Tarea 6
143751

① Calculo de R de ruina $\psi[C_0]$ si $X \sim exp(X|1/\mu)$ con $E[X]=\mu$.

Datos

- la $F_X(x) = 1 - e^{-x/\mu}$, $x \geq 0$
- coef. de lündberg es: $1+(1+\theta)\mu r = M_X(r)$
- $M_X(r) = (1-\mu r)^{-1}$, $t < 1/\mu$

Si igualas a despejas $\Rightarrow$

$$1+(1+\theta)\mu r = (1-\mu r)^{-1}$$

$$1+(\mu+\mu\theta)r = (1-\mu r)^{-1}$$

$$1+(\mu+\mu\theta)r - (1-\mu r)^{-1} = 0$$

$$1-\mu r+(1-\mu r)(\mu+\mu\theta)r - 1 = 0$$

$$1-\mu r+\mu r+\mu\theta r-\mu^2r^2-\mu^2r^2\theta - 1 = 0$$

$$\mu\theta r - \mu^2 r^2 - \mu^2 r^2\theta = 0$$

$$\theta - \mu r - \mu r\theta = 0$$

$$r = \frac{\theta}{\mu(1+\theta)} \Rightarrow r\ \exists \Rightarrow$$

$$\psi[C_0] = \frac{exp\{-rC_0\}}{\mathbb{E}_{F_{S_t}}[exp\{-r(\tilde{U})\}|T<\infty]} \quad \exists$$

Por convoluciones, la Σ de n variables aleat que se distribuyen exponencial se distribuye Gamma(n, μ) $\Rightarrow$

$$F(x) = 1 - \sum_{k=0}^{n-1} e^{-x/\mu} \cdot \frac{\left(\frac{x}{\mu}\right)^k}{k!} \quad x > 0$$

$$\Rightarrow 1 - \psi[C_0] = \frac{\theta}{1+\theta} \sum_{n=0}^{\infty} \frac{1}{(1+\theta)^n} F^{(n)*}(C_0)$$

$$= \left[\frac{\theta}{1+\theta}\right]\left[1 + \sum_{n=1}^{\infty} \frac{1}{(1+\theta)^n} \cdot \left[1 - \sum_{k=0}^{n-1} e^{-C_0/\mu} \cdot \frac{\left(\frac{C_0}{\mu}\right)^k}{k!}\right]\right]$$

② Considere $F_X(x) = Pa(x|\alpha)$ con $f_X(x) = (1+x)^{-\alpha}$ $\mathbb{I}_{(0,\infty)}(x)$ y $\alpha > 0$. Muestre que la Inecuación 2.4 no está definida

$$F_X = \int_0^x f_\alpha(\alpha)\,d\alpha = \int_0^x (1-u)^\alpha\, \mathbb{I}_{(0,\infty)}(u)\,du \qquad \begin{matrix} v = 1-u \\ dv = -du \end{matrix}$$

$$= \int v^\alpha\,dv = \frac{-v^{\alpha+1}}{\alpha+1} \Rightarrow \left.\frac{-(1-u)}{\alpha+1}\right|_0^x = \frac{1}{\alpha+1}\left[1-(1-x)^{\alpha+1}\right]$$

$$\Rightarrow \int_0^\infty \exp^{r(x)} \left[1-(1-x)^{\alpha+1}\right]\left[\frac{1}{\alpha+1}\right]dx$$

$$= \frac{1}{\alpha+1} \int_0^\infty \exp^{rx}\left[1-(1-x)^{\alpha+1}\right]dx$$

diverge

$$= \int_0^\infty e^{rx}\,dx - \int_0^\infty e^{rx}(1-x)^{\alpha+1}\,dx = \text{no } \exists$$