Exercise: LU of a penta-diagonal matrix

Consider the matrix

$$\begin{pmatrix} 2 & 0 & -1 & & & \\ 0 & 2 & 0 & -1 & & \\ -1 & 0 & 2 & 0 & -1 & \\ & -1 & 0 & 2 & 0 & -1 \\ & & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

Describe the *LU* factorization of this matrix:

- Convince yourself that there will be no fill-in. Give an inductive proof of this.
- What does the graph of this matrix look like? (Find a tutorial on graph theory. What is a name for such a graph?)
- Can you relate this graph to the answer on the question of the fill-in?

Exercise: LU of a band matrix

Suppose a matrix *A* is banded with *halfbandwidth p*:

$$a_{ij} = 0 \quad \text{if } |i-j| > p$$

Derive how much space an LU factorization of *A* will take if no pivoting is used. (For bonus points: consider partial pivoting.)

Can you also derive how much space the inverse will take? (Hint: if $A = LU$, does that give you an easy formula for the inverse?)