ACT-11302 Cálculo Actuarial III
Primavera 2019
Tarea 06
Fecha de entrega: 11/Abr/2019

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Consideren la clase del martes 9 de abril de 2019.

1. Calcula la probabilidad de ruina $\Psi(C_0)$ cuando $X \sim \text{Exp}(x|1/\mu)$, con $\mathbb{E}(X) = \mu$.
2. Considera $F_X(x) = \text{Pa}(x|\alpha)$, con densidad $f_X(x) = (1+x)^{-\alpha}\mathbb{I}(0,\infty)(x)$, y $\alpha > 0$. Muestra que para valores de $\alpha > 0$ (caso con colas pesadas) la integral de la Sección 2.4. de las notas no está definida.

$$M_X(r) = \frac{1/\mu}{1/\mu - r}$$

① $X \sim Exp(x \mid 1/\mu)$, $E[X] = \mu$

$$\Psi_r(C_0) = \frac{\exp(-r\,C_0)}{E_{F_{st}}\cdot(\exp(-r\,C_T) \mid T<\infty)} \leq \exp(-r\,C_0)$$

$$= \frac{1/\mu}{\frac{1-\mu r}{\mu}} = \frac{1}{1-\mu r}$$

Coef. de Lundberg: $1+(1+\theta)\mu r = M_X(r) = \frac{1}{1-\mu r}$, $r>0$

$\Rightarrow (1-\mu r) + (1-\mu r)(1+\theta)\mu r = 1-\mu r + (1-\mu r + \theta - \mu r\theta)\mu r = 1$

$\Rightarrow 1 - \cancel{\mu r} + \cancel{\mu r} - (\mu r)^2 + \theta\mu r - (\mu r)^2\theta = \cancel{1} - (\theta+1)(\mu r)^2 + \theta\mu r = \cancel{1}$

$\Rightarrow -\mu^2(1+\theta)r^2 + \theta\mu r = 0 \Rightarrow r[\theta\mu - \mu^2(1+\theta)r] = 0$

$\Rightarrow \mu^2(1+\theta)r = \theta\mu \Rightarrow r = \frac{\theta\mu}{\mu^2(1+\theta)} = \frac{\theta}{\mu(1+\theta)} > 0$

$\boxed{r>0}$

$$\therefore \Psi_r(C_0) = \frac{\exp\left(-\frac{\theta C_0}{\mu(1+\theta)}\right)}{E_{F_{st}}\cdot\exp\left(-\frac{\theta C_T}{\mu(1+\theta)} \mid T<\infty\right)} \leq \exp\left(-\frac{\theta C_0}{\mu(1+\theta)}\right)$$

Como r existe $\Rightarrow E_{F_{st}}\exp\left(\frac{-\theta C_T}{\mu(1+\theta)} \mid T<\infty\right) > 1$

$\Rightarrow$ la cota superior a $\Psi_r(C_0)$ es $\exp\left(\frac{-\theta C_0}{\mu(1+\theta)}\right)$

② $F_X(x) = \text{Pareto}(\alpha)$, $f_X(x) = (1+x)^{-\alpha} \mathbb{1}_{(0,\infty)}(x)$, $\alpha>0$

Mostrar que para $\alpha>0$, $\int_0^\infty \exp\langle rx \rangle F_X(dx)$ está indefinida.

Sabemos que:

$$F_X(x) = \int_0^x (1+s)^{-\alpha} ds = \int_1^{1+x} u^{-\alpha} du = \left[\frac{u^{-\alpha+1}}{-\alpha+1} \right]_1^{1+x}$$

$\boxed{u = 1+s \quad s = u-1 \quad \frac{du}{ds} = 1}$

$$= \frac{(1+x)^{-\alpha+1}}{-\alpha+1} - \frac{1}{-\alpha+1} = \frac{(1+x)^{-\alpha+1} - 1}{-\alpha+1}$$

$$= \frac{(1+x)^{1-\alpha} - 1}{1-\alpha}$$

Ahora,

$$\int_0^\infty \frac{e^{rx}[(1+x)^{1-\alpha} - 1]}{1-\alpha} dx = \underbrace{\int_0^\infty \frac{e^{rx}(1+x)^{1-\alpha}}{1-\alpha} dx}_{①} - \underbrace{\int_0^\infty \frac{e^{rx}}{1-\alpha} dx}_{②}$$

$$②: \int_0^\infty \frac{e^{rx}}{1-\alpha} dx = \frac{1}{r(1-\alpha)} \int_0^\infty r e^{rx} dx = \frac{1}{r(1-\alpha)} \left[e^{rx} \right]_0^\infty$$

$$\underset{\boxed{r>0}}{=} \frac{1}{r(1-\alpha)} \left[\infty - 1 \right] \longrightarrow \infty \quad (\text{converge})$$

$\therefore$ Como una parte de la integral converge, la integral completa no está definida.