

EST-46114

= Distribución gaussiana =

$\mathbf{x} \in \mathbb{R}^p$, an

$$\begin{aligned} f(\mathbf{x} | \mu, \Sigma) &= (2\pi)^{-p/2} |\Sigma|^{-1/2} \\ &\quad \times \exp \left\{ -\frac{1}{2} (\mathbf{x} - \mu)' \Sigma^{-1} (\mathbf{x} - \mu) \right\} \\ &= (2\pi)^{-p/2} |\Lambda|^{1/2} \\ &\quad \times \exp \left\{ -\frac{1}{2} (\mathbf{x} - \mu)' \Lambda (\mathbf{x} - \mu) \right\} \end{aligned}$$

an Σ - matriz covarianzas

$\Lambda = \Sigma^{-1}$ - matriz de precisión.

Caso $p=2$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{pmatrix}$$

$$\Rightarrow \Lambda = \begin{pmatrix} \frac{1}{\sigma_1^2 (1-\rho^2)} & -\frac{\rho}{\sigma_1 \sigma_2 (1-\rho^2)} \\ -\frac{\rho}{\sigma_1 \sigma_2 (1-\rho^2)} & \frac{1}{\sigma_2^2 (1-\rho^2)} \end{pmatrix}$$

donde $\text{cov}(\mathbf{x}_1, \mathbf{x}_2) = \rho \sigma_1 \sigma_2$, ie.

$$\rho = \text{corr}(\mathbf{x}_1, \mathbf{x}_2).$$

Caso $p=2$

$$f_{X_2|X_1}(x_2|x_1) = N(x_2 | \mu_2(x_1), \sigma_2^2(x_1))$$

$$\text{donde } \mu_2(x_1) = \mu_1 - \frac{\rho\sigma_2}{\sigma_1}(x_1 - \mu_1)$$

$$\sigma_2(x_1) = \sigma_2(1 - \rho^2)^{1/2}$$

$$\text{donde } \rho = \text{corr}(X_1, X_2).$$

i.e.

$$f_{X_2|X_1}(x_2|x_1) = (2\pi)^{-1/2} \left[\sigma_2^2(1 - \rho^2) \right]^{-1/2} \\ \times \exp \left\{ -\frac{1}{2\sigma_2^2(1 - \rho^2)} \times \right. \\ \left. \times \left(x_2 - \mu_2 - \frac{\rho\sigma_2}{\sigma_1}(x_1 - \mu_1) \right)^2 \right\}$$