AGREGACIÓN

$N_1, \ldots, N_q$ v.a.i.i.d $\qquad P_o(N_j = n_j / \lambda_j)$

- $N = \sum_{j=1}^{q} N_j \sim$ Poisson
- N tiene tasa de intensidad $\lambda = \sum_{j=1}^{q} \lambda_j$

Dado que N_i son independientes

$$\Leftrightarrow N = \sum_{j=1}^{q} N_j \Leftrightarrow m_N(t) \Leftrightarrow E(e^{tN}) = E(e^{t \sum N_j})$$

$$\Leftrightarrow E(e^{t(N_1 + N_2 + \cdots + N_q)}) \Leftrightarrow E(e^{tN_1} e^{tN_2} \cdots e^{tN_q})$$

$$\Leftrightarrow E(e^{tN_1}) \cdots E(e^{tN_q})$$

$$\Leftrightarrow e^{\lambda_1(e^t - 1)} \cdots e^{\lambda_q(e^t - 1)}$$

$$\Leftrightarrow e^{(\lambda_1 + \cdots + \lambda_q)(e^t - 1)}$$

$$\Leftrightarrow N = \sum_{j=1}^{q} N_j \sim \text{Poisson}(\lambda) \quad \text{donde } \lambda = \sum_{j=1}^{q} \lambda_j$$

con $\lambda_j > 0$

3.- BINOMIAL NEGATIVA COMO MEZCLA DE POISSON GAMMA

$Y_1 \sim Poisson(\lambda)$
$Y_2 \sim Gamma(\alpha, \beta)$

$$P(N=n) = \int_0^\infty Po(n|\lambda)\, Ga(\lambda|a,b)\, d\lambda$$

$$= \int_0^\infty \frac{\lambda^n e^{-\lambda}}{n!} \cdot \frac{\lambda^{\alpha-1} e^{-\lambda/\beta}}{\Gamma(\alpha)\beta^\alpha} d\lambda$$

$$= \frac{1}{\Gamma(\alpha)\beta^\alpha n!} \int_0^\infty \frac{\lambda^{n+\alpha-1} e^{-\lambda(1+1/\beta)}}{} d\lambda \frac{\Gamma(n+\alpha)\left(\frac{\beta}{\beta+1}\right)^{n+\alpha}}{\Gamma(n+\alpha)\left(\frac{\beta}{\beta+1}\right)^{n+\alpha}}$$

$$= \left(\frac{\beta}{\beta+1}\right)^{n+\alpha} \frac{\Gamma(n+\alpha)}{\Gamma(\alpha)\beta^\alpha n!} \cancel{\int_0^\infty \frac{\lambda^{n+\alpha-1} e^{-\lambda(1+1/\beta)}}{\Gamma(n+\alpha)\left(\frac{\beta}{\beta+1}\right)^{n+\alpha}}} \to 1$$

$$Gamma\left(n+\alpha, \frac{\beta}{\beta+1}\right)$$

$$= \left(\frac{\beta}{\beta+1}\right)^{n+\alpha} \frac{\Gamma(n+\alpha)}{\Gamma(\alpha)\beta^\alpha n!} \cdot \left(\frac{\beta}{\beta+1}\right)^{n+\alpha} \frac{n+\alpha-1!}{\alpha-1!\ n!\ \beta^\alpha}$$

$$= \frac{\beta^n}{(\beta+1)^{n+\alpha}} \left(\frac{(n+\alpha-1)!}{(\alpha-1)!\ n!}\right) = \left(\frac{1}{\beta+1}\right)^\alpha \left(\frac{\beta}{\beta+1}\right)^n \binom{n+\alpha-1}{\alpha-1}$$

$$\sim BinNeg\left(\alpha, \frac{1}{\beta+1}\right)$$