TAREA 6

Calcula la prob de ruina $\psi(c_0)$ cuando $X \sim Exp(x|1/\mu)$ con $E(X) < \mu$

Función generadora $M_x(t) = (1-\mu t)^{-1}$; $t < \mu^{-1}$ para que esté definida

$\Rightarrow$ coef. Lundberg. $M_x(r) = 1 + (1+\theta)\mu r$

$$M_x(r) = 1 + (1+\theta)\mu r = (1-\mu r)^{-1} \qquad r < \mu^{-1}$$

$$(1-\mu r)(1+(1+\theta)\mu r) = 1$$
$$1 + (1+\theta)\mu r - \mu r - (1+\theta)\mu r^2 = 1$$
$$-(1+\theta)(\mu r)^2 + \theta\mu r + 0 = 0$$

$$r = \frac{\mu\theta \pm \sqrt{\theta^2\mu^2 - 4(-(1+\theta))\cdot 0 \cdot \mu}}{2(-(1+\theta)\mu)} = \frac{\mu\theta \pm \theta\mu}{-2(1-\theta)\mu}$$

$$r = \frac{2\mu\theta}{-2(1-\theta)\mu} = \frac{\theta}{-(1-\theta)} \Rightarrow \psi(c_0) = \frac{e^{\frac{\theta}{1-\theta}c_0}}{E_\pi(e^{\frac{\theta}{1-\theta}r} \mid T<\infty)}$$
$$\leq \exp\left(\left(\frac{\theta}{(1-\theta)}\right)c_0\right)$$

Considera $F_x(x) = Pa(x|\alpha)$ con densidad $f_x(x) = (1+x)^{-\alpha}\, \mathbb{1}_{(0,\infty)}$ y $\alpha > 0$. Muestra que para $\alpha > 0$ no está definida

$$F_x(x) = \int_0^x (1+x)^{-\alpha}\,dx = \int_1^{1+x} \frac{1}{(1+x)^\alpha}\,dx \qquad a = 1+x \quad da = dx$$

$$= \int_1^{1+x} \frac{1}{a^\alpha}\,da = \int_1^{1+x} a^{-\alpha}\,da = \left.\frac{a^{-\alpha+1}}{-\alpha+1}\right|_1^{1+x} = \frac{(1+x)^{-\alpha+1}}{-\alpha+1} - \frac{1^{-\alpha+1}}{-\alpha+1}$$

$$= \frac{1}{-\alpha+1}\left((1+x)^{-\alpha+1} - 1^{-\alpha+1}\right)$$

ADEMÁS

$$\int_0^\infty \exp^{(r)(x)} \left[\frac{1}{-\alpha+1} \left((1+x)^{-\alpha+1} - 1^{-\alpha+1} \right) \right] dx$$

$$= \frac{1}{-\alpha+1} \int_0^\infty e^{rx} \left((1+x)^{-\alpha+1} - 1^{-\alpha+1} \right) dx$$

$$= \frac{1}{-\alpha+1} \left[\int_0^\infty e^{rx} (1+x)^{-\alpha+1}\, dx - \int_0^\infty 1^{-\alpha+1}\, dx \right]$$

$$= \frac{1}{-\alpha+1} \left[\int_0^\infty e^{rx} (1+x)^{-\alpha+1} dx - (1^{-\alpha+1}) \cdot x \Big|_0^\infty \right]$$