Distribucion Gaussiana - Univariada.

$x_1, \dots, x_n \leftarrow$ datos de MEMO

$$\underline{X}|\mu \sim N(x|\mu, \sigma^2)$$
$$\mu \sim N(\mu|\mu_0, \tau^2)$$

con σ^2 y τ^2 conocidas.

$\boxed{\text{Posterior}}$ bajo independencia condicional

$$\pi(\mu|x_1, \dots, x_n) \propto \prod_{i=1}^{n} N(x_i|\mu, \sigma^2) \cdot N(\mu|\mu_0, \tau^2)$$

$$\propto \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2\right\} \exp\left\{-\frac{1}{2\tau}(\mu-\mu_0)^2\right\}$$

$$\propto \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i^2 - 2\mu x_i + \mu^2) - \frac{1}{2\tau^2}(\mu - 2\mu\mu_0 + \mu_0^2)\right\}$$

$$\propto \exp\left\{-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n} x_i^2 - 2\mu\sum_{i=1}^{n} x_i + n\mu^2\right) - \frac{1}{2\tau^2}(\mu^2 - 2\mu\mu_0 + \mu_0^2)\right\}$$

[Notacion: $\sum_{i=1}^{n} x_i = n\bar{x}$]

$$\propto \exp\left\{-\frac{1}{2}\left[\left(\frac{n}{\sigma^2} + \frac{1}{\tau^2}\right)\mu^2 - 2\left(\frac{n}{\sigma^2}\bar{x} + \frac{1}{\tau^2}\mu_0\right)\mu\right]\right\}$$

[completando el cuadrado]

$$a = \frac{n}{\sigma^2} + \frac{1}{\tau^2}$$

$$b = \frac{n\bar{x}}{\sigma^2} + \frac{\mu_0}{\tau^2}$$

para $-\frac{1}{2}(a\mu^2 - 2b\mu)$

MEMO

i.e.

$$-\frac{1}{2}\left(a\mu^2 - 2b\mu + \frac{b^2}{a^2} - \frac{b^2}{a^2}\right) =$$

$$= -\frac{a}{2}\left(\mu^2 - 2\frac{b}{a}\mu + \frac{b^2}{a^2}\right) + \frac{1}{2}\frac{b^2}{a^2}$$

Retomando proporcionalidad:

$$\propto \exp\left\{-\frac{a}{2}\left(\mu - \frac{b}{a}\right)^2\right\},$$

$$\propto \exp\left\{-\frac{1}{2}\left(\frac{n}{\sigma^2} + \frac{1}{\tau^2}\right)\left(\mu - \frac{n\bar{x}/\sigma^2 + \mu_0/\tau_2}{\frac{n}{\sigma^2} + \frac{1}{\tau^2}}\right)^2\right\}$$

i.e.

$$\mu \mid x_1, \dots, x_n \sim N(\mu \mid \mu_n, \sigma_n^2)$$

$$\mu_n = \sigma_n^2\left(\frac{n\bar{x}}{\sigma^2} + \frac{\mu_0}{\tau^2}\right)$$

$$\sigma_n^2 = \left(\frac{n}{\sigma^2} + \frac{1}{\tau^2}\right)^{-1}.$$