

Tarea 04. Cálculo Actuarial III.

1. Deriven la modificación de la distribución $Po(n; \lambda)$ en la que
 $Q(N_t=0) = 0$ y $Q(N_t=1) = 1/3$

Átomos a modificar 40,14

Átomos fijos 42,34, - 4

$$P(N_t=n) = \frac{\lambda^n e^{-\lambda}}{n!} \mathbb{I}(n \geq 2)$$

2. A modification para $n < 2$

$$\frac{\tilde{P}(N_t=n)}{1 - \tilde{P}(N_t \leq 1)} \quad \text{con } \lambda = 40,14 \Rightarrow \frac{\tilde{P}(N_t=n)}{1 - P(N_t \leq 1)}$$

$$\frac{P(N_t=n)}{1 - P(N_t \leq 1)} = \frac{\frac{e^{-\lambda} \lambda^n}{n!}}{1 - \sum_{k=0}^1 \frac{e^{-\lambda} \lambda^k}{k!}} = \frac{\frac{e^{-\lambda} \lambda^n}{n!} \mathbb{I}(n \geq 2)}{1 - (e^{-\lambda} + e^{-\lambda} \lambda)}$$

3. verificación

$$q_n = \frac{P(N_t=n)}{1 - P(N_t \leq 1)} \quad ; \quad \tilde{P}(N_t \leq 1) = 1 - \tilde{P}(N_t=0) - \tilde{P}(N_t=1)$$

$$= 1 - e^{-\lambda} - e^{-\lambda} \lambda$$

$$\tilde{P}(N_t \leq 1) = 1 - 0 - 1/3 = 2/3$$

$$\Rightarrow q_n = \left(\frac{2}{3}\right) \cdot \frac{(e^{-\lambda} \lambda^n / n!)}{(1 - e^{-\lambda} - e^{-\lambda} \lambda)} \mathbb{I}(n \geq 2)$$

2. Deriven la modificación en la que

$$(N_t | N_t \leq 15) \sim P_0(n | 30)$$

$$(N_t | N_t > 15) \sim \text{Bin}(n | 100, 1/3)$$

átomos fijos = $\{0, 1, \dots, 15\}$

Átomos x modificar = $\{16, 17, 18, \dots, 100\}$

Tomamos Poisson como base

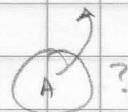
$$P_n = P_n = \frac{\lambda^n e^{-\lambda}}{n!} \mathbb{I}(n \leq 15)$$

meta a modificar y redistribuir a la binomial

$$1 - \sum_{k=0}^{15} \frac{\lambda^k e^{-\lambda}}{k!}$$

-) condicionamos en todos los átomos, bin con $n \in (15, 100]$ (Binomial)

$$\frac{\tilde{P}(N_t = n)}{1 - \tilde{P}(N_t \in A)} = \frac{\binom{100}{n} \theta^n (1-\theta)^{100-n} \mathbb{I}(15 < n \leq 100)}{1 - \sum_{k=16}^{100} \binom{100}{k} \theta^k (1-\theta)^{100-k}}$$



ii) reescalamos

$$P_n = \frac{\tilde{P}(N_t \leq 15)}{1 - \tilde{P}(N_t \in A)} \cdot \frac{P(N_t = n)}{1 - \tilde{P}(N_t \in A)} = \left[1 - \sum_{k=0}^{15} \frac{\lambda^k e^{-\lambda}}{k!} \right] \left[\frac{\binom{100}{n} \theta^n (1-\theta)^{100-n}}{1 - \sum_{k=16}^{100} \binom{100}{k} \theta^k (1-\theta)^{100-k}} \right] \mathbb{I}(15 < n \leq 100)$$

$$\Rightarrow \mathbb{Q}(N_t = n) = \begin{cases} \frac{e^{-30} 30^n}{n!} \mathbb{I}(n \leq 15) \\ \left[1 - \sum_{k=0}^{15} \frac{\lambda^k e^{-\lambda}}{k!} \right] \left[\frac{\binom{100}{n} (1/3)^n (2/3)^{100-n}}{1 - \sum_{k=16}^{100} \binom{100}{k} (1/3)^k (2/3)^{100-k}} \right] \mathbb{I}(n > 15) \end{cases}$$