Parte 1

$$\begin{pmatrix} \sigma_1^2 & \rho\,\sigma_1\sigma_2 \\ \rho\,\sigma_1\sigma_2 & \sigma_2 \end{pmatrix}$$

$$E[X_1 \mid X_2] = \mu_1 + \rho\,\sigma_1\sigma_2 \frac{1}{\sigma_2^2}(x_2 - \mu_2)$$

$$= \underbrace{\mu - \rho \frac{\sigma_1}{\sigma_2}\mu_2}_{B_0} + \underbrace{\rho \frac{\sigma_1}{\sigma_2}}_{B_1} x_2$$

Ojo que:

$$E[X_1 \mid X_2] = B_0 + B_1 X_2$$

$$\min_{B_0\, B_1} E[(X_1 - B_0 - B_1 X_2)^2]$$

$$E(2(X_1 - B_0 - B_1 X_2)(-1)] = 0 \quad (1)$$

$$E[2(X_1 - B_0 - B_1 X_2)(-X_2)] = 0 \quad (2)$$

De ① $E[X_1] - E[B_0] - E[B_1 X_2] = 0$

$$\mu_1 - B_0 - B_1 \mu_2 = 0$$

$$B_0 = \mu_1 - B_1 \mu_2$$

De ②

$$E[X_1 X_2] - B_0 E[X_2] - B_1 E[X_2^2] = 0$$

$$\operatorname{Cov}(X_1, X_2) = E[X_1 X_2] - E[X_1] E[X_2]$$

$$\rho \sigma_1 \sigma_2 = E[X_1 X_2] - \mu_1 \mu_2$$

$$E[X_1 X_2] = \rho \sigma_1 \sigma_2 + \mu_1 \mu_2$$

$$\operatorname{Var}(X_2) = E[X_2^2] - E[X_2])^2$$

$$\sigma_2^2 = E[X_2^2] - \mu^2$$

$$E[X_2^2] = \sigma_2^2 + \mu_2^2$$

$$\rho \sigma_1 \sigma_2 + \mu_1 \mu_2 - (\mu_1 - B_1 \mu_2)\mu_2 - B_1(\sigma_2^2 + \mu_2^2) = 0$$

$$\rho \sigma_1 \sigma_2 + \cancel{\mu_1 \mu_2} - \cancel{\mu_1 \mu_2} + \cancel{B_1 \mu_2^2} - B_1 \sigma_2^2 - \cancel{B_1 \mu_2^2}$$

$$\rho \sigma_1 \sigma_2 = B \sigma_2^2 \qquad B_1 = \rho \frac{\sigma_1}{\sigma_2}$$

$$B_0 = \mu_1 - \rho \frac{\sigma_1}{\sigma_2} \mu_2 \qquad \therefore \; B_0 = \alpha_0, \; B_1 = \alpha_1$$