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Tarea 4

1 $P_o(n|\lambda)$

$Q(N_t=0)=0$

$Q(N_t=1)=1/3$

$q_0=0$

$q_1=1$

Sea base q_0

$$\text{Poisson}(n|\lambda) = \frac{e^{-\lambda}\lambda^n}{n!} \, I_{\{0,1,\dots\}}(n)$$

Masa modificada

1ra parte

$$\frac{P(N_t=1)}{1-P(N_t=0)} = \frac{\frac{1}{3}}{1-1/3} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$q_n = \cancel{q}(N_t=1) = 1\left(\frac{1}{2}\right)$$

2da parte

$$\frac{P(N_t=n)}{1-P(N_t=4)} = \frac{e^{-\lambda}\lambda^n}{n!}$$

Scribe

$$\frac{P(N_t=n)}{1-P(N_t=1)} = \frac{\frac{e^{-\lambda}\lambda^n}{n!}}{1-(e^{-\lambda}+e^{-\lambda}\lambda)} = \frac{e^{-\lambda}\lambda^n}{n!(1-e^{-\lambda}-\lambda e^{-\lambda})}$$

$$q_n = Q(N_t=n) = (1-1/3)\,\frac{e^{-\lambda}\lambda^n}{n!(1-e^{-\lambda}-\lambda e^{-\lambda})}$$

$$Q(N_t=n) = \begin{cases} 0 & \text{si } N_t=0 \\ 1/3 & \text{si } N_t=1 \\ \frac{2}{3}\frac{e^{-\lambda}\lambda^n}{n!(1-e^{-\lambda}-\lambda e^{-\lambda})} & N_t \geq 2 \end{cases}$$

2.

$$N_t \mid N_t \leq 15 \sim Poi(n \mid 30)$$

$$N_t \mid N_t > 15 \sim Bin(n \mid 100, 1/3)$$

Poisson como base

Atomos Fijos = $\{0, 1, 2, \dots, 14\}$

Atomos a modifica $(n > 15)$

$$q_n = p_n = \frac{e^{-30}\,30^n}{n!} \quad I_{\{0,1,\dots,14\}}$$

$$\left(1 - \sum_{k=0}^{14} e^{-30} \frac{30^n}{n!}\right)$$

$$\frac{P(N_t = n)}{1 - P(N_t = A)} = \frac{\binom{100}{n} \theta^n (1-\theta)^{100-n}}{1 - \sum_{k=0}^{14} \binom{100}{k} \theta^k (1-\theta)^{100-k}}$$

$$= \frac{Bin(n \mid 100, 1/3)}{1 - \sum_{k=0}^{14} Bin(n \mid 100, 1/3)}$$

$$Q(N_t = n) = \begin{cases} e^{-30} \dfrac{30^n}{n!} & I_{\{0, 1, \ldots, 14\}} \\ \left(1 - \sum_{k=0}^{14} e^{-30} \dfrac{30^n}{n!}\right) \left(\dfrac{\binom{100}{n} \theta^n (1-\theta)^{100-n}}{1 - \sum_{k=0}^{14} \binom{100}{k} \theta^k (1-\theta)^{100-k}} \right) & I_{n \geq 15} \end{cases}$$

$S = N_1 + N_2 + N_3 + N_4$

$Ber(n|\theta)$

$Bin(s|4,\theta)$

Sol

$$p^{*(4)}(s) = P(N_1 + N_2 + N_3 + N_4 = s)$$

$$= \sum_{n \in N} p^{*(3)}(s-n) P(N_4 = n)$$

$$p^{*(2)} = \sum_{n=0}^{1} P(N_1 + N_2 = s, N_2 = n)$$

$$= \sum_{n=0}^{1} P(N_1 = s-n, N_2 = n)$$

$$= \sum_{n=0}^{1} P(N_1 = s-n) P(N_2 = n)$$

$$= \sum_{n=0}^{1} Ber(s-n|\theta) Ber(n|\theta) = \sum_{n=0}^{1} \theta^{s-n} (1-\theta)^{1-s+n} \theta^{n} (1-\theta)^{1-n}$$

$$= \sum_{n=0}^{1} \theta^{s} (1-\theta)^{2-s}$$

$$p^{*(3)} = \sum_{n=0}^{1} P(S_{12} = s-n) P(N_3 = n)$$

$$= \sum_{n=0}^{1} \sum_{n=0}^{1} \theta^{s-n} (1-\theta)^{2-s+n} \theta^{n} (1-\theta)^{1-n}$$

$$= \sum_{n=0}^{1} \theta^{s} (1-\theta)^{3-s}$$

Scribe

$$P^{*(4)}(S) = \sum_{n=0}^{1} \Theta^{3}(1-\Theta)^{3-S} \; \Theta^{n}(1-\Theta)^{1-S}$$

$$= \sum_{n\in N} \Theta^{4}(1-\Theta)^{4-S}$$