Tarea 4 | 158528

(1) $P_o(n|\lambda)$

$Q(N_t=0)=0$

$Q(N_t=1)=1/3$

Poisson base $\rightarrow P_o = e^{-\lambda}\frac{\lambda^n}{n!}\mathbb{1}^{(n)}_{\{0,\dots,\infty\}}$

$$\frac{P(N_t=n)}{1-P(N_t\in A)} = \frac{e^{-\lambda}\frac{\lambda^n}{n!}}{1-e^{-\lambda}-e^{-\lambda}\lambda}$$

$$Q(N_t) = 1-Q(N_t=0)-Q(N_t=1) = 1-0-\frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow \frac{2}{3}\frac{P(N_t=n)}{1-P(N_t\in A)} = \frac{2}{3}\frac{e^{-\lambda}\frac{\lambda^n}{n!}}{1-e^{-\lambda}-e^{-\lambda}\lambda}$$

(2) Modificación

$N_t | N_t \leq 15 \sim P_o(n|30)$

$N_t | N_t > 15 \sim Bin(n|100, 1/3)$

$P_n = e^{-\lambda}\frac{\lambda^n}{n!}\mathbb{1}^{(n)}_{\{0,\dots,\infty\}}$

$\tilde{P}_n = \binom{100}{n}\theta^n(1-\theta)^{100-n}\mathbb{1}^{(n)}_{\{0,\dots,\infty\}}$

Si tomamos la Poisson como base

$$\Rightarrow q_n = P_n = \frac{e^{-\lambda}\lambda^n}{n!}\mathbb{1}^{(n)}_{\{0,\dots,15\}} = 1-\sum_{n=0}^{15}e^{-\lambda}\frac{\lambda^n}{n!}$$

$$\frac{P_n(N_t=n)}{\tilde{P}_n(N_t\in A)} = \frac{\binom{100}{n}\theta^n(1-\theta)^{100-n}}{1-\sum_{k=0}^{15}\binom{100}{n}\theta^n(1-\theta)^{100-n}}\mathbb{1}^{(n)}_{\{16,\dots,\infty\}}$$

$\Rightarrow$

$$Q(N_t=n) = \begin{cases} e^{-\lambda}\dfrac{\lambda^n}{n!} \quad \mathbb{1}^{(n)}_{\{0,\dots,15\}} \\ \\ \left(1-\sum_{n=0}^{15} e^{-\lambda}\dfrac{\lambda^n}{n!}\right)\left(\dfrac{\binom{100}{n}\theta^n(1-\theta)^{100-n}}{\left(1-\sum_{k=0}^{15}\binom{100}{n}\theta^n(1-\theta)^{100-n}\right)}\right) \mathbb{1}^{(n)}_{\{16,\dots,\infty\}} \end{cases}$$