Octavio Fuentes 150792

Pregunta 1

$$F(x) = 1 - e^{-x/\mu}, \quad x \geq 0$$

$$\Rightarrow M_x(t) = (1 - \mu t)^{-1}, \quad t < \mu^{-1}$$

Sabemos que, por el coeficiente de Lundberg:

$$1 + (1+\theta)\mu r = M_x(r)$$

$\Rightarrow$

$$1 + (1+\theta)\mu r = (1 - \mu r)^{-1}$$

$$\Rightarrow (1 - \mu r) + (1+\theta)\mu r (1 - \mu r) = 1$$

$$\Rightarrow -\mu r + (1+\theta)\mu r - (1+\theta)\mu r^2 = 0$$

$$\Rightarrow -1 + (1+\theta) - (1+\theta)\mu r = 0$$

$$\Rightarrow r = \frac{\theta}{(1+\theta)\mu}$$

$\Rightarrow$

$$\Psi(C_0) = \frac{\exp\left(-\frac{\theta}{(1+\theta)\mu} C_0\right)}{E_{F_S}\left(\exp\left(-\frac{\theta}{(1+\theta)\mu} C_T\right) \middle| T < \infty\right)} \leq \exp\left(-\frac{\theta}{(1+\theta)\mu} C_0\right)$$

$\Rightarrow$

$$E_{F_{ST}}\left(\exp\left(-\frac{\theta}{(1+\theta)\mu} C_T\right) \middle| T < \infty\right) > 1$$

Pregunta 2

$F_X(x) = P_a(x \mid \alpha)$

$f_X(x) = (1+x)^{-\alpha} \; I_{(0,\infty)}(x) \;, \quad \alpha > 0$

Primero, tenemos que obtener r

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx = \int_0^{\infty} e^{tx} (1+x)^{-\alpha}\, dx$$

$$F_X(x) = \int_0^x f_X(t)\, dt = \int_0^x (1+t)^{-\alpha}\, dt = \frac{(1+x)^{1-\alpha} - 1}{1-\alpha} \;, \quad \alpha > 0$$

$$\int_0^{\infty} \exp(rx) F_X(x)\, dx = \int_0^{\infty} \exp(rx) \frac{(1+x)^{1-\alpha} - 1}{1-\alpha}\, dx = *$$

$$\frac{1 - F_X(x)}{\exp(-rx)} \leq E_{F_X}(\exp(rx))$$

$\Rightarrow$

$$\frac{1 - \dfrac{(1+x)^{1-\alpha} - 1}{1-\alpha}}{\exp(rx)} = \frac{1-\alpha - (1+x)^{1-\alpha} + 1}{\exp(-rx)(1-\alpha)}$$

$$* = \left(\frac{1}{1-\alpha}\right)\left(\underbrace{\int_0^{\infty} e^{rx}(1+x)^{1-\alpha}\, dx}_{A} - \underbrace{\int_0^{\infty} e^{rx}\, dx}_{B}\right)$$

$\Rightarrow$

A $\left.\dfrac{e^{rx}}{r}\right|_0^{\infty} = \infty$

B $\int_0^{\infty} e^{rx}(1+x)^{1-\alpha}\, dx \Rightarrow$ no converge