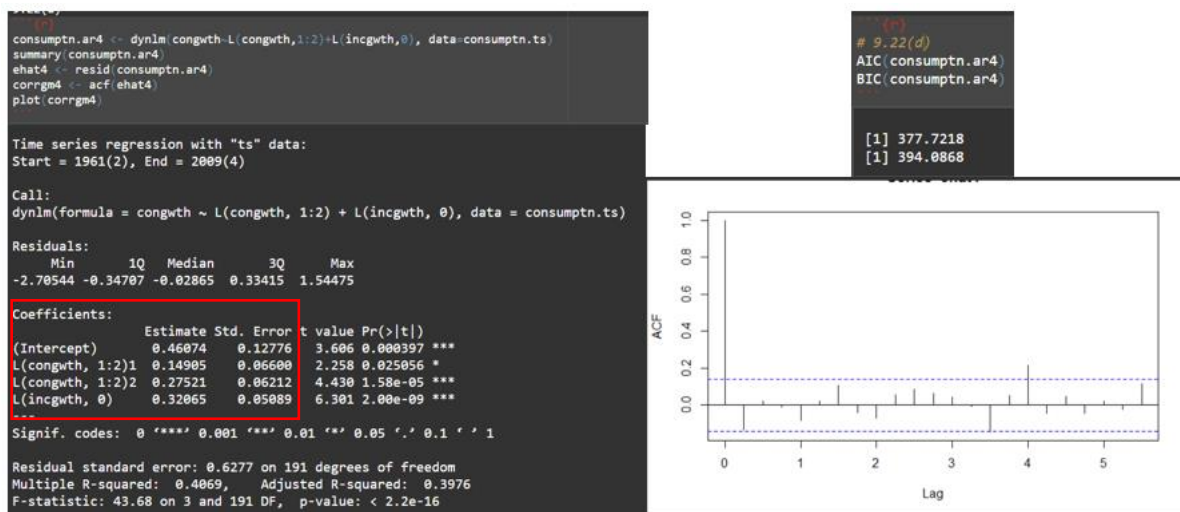


The estimated model is

$\widehat{\text{CONGWTH}}_t = 0.67820 + 0.26882 \cdot \text{CONGWTH}_{t-1} + 0.34953 \cdot \text{INCGWTH}_t$

The estimate  $\hat{\theta}_1 = 0.26882$  is significantly different from zero at the 5% significance level ( $t = 0.26882/0.06413 = 4.1918 > t_{(0.975,193)} = 1.97$ ), The *LM* test rejects the null hypothesis that the errors are not serially correlated. The AIC, SC is smaller than part(b) suggest part(c) is an improvement. We conclude that the model is improved, but it is still not satisfactory.

**(d) Add the variable  $\text{CONGWTH}_{t-2}$  to the model in part (c) and re-estimate. Is this model an improvement over that in part (c)? Is the estimate for  $\theta_2$  (the coefficient of  $\text{CONGWTH}_{t-2}$ ) significantly different from zero? Have the values for the AIC and the SC gone down? Has serial correlation in the errors been eliminated?**



The estimated model is

$$\widehat{\text{CONGWTH}}_t = 0.4607 + 0.1490 \cdot \text{CONGWTH}_{t-1} + 0.27521 \cdot \text{CONGWTH}_{t-2} + 0.34953 \cdot \text{INCGWTH}_t$$

The estimate  $\hat{\theta}_2 = 0.27521$  is significantly different from zero at the 5% significance level ( $t = 0.27521/0.06212 = 4.43 > t_{(0.975,191)} = 1.97$ ), the lower values of AIC and BIC suggest this model is an improvement. In the correlogram of the residuals above suggesting that serially correlated errors are still existence. We conclude that adding  $\text{CONGWTH}_{t-2}$  has improved the model, but the existence of serially correlated errors point out that it is still not satisfactory.

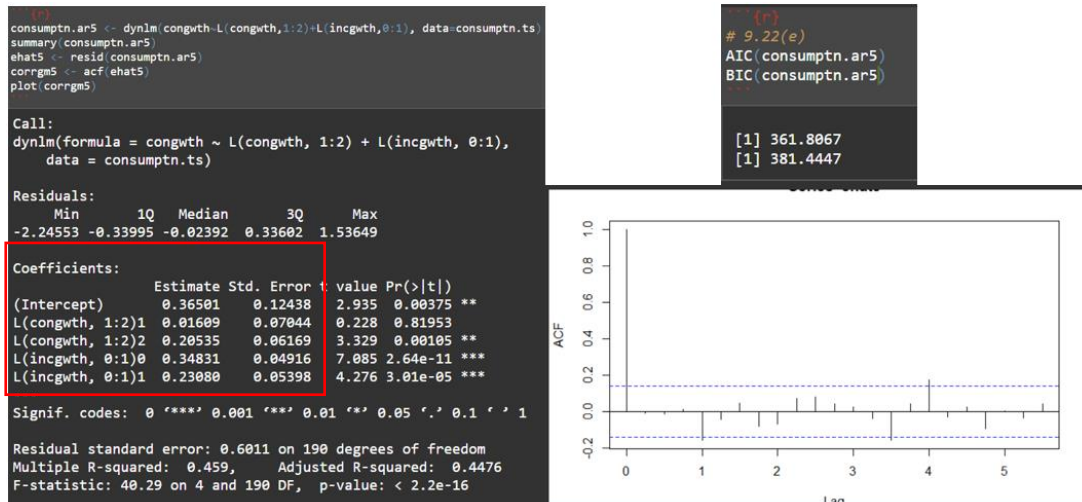
**(e) Add the variable  $\text{INCGWTH}_{t-1}$  to the model in part (d) and re-estimate. Is this model an improvement over that in part (d)? Is the estimate for  $\delta_1$  (the coefficient of  $\text{INCGWTH}_{t-1}$ ) significantly different from zero? Have the values for the AIC and the SC gone down? Has serial correlation in the errors been eliminated?**

The estimated model is

$$\widehat{\text{CONGWTH}}_t = 0.3650 + 0.0161 \cdot \text{CONGWTH}_{t-1} + 0.2054 \cdot \text{CONGWTH}_{t-2} + 0.3483 \cdot \text{INCGWTH}_t + 0.2308 \cdot \text{INCGWTH}_{t-1}$$

The estimate  $\hat{\delta}_1 = 0.2308$  is significantly different from zero at the 5% significance level ( $t = 0.2308/0.0540 = 4.27 > t_{(0.975,190)} = 1.97$ ), the lower values of AIC and BIC suggest this model is an improvement. In the correlogram of the residuals below suggesting that serially correlated errors are not much. We conclude that adding  $\text{INCGWTH}_{t-1}$  has improved the model, and serially correlated is no longer a problem.





(f) Does the addition of  $CONGWTH_{t-3}$  or  $INCGWTH_{t-2}$  improve the model in part (e)?

```
9.22(f)
(f)
consumptn.ar6 <- dynlm(congwth~L(congwth,1:3)+L(incgwth,0:2), data=consumptn.ts)
AIC(consumptn.ar6)
BIC(consumptn.ar6)

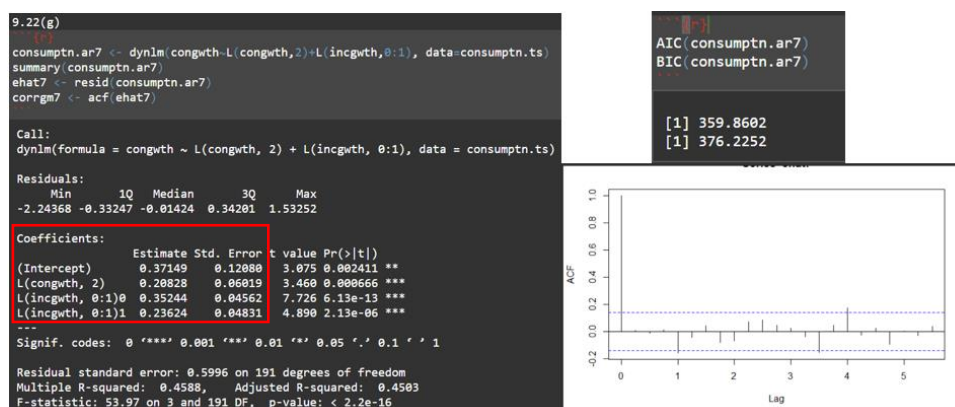
[1] 364.1762
[1] 390.3191
```

According to the result of AIC and BIC, the higher values of AIC and BIC suggest this model is not an improvement of part (e).

(g) Drop the variable  $CONGWTH_{t-1}$  from the model in part (e) and re-estimate. Why might you consider dropping this variable? The model you should be estimating is

$$CONGWTH_t = \delta + \theta_2 CONGWTH_{t-2} + \delta_0 INCGWTH_t + \delta_1 INCGWTH_{t-1} + v_t \quad (9.94)$$

Does this model have lower AIC and SC values than that in (e)? Is there any evidence of serially correlated errors?



The estimated model is

$$CONGWTH_t = 0.3715 + 0.2083 \cdot CONGWTH_{t-2} + 0.3524 \cdot INCGWTH_t + 0.2362 \cdot INCGWTH_{t-1}$$

The AIC and BIC values are lower than those for the model estimated in part (e). In the correlogram of the residuals above suggesting that serially correlated errors are not much.