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Tarea 04. Cálculo Actuarial III.

1. Deriven la modificación de la distribución $Po(n|\lambda)$ en la que $Q(N_t=0)=0 \wedge Q(N_t=1)=1/3$

Átomos a modificar $\{0,1\}$

Átomos fijos $\{2,3,4,\dots\}$

$$P(N_t=n) = \frac{\lambda^n e^{-\lambda}}{n!} \mathbb{I}(n\geq 2)$$

·) Se modifica para $n<2$

$$\frac{\tilde{P}(N_t=n)}{1-\tilde{P}(N_t\in A)} \quad \text{con } A=\{0,1\} \Rightarrow \frac{\tilde{P}(N_t=n)}{1-P(N_t<2)}$$

$$\frac{P(N_t=n)}{1-P(N_t<2)} = \frac{\frac{e^{-\lambda}\lambda^n}{n!}}{1-\sum_{k=0}^{1}\frac{e^{-\lambda}\lambda^k}{k!}} = \frac{\frac{e^{-\lambda}\lambda^n}{n!}\mathbb{I}(n\geq 2)}{1-[e^{-\lambda}+e^{-\lambda}\lambda]}$$

··) reescalamos

$$q_n = \tilde{P}(N_t<2)\cdot\frac{P(N_t=n)}{P(N_t<2)} \quad ; \quad \tilde{P}(N_t<2) = 1-\tilde{P}(N_t=0)-\tilde{P}(N_t=1)$$
$$= 1-e^{-\lambda}-e^{-\lambda}\lambda$$

$$\tilde{P}(N_t<2) = 1-0-1/3 = 2/3$$

$$\Rightarrow q_n = \left(\frac{2}{3}\right)\cdot\frac{(e^{-\lambda}\lambda^n/n!)}{(1-e^{-\lambda}-e^{-\lambda}\lambda)}\mathbb{I}(n\geq 2)$$

2. Deriven la modificación en la que

$$(N_t \mid N_t \le 15) \sim Po(n \mid 30)$$
$$(N_t \mid N_t > 15) \sim Bin(n \mid 100, 1/3)$$

átomos fijos $= \{0, 1, \dots, 15\}$

Átomos × modificar $= \{16, 17, 18, \dots, 100\}$

Tomamos Poisson como base

$$q_n = p_n = \frac{\lambda^n e^{-\lambda}}{n!} \mathbb{I}(n \le 15)$$

masa a modificar y redistribuir a la binomial $1 - \sum_{k=0}^{15} \frac{\lambda^k e^{-\lambda}}{k!}$

•) Condicionamos en todos los átomos, bin con $n \in (15, 100]$ (Binomial)

A ?

$$\frac{\tilde{P}(N_t = n)}{1 - \tilde{P}(N_t \in A)} = \frac{\binom{100}{n} \theta^n (1-\theta)^{100-n} \quad \mathbb{I}(15 < n \le 100)}{1 - \sum_{k=16}^{100} \binom{100}{k} \theta^k (1-\theta)^{100-k}}$$

••) Reescalamos

$$q_n = \tilde{P}(N_t \le 15) \cdot \frac{P(N_t = n)}{1 - P(N_t \in A)} = \left[1 - \sum_{k=0}^{15} \frac{\lambda^k e^{-\lambda}}{k!}\right] \left[\frac{\binom{100}{n} \theta^n (1-\theta)^{100-n}}{1 - \sum_{k=0}^{15} \binom{100}{k} \theta^k (1-\theta)^{100-k}}\right] \mathbb{I}(15 \le n \le 100)$$

$$\Rightarrow Q(N_t = n) = \begin{cases} \dfrac{e^{-30}\, 30^n}{n!} \quad \mathbb{I}(n \le 15) \\ \left[1 - \sum_{k=0}^{15} \dfrac{\lambda^k e^{-\lambda}}{k!}\right] \left[\dfrac{\binom{100}{n} (1/3)^n (2/3)^{100-n}}{1 - \sum_{k=0}^{14} \binom{100}{k} (1/3)^k (2/3)^{100-k}}\right] \quad \mathbb{I}(n > 15) \end{cases}$$