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Specification and Estimation of Bayesian Dynamic Factor Models: A Monte Carlo Analysis with an Application to Global House Price Comovement *

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Abstract

We compare methods to measure comovement in business cycle data using multi-level dynamic factor models. To do so, we employ a Monte Carlo procedure to evaluate model performance for different specifications of factor models across three different estimation procedures. We consider three general factor model specifications used in applied work. The first is a single-factor model, the second a two-level factor model, and the third a three-level factor model. Our estimation procedures are the Bayesian approach of Otrok and Whiteman (1998), the Bayesian state space approach of Kim and Nelson (1998) and a frequentist principal components approach. The latter serves as a benchmark to measure any potential gains from the more computationally intensive Bayesian procedures. We then apply the three methods to a novel new dataset on house prices in advanced and emerging markets from Cesa-Bianchi, Cespedes, and Rebucci (2015) and interpret the empirical results in light of the Monte Carlo results. [JEL codes: C3]

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1 Introduction

Dynamic factor models have gained widespread use in analyzing business cycle comovement. The literature began with the Sargent and Sims (1977) analysis of U.S. business cycles. Since then the dynamic factor framework has been applied to a long list of empirical questions. For example, Engle and Watson (1981) study metropolitan wage rates, Forni and Reichlin (1998) analyze industry level business cycles, Stock and Watson (2002) forecast the U.S. economy, while Kose, Otrok and Whiteman (2003) study international business cycles. It is clear that dynamic factor models have become a standard tool to measure comovement, a fact that has become increasingly true as methods to deal with large datasets have been developed and the profession has gained interest in the "Big Data" movement.

Estimation of this class of models has evolved significantly since the original frequency domain methods of Geweke (1977) and Sargent and Sims (1977). Stock and Watson (1989) adopted a state-space approach and employed the Kalman filter to estimate the model. Stock and Watson (2002) utilized a two-step procedure whereby the unobserved factors are computed from the principal components of the data. Forni, Hallin, Lippi, and Reichlin (2000) compute the eigenvector-eigenvalue decomposition of the spectral density matrix of the data frequency by frequency, inverse-Fourier transforming the eigenvectors to create polynomials which are then used to construct the factors. This latter approach is essentially a dynamic version of principal components. A large number of refinements to these methods have been developed for frequentist estimation of large-scale factor models since the publication of these papers.

A Bayesian approach to estimating dynamic factor models was developed by Otrok and Whiteman (1998), who employed a Gibbs sampler. The key innovation of their paper was to derive the distribution of the factors conditional on model parameters that is needed for the Gibbs sampler. Kim and Nelson (1998) also developed a Bayesian approach using a state-space procedure that employs the Carter-Kohn approach to filtering the state-space model. The key difference between the two approaches is that the Otrok-Whiteman procedure can be applied to large datasets, while, because of computational constraints, the Kim-Nelson method cannot. The Bayesian approach in both papers is particularly useful when one wants to impose 'zero' restrictions on the factor loading matrix to identify group specific factors. In addition, both approaches, because they are Bayesian, draw inference conditional on the size of the dataset at hand; the classical approaches discussed above generally rely on asymptotics. While this is a not a problem when the factors are estimated on large datasets, for smaller datasets—or multi-level factor models where some levels have few time series, it may be problematic. Lastly, the Bayesian approach is the only framework that can handle the case of multi-level factor models when the variables are not assigned to groups a priori (e.g., Francis, Owyang, and Savasçin 2014).

In this paper, we compare the accuracy of the two Bayesian approaches and a multi-step principal components estimator. In particular, we are interested in the class of multi-level factor models where one imposes various 'zero' restrictions to identify group-specific factors (e.g. regional factors). To be concrete, we will label these models as in the international business cycle literature, although the models have natural applications to multi-sector closed economies or to models that mix real and financial variables. We perform Monte Carlo experiments using three different models of increasing complexity. The first model is the ubiquitous single factor model. The second is a two-level factor model that we interpret as a world-country factor model. In this model, one (world) factor affects all of the series; the other factors affect non-overlapping subsets of the series. The third is a three-level factor model that we interpret as world-region-country factor model.

For each model, we first generate a random set of model coefficients. Using the coefficients we generate 'true' factors and data sample. We then apply each estimation procedure to the simulated data to extract factors and model coefficients. We then repeat this sequence many times, starting with a new draw for the model parameters each time. The Bayesian estimation approach is a simulation based Markov Chain Monte Carlo (MCMC) estimator, making the estimate of one model non-trivial in terms of time; however, modern computing power makes Monte Carlo study of Bayesian factor models feasible.

In this sense, our paper provides a complementary study to Breitung and Eickmeier (2014) who employ a Monte Carlo analysis of various frequentist estimators of multi-level factor models with their new sequential least squares estimator. There are three key differences in our Monte Carlo procedures with that of Breitung and Eickmeier (2014). First, they study a fixed and constant set of parameters. As they note in their paper, the accuracy of the factor estimates can depend on the variance of the factors (or more generally the signal to noise ratio). To produce a general set of results that abstracts away from any one or two parameter settings, we randomly draw new parameters for each simulation. A second difference is that the number of observations in each of the levels of their factor model is always large enough to expect the asymptotics to hold. In our model specification, we combine levels where the cross-sections are both large and small, which is often the case in applied work. Third, we include in our study measures of uncertainty in factor estimates while Breitung and Eickmeier (2014) focus on the accuracy of the mean of an estimate. Taken together the two papers provide a comprehensive Monte Carlo analysis of the accuracy of a wide range of the procedures used for a number of different model specifications and sizes.

Our evaluation mainly focuses on the three key features of the results that are important in applied work with factor models. The first is the accuracy of the approaches in estimating the 'true' factors as measured by the correlation of the posterior mean factor estimate with the truth. The second is the extent to which the methods characterize the amount of uncertainty in factor estimates. To do so, we measure the width of the posterior coverage interval as well as count how many times the true factor lies in the posterior coverage interval. The third is the correspondence of the estimated variance decomposition with the true variance decomposition implied by the population parameters.[1] In simulation work, we compare two ways to measure the variance decomposition in finite samples. The first takes the estimated factors, orthogonalizes them draw-by-draw, and computes the decomposition based on a regression on the orthogonalized factors (i.e., not the estimated factor loadings).[2] The second takes each draw of the model parameters and calculates the implied variance decomposition. While the factors are assumed to be orthogonal, this is not imposed in the estimation procedures, which could bias a model where the factors have some correlation in finite samples.

We find that, for the one factor model, the three methods do equally well at estimating a factor that is correlated with the true factor. For models with multiple levels, however, the Kalman-filtered state-space method typically does a better job at identifying the true factor. As the number of levels increases, the Otrok-Whiteman procedure—which redraws the factor at each Gibbs iteration—estimates a factor more highly correlated with the true factor than does PCA, which estimates the factor ex ante. We find that both the state-space and Otrok-Whiteman procedures provide fairly

[1] One could also consider the accuracy of other model parameters. However, factor analysis has tended to focus on the variance decomposition because it is this output that is most useful in telling an economic story about the data. In addition, since the scale of a factor model is not identified, the factor loading is not as of as much interest as the scale independent variance decomposition.

[2] This is the procedure in Kose, Otrok and Whiteman (2003, 2008), and Kose, Otrok Prasad (2012).

accurate, albeit conservative, estimates of the percentage of the total variance explained by the factors. PCA, on the other hand, tends to overestimate the contribution of the factors.

When we apply the three procedures to house price data in advanced and emerging markets we find that there does exist a world house price cycle that is both pervasive and quantitatively important. We find less evidence of a widely important additional factor for advanced economies or or for emerging markets. Consistent with the Monte Carlo results we find that all three methods deliver the same global factor. We also find that the Kalman Filter and Otrok-Whiteman procedures deliver similar regional factors, which is virtually uncorrelated with the PCA regional factor. The PCA method provides estimates of variance decompositions that are greater than the Bayesian procedures, which is also consistent with the Monte Carlo evidence. Lastly, the parametric variance decompositions are uniformly greater than the factor based estimates, which is also consistent with the Monte Carlo evidence.

The outline of this chapter is as follows: Section 2 describes the empirical model and outlines its estimation using the three techniques—a Bayesian version of principal components analysis, the Bayesian procedure of Otrok and Whiteman, and a Bayesian version of the state-space estimation of the factor—we study. Section 3 outlines the Monte Carlo experiments and describes the methods we use to evaluate the three methods. In this section, we also present the results from the Monte Carlo experiments. Section 4 applies the methods to a dataset on house prices in Advanced and Emerging Market Economies. Section 5 offers some conclusions.

2 Specification and Estimation of the Dynamic Factor Model

In the prototypical dynamic factor model, all comovement among variables in the dataset is captured by a set of M latent variables, F_t. Let Y_t denote an $(N \times 1)$ vector of observable data. The dynamic factor model for this set of time series can be written as:

$$Y_t = \beta F_t + \Gamma_t, \tag{1}$$

$$\Gamma_t = \Psi(L)\,\Gamma_{t-1} + U_t, \tag{2}$$

with $E_t\left(U_t U_t'\right) = \Omega$,

$$F_t = \Phi\left(L\right) F_{t-1} + V_t, \tag{3}$$

with $E_t\left(V_t V_t'\right) = I_M$. Vector Γ_t is a $(N \times 1)$ vector of idiosyncratic shocks which captures movement in each observable series specific to that time series. Each element of Γ_t is assumed to follow an independent AR(q) process, hence $\Psi(L)$ is a block diagonal lag polynomial matrix and Ω is a covariance matrix that is restricted to be diagonal. The latent factors are denoted by the $(M \times 1)$ vector F_t, whose dynamics follow an AR(p) process. The $(N \times M)$ matrix β contains the factor loadings which measure the response (or sensitivity) of each observable variable to each factor. With estimated factors and factor loadings, we are then able to quantify the extent to which the variability in the observable data is common. Our one factor model sets β to a vector of length M, implying all variables respond to this factor.

In multiple factor models, it is often useful to impose zero restrictions on β in order to give an economic interpretation to the factors. The Bayesian approach also allows (but does not require) the imposition of restrictions on the factor loadings such that the model has a multi-level structure as a special case. For example, Kose, Otrok, and Whiteman (2008) impose zero restrictions on β to separate out world and country factors. They use a dataset on output, consumption and investment for G-7 countries to estimate a model with 1 common (world) factor and 7 country-specific factors. Identification of the country factors is obtained by only allowing variables within each country to load on a particular factor, which we then label as the country factor. For the G-7 model, the β matrix (of dimension 21×24 when estimating the model with 3 dataseries) is:

$$
\begin{bmatrix}
\beta^{G7}_{US,Y} & 0 & 0 & \beta^{US}_{US,Y} & 0 & 0 & 0 & \cdots & 0 \\
\beta^{G7}_{US,C} & 0 & 0 & \beta^{US}_{US,C} & 0 & 0 & 0 & \cdots & 0 \\
\beta^{G7}_{US,I} & 0 & 0 & \beta^{US}_{US,I} & 0 & 0 & 0 & \cdots & 0 \\
\beta^{G7}_{Fr,Y} & 0 & 0 & 0 & 0 & 0 & \beta^{Fr}_{Fr,Y} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\beta^{G7}_{UK,Y} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \beta^{UK}_{UK,Y} \\
\beta^{G7}_{UK,C} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \beta^{UK}_{UK,C} \\
\beta^{G7}_{UK,I} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \beta^{UK}_{UK,I}
\end{bmatrix}.
$$

Here, all variables load on the first (world) factor while only U.S. variables load on the second (U.S. country) factor.

The three-level model adds an additional layer to the model to include world, region, and country-level factors. In this setup all countries within a given region load on the factor specific to that region in addition to the world and country factors. The objective of all three econometric procedures is to estimate the factors and parameters of this class of models as accurately as possible.

2.1 The Otrok-Whiteman Bayesian Approach

Estimation of dynamic factor models is difficult when the factors are unobservable. If, contrary to assumption, the dynamic factors were observable, analysis of the system would be straightforward; because they are not, special methods must be employed. Otrok and Whiteman (1998) developed a procedure based on an innovation in the Bayesian literature on missing data problems, that of "data augmentation" (Tanner and Wong, 1987). The essential idea is to determine posterior distributions for all unknown parameters conditional on the latent factor and then determine the conditional distribution of the latent factor given the observables and the other parameters. That is, the observable data are "augmented" by samples from the conditional distribution for the factor given the data and the parameters of the model. Specifically, the joint posterior distribution for the unknown parameters and the unobserved factor can be sampled using a Markov Chain Monte Carlo procedure on the full set of conditional distributions. The Markov chain samples sequentially from the conditional distributions for (parameters|factors) and (factors|parameters) and, at each stage, uses the previous iterate's drawing as the conditioning variable, ultimately yields drawings from the joint distribution for (parameters, factors). Provided samples are readily generated from each conditional distribution, it is possible to sample from otherwise intractable joint distributions. Large cross-sections of data present no special problems for this procedure since natural ancillary assumptions ensure that the conditional distributions for (parameters|factors) can be sampled equation by equation; increasing the number of variables has a small impact on computational time.

When the factors are treated as conditioning variables, the posterior distributions for the rest of the parameters are well known from the multivariate regression model; finding the conditional distribution of the factor given the parameters of the model involves solving a "signal extraction" problem. Otrok and Whiteman (1998) used standard multivariate normal theory to determine the conditional distribution of the entire time series of the factors, $(F_1, ..., F_T)$ simultaneously. Details on these distributions are available in Otrok and Whiteman (1998). The extension to multi-level models was developed in Kose, Otrok and Whiteman (2003). Their procedure samples the factor with a sequence of factors by level. For example, in the world-country model we first sample from the conditional distribution of (world factor|country factors, parameters), then from the conditional distribution of (country factors|world factor, parameters).

It is important to note that in the step where the unobserved factors are treated as data, the Gibbs sampler does in fact take into account the factor estimates' uncertainty when estimating the parameters. This is because we sequentially sample from the conditional posteriors a large number of times. In particular, when the cross-section is small, the procedure will accurately measure uncertainty in factor estimates, which will then affect the uncertainty in the parameters estimates. A second important feature of the Otrok and Whiteman procedure is that it samples from the conditional posteriors of the parameters sequentially by equation; thus, as the number of series increases, the increases in computational time is only linear.

2.2 The Kim-Nelson Bayesian State-Space Approach

A second approach to estimation follows Kim and Nelson (1998). As noted by Stock and Watson (1989), the set of equations (1) – (3) comprises a state-space system where (1) corresponds to the measurement equation and (2) and (3) corresponds to the state transition equation. One approach to estimating the model is to use the Kalman filter. Kim and Nelson instead combine the state-space structure with a Gibbs sampling procedure to estimate the parameters and factors. To implement this idea, we use the same conditional distribution of parameters given the factors as in Otrok and Whiteman (2008). This allow us to focus on the differences in drawing the factors across the two Bayesian procedures. To draw the factors conditional on parameters, we use the Kim-Nelson state-space approach.

In the state-space setup, the F_t vector contains both contemporaneous values of the factors as well as lags. The lags of the factor enter the state equation (3) to allow for dynamics in each factor. Let M be the number of factors $(M < N)$ and p be the order of the autoregressive process each factor follows, then we can define $k = Mp$ as the dimension of the state vector. F_t is then an $(k \times 1)$ vector of unobservable factors (and its lags) and $\Phi(L)$ is a matrix lag polynomial governing the evolution of these factors.

Two issues arise concerning the feasibility of sampling from the implied conditional distribution. The first has to do with the structure of the state space for higher-order autoregressions; the second has to do with the dimension of the state in the presence of idiosyncratic dynamics. To understand the first issue, note that, because the state is Markov, it is advantageous to carry the sequential conditioning argument one step further: Rather than drawing simultaneously from the distribution for $(F_1, ..., F_T)$, one samples from the $T-$conditional distributions $(F_j | F_1, ..., F_{j-1}, F_{j+1}, ..., F_T)$ for $j = 1, \ldots, T$. If F_t itself is autoregressive of order 1, then only adjacent values matter in the conditional distribution, which simplifies matters considerably.

When the factor itself is of a higher order, say an autoregression of order $p^{\dagger}$, one defines a new $p^{\dagger}$-dimensional state $X_t = [F_t, F_{t-1}, \ldots, F_{t-p^{\dagger}+1}]$, which in turn has a first-order vector autoregressive representation. The issue arises in the way the sequential conditioning is done in sampling from the distribution for the factor. Note that in $(X_t | X_{t-1}, X_{t+1})$, there is in fact no uncertainty at all about X_t. Samples from this sequence of conditionals actually only involve factors at the ends of the data set. Thus, this "single move" sampling (a version of which was introduced Carlin, Polson, and Stoffer, 1992) does not succeed in sampling from the joint distribution in cases where the state has been expanded to accommodate lags. Fortunately, an ingenious procedure to carry out "multimove" sampling was introduced by Carter and Kohn (1994). Subsequently more efficient multimove samplers were introduced by de Jong and Sheppard (1995) and Durbin and Koopman (2002). We follow Kim and Nelson (1998) in their Bayesian implementation of a dynamic factor model and use Carter and Kohn (1994). In our analysis of the three econometric procedures we will not be focusing on computational time.

The second issue arises because, while the multimode samplers solves the "big-T" curse of dimensionality, it potentially reintroduces the "big-N" curse when the cross section is large. The reason is that the matrix calculations in the algorithm may be of the same dimension as that of the state vector. When the idiosyncratic errors u_t have an autoregressive structure, the natural formulation of the state vector involves augmenting the factor(s) and their lags with contemporaneous and lagged values of the errors (see Kim and Nelson, 1998; 1999, chapter 3). For example, if each observable variable is represented using a single factor that is AR(p) and an error that is AR(q), the state vector would be of dimension $p + Nq$, which is problematic for large N.

An alternative formulation of the state due to Quah and Sargent (1993) and Kim and Nelson (1999, chapter 8) avoids the "big-N" problem by isolating the idiosyncratic dynamics in the observation equation. To see this, suppose we have N observable variables, y_n for $n = 1, ..., N$, and M unobserved dynamic factors, f_m for $m = 1, ..., M$, which account for all of the comovement in the observable variables. The observable time series are described by the following version of (1):

$$y_{n,t} = a_n + b_n f_t + \gamma_{nt}; \tag{4}$$

where

$$\gamma_{nt} = \psi_{n,1}\gamma_{n,t-1} + \ldots + \psi_{n,q}\gamma_{n,t-q} + u_{nt} \tag{5}$$

with $u_{nt} \sim iidN(0, \sigma_n^2)$. The factors evolve as independent AR(p) processes:

$$f_{mt} = \phi_{m1} f_{m,t-1} + ... + \phi_{mp} f_{m,t-p} + v_{mt}, \tag{6}$$

where $v_{mt} \sim iidN\,(0, 1)$. Suppose for illustration that $M = 1$ and $q \geq p$. The "big-N" version of the state space form for (3)-(5) is

$$Y_t = HF_t, \tag{7}$$

$$F_t = BF_{t-1} + E_t, \tag{8}$$

where $Y_t = (y_{1t}, ..., y_{nt})'$,

$$E_t = (u_t, 0, ..., 0, u_{1,t}, 0, ..., 0, u_{2,t}, ..., 0)',$$

and

$$F_t = \left[f_t, f_{t-1}, ..., f_{t-p+1}, \gamma_{1,t}, \gamma_{1,t-1}, ..., \gamma_{n,t}, \gamma_{n,t-1}, ..., \gamma_{n,t-p+1}\right]'.$$

Here, B is block diagonal with the companion matrix having first row $\phi_1, \phi_2, ..., \phi_p$ in the $(1,1)$ block; the companion matrix with first row $\psi_{11}, \psi_{12}, ..., \psi_{1n}$ in the $(2,2)$ block; etc.; with the companion matrix having first row $\psi_{n1}, ..., \psi_{nq}$ in the southeastern-most block. The matrix H is 0 except for $(b_1, ..., b_n)'$ in the first column, and 1's in the columns and rows corresponding to $\gamma_{1,t}$, $\gamma_{2,t}$, etc. in F_t.

Alternatively, a system with a lower-dimension state can be obtained by operating on both sides of (4) by $\left(1 - \psi_{n,1} - ... - \psi_{n,q}L^{t-q}\right)$ to get

$$y^*_{n,t} = a^*_n + b_n(\left(1 - \psi_{n,1} - ... - \psi_{n,q}L^{t-q}\right) f_t + u_{nt}, \tag{9}$$

where $y^*_{n,t} = y_{n,t} - \psi_{n,1}y_{n,t-1} - \ldots - \psi_{n,q}y_{n,t-q}$, $a^*_n = \left(1 - \psi_{n,1} - ... - \psi_{n,q}\right) a_n$. This yields the state-space system

$$Y^*_t = A^*D_t + H^*F_t + U_t, \tag{10}$$

$$F_t = BF_{t-1} + E_t, \tag{11}$$

where $Y^*_t = (y^*_{1t}, \ldots, y^*_{nt})'$, $U_t = (u_{1t}, u_{2t}, \ldots, u_{nt})'$, $F^*_t = (f_t, f_{t-1}, \ldots, f_{t-p})'$, $E^*_t = (e_t, 0, \ldots, 0)'$, the nth row of H^* is $\left(b_n, -\psi_{n,1}b_n, ..., -\psi_{n,q}b_n\right)$, and B^* has $\phi_1, \phi_2, ..., \phi_q, 0, ..., 0$ in the first row and 1's on the first subdiagonal. (The extra $q - p + 1$ columns of zeros in B^* accommodate the lags of the factor introduced into the measurement equation by the transformation to serially uncorrelated residuals.) This model has a $l \times m$ dimensional state vector, so N no longer impacts the size of the state vector. Jungbacker, Koopman and van der Wal (2011) have a discussion comparing the state space formulation with quasi-differencing (as above) with a formulation that adds the idiosyncratic error terms directly into the state vector. As they note, both formulations will lead to the same answer if the filters are properly normalized.

2.3 Principal Components

A third approach to estimating the latent factors employs Principal Components Analysis (hereafter, PCA) which solves an eigenvector-eigenvalue problem to extract the factors before conditioning on said factors. The latter conditioning step treats the extracted factors as observable variables. Thus, PCA identifies the common movements in the cross-sectional data without imposing any additional model structure. The advantage of PCA is that it is simple to use and has been shown, under certain conditions, to produce a consistent approximation of the filtered (i.e., direct) estimate of the factors. An expansive literature has assessed the potential applications of factor modelling techniques and principal components analysis. Bai and Ng (2008) gives a detailed survey of the asymptotic properties of static factor models and dynamic factor models expressed in static form. Stock and Watson (2002) delivers key results suggesting that the method of asymptotic PCA consistently estimates the true factor space.

Consider the collection of data at time t, $Y_t = (y_{1t}, ..., y_{Nt})'$ to be a random $(N \times 1)$ vector with sample mean and covariance $\overline{y}$ and S, respectively. Normalizing the data to have mean zero, PCA results in the transformation:

$$F_{t,(i)} = (Y_t - \mathbf{1}\overline{y}')g_{(i)},$$

$i = 1, ..., N$ and where 1 is an $(N \times 1)$ vector of ones. The vector $g_{(i)}$ denotes the standardized eigenvector corresponding to the i^{th} largest eigenvalue of S $(S = GVG')$ where V is a matrix with the eigenvalues of S in descending order along the diagonal. G is an orthogonal matrix of principal component loadings with columns $g_{(i)}$. Thus, $g_{(1)}$ corresponds to the largest eigenvalue associated with the first principal component $F_{t,(1)} = (Y_t - 1\overline{y}')g_{(1)}$.

When extracting static principal components, we find the standardized eigenvalues of the sample covariance matrix and treat the corresponding standardized eigenvectors as the factor loadings relating the static factors to the observable data. In order to impose the structure associated with world, region, and country factors, we extract each factor from the data assumed to load upon each factor. For the multi-level factor models, we extract the first factor, F^W, with its associated eigenvector, g_W, from the entire $(T \times N)$ dataset. Subsequently, letting Y_{n_m} for $n_m = 1, ..., m$ correspond to the observable series which load upon the second-level factor m, we adjust the data to remove the first factor: $\widetilde{Y}_{n_m} = Y_{n_m} - F^W g_W$. Next, we extract the first principal component from this set of $\widetilde{Y}_{n_m}$, $n_m = 1, ..., m$. We perform a similar adjustment for the three-level factor model. Note that uncertainty in estimating F^W is not taken into account in this procedure.

These standardized principal component estimates serve as the latent factor estimates. As in the previous estimation methods, we condition on the latent factor estimates and apply Bayesian estimation to obtain the parameters of the model. However, using the PCA method, we extract the principal components outside of the Gibbs sampler and then treat the unobserved factors as data when we sample from the conditional posterior distributions of the parameters. By construction then, the PCA approach will underestimate the uncertainty in variance decompositions.

3 Monte Carlo Evaluation

The three methods presented in the previous section are evaluated using Monte Carlo experiments. For each of the three models (1-factor, 2-factor, 3-factor), we generate 1000 sets of true data that includes a set of true factors by simulating U_t and V_t from multivariate normal distributions and then applying equations (1) - (3). The true data consist of 100 time series observations forming a balanced panel. For the one factor case, we generate 21 series of data. For the two factor case, we generate 3 series of data for each of 7 countries. For the three factor case, we generate a small-region model with 3 series of data for each of 8 countries broken up into 2 equally-sized regions. Additionally, we generate a large-region model with 3 series of data for each of 16 countries, again broken into 2 equally-sized regions.

In order to assess the methods across a wide range of model parameterizations, we redraw the model parameters at each Monte Carlo iteration. The covariance matrices of each of the innovation processes are fixed. All shocks are normalized to have unit variance and are assumed orthogonal. The AR parameters are drawn from univariate normal distributions with decreasing means for higher lag orders:

$$\Phi_{mi} \sim N\left(0.15(p+1-i), (0.1)^2\right),$$
$$\Psi_{ni} \sim N\left(0.15(q+1-i), (0.1)^2\right),$$

where p and q are the lag orders of factor and innovation AR processes, respectively. The AR parameters for each process are constrained to be stationary; we redraw the parameters if stationarity of the lag polynomial is violated. The factor loadings are also drawn from normal distributions:

$$\beta_{nk} \sim N\left(1, (0.25)^2\right),$$

where the multi-level model zero restrictions on the factor loadings are appropriately applied.

We then estimate the model using the three methods described above. We assume that the number of factors and, in the three factor case, the number and composition of the regions are ex ante known.

3.1 Priors for Estimation

The estimation in our Monte Carlo exercises are Bayesian and requires a prior. The prior for the model parameters are generally weakly informative, with the exception that we impose stationarity on the dynamic components. In the dynamics for the observable data, the prior on the constant, β_{n0}, and each factor's loading, β_{nm}, for $m = 1, ..., M$, for each country $n = 1, ..., N$ is:

$$\begin{bmatrix} \beta_{n0} & \beta_{n1} & \cdots & \beta_{nM} \end{bmatrix}' \sim N\left(\mathbf{0}_{(M+1)x1}, diag\left(\left[1, 10 \times \mathbf{1}_{(1\times M)}\right]\right)\right), \tag{12}$$

where $1_{(1\times M)}$ is a $(1 \times M)$ vector of ones. The prior for the autoregressive parameters of the factors and of the innovations in the measurement equation are truncated normal. In particular, the AR parameters are assumed to be a multivariate standard normal truncated to maintain stationarity. The prior for the innovation variances in the measurement equation is Inverted Gamma, parameterized as $IG(0.05 * T, 0.25^2)$. The factor innovation is normalized to have unit variance and is not estimated. These priors are fairly diffuse. In work on actual datasets (e.g. Kose, Otrok and Whiteman 2008), we have experimented with different priors and we did not find the results to be sensitive to changes in these priors. Including sensitivity to the prior in our Monte Carlo work would be computationally very expensive.

3.2 Accuracy of Factor Estimates

The first metric for assessing the three estimation methods outlined above is to determine the accuracy of the factor estimates. For each simulation, we have the true values of the factors. In estimation, for each iteration of the Gibbs sampler, we produce a draw from the conditional distribution of the factor. To assess the accuracy of each method, we compare the correlation of the true factor with each draw of the sampler to form a distribution for the correlation. Figure 1 shows the distribution of this correlation for the world factor in the 1-factor case. For the 2-factor case, we show the distribution of the correlation for the world factor in Figure 2. For the country factor, we compute the correlation for each country, then take the average correlation across countries and report this in one PDF in Figure 3. For the 3-factor case with small regions, we show the correlation distribution for the world factor in Figure 4, the average correlation across regional factors in Figure 5, and average across countries in Figure 6. For the 3-factor case with large regions, these same plots are shown in Figures 7-9.

In the 1-factor case, all three methods produce similar results with correlations very close to 1. However, when the model is extended to include additional factors, the accuracy of the factor estimates for each of the methods deteriorates, even for the factor estimated across all of the series (the world factor). In addition, for higher level models, the average correlation between the true factor and the estimated factor falls considerably. Because the world factor is computed with a larger number of series, we expect the factor to be estimated more accurately.

In each case, the factors that affect smaller number of series are more accurately estimated using the Kalman filter. In most cases, the Otrok-Whiteman procedure performs similarly to PCA. One difference occurs when estimating the country factor for the 3-factor model. In this case, the country factor is estimated with only 3 series with two additional layers (factors) contributing to the uncertainty of the estimates. In this case, the Otrok-Whiteman procedure outperforms PCA but continues to be outperformed by the state-space method. This last result may stem from the fact that the state space imposes orthogonality across the factor estimates, while Otrok-Whiteman procedure assumes it but does not impose it.

3.3 Uncertainty in Factor Estimates

A second method for evaluation of the estimation procedures outlined in the previous section is to determine the uncertainty in the estimates of the factor. To do this, we construct the average uncertainty in the factor estimates for the procedures. Our measure of average uncertainty is computed by determining the width of the 68- and 90-percent coverage intervals of the factor for each time period. These intervals are computed for each Monte Carlo iteration across all saved draws of the Gibbs sampler. We then compute the average of these intervals across time and across Monte Carlo iterations and report the numbers in the top panel of Table 1. In the principal components case, the factor is determined outside the Gibbs sampler, and, thus, does not have a small-sample uncertainty measure. We see this as a limitation of this approach as uncertainty in the factor estimate should be part of interpreting the importance of the factor. In each of the three models, the Otrok-Whiteman procedure has, on average, narrower coverages than the state-space method. In particular, the Otrok-Whiteman procedure yields about 20-30 percent narrower bands than the Kalman filter. Since there is no 'true' measure of uncertainty our interpretation is that there are precision gains associated with drawing the factors directly from their distribution as opposed to simulating them in state-space model.

3.4 Accuracy in Variance Decompositions

A third metric that can be used to evaluate the three estimation procedures is to analyze the accuracy of the variance explained by each of the estimated factors—i.e., do the estimated factors explain more or less of the variation explained by the true factors? We measure the variance decomposition in two ways. First, we compute a measure of the variance decomposition using a parameter-based method. This measure uses the draw of the set of model parameters at each Gibbs iteration to calculate the implied variance decomposition. The variance of the factor and idiosyncratic component is constructed using the parameters governing the time series process. These variances, along with the factor loadings are then used to construct the implied variance of the observable data and the resulting variance decomposition. This procedure relies on the accuracy of the estimates of the full set of the true parameters. In the second procedure, at each iteration of the Gibbs sampler, we orthogonalize the set of estimated factors and then compute the variance decomposition based on a regression of the observable data on the orthogonalized factors. This factor-based method is similar to the approach used in Kose, Otrok and Whiteman (2003, 2008), and Kose, Otrok, and Prasad (2012). Because the Otrok-Whiteman procedure does not impose orthogonality when estimating the factors themselves, the explained variances could be biased if the factors exhibit some finite sample correlation. The orthogonalization procedure addresses this issue.

To assess accuracy we first compute the posterior mean of the variance explained by each of the estimated factors across Gibbs iterations. Next, we compute the variance of the data explained by the true factors within each simulated dataset. Finally, we take the difference between the true and estimated variance decompositions for each Monte Carlo iteration. To measure the total size of the bias, we compute the absolute value of the bias. The top panel of Figure 10 plots the pdfs of the absolute value difference between the true and the estimated factor-based variance decomposition for all Monte Carlo iterations for the three estimation methods applied to the one-factor model. The Kalman filter and Otrok-Whiteman methods produce almost identical results with a nearly identical PDFs. The PCA approach has a distribution that lies to the right (larger absolute error). For parameter-based estimates, the bulk of the distribution is in the same area as the factor based, but has a noticeably larger tail. This larger tail indicates that the parameter-based estimates can occasionally yield large errors in the variance decomposition. Experience tells us that this is driven by nearly non-stationary parameter sets. Small variations in parameters can yield large difference in implied variance when near a unit root. In this sense, the factor-based estimates of variance decompositions are more robust to the underlying model parameters.

Table 2 shows that the mean of the absolute error in the variance decomposition between the true and estimated variance decompositions is 0.092 for the Kalman filter and Otrok-Whiteman methods. The PCA method slightly overestimates the variance attributed to the world factor and as a result, slightly underestimates the variance attributed to the idiosyncratic component. The mean of this distribution, 0.095, is slightly larger in magnitude than that of the other two methods, though the difference is negligible. Average errors, then, appear similar across methods. In terms of the sign of the bias, the factor-based estimates do better in all three cases .The Otrok-Whiteman method yields the same number of positive and negative biases regardless of method. For both the PCA and Kalman Filter methods, the number of positive/negative biases goes to a much more asymmetric distribution with a 70/30 split for the parametric case. Taking all these results together, it appears that the factor-based variance decompositions are less likely to lead to significantly wrong answers.

For the two-factor model, the top panel of Figure 11 illustrates that the Kalman filter and Otrok-Whiteman again produce similar results when using the factor-based variance decomposition. For the country factors, which are the addition relative to Figure 10, we see that the dispersion is less diffuse for all 3 methods with the factor based approach. With the parameter-based approach, the absolute errors are both larger in mean, more diffuse, and exhibits a larger right tail. This confirms the results of the one-factor model that the factor-based variance decompositions are more robust then then parametric. From Table 3, we can see that the three methods have similar mean absolute errors. However, in this case the PCA approach has more symmetric errors than either Bayesian which both have more positive errors than negative.

For the three-factor model with small regions, the top panel of Figure 12 shows the absolute biases resulting from the factor-based variance decomposition. Table 4 gives the mean and percent of positive and negative biases. For the world factor and the idiosyncratic component, the Kaman filter and Otrok-Whiteman methods produce very similar similar results for all three types of factors. The Bayesian approaches do better than PCA for all three layers of factors. This can all be seen by the means reported in Table 4. The differences in performance for this more complex model is striking. The mean bias for the world factor is 70% higher with PCA then the Kalman filter. The parametric estimates do better for the PCA case then the factor based estimates. A reversal of previous results. For the Bayesian approach, the factor based estimates were again superior. Table 5 and Figure 13 report the same facts for the larger country model. As can be seen, there is no real difference in the relative performance of the procedures. This indicates that additional cross-section data will likely not change are results, or in the case of the PCA method the smaller model is big enough for asymptotics to apply.

4 An Application to Global House Prices

The recent financial crisis was centered around a global housing price collapse. This has heightened interest in the nature of house price fluctuations. Most work on the issue has been at the national level. One recent exception is Hirata, Kose, Otrok, and Terrones (2013) who use principal-component–based factor models to study house price for a panel of advanced economies. A new dataset by Cesa-Bianchi, Cespedes, and Rebucci (2015) develops a set of comparable house prices for both emerging markets and advanced economies. Their contribution is to build the dataset, analyze the dataset in terms of moments of house prices and their comovement with the macroeconomy. They then use a panel VAR to understand the differential role of liquidity shocks on house prices across countries in regions defined as advanced and emerging. Here, we extend their work by using multi-layer factor models to measure the importance of world and regional cycles in house prices across advanced and emerging markets. Following the theme of this paper, we will apply all three methods to estimate the factors.

In Figure 14, we plot estimates of the world factor for house prices. The global house price factor exhibits a long and fairly steady growth cycle from 1991 to 2005, when a modest decline begins, followed by a sharp drop in 2007. The global house cycle appears to have little high frequency volatility. The Otrok-Whiteman posterior coverage intervals are tighter than that of the Kalman Filter. This result is consistent with the Monte Carlo results which showed more precise coverage intervals for Otrok-Whiteman. Our Monte Carlo results also showed that the common world factor was generally very similar across procedures. The lower right panel of Figure 14 plots the means of the factors on the same graph (though PCA is plotted a a different scale). It is clear that they all deliver the exact same message about global house prices.

The factor for advanced economies is plotted in Figure 15. Posterior coverage intervals for Otrok-Whiteman are again tighter than the Kalman filter. There now appears to be less agreement on the regional factor. The correlation of the PCA factor with Otrok-Whiteman is only 0.19. The correlation between Otrok-Whiteman and the Kalman Filter is 0.62. This is also consistent with our Monte Carlo results in that the two Bayesian procedures deliver similar factors, and we found that in multi-layer models the Bayesian procedures did a better job at finding the true factor. The variance decompositions in Table 6 show that the regional factor is not that important quantitatively, which means there is not a lot of information in the data in the cycle. It is then not surprising that we may get somewhat different estimates of this factor.

The emerging markets factor is plotted in Figure 16. Consistent with the results for advanced economies, there is little relationship between the PCA factor and the Bayesian factors. The correlation of PCA with the Bayesian estimates is 0, while the correlation between the Bayesian estimates is again 0.6. Taken together, Figures 14-16 tell an interesting story about the evolution of house prices. In the pre-crisis period, the emerging markets factor is quite volatile, while in the crisis period the factor does not move very much. In contrast, the advanced economies factor is fairly smooth in the pre-crisis period while it drops in the crisis period. Because the world factor also drops in this period, the two factors imply that the crash in advanced economies was in fact worse than in emerging markets. The world factor shows that all house prices fell in the crisis, the regional factors indicate a greater drop in advanced economies, while in emerging markets there was relative calm.

The variance decompositions in Table 6 shows patterns that would be expected based on our Monte Carlo results. The parametric results are all greater than the factor-based estimates. Our Monte Carlo results suggest the factor based results are more accurate (and conservative) so we will base our discussion on them. Figure 17 plots the variance decompositions from the three models for each country. Its clear that PCA lies above the two Bayesian procedures, consistent with the Monte Carlo results. All three methods show an important role for the world factor, though the PCA perhaps overstates the importance.

For advanced economies their is an important role for the global factor, both PCA and Otrok-Whiteman method attribute about 1/3 of fluctuations to this factor. The advanced regional factor plays a much smaller role, accounting for about 10% of the variation on average. The median variance decompositions are even smaller, suggesting that this factor is only affecting a small number of countries.

Cesa-Bianchi et al (2015) report average correlations that are greater for advanced economies then for emerging economies. Our results are consistent with these results. The world factor plays a much smaller role for emerging markets as the lower average variance decompositions show. It is also clear that the emerging market comovement is largely through the world factor. The median variance decomposition for the emerging markets factors is only 1%. We conclude then that regional factors are not important in understanding house price movements in either advanced or emerging markets. There is a common global cycle of significant importance.

5 Conclusion and Discussion

In this paper, we use Monte Carlo methods to analyze the accuracy of three methods to estimate dynamic factor models. All three methods worked well in the one factor case. It also is apparent that the factor-based estimates of variance decompositions are better than parametric based estimates. Our experience tells us that this is due to the fact that variances can blow up with parameter estimates near the unit root. In applied work, we have found this to often be true. As model complexity increased the Bayesian approaches yielded more accurate results, which in the case of the three-layer model were substantially different.

This latter results was perhaps unsurprising in sign, though the magnitude of the difference was surprisingly large. The Kalman-filter based method would expected to be most accurate as we are directly drawing from the likelihood of the model. The gains over the Otrok-Whiteman method were surprisingly small. This is goods news in that the Otrok-Whiteman method is more computationally efficient in dealing with large datasets. It is useful to know that its accuracy is a good as the state-space approach. Both the PCA and Otrok-Whiteman approach share an iterative approach to estimation. They differ in that the PCA approach conditions in one direction, while the Otrok-Whiteman conditions (repeatedly) in multiple directions (i.e., world is drawn conditional on country, country then drawn conditional on world). While this is clearly costly in computational terms, it yields more accurate estimates of the importance of factors in complex factor settings.

There are a number of choices to be made in applied work on factor models. The PCA approach will always be best to get quick answers, and in the one factor case there seems to be no accuracy gains from the more complicated methods. On the other hand, the Bayesian methods naturally yield measures of factor uncertainty, which should always accompany applied work in order to establish the statistical legitimacy of the results. In this paper, we have not reported information on computational time, as for any specific application the computation time is not large. Here, for each model type, we have estimated 1000 different applications (i.e. each model parameter drawn). It follows that estimating one is not a problem for modern computers.

In our application, we did find that PCA yielded factors that appeared to have greater importance then the Bayesian methods. In economic terms though, all methods delivered a similar story in that house price comovement across advanced and emerging markets is through a common factor, and not through group specific factors.

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A Tables and Figures

Table 1: Mean Width of Posterior Coverage Intervals			
	PCA	**Kalman Filter**	**Otrok-Whiteman**
One-Factor Model			
World Factor: 68% Interval	0.000	0.585	0.426
World Factor: 90% Interval	0.000	0.967	0.706
Two-Factor Model			
World Factor: 68% Interval	0.000	0.875	0.622
Country Factor: 68% Interval	0.000	1.386	0.983
World Factor: 90% Interval	0.000	1.450	1.004
Country Factor: 90% Interval	0.000	2.352	1.650
Three-Factor Model: 8 Country			
World Factor: 68% Interval	0.000	1.007	0.657
Region Factor: 68% Interval	0.000	1.358	0.976
Country Factor: 68% Interval	0.000	1.772	0.990
World Factor: 90% Interval	0.000	1.672	1.055
Region Factor: 90% Interval	0.000	2.270	1.590
Country Factor: 90% Interval	0.000	2.998	1.654
Three-Factor Model: 16 Country			
World Factor: 68% Interval	0.000	0.746	0.494
Region Factor: 68% Interval	0.000	1.102	0.770
Country Factor: 68% Interval	0.000	1.425	0.924
World Factor: 90% Interval	0.000	1.238	0.782
Region Factor: 90% Interval	0.000	1.836	1.241
Country Factor: 90% Interval	0.000	2.398	1.547

Table 2: PDF of Variance Decomposition Biases - One-Factor Model			
	PCA	**Kalman Filter**	**Otrok-Whiteman**
Factor-Based Variance Decomposition			
Mean of PDF			
World Factor	0.095	0.092	0.092
Idiosyncratic Factor	0.095	0.092	0.092
Percent of Positive Biases			
World Factor	0.417	0.486	0.489
Idiosyncratic Factor	0.583	0.514	0.511
Percent of Negative Biases			
World Factor	0.583	0.514	0.511
Idiosyncratic Factor	0.417	0.486	0.489
Parameter-Based Variance Decomposition			
Mean of PDF			
World Factor	0.089	0.094	0.083
Idiosyncratic Factor	0.089	0.094	0.083
Percent of Positive Biases			
World Factor	0.674	0.306	0.482
Idiosyncratic Factor	0.326	0.694	0.518
Percent of Negative Biases			
World Factor	0.326	0.694	0.518
Idiosyncratic Factor	0.674	0.306	0.482

Table 3: PDF of Variance Decomposition Biases - Two-Factor Model			
	PCA	**Kalman Filter**	**Otrok-Whiteman**
Factor-Based Variance Decomposition			
Mean of PDF			
World Factor	0.142	0.159	0.155
Country Factor	0.121	0.110	0.130
Idiosyncratic Factor	0.120	0.216	0.217
Percent of Positive Biases			
World Factor	0.401	0.838	0.818
Country Factor	0.443	0.704	0.679
Idiosyncratic Factor	0.752	0.094	0.156
Percent of Negative Biases			
World Factor	0.599	0.162	0.182
Country Factor	0.557	0.296	0.321
Idiosyncratic Factor	0.248	0.906	0.844
Parameter-Based Variance Decomposition			
Mean of PDF			
World Factor	0.161	0.131	0.137
Country Factor	0.153	0.138	0.162
Idiosyncratic Factor	0.070	0.079	0.104
Percent of Positive Biases			
World Factor	0.457	0.318	0.313
Country Factor	0.538	0.653	0.659
Idiosyncratic Factor	0.519	0.512	0.492
Percent of Negative Biases			
World Factor	0.543	0.682	0.687
Country Factor	0.462	0.347	0.341
Idiosyncratic Factor	0.481	0.488	0.508

Table 4: PDF of Variance Decomposition Biases - Three-Factor Model, 8 Country			
	PCA	**Kalman Filter**	**Otrok-Whiteman**
Factor-Based Variance Decomposition			
Mean of PDF			
World Factor	0.221	0.144	0.155
Region Factor	0.200	0.180	0.179
Country Factor	0.162	0.127	0.140
Idiosyncratic Factor	0.269	0.198	0.205
Percent of Positive Biases			
World Factor	0.264	0.643	0.614
Region Factor	0.251	0.199	0.236
Country Factor	0.586	0.617	0.604
Idiosyncratic Factor	0.860	0.548	0.559
Percent of Negative Biases			
World Factor	0.736	0.357	0.386
Region Factor	0.749	0.801	0.764
Country Factor	0.414	0.383	0.396
Idiosyncratic Factor	0.140	0.452	0.441
Parameter-Based Variance Decomposition			
Mean of PDF			
World Factor	0.184	0.172	0.191
Region Factor	0.205	0.171	0.183
Country Factor	0.159	0.134	0.151
Idiosyncratic Factor	0.065	0.062	0.083
Percent of Positive Biases			
World Factor	0.408	0.368	0.329
Region Factor	0.542	0.551	0.613
Country Factor	0.574	0.599	0.604
Idiosyncratic Factor	0.438	0.488	0.491
Percent of Negative Biases			
World Factor	0.592	0.632	0.671
Region Factor	0.458	0.449	0.387
Country Factor	0.426	0.401	0.396
Idiosyncratic Factor	0.562	0.513	0.509

Table 5: PDF of Variance Decomposition Biases - Three-Factor Model, 16 Country			
	PCA	**Kalman Filter**	**Otrok-Whiteman**
Factor-Based Variance Decomposition			
Mean of PDF			
World Factor	0.208	0.129	0.149
Region Factor	0.177	0.175	0.173
Country Factor	0.142	0.114	0.129
Idiosyncratic Factor	0.273	0.195	0.204
Percent of Positive Biases			
World Factor	0.270	0.690	0.650
Region Factor	0.280	0.188	0.226
Country Factor	0.480	0.617	0.604
Idiosyncratic Factor	0.864	0.533	0.543
Percent of Negative Biases			
World Factor	0.730	0.310	0.350
Region Factor	0.720	0.812	0.774
Country Factor	0.520	0.383	0.396
Idiosyncratic Factor	0.136	0.467	0.457
Parameter-Based Variance Decomposition			
Mean of PDF			
World Factor	0.176	0.172	0.180
Region Factor	0.201	0.150	0.162
Country Factor	0.150	0.126	0.140
Idiosyncratic Factor	0.061	0.057	0.079
Percent of Positive Biases			
World Factor	0.411	0.277	0.279
Region Factor	0.549	0.538	0.613
Country Factor	0.575	0.685	0.634
Idiosyncratic Factor	0.447	0.568	0.509
Percent of Negative Biases			
World Factor	0.589	0.723	0.721
Region Factor	0.451	0.462	0.387
Country Factor	0.425	0.315	0.366
Idiosyncratic Factor	0.553	0.432	0.491

Table 6: Variance Decompositions

	PCA						Kalman Filter						Otrok-Whiteman					
	Factor-Based			Parameter-Based			Factor-Based			Parameter-Based			Factor-Based			Parameter-Based		
	World	Region	Idio.	World	Region	Idio.	World	Region	Idio.	World	Region	Idio.	World	Region	Idio.	World	Region	Idio.
Advanced Economies																		
United States	0.612	0.023	0.365	0.780	0.064	0.156	0.207	0.130	0.663	0.075	0.110	0.816	0.386	0.048	0.567	0.086	0.059	0.856
Australia	0.370	0.041	0.589	0.691	0.186	0.122	0.186	0.127	0.687	0.363	0.164	0.473	0.360	0.032	0.608	0.548	0.016	0.436
Austria	0.168	0.003	0.829	0.221	0.236	0.543	0.132	0.137	0.730	0.207	0.179	0.614	0.270	0.510	0.220	0.312	0.584	0.104
Belgium	0.523	0.000	0.477	0.842	0.061	0.097	0.120	0.185	0.695	0.191	0.200	0.610	0.241	0.065	0.694	0.244	0.114	0.642
Canada	0.219	0.314	0.467	0.263	0.671	0.066	0.101	0.248	0.651	0.261	0.274	0.465	0.155	0.086	0.759	0.383	0.033	0.584
Denmark	0.487	0.019	0.494	0.858	0.035	0.107	0.161	0.115	0.724	0.355	0.176	0.469	0.301	0.015	0.684	0.536	0.005	0.459
Finland	0.364	0.246	0.390	0.442	0.467	0.090	0.222	0.118	0.660	0.309	0.145	0.547	0.448	0.008	0.543	0.487	0.012	0.501
France	0.657	0.045	0.298	0.735	0.055	0.211	0.206	0.230	0.565	0.126	0.063	0.812	0.414	0.067	0.519	0.179	0.004	0.817
Germany	0.446	0.035	0.519	0.529	0.175	0.296	0.162	0.083	0.755	0.159	0.099	0.742	0.279	0.018	0.703	0.243	0.073	0.684
Greece	0.487	0.300	0.213	0.468	0.499	0.033	0.176	0.078	0.746	0.042	0.023	0.935	0.250	0.009	0.741	0.038	0.005	0.957
Ireland	0.561	0.265	0.174	0.588	0.361	0.051	0.238	0.097	0.665	0.191	0.084	0.725	0.395	0.007	0.597	0.207	0.007	0.786
Italy	0.443	0.017	0.540	0.149	0.148	0.703	0.130	0.158	0.712	0.013	0.008	0.979	0.199	0.119	0.683	0.011	0.001	0.988
Japan	0.062	0.015	0.923	0.293	0.239	0.468	0.013	0.031	0.956	0.074	0.035	0.891	0.038	0.037	0.926	0.132	0.006	0.862
Luxembourg	0.445	0.015	0.540	0.700	0.103	0.197	0.144	0.147	0.709	0.187	0.219	0.594	0.230	0.134	0.635	0.152	0.124	0.724
Malta	0.212	0.039	0.749	0.665	0.091	0.243	0.084	0.062	0.853	0.227	0.109	0.664	0.122	0.004	0.873	0.347	0.012	0.641
Netherlands	0.221	0.358	0.421	0.438	0.241	0.322	0.125	0.058	0.817	0.030	0.044	0.926	0.210	0.058	0.733	0.065	0.012	0.923
New Zealand	0.336	0.070	0.594	0.460	0.335	0.205	0.120	0.128	0.752	0.182	0.113	0.705	0.229	0.013	0.758	0.249	0.008	0.743
Norway	0.267	0.162	0.571	0.496	0.427	0.077	0.166	0.101	0.733	0.344	0.188	0.469	0.362	0.010	0.628	0.572	0.021	0.407
Portugal	0.006	0.329	0.666	0.239	0.462	0.299	0.020	0.035	0.944	0.049	0.048	0.904	0.002	0.089	0.909	0.056	0.043	0.901
Spain	0.749	0.043	0.208	0.759	0.154	0.088	0.263	0.163	0.574	0.159	0.058	0.783	0.418	0.045	0.537	0.196	0.005	0.799
Sweden	0.535	0.073	0.392	0.867	0.052	0.081	0.261	0.171	0.568	0.438	0.234	0.327	0.500	0.045	0.455	0.633	0.028	0.339
Switzerland	0.046	0.635	0.319	0.061	0.875	0.064	0.036	0.073	0.891	0.087	0.047	0.866	0.097	0.017	0.886	0.121	0.007	0.872
United Kingdom	0.733	0.027	0.240	0.928	0.026	0.046	0.373	0.179	0.448	0.583	0.259	0.159	0.738	0.013	0.249	0.870	0.016	0.113
Mean	**0.389**	**0.134**	**0.477**	**0.542**	**0.259**	**0.198**	**0.159**	**0.124**	**0.717**	**0.202**	**0.125**	**0.673**	**0.289**	**0.063**	**0.648**	**0.290**	**0.052**	**0.658**
Median	**0.443**	**0.043**	**0.477**	**0.529**	**0.186**	**0.122**	**0.161**	**0.127**	**0.712**	**0.187**	**0.110**	**0.705**	**0.270**	**0.037**	**0.683**	**0.243**	**0.012**	**0.724**
Emerging Economies																		
Hong Kong	0.051	0.566	0.383	0.149	0.723	0.128	0.095	0.036	0.869	0.208	0.019	0.774	0.070	0.014	0.916	0.156	0.008	0.836
Argentina	0.079	0.343	0.578	0.229	0.626	0.146	0.090	0.147	0.763	0.266	0.070	0.664	0.139	0.052	0.809	0.285	0.004	0.711
Chile	0.000	0.010	0.990	0.104	0.200	0.696	0.012	0.553	0.436	0.084	0.467	0.449	0.004	0.734	0.262	0.049	0.776	0.175
Colombia	0.067	0.157	0.776	0.155	0.440	0.405	0.078	0.007	0.914	0.073	0.007	0.921	0.054	0.007	0.938	0.050	0.018	0.932
Croatia	0.006	0.142	0.852	0.175	0.364	0.461	0.009	0.005	0.986	0.082	0.008	0.910	0.026	0.019	0.955	0.144	0.009	0.847
Korea	0.078	0.026	0.897	0.204	0.179	0.616	0.055	0.058	0.887	0.075	0.026	0.899	0.062	0.005	0.933	0.045	0.005	0.950
Malaysia	0.216	0.378	0.406	0.141	0.621	0.239	0.107	0.013	0.880	0.056	0.005	0.940	0.200	0.033	0.767	0.071	0.001	0.928
Singapore	0.033	0.368	0.599	0.117	0.651	0.232	0.067	0.011	0.923	0.177	0.007	0.817	0.023	0.018	0.959	0.089	0.003	0.908
South Africa	0.705	0.098	0.197	0.830	0.120	0.050	0.226	0.006	0.768	0.251	0.003	0.746	0.447	0.002	0.551	0.367	0.001	0.632
Uruguay	0.003	0.072	0.924	0.069	0.463	0.468	0.006	0.055	0.939	0.037	0.045	0.918	0.001	0.009	0.990	0.026	0.007	0.967
Mean	**0.124**	**0.216**	**0.660**	**0.217**	**0.439**	**0.344**	**0.074**	**0.089**	**0.837**	**0.131**	**0.066**	**0.804**	**0.103**	**0.089**	**0.808**	**0.128**	**0.083**	**0.789**
Median	**0.059**	**0.150**	**0.688**	**0.152**	**0.451**	**0.322**	**0.072**	**0.025**	**0.883**	**0.083**	**0.013**	**0.858**	**0.058**	**0.016**	**0.925**	**0.080**	**0.006**	**0.877**

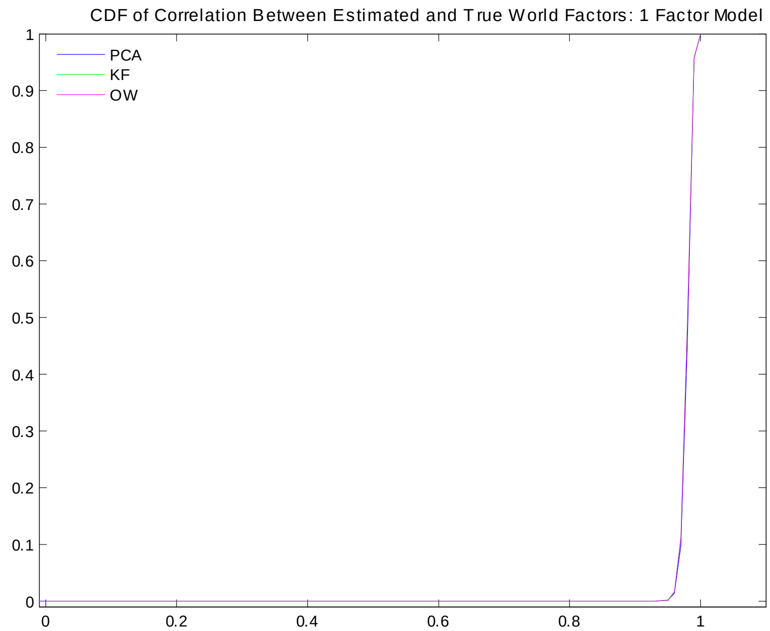

Figure 1: CDF of the correlation between the true and estimated world factor in the one-factor model, over 1000 MC simulations.

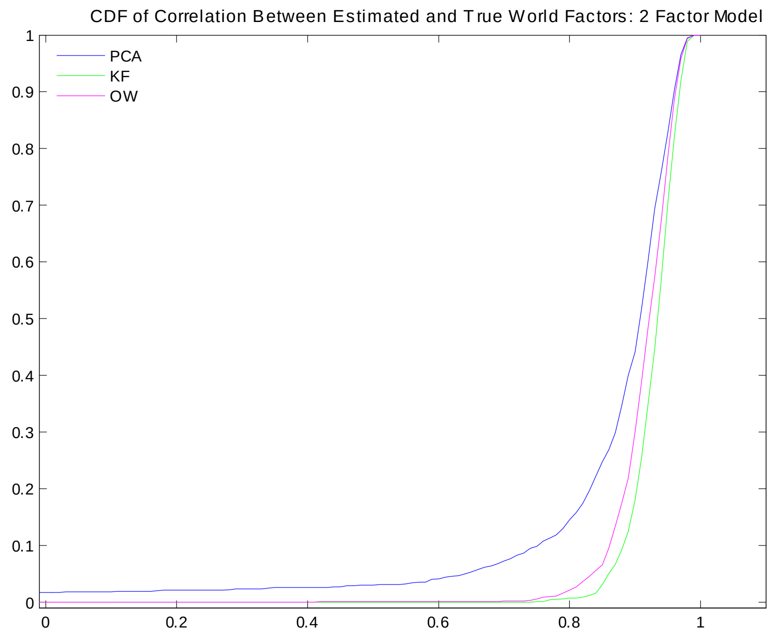

Figure 2: CDF of the correlation between the true and estimated world factor in the two-factor model, over 1000 MC simulations.

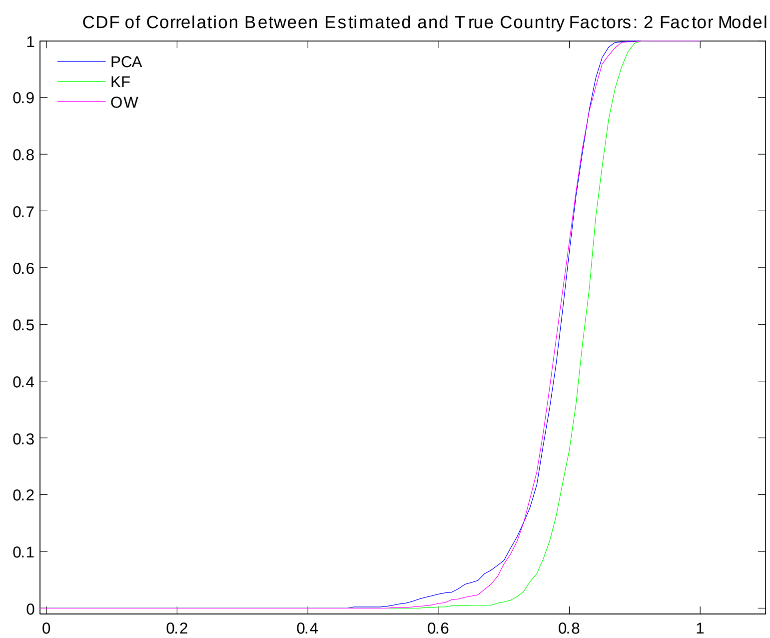

Figure 3: CDF of the correlation between the true and estimated country factors in the two-factor model, over 1000 MC simulations. The correlations are averaged across countries.

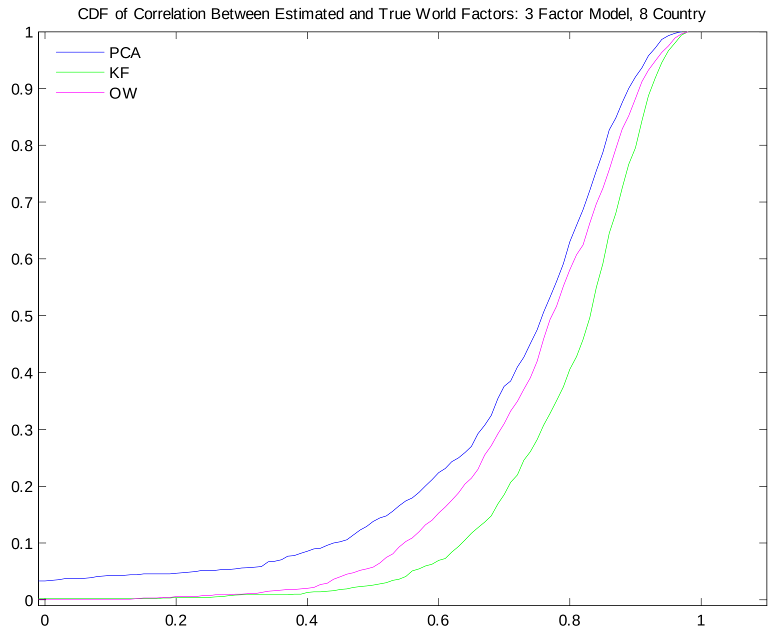

Figure 4: CDF of the correlation between the true and estimated world factor in the three-factor model with small regions, over 1000 MC simulations. The datasets consist of 8 countries broken into two equally-sized regions.

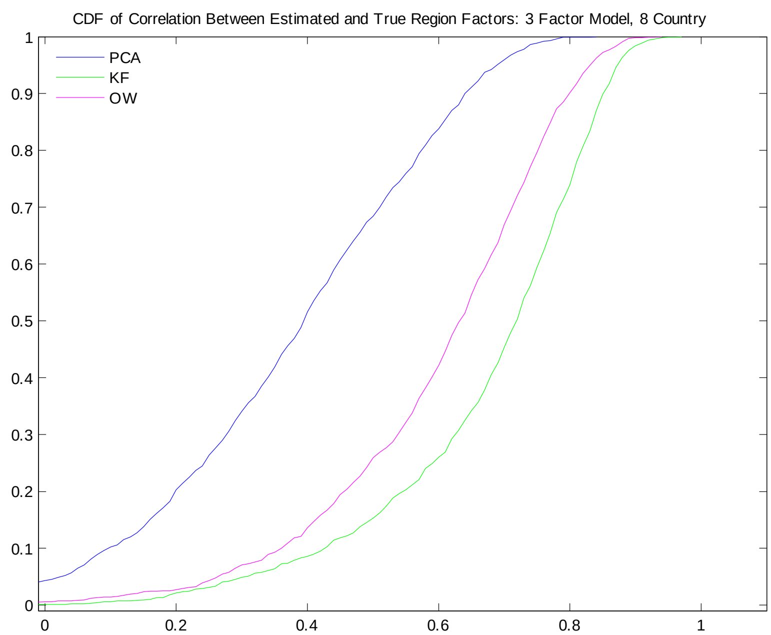

Figure 5: CDF of the correlation between the true and estimated region factor in the three-factor model with small regions, over 1000 MC simulations. The datasets consist of 8 countries broken into two equally-sized regions. The correlations represent the average across regions.

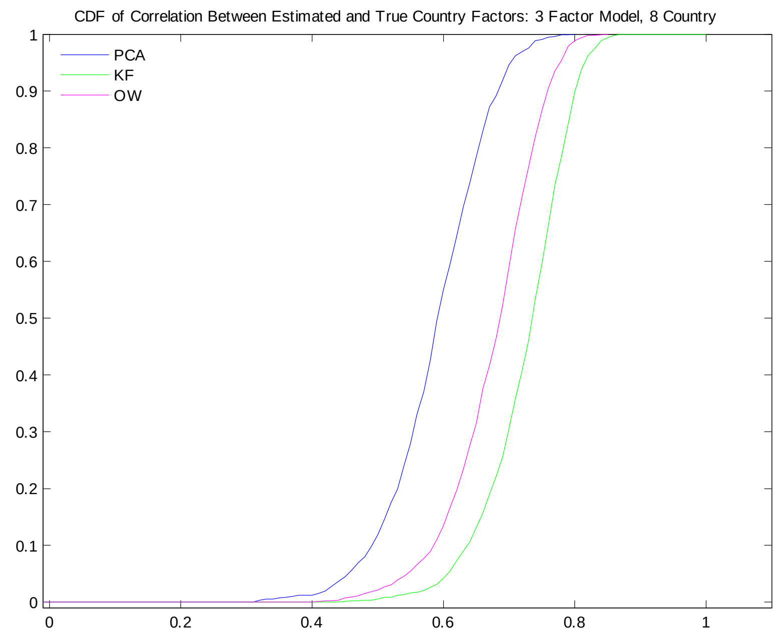

Figure 6: CDF of the correlation between the true and estimated country factors in the three-factor model with small regions, over 1000 MC simulations. The datasets consist of 8 countries broken into two equally-sized regions. The correlations represent the average across countries.

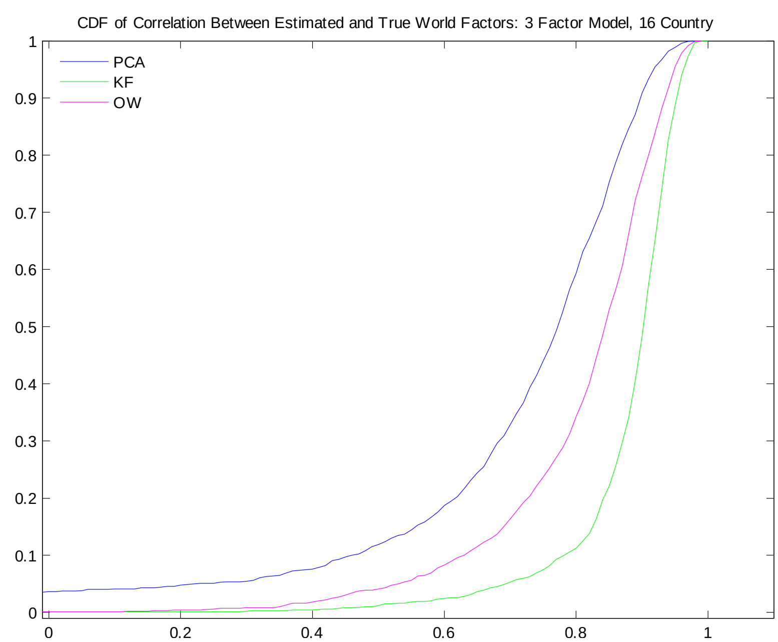

Figure 7: CDF of the correlation between the true and estimated world factor in the three-factor model with large regions, over 1000 MC simulations. The datasets consist of 16 countries broken into two equally-sized regions.

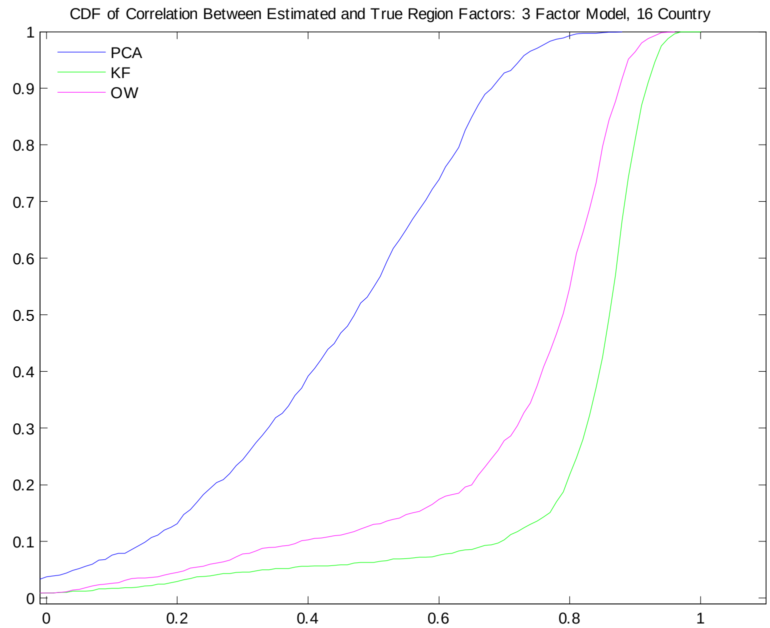

Figure 8: CDF of the correlation between the true and estimated region factor in the three-factor model with large regions, over 1000 MC simulations. The datasets consist of 16 countries broken into two equally-sized regions. The correlations represent the average across regions.

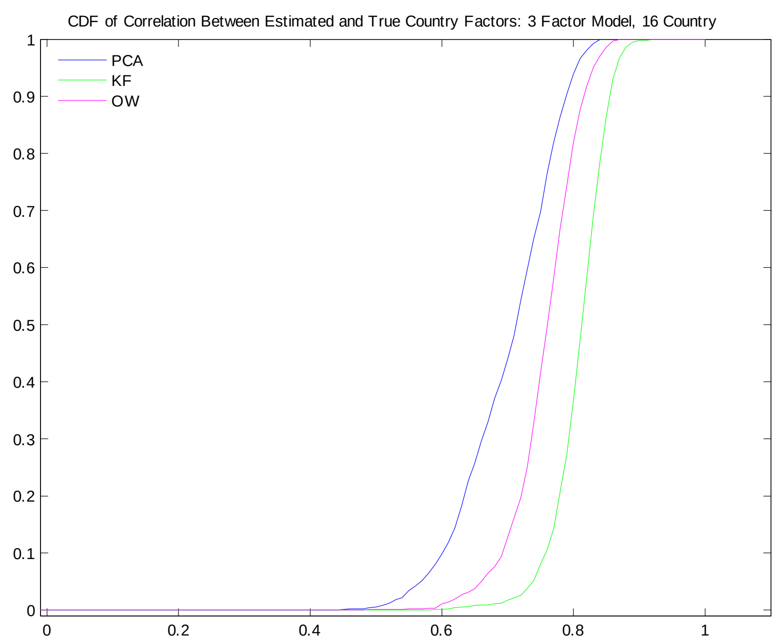

Figure 9: CDF of the correlation between the true and estimated country factors in the three-factor model with large regions, over 1000 MC simulations. The datasets consist of 16 countries broken into two equally-sized regions. The correlations represent the average across countries.

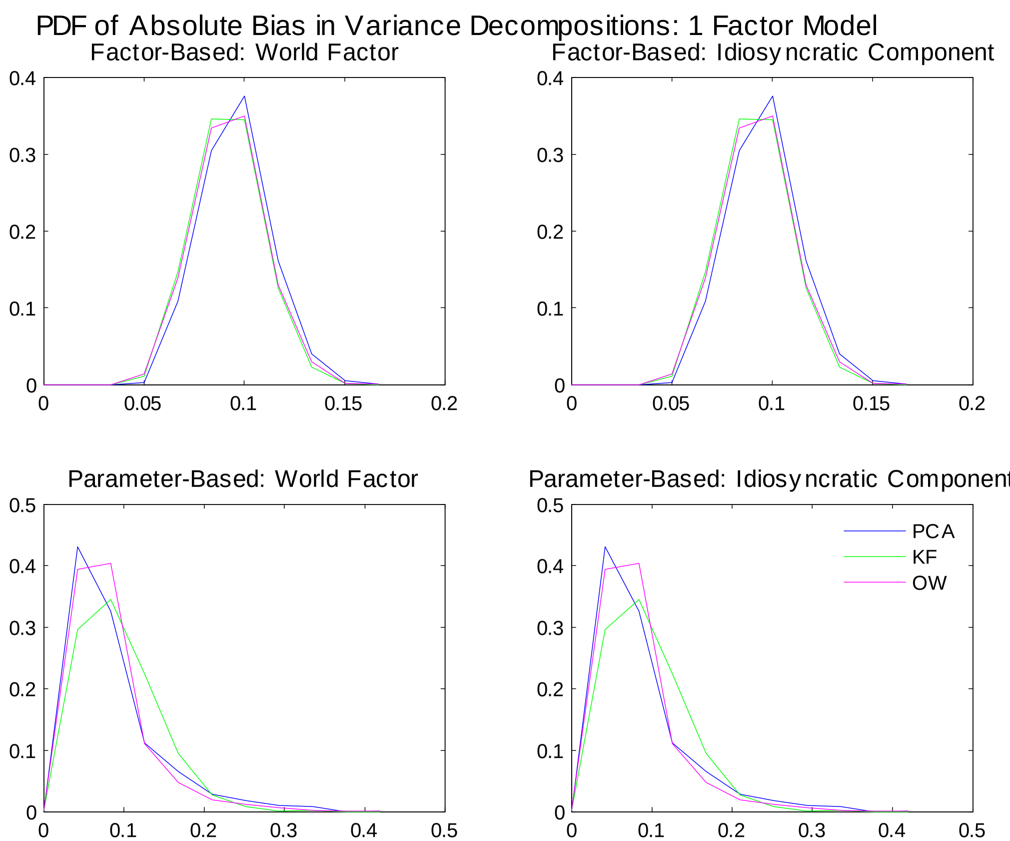

Figure 10: PDF of the mean absolute bias in the true versus the estimated variance decompositions for the one-factor model. The top row shows the factor-based variance decomposition for the world factor and the idiosyncratic component. The bottom row shows the parameter-based decompositions.

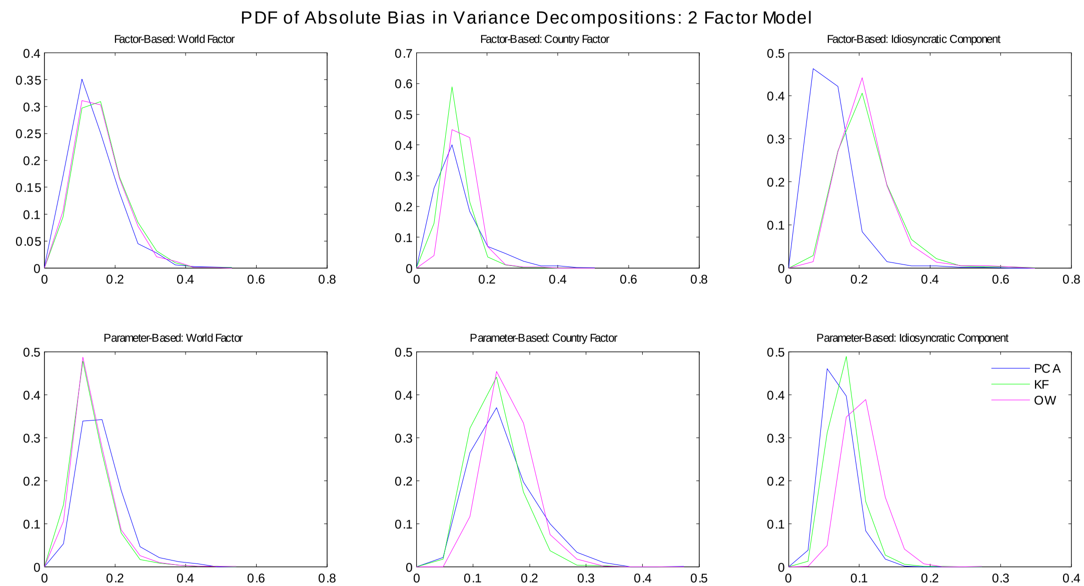

Figure 11: PDF of the mean absolute bias in the true versus the estimated variance decompositions for the two-factor model. The top row shows the factor-based variance decomposition for the world and country factors and the idiosyncratic component. The bottom row shows the parameter-based decompositions.

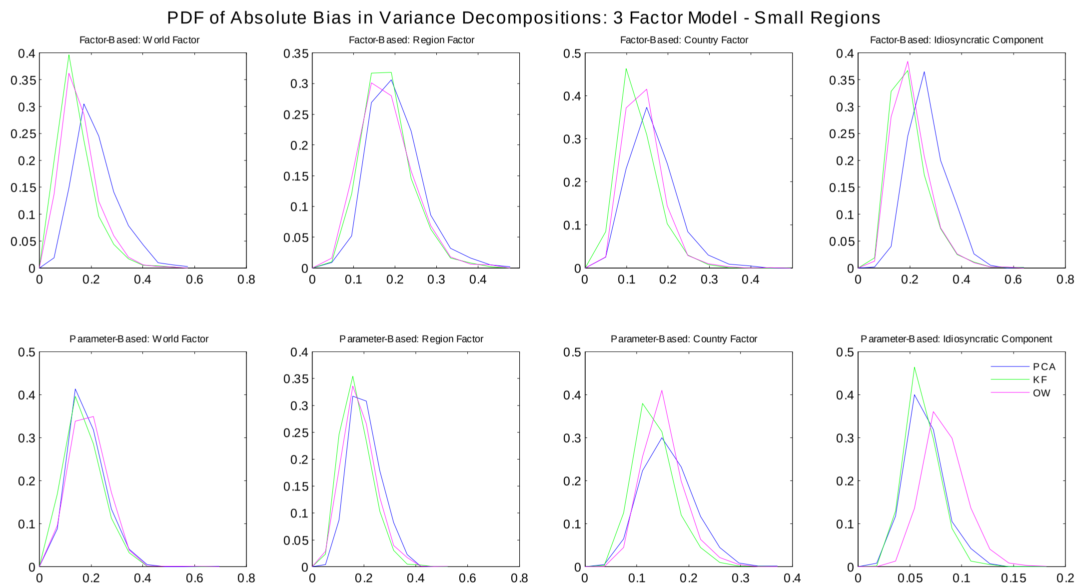

Figure 12: PDF of the mean absolute bias in the true versus the estimated variance decompositions for the three-factor model with small regions. The top row shows the factor-based variance decomposition for the world, region, and country factors and the idiosyncratic component. The bottom row shows the parameter-based decompositions.

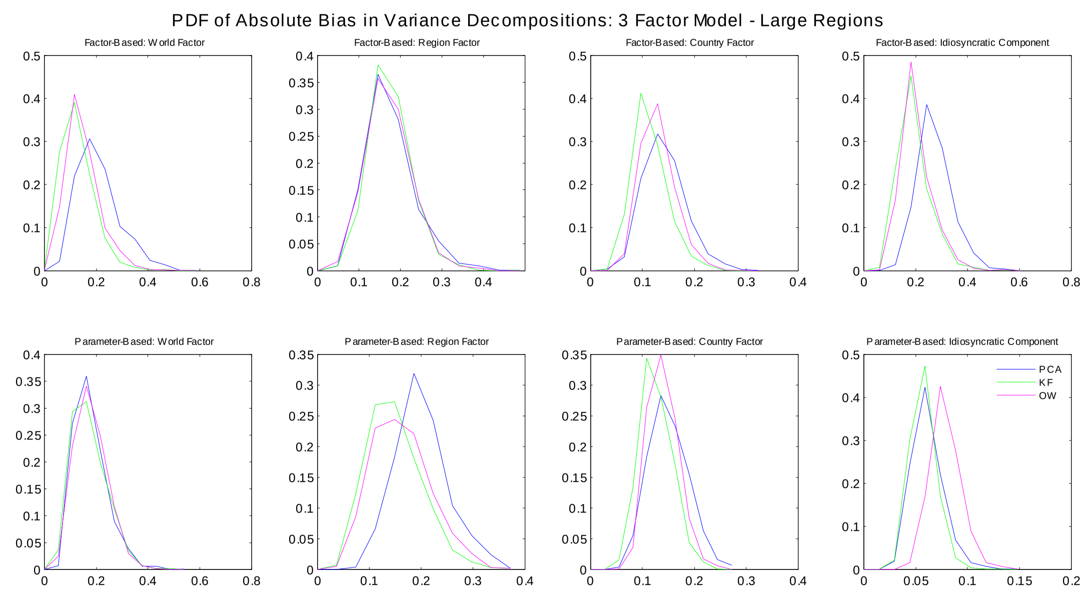

Figure 13: PDF of the mean absolute bias in the true versus the estimated variance decompositions for the three-factor model with large regions. The top row shows the factor-based variance decomposition for the world, region, and country factors and the idiosyncratic component. The bottom row shows the parameter-based decompositions.

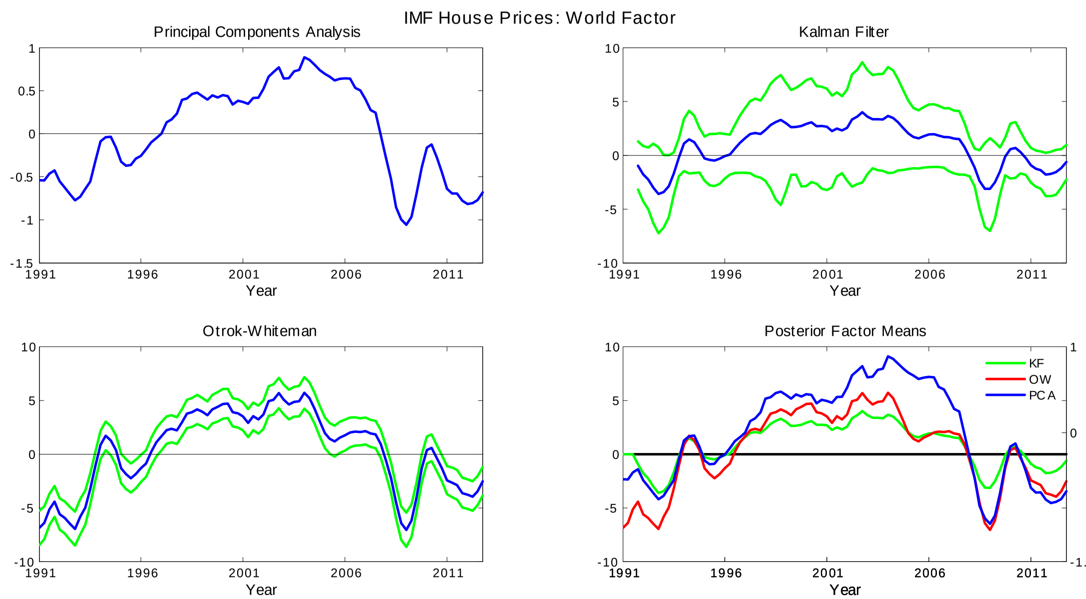

Figure 14: World factors extracted from IMF real house price data in advanced and emerging economies using three estimation methods: Principal Components Analysis and Bayesian Estimation with Kalman Filtering or the Otrok-Whiteman method. All plots show the posterior mean factor estimates. The Kalman Filter and Otrok-Whiteman plots also include the 68% posterior coverage interval.

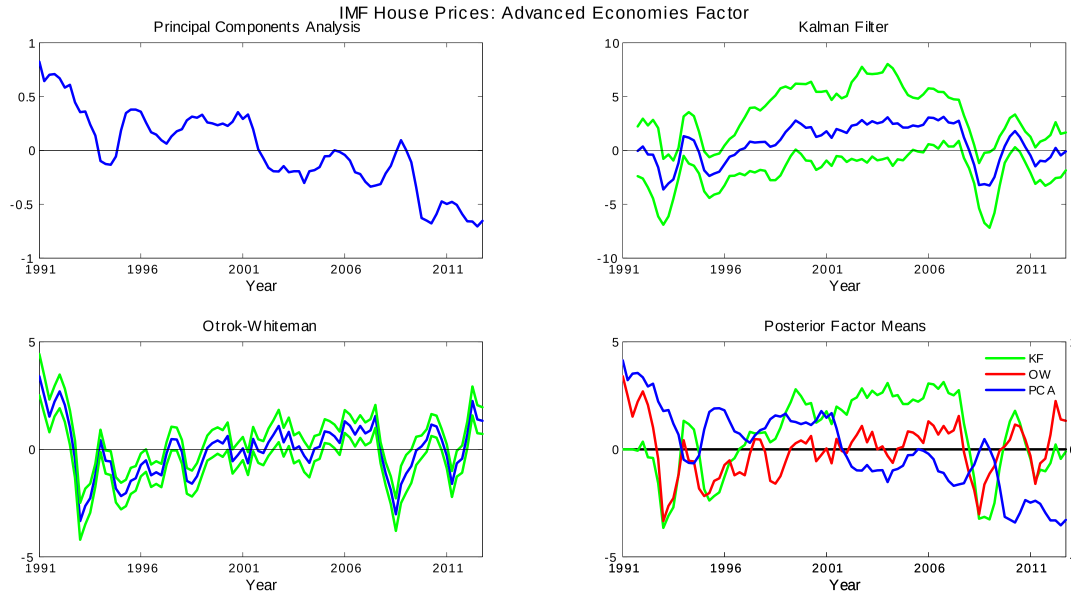

Figure 15: Regional factors extracted from IMF real house price data in advanced economies using three estimation methods: Principal Components Analysis and Bayesian Estimation with Kalman Filtering or the Otrok-Whiteman method. All plots show the posterior mean factor estimates. The Kalman Filter and Otrok-Whiteman plots also include the 68% posterior coverage interval.

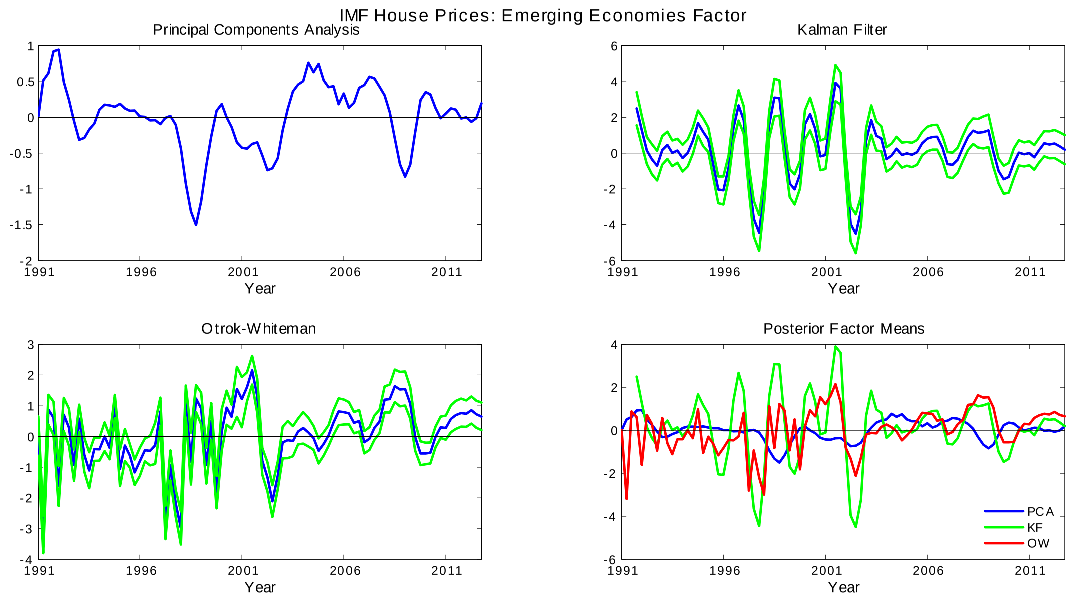

Figure 16: Regional factors extracted from IMF real house price data in emerging economies using three estimation methods: Principal Components Analysis and Bayesian Estimation with Kalman Filtering or the Otrok-Whiteman method. All plots show the posterior mean factor estimates. The Kalman Filter and Otrok-Whiteman plots also include the 68% posterior coverage interval.

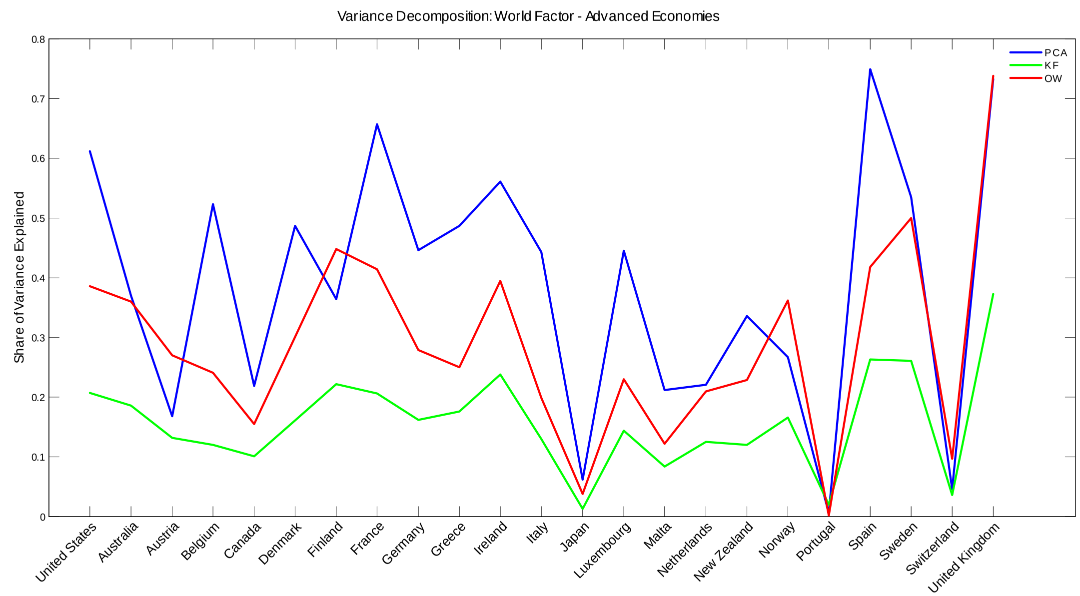

Figure 17: Variance Decomposition of World Factor for Advanced Economies: The world factor is extracted from IMF real house price data in advanced and emerging economies using three estimation methods: Principal Components Analysis and Bayesian Estimation with Kalman Filtering or the Otrok-Whiteman method. The plot shows the share of variation in each country's house prices that is attributable to the world factor.