National University of Singapore
School of Computing
CS2109S: Introduction to AI and Machine Learning
Semester I, 2023/2024

Tutorial 2
Informed Search

These questions will be discussed during the tutorial session. Please be prepared to answer them.

Summary of Key Concepts

In this tutorial, we will discuss and explore the following key learning points/lessons from Lecture:

1. Heuristic functions
   (a) Formulation of heuristic functions
   (b) Admissible and consistent heuristics
   (c) Dominance
2. A* Search
   (a) Optimality of tree search vs graph search

Problems

1. Pacman is a maze action video game. For this question, we will take a look at a simplified version of Pacman. In this version, Players control Pacman, with the objective of eating all the pellets in the maze while executing the least amount of moves. More formally, the initial state is a game state where Pacman is at a starting location and pellets are scattered around the maze (refer to Figure 1). Available actions are 4-directional movement unless blocked by walls, and if Pacman moves to a square with a pellet, he will automatically eat it with no additional cost (each movement has a cost of 1). The goal state is when Pacmac has eaten all the pellets. Recall that A* tree search finds an optimal solution with an admissible heuristic. Thus, devise a non-trivial admissible heuristic for this problem.

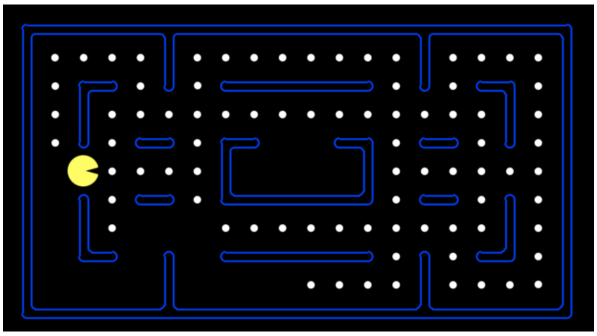

Figure 1: A state of Pacman.

2. We learnt that for route-finding problems, a simple admissible heuristic is the straight line distance. More formally, given a graph $G = (V, E)$ where each node v_n having coordinates (x_n, y_n), each edge (v_i, v_j) having weight equals to the distance between v_i and v_j, and a unique goal node v_g with coordinates (x_g, y_g), the straight line distance heuristic is given by:

$$h_{SLD}(n) = \sqrt{(x_n - x_g)^2 + (y_n - y_g)^2}$$

Consider the following two heuristic functions

$$h_1(n) = max\{|x_n - x_g|, |y_n - y_g|\}$$

$$h_2(n) = |x_n - x_g| + |y_n - y_g|$$

(a) Is $h_1(n)$ an admissible heuristic? If yes, prove it; otherwise, provide a counter-example.

(b) Is $h_2(n)$ an admissible heuristic? If yes, prove it; otherwise, provide a counter-example.

(c) Out of $h_1(n)$, $h_2(n)$ and $h_{SLD}(n)$, which heuristic function would you choose for A* search? Justify your answer.

3. (a) Given that a heuristic h is such that $h(G) = 0$, where G is any goal state, prove that if h is consistent, then it must be admissible. (Hint: Think of the path the algorithm takes)

(b) Give an example of an admissible heuristic function that is not consistent.

4. Refer to the Figure 2 below. Apply the A* search algorithm to find a path from Fagaras to Craiova where $h(n) = |h_{SLD}(\text{Craiov}) - h_{SLD}(n)|$ and $h_{SLD}(n)$ is the straight-line distance from any city n to Bucharest given in Figure 4.1.

(a) Trace the A* search algorithm by showing the series of search trees as each node is expanded, based on the `TREE-SEARCH` algorithm below.

(b) Prove that $h(n)$ is an admissible heuristic.

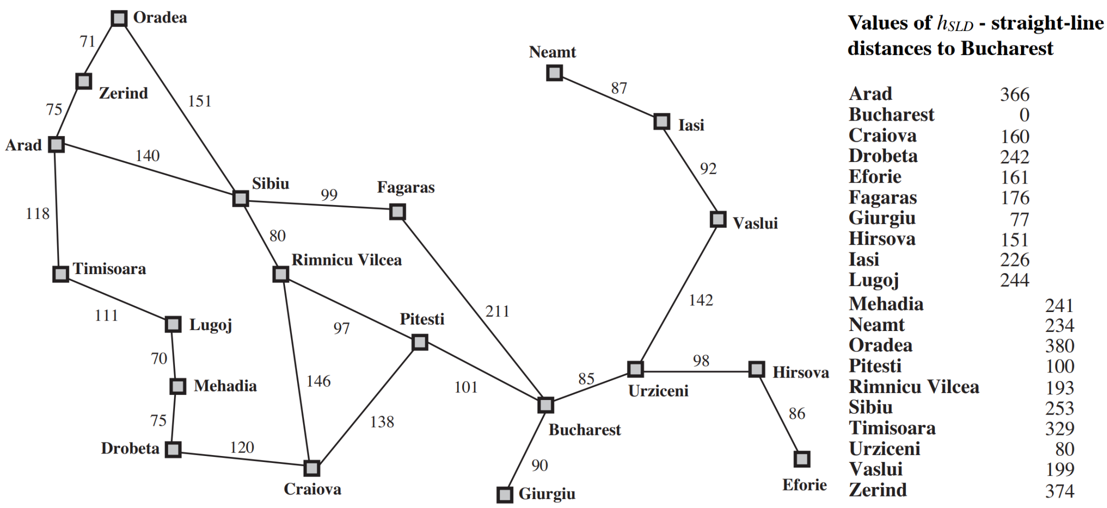

Figure 2: Graph of Romania.

function TREE-SEARCH(*problem, fringe*) **returns** a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if *fringe* is empty **then return** failure
    node ← REMOVE-FRONT(*fringe*)
    if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **return** *node*
    fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

Figure 3: Tree Search Algorithm.

5. You have learned before that A* using graph search is optimal if $h(n)$ is consistent. Does this optimality still hold if $h(n)$ is admissible but inconsistent? Using the graph in Figure 4, let us now show that A* using graph search returns a non-optimal solution path from start node S to goal node G when using an admissible but inconsistent $h(n)$. We assume that $h(G) = 0$. Then, show that tree search will return the optimal solution with the same heuristic.

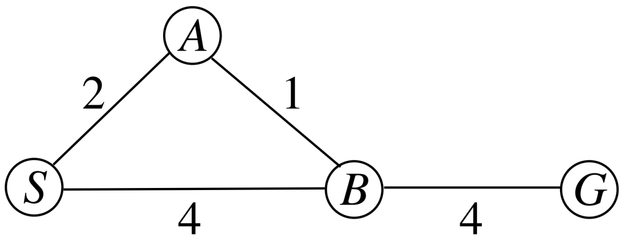

Figure 4: Graph.

6. (Extra & Optional) Would A* work with negative edge weights? Assume no negative cycles. If yes, prove it; otherwise, provide a counterexample. What if there are negative cycles?