

Appendix A: Derivation for DCFNet

We derive the backward formulas for optimizing the CNN parameters θ for DCFNet. We treat the elements in $\mathbf{g}_\theta(\mathbf{z})$ as variables which are relevant with $\varphi^l(\mathbf{z})$ and $\varphi^l(\mathbf{x})$ and simplify $\partial \mathbf{g}_\theta(\mathbf{z})$ as $\partial \mathbf{g}$. It is assumed that the channels are independent. By using the chain rule to return the errors to the two branches of the network individually, we can obtain:

$$\begin{aligned} \frac{\partial L(\theta)}{\partial \theta} &= \sum_{l=1}^D \frac{\partial L(\theta)}{\partial \varphi^l(\mathbf{z})} \frac{\partial \varphi^l(\mathbf{z})}{\partial \theta} + \sum_{l=1}^D \frac{\partial L(\theta)}{\partial \varphi^l(\mathbf{x})} \frac{\partial \varphi^l(\mathbf{x})}{\partial \theta} + 2\gamma\theta \\ &= \sum_{l=1}^D \text{F}^{-1} \left(\frac{\partial L(\theta)}{\partial \hat{\varphi}^l(\mathbf{z})^*} \right) \frac{\partial \varphi^l(\mathbf{z})}{\partial \theta} + \sum_{l=1}^D \text{F}^{-1} \left(\frac{\partial L(\theta)}{\partial \hat{\varphi}^l(\mathbf{x})^*} \right) \frac{\partial \varphi^l(\mathbf{x})}{\partial \theta} + 2\gamma\theta. \end{aligned} \quad (\text{a})$$

According to (a), it is necessary to compute $\partial L / \partial(\varphi^l(\mathbf{z}))$ and $\partial L / \partial(\varphi^l(\mathbf{x}))$. We start with $\partial L / \partial \mathbf{g}$.

According to (4) and (5) in the main paper,

$$\frac{\partial L}{\partial \mathbf{g}} = 2 \left(\sum_{l=1}^D \mathbf{w}^l \star \varphi^l(\mathbf{x}) - \mathbf{y} \right). \quad (\text{b})$$

The chain rule is a little complicated since the intermediate variables are complex-valued variables. The definitions of the discrete Fourier transform and inverse discrete Fourier transform are used to derive their gradients [29]. Let N be the number of components in vector \mathbf{g} . It is apparent that

$$\hat{\mathbf{g}}_f = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \mathbf{g}_n e^{-j \frac{2\pi}{N} n f} = F_{n \rightarrow f}(\mathbf{g}_n), \quad (\text{c})$$

$$\mathbf{g}_n = \frac{1}{\sqrt{N}} \sum_{f=0}^{N-1} \hat{\mathbf{g}}_f e^{-j \frac{2\pi}{N} n f} = F_{f \rightarrow n}^{-1}(\hat{\mathbf{g}}_f), \quad (\text{d})$$

with \mathbf{g}_n as the discrete time signal, $\hat{\mathbf{g}}_f$ as the transformed signal in the frequency domain, and n and f

having the range $\{0, \dots, N\}$. Partial derivatives of $\hat{\mathbf{g}}_f$ with respect to \mathbf{g}_n and \mathbf{g}_n^* are:

$$\begin{cases} \frac{\partial \hat{\mathbf{g}}_f}{\partial \mathbf{g}_n} = \frac{1}{\sqrt{N}} e^{-j \frac{2\pi}{N} n f}, \\ \frac{\partial \hat{\mathbf{g}}_f}{\partial \mathbf{g}_n^*} = 0, \end{cases} \quad (\text{e})$$

Partial derivatives of \mathbf{g}_n with respect to $\hat{\mathbf{g}}_f$ and $\hat{\mathbf{g}}_f^*$ are:

$$\begin{cases} \frac{\partial \mathbf{g}_n}{\partial \hat{\mathbf{g}}_f} = \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} n f}, \\ \frac{\partial \mathbf{g}_n}{\partial \hat{\mathbf{g}}_f^*} = 0. \end{cases} \quad (\text{f})$$

Then, we can obtain the following inferences:

$$\frac{\partial L}{\partial \mathbf{g}_n^*} = \sum_{f=0}^{N-1} \left(\left(\frac{\partial L}{\partial \hat{\mathbf{g}}_f^*} \right)^* 0 + \frac{1}{\sqrt{N}} \frac{\partial L}{\partial \hat{\mathbf{g}}_f^*} \left(e^{-j \frac{2\pi}{N} n f} \right)^* \right) = \frac{1}{\sqrt{N}} \sum_{f=0}^{N-1} \frac{\partial L}{\partial \hat{\mathbf{g}}_f^*} e^{j \frac{2\pi}{N} n f} = F^{-1} \left(\frac{\partial L}{\partial \hat{\mathbf{g}}_f^*} \right), \quad (\text{g})$$

$$\frac{\partial L}{\partial \hat{\mathbf{g}}_f^*} = \sum_{n=0}^{N-1} \left(\left(\frac{\partial L}{\partial \mathbf{g}_n^*} \right)^* 0 + \frac{1}{\sqrt{N}} \frac{\partial L}{\partial \mathbf{g}_n^*} \left(e^{j \frac{2\pi}{N} n f} \right)^* \right) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \frac{\partial L}{\partial \mathbf{g}_n^*} e^{-j \frac{2\pi}{N} n f} = F \left(\frac{\partial L}{\partial \mathbf{g}_f^*} \right). \quad (\text{h})$$

Since \mathbf{g} is the correlation response which is a real-valued vector, it holds that $\mathbf{g} = \mathbf{g}^*$. The discrete Fourier transform and inverse discrete Fourier transform of the derivatives with respect to \mathbf{g} have the following relations:

$$\begin{cases} \hat{\mathbf{g}} = F(\mathbf{g}), \\ \frac{\partial L}{\partial \hat{\mathbf{g}}^*} = F \left(\frac{\partial L}{\partial \mathbf{g}^*} \right) = F \left(\frac{\partial L}{\partial \mathbf{g}} \right), \\ \frac{\partial L}{\partial \mathbf{g}} = \frac{\partial L}{\partial \mathbf{g}^*} = F^{-1} \left(\frac{\partial L}{\partial \hat{\mathbf{g}}^*} \right). \end{cases} \quad (\text{i})$$

Since the operations in the forward pass process only contain the element-based Hadamard product and division, we calculate the derivative in (i) per pixel (u, v) in the frequency domain:

$$\frac{\partial L}{\partial \hat{\mathbf{g}}_{uv}^*} = \left(F \left(\frac{\partial L}{\partial \mathbf{g}} \right) \right)_{uv}. \quad (\text{j})$$

For the back-propagation of the tracking branch, the partial differential $\partial L / \partial(\phi^l(\mathbf{z}))$ for \mathbf{z} is required to compute. According to (4) in the main paper,

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*(\mathbf{z})}{\partial(\phi_{uv}^l(\mathbf{z}))^*} = \hat{\mathbf{w}}_{uv}^l. \quad (\text{k})$$

Then,

$$\frac{\partial L}{\partial(\phi_{uv}^l(\mathbf{z}))^*} = \frac{\partial L}{\partial \hat{\mathbf{g}}_{uv}^*} \frac{\partial \hat{\mathbf{g}}_{uv}^*(\mathbf{z})}{\partial(\phi_{uv}^l(\mathbf{z}))^*} = \frac{\partial L}{\partial \hat{\mathbf{g}}_{uv}^*} \hat{\mathbf{w}}_{uv}^l, \quad (\text{l})$$

where $\partial L / \partial \hat{\mathbf{g}}_{uv}^*$ is computed using (b) and (j). It holds that

$$\frac{\partial L}{\partial \phi^l(\mathbf{z})} = \mathbf{F}^{-1} \left(\frac{\partial L}{\partial (\hat{\phi}^l(\mathbf{z}))^*} \right). \quad (\text{m})$$

For the back-propagation of the branch of learning the correlation filter, the partial differential $\partial L / \partial (\phi^l(\mathbf{x}))$ for \mathbf{x} is required to compute. We compute $\partial L / \partial \hat{\phi}_{uv}^l(\mathbf{x})$ and $\partial L / \partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*$ independently, and then combine them to compute $\partial L / \partial (\phi^l(\mathbf{x}))$. It is apparent that

$$\begin{cases} \frac{\partial L}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} = \frac{\partial L}{\partial \hat{\mathbf{g}}_{uv}^*} \frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\phi}_{uv}^l(\mathbf{x})}, \\ \frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} = \frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\mathbf{w}}_{uv}^l} \frac{\partial \hat{\mathbf{w}}_{uv}^l}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} = (\hat{\phi}_{uv}^l(\mathbf{z}))^* \frac{\partial \hat{\mathbf{w}}_{uv}^l}{\partial \hat{\phi}_{uv}^l(\mathbf{x})}, \end{cases} \quad (\text{n})$$

where according to (4) in the main paper,

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\mathbf{w}}_{uv}^l} = (\hat{\phi}_{uv}^l(\mathbf{z}))^*. \quad (\text{o})$$

Let μ be the denominator of (3) in the main paper:

$$\mu = \sum_{k=1}^D \hat{\phi}_{uv}^k(\mathbf{x})(\hat{\phi}_{uv}^k(\mathbf{x}))^* + \lambda. \quad (\text{p})$$

According to (3) in the main paper,

$$\frac{\partial \hat{\mathbf{w}}_{uv}^l}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} = \frac{\hat{\mathbf{y}}_{uv}^* \mu - \hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x})(\hat{\phi}_{uv}^l(\mathbf{x}))^*}{\mu^2}. \quad (\text{q})$$

Substitution of (q) into (n) yields

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} = \frac{\hat{\mathbf{y}}_{uv}^* \mu (\hat{\phi}_{uv}^l(\mathbf{z}))^* - \hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x})(\hat{\phi}_{uv}^l(\mathbf{x}))^* (\hat{\phi}_{uv}^l(\mathbf{z}))^*}{\mu^2}. \quad (\text{r})$$

According to (3) in the main paper,

$$\mu = \frac{\hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^*}{\hat{\mathbf{w}}_{uv}^l}, \quad (\text{s})$$

and $\mu \hat{\mathbf{w}}_{uv}^l = \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^*$. Substitution of (s) into one μ in the denominator of (r) yields

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} = \frac{\hat{\mathbf{y}}_{uv}^* \mu (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l - \hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x})(\hat{\phi}_{uv}^l(\mathbf{x}))^* (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l}{\mu \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^*}. \quad (\text{t})$$

Replacing $\mu \hat{\mathbf{w}}_{uv}^l$ in the numerator of (t) with $\hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^*$ yields

$$\begin{aligned} \frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} &= \frac{\hat{\mathbf{y}}_{uv}^* (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^* - \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^* (\hat{\phi}_{uv}^l(\mathbf{x}))^* (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l}{\mu \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^*} \\ &= \frac{\hat{\mathbf{y}}_{uv}^* (\hat{\phi}_{uv}^l(\mathbf{z}))^* - (\hat{\phi}_{uv}^l(\mathbf{x}))^* (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l}{\mu}, \end{aligned} \quad (\text{u})$$

where $\hat{\phi}_{uv}^l(\mathbf{x}) \hat{\mathbf{y}}_{uv}^*$ in the denominator and the numerator is canceled. Substitution of (p) into (u) yields

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\phi}_{uv}^l(\mathbf{x})} = \frac{\hat{\mathbf{y}}_{uv}^* (\hat{\phi}_{uv}^l(\mathbf{z}))^* - (\hat{\phi}_{uv}^l(\mathbf{x}))^* (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l}{\sum_{k=1}^D \hat{\phi}_{uv}^k(\mathbf{x}) (\hat{\phi}_{uv}^k(\mathbf{x}))^* + \lambda}. \quad (\text{v})$$

Substitution of (v) and (j) into the upper part of (n) yields the solution for the partial differential

$\partial L / \partial \hat{\phi}_{uv}^l(\mathbf{x})$. We derive $\partial L / \partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*$ below. It is apparent that

$$\begin{cases} \frac{\partial L}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*} = \frac{\partial L}{\partial \hat{\mathbf{g}}_{uv}^*} \frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*}, \\ \frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*} = \frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial \hat{\mathbf{w}}_{uv}^l} \frac{\partial \hat{\mathbf{w}}_{uv}^l}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*}. \end{cases} \quad (\text{w})$$

According to (3) in the main paper,

$$\frac{\partial \hat{\mathbf{w}}_{uv}^l}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*} = - \frac{\hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\phi}_{uv}^l(\mathbf{x})}{\mu^2}. \quad (\text{x})$$

Substitution of (o) and (x) into the lower part of (w) yields

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*} = - \frac{\hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\phi}_{uv}^l(\mathbf{x}) (\hat{\phi}_{uv}^l(\mathbf{z}))^*}{\mu^2}. \quad (\text{y})$$

Substitution of (s) into one μ in the denominator of (y) yields

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*} = - \frac{\hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x}) \hat{\phi}_{uv}^l(\mathbf{x}) (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l}{\mu \hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x})} = - \frac{\hat{\phi}_{uv}^l(\mathbf{x}) (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l}{\mu}, \quad (\text{z})$$

where $\hat{\mathbf{y}}_{uv}^* \hat{\phi}_{uv}^l(\mathbf{x})$ in the denominator and the numerator is canceled. Substitution of (p) into (z) yields

$$\frac{\partial \hat{\mathbf{g}}_{uv}^*}{\partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*} = \frac{-\hat{\phi}_{uv}^l(\mathbf{x}) (\hat{\phi}_{uv}^l(\mathbf{z}))^* \hat{\mathbf{w}}_{uv}^l}{\sum_{k=1}^D \hat{\phi}_{uv}^k(\mathbf{x}) (\hat{\phi}_{uv}^k(\mathbf{x}))^* + \lambda}. \quad (\text{A})$$

Substitution of (j) and (A) into the upper part of (w) yields the solution for the partial differential

$\partial L / \partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*$. Combination of $\partial L / \partial \hat{\phi}_{uv}^l(\mathbf{x})$ and $\partial L / \partial (\hat{\phi}_{uv}^l(\mathbf{x}))^*$ yields

$$\frac{\partial L}{\partial \phi^l(\mathbf{x})} = \mathbf{F}^{-1} \left(\frac{\partial L}{\partial (\hat{\phi}^l(\mathbf{x}))^*} + \left(\frac{\partial L}{\partial \hat{\phi}^l(\mathbf{x})} \right)^* \right). \quad (\text{B})$$

Appendix B: Derivation for Convolutional-Deconvolutional DCFNet

The parameters θ of the CNN in the convolutional-deconvolutional DCFNet are optimized by minimizing (32) in the main paper. The derivatives of L_{low} in (32) in the main paper are obtained:

$$\frac{\partial L_{low}}{\partial \hat{g}^*(\mathbf{z}_s)} = 2(\hat{g}(\mathbf{z}_s) - \hat{\mathbf{y}}_s). \quad (C)$$

The back-propagation of the tracking branch for search image \mathbf{z} is represented by:

$$\frac{\partial L_{low}}{\partial \phi^l(\mathbf{z}_s)} = \mathbf{F}^{-1} \left(\frac{\partial L_{low}}{\partial \hat{g}^*(\mathbf{z}_s)} \odot \hat{\mathbf{w}}^{l*} \right). \quad (D)$$

For the back-propagation of the learning branch for the target image \mathbf{x} , according to (31) in the main paper, the gradient of back-propagation of the filter \mathbf{w} is computed by:

$$\frac{\partial L_{low}}{\partial \hat{\mathbf{w}}^l} = \sum_{s=1}^S \frac{\partial L_{low}}{\partial \hat{g}^*(\mathbf{z}_s)} \odot (\hat{\phi}^l(\mathbf{z}_s))^*. \quad (E)$$

Let μ be the denominator of $\hat{\mathbf{w}}$ in (30) in the main paper:

$$\mu = \sum_{s=1}^S (\hat{\phi}^l(\mathbf{x}_s^0))^* \odot \hat{\phi}^l(\mathbf{x}_s^0) + \lambda_1 + \lambda_2 \sum_{i=1}^k (\hat{\phi}^l(\mathbf{x}^i))^* \odot \hat{\phi}^l(\mathbf{x}^i). \quad (F)$$

We compute the gradients for the object image patch's features $\phi(\mathbf{x}_s^0)$ and the context image patch's features $\phi(\mathbf{x}^i)$ as follows: According to (30) in the main paper, the following partial differentials hold:

$$\frac{\partial \hat{\mathbf{w}}^l}{\partial (\hat{\phi}^l(\mathbf{x}_s^0))^*} = \frac{\mu \hat{\mathbf{y}}_s - \hat{\phi}^l(\mathbf{x}_s^0) \odot (\hat{\phi}^l(\mathbf{x}_s^0))^* \odot \hat{\mathbf{y}}_s}{\mu^2} = \frac{\hat{\mathbf{y}}_s - \hat{\phi}^l(\mathbf{x}_s^0) \odot \hat{\mathbf{w}}^l}{\mu}, \quad (G)$$

$$\frac{\partial \hat{\mathbf{w}}^l}{\partial \hat{\phi}^l(\mathbf{x}_s^0)} = \frac{-\hat{\phi}^l(\mathbf{x}_s^0)^* \odot \hat{\phi}^l(\mathbf{x}_s^0)^* \odot \hat{\mathbf{y}}_s}{\mu^2} = \frac{-\hat{\phi}^l(\mathbf{x}_s^0)^* \odot \hat{\mathbf{w}}^l}{\mu}, \quad (H)$$

$$\frac{\partial \hat{\mathbf{w}}^l}{\partial (\hat{\phi}^l(\mathbf{x}^i))^*} = \frac{\sum_{s=1}^S -(\hat{\phi}^l(\mathbf{x}^i))^* \odot (\hat{\phi}^l(\mathbf{x}_s^0))^* \odot \hat{\mathbf{y}}_s}{\mu^2} = \frac{\sum_{s=1}^S -(\hat{\phi}^l(\mathbf{x}_s^0))^* \odot \hat{\mathbf{w}}^l}{\mu}, \quad (I)$$

$$\frac{\partial \hat{\mathbf{w}}^l}{\partial \hat{\phi}^l(\mathbf{x}^i)} = \frac{\sum_{s=1}^S -(\hat{\phi}^l(\mathbf{x}^i))^* \odot (\hat{\phi}^l(\mathbf{x}_s^0))^* \odot \hat{\mathbf{y}}_s}{\mu^2} = \frac{\sum_{s=1}^S -(\hat{\phi}^l(\mathbf{x}_s^0))^* \odot \hat{\mathbf{w}}^l}{\mu}. \quad (J)$$

Let $\text{Re}(\cdot)$ be the real part of a complex-valued matrix. The back-propagation of the learning branch is carried out by:

$$\begin{aligned}
\frac{\partial L_{low}}{\partial \hat{\phi}^l(\mathbf{x}_s^0)} &= \mathbf{F}^{-1} \left(\frac{\partial L_{low}}{\partial \hat{\mathbf{w}}^l} \frac{\partial \hat{\mathbf{w}}}{\partial (\hat{\phi}^l(\mathbf{x}_s^0))^*} + \left(\frac{\partial L_{low}}{\partial \hat{\mathbf{w}}^l} \frac{\partial \hat{\mathbf{w}}^l}{\partial \hat{\phi}(\mathbf{x}_s^0)} \right)^* \right) \\
&= \mathbf{F}^{-1} \left(\frac{\partial L_{low}}{\partial \hat{\mathbf{w}}^l} \odot \frac{\hat{\mathbf{y}}_s^* - 2\text{Re}((\hat{\phi}^l(\mathbf{x}_s^0))^* \odot \hat{\mathbf{w}}^l) \mathbf{e}}{\mu} \right),
\end{aligned} \tag{K}$$

$$\begin{aligned}
\frac{\partial L_{low}}{\partial \hat{\phi}^l(\mathbf{x}^i)} &= \mathbf{F}^{-1} \left(\frac{\partial L_{low}}{\partial \hat{\mathbf{w}}^l} \frac{\partial \hat{\mathbf{w}}^l}{\partial (\hat{\phi}^l(\mathbf{x}^i))^*} + \left(\frac{\partial L_{low}}{\partial \hat{\mathbf{w}}^l} \frac{\partial \hat{\mathbf{w}}^l}{\partial \hat{\phi}(\mathbf{x}^i)} \right)^* \right) \\
&= \mathbf{F}^{-1} \left(\frac{\partial L_{low}}{\partial \hat{\mathbf{w}}^l} \odot \frac{-2\text{Re}((\hat{\phi}^l(\mathbf{x}^i))^* \odot \hat{\mathbf{w}}^l) \mathbf{e}}{\mu} \right).
\end{aligned} \tag{L}$$