TAREA 4

1.- Modificación de $P_0(n|\lambda)$ /

$Q(N_t=0)=0$,
$Q(N_t=1)=1/3$

- Atomos por modificar {0,1}
- Atomos fijos {2,3,...}

$$\Rightarrow \tilde{P}(N_t=n) = \frac{e^{-\lambda}\lambda^n}{n!} \quad \mathbb{I}(n\geq 2)$$

$\Rightarrow$ Tenemos que modificar $n<2$

$$\left(1-\sum_{k=2}^{\infty}\frac{e^{-\lambda}\lambda^n}{n!}\right)$$

ó

$$\left(\sum_{k=0}^{1}\frac{e^{-\lambda}\lambda^k}{k!}\right)$$

$$\Rightarrow q_0 = \frac{e^{-\lambda}\lambda^0}{0!}\cdot\frac{0}{\frac{e^{-\lambda}\lambda^0}{0!}} = 0$$

$$\Rightarrow q_1 = \frac{e^{-\lambda}\lambda^1}{1!}\cdot\frac{1/3}{\frac{e^{-\lambda}\lambda^1}{1!}} = 1/3$$

$$\Rightarrow Q(N_t=n) = \begin{cases} 0 & \text{si } n=0 \\ 1/3 & \text{si } n=1 \\ \frac{e^{-\lambda}\lambda^n}{n!}\cdot\frac{\frac{2}{3}}{\sum_{k=2}^{\infty}\frac{e^{-\lambda}\lambda^k}{k!}} & \text{si } \mathbb{I}(n\geq 2) \end{cases}$$

2.- $N_t \mid N_t \leq 15 \sim Po(n \mid 30)$

$N_t \mid N_t > 15 \sim Bin(n \mid 100, 1/3)$

> Poisson como base

Atomos fijos $= \{0, 1, \dots, 14\}$
Atomos por modificar $= \{n \geq 15\}$

$$\Rightarrow Q_n = P_n = \left(\frac{e^{-30}\, 30^n}{n!} \right) \mathbb{I}_{\{0,\dots,14\}}^{(n)} \leftarrow P(N_t < 15)$$

Por lo que tenemos que modificar

$$\left(1 - \sum_{n=0}^{14} \frac{e^{-30}\, 30^n}{n!} \right)$$

En la binomial $(n > 15)$

$$\frac{P(N_t = n)}{P(N_t > 15)} = \frac{P(N_t = n)}{1 - P(N_t \leq 15)} = \frac{Bin(n \mid 100, 1/3)}{1 - \sum_{K=0}^{14} Bin(n \mid 100, 1/3)} = \frac{\binom{100}{n} (1/3)^n (2/3)^{100-n}}{1 - \sum_{K=0}^{100} \binom{100}{K} (1/3)^K (2/3)^{100-K}}$$

..) Reescalamos

$$Q(N_t = n) = \tilde{P}(N_t > 15) \frac{P(N_t = n)}{P(N_t > 15)}$$

$$= \left(1 - \sum_{K=0}^{14} \frac{e^{-30}\, 30^K}{K!} \right) \frac{\binom{100}{n} \left(\frac{1}{3}\right)^n (2/3)^{100-n}}{1 - \sum_{K=0}^{14} \binom{100}{K} \left(\frac{1}{3}\right)^K \left(2/3\right)^{100-K}}$$

$$\Rightarrow Q(N_t = n) = \begin{cases} \dfrac{e^{-30}\, 30^n}{n!}, & \mathbb{I}(n < 15) \\[2ex] \left(1 - \sum_{K=0}^{14} \dfrac{e^{-30}\, 30^K}{K!} \right) \dfrac{\binom{100}{n} \left(\frac{1}{3}\right)^n (2/3)^{100-n}}{1 - \sum_{K=0}^{14} \binom{100}{K} \left(\frac{1}{3}\right)^K \left(\frac{2}{3}\right)^{100-K}} & \mathbb{I}(n \geq 15) \end{cases}$$