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Cálculo Actuarial III

Considera $F_X(x) = Pa(x|\alpha)$ con densidad $f_X(x) = (1+x)^{-\alpha} \mathbb{1}_{(0,\infty)}(x)$ y $\alpha > 0$. Muestra que para $\alpha > 0$ la integral 2.4 no está definida.

$$F_x = \int_0^x f_a(a)\,da = \int_0^x (1-a)^{\alpha} \mathbb{1}_{(0,\infty)}(a)\,da \qquad \text{con} \quad \begin{array}{l} u = 1-a \\ du = -da \end{array}$$

$$-\int u^{\alpha}\,du = -\frac{u^{\alpha+1}}{\alpha+1} \qquad \Rightarrow \qquad -\frac{(1-a)^{\alpha+1}}{\alpha+1}\Bigg|_0^x = \frac{1}{\alpha+1}\left(1-(1-x)^{\alpha+1}\right)$$

$$\Rightarrow \int_0^{\infty} \exp^{(r(x))}\left[1-(1-x)^{\alpha+1}\right]\left[\frac{1}{\alpha+1}\right]dx = \frac{1}{\alpha+1}\int_0^{\infty} e^{rx}\left[1-(1-x)^{\alpha+1}\right]dx$$

$$= \underbrace{\int_0^{\infty} e^{rx}\,dx}_{①} - \underbrace{\int_0^{\infty} e^{rx}(1-x)^{\alpha+1}\,dx}_{②}$$

$$① = \frac{e^{rx}}{r}\Bigg|_0^{\infty} = \infty$$

$$② = \int_0^{\infty} e^{rx}(1-x)^{\alpha+1}\,dx \quad \rightarrow \text{ esta integral diverge}$$

$\therefore$ para $\alpha + 1$ no está bien definida.

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Calcula la probabilidad de ruina $\Psi(\omega)$ cuando $X \sim Exp(x|1/\mu)$ con $E(X)=\mu$

$$F_X = 1 - e^{-x/\mu}$$

$$M_X(t) = (1-\mu t)^{-1} \quad t<\mu^{-1}$$

Por el coeficiente de Lündberg: $1 + (1+\theta)\mu r = M_X(r)$

$$\Rightarrow \quad 1 + (1+\theta)\mu r = (1+\mu r)^{-1}$$

$$\Rightarrow \quad 1 + (1+\theta)\mu r = \frac{1}{(1-\mu r)}$$

$$\Rightarrow \quad (1-\mu r) + (1-\mu r)(1+\theta)\mu r = 1$$

$$\Rightarrow \quad -\mu r + (1+\theta - \mu r - \mu r\theta)\mu r = 0$$

$$\cancel{-\mu r} + \cancel{\mu r} + \theta\mu r - \mu^2 r^2 - \mu^2 r^2\theta = 0$$

$$r^2(-\mu^2 - \mu^2\theta) + \theta\mu r = 0$$

$$r(-\mu^2 r - \mu^2\theta r + \theta\mu) = 0$$

$$-\mu^2 r - \mu^2\theta r + \theta\mu = 0$$

$$r(-\mu^2 - \mu^2\theta + \mu\theta) = 0$$