OpenFoam Assignment | The Lid-Driven Cavity

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Abstract

–SAMPLE ABSTRACT–

Keywords: computational fluid dynamics CFD incompressible Paraview R Python coe347 spring 2022

1. Implementation

We implement all simulation with OpenFoam, analysis with Paraview and Python3, and documentation code in R Xie, Dervieux, and Riederer (2020).

2. Nondimensional Form of the Navier-Stokes Equations for the Lid-Cavity Problem

The fluid flow in the lid-driven cavity problem is assumed to be 2D with constant density and dynamic viscosity, thus allowing us to consider the incompressible, steady form of the Navier-Stokes equations for continuity and momentum.

Let L indicate the length of the cavity wall, U the speed of the lid, the kinematic viscosity of the fluid, the density of the fluid, and P the pressure. The variables x and y indicate the coordinate system and u and v represent the fluid velocity components in the x and y directions respectively.

The reference quantities can be non-dimensionalized as follows:

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Nondimensional variables

$$\tilde{u} = \frac{u}{U}, \quad \tilde{v} = \frac{v}{U}, \quad \tilde{p} = \frac{P}{\rho U^2} \quad \longrightarrow \quad P = \tilde{p} \rho U^2$$

$$\tilde{x} = \frac{x}{L}, \quad \tilde{y} = \frac{y}{L}, \quad \tilde{t} = t\frac{U}{L}$$

$$\tilde{\nabla} = \frac{\partial}{\partial \tilde{x}}\vec{i} + \frac{\partial}{\partial \tilde{y}}\vec{j}, \quad \tilde{t} = t\frac{U}{L}$$

or

$\tilde{x} = \frac{x}{L}$, $\tilde{y} = \frac{y}{L}$, $\tilde{u} = \frac{u}{U}$, $\tilde{v} = \frac{v}{U}$, $\tilde{p} = \frac{P}{\rho U^2}$, and $\tilde{t} = \frac{tU}{L}$.

We will now substitute the nondimensional terms into the incompressible, steady form of the Navier-Stokes equations, as given below.

X and Y Component of Momentum Eq.:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Continuity Equation

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (m)$$

The resulting nondimensional continuity equation is given by the following:

Find nondimensional form of the continuity equation:

From (3),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The nondimensional variables,

$$\frac{\partial u}{\partial x} = \frac{\partial (U\tilde{u})}{\partial (L\tilde{x})} = \frac{U}{L}\frac{\partial \tilde{u}}{\partial \tilde{x}}$$

$$\frac{\partial v}{\partial y} = \frac{\partial (U\tilde{v})}{\partial (L\tilde{y})} = \frac{U}{L}\frac{\partial \tilde{v}}{\partial \tilde{y}}$$

Plug into (7),

$$\boxed{\frac{U}{L}\left[\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}}\right] = 0}$$

Nondimensional form of continuity

Given that U,L are constants, we further get:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

Similarly, the nondimensional form of the momentum equation in the x and y directions are derived as follows:

Find nondimensional form of x & y components of momentum equation:

From (1),

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

The nondimensional variables,

$$\frac{du}{dx} = \frac{U}{L}\frac{\partial \tilde{u}}{\partial \tilde{x}}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{U}{L}\frac{\partial}{\partial \tilde{x}}\left(\frac{\partial \tilde{u}}{\partial \tilde{x}}\right)\frac{\partial \tilde{x}}{\partial x} = \frac{U}{L^2}\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2}$$

$$\frac{\partial v}{\partial y} = \frac{U}{L}\frac{\partial \tilde{v}}{\partial \tilde{y}}$$

$$\frac{\partial^2 v}{\partial y^2} = \frac{U}{L^2}\frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}$$

** $\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ in 2D

and $\tilde{\nabla} = \frac{\partial}{\partial \tilde{x}}\hat{i} + \frac{\partial}{\partial \tilde{y}}\hat{j}$ in 2D

Plug into (1),

$$\rho\left(\left(\frac{U}{L}\right)\left(\frac{U}{L}\right)\frac{\partial \tilde{u}}{\partial \tilde{t}} + (\tilde{u}U)\left(\frac{U}{L}\frac{\partial \tilde{u}}{\partial \tilde{x}}\right) + (\tilde{v}U)\left(\frac{U}{L}\frac{\partial \tilde{v}}{\partial \tilde{y}}\right)\right) = \frac{-\partial p}{\partial(\tilde{x}L)} + \mu\left(\frac{U}{L^2}\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{U}{L^2}\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)$$

$$\underline{\rho\frac{U^2}{L}\left(\frac{\partial \tilde{u}}{\partial t^2} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}\right) = -\frac{\rho U^2}{L}\frac{\partial \tilde{p}}{\partial \tilde{x}} + \mu\frac{U}{L^2}\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)}$$

Simplifying, we get for the x-momentum-balance:

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{\mathbf{u}}\cdot(\tilde{\nabla}\tilde{u}) \ = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\nu}{UL}\Delta\tilde{\mathbf{u}}$$

Likewise, for the y-momentum-balance:

$$\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{\mathbf{u}}\cdot(\tilde{\nabla}\tilde{v}) \ = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \frac{\nu}{UL}\Delta\tilde{\mathbf{u}}$$

The complete form is listed below for convenience:

$$\frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + (\tilde{\mathbf{u}}\cdot\tilde{\nabla})\tilde{\mathbf{u}} \ = -\tilde{\nabla}\tilde{p} + \frac{1}{Re}\tilde{\Delta}\tilde{\mathbf{u}} + \frac{1}{Fr^2}\hat{g}.$$

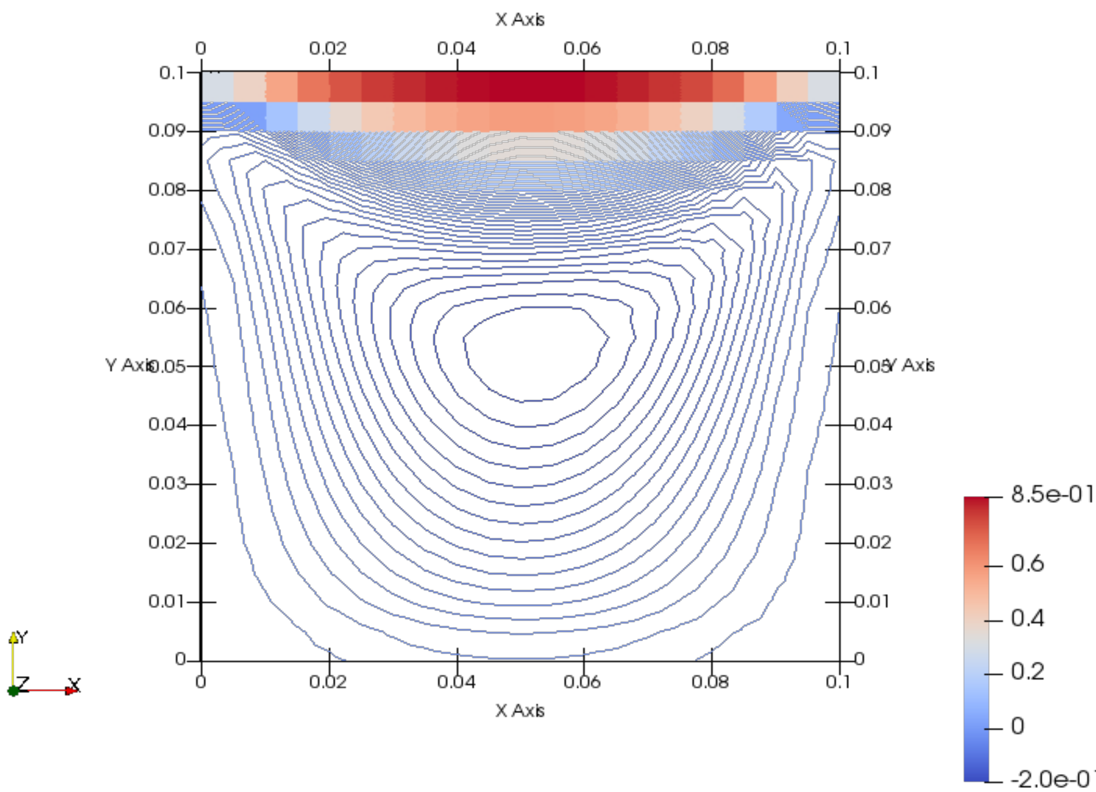
where $\tilde{\Delta}\tilde{\mathbf{u}} = \tilde{\nabla}^2\tilde{\mathbf{u}}$, $\mathrm{Re} = \dfrac{UL}{\nu} = \dfrac{\rho UL}{\mu}$, and $\mathrm{Fr} = \dfrac{U}{\sqrt{gL}}$ as commonly notated.

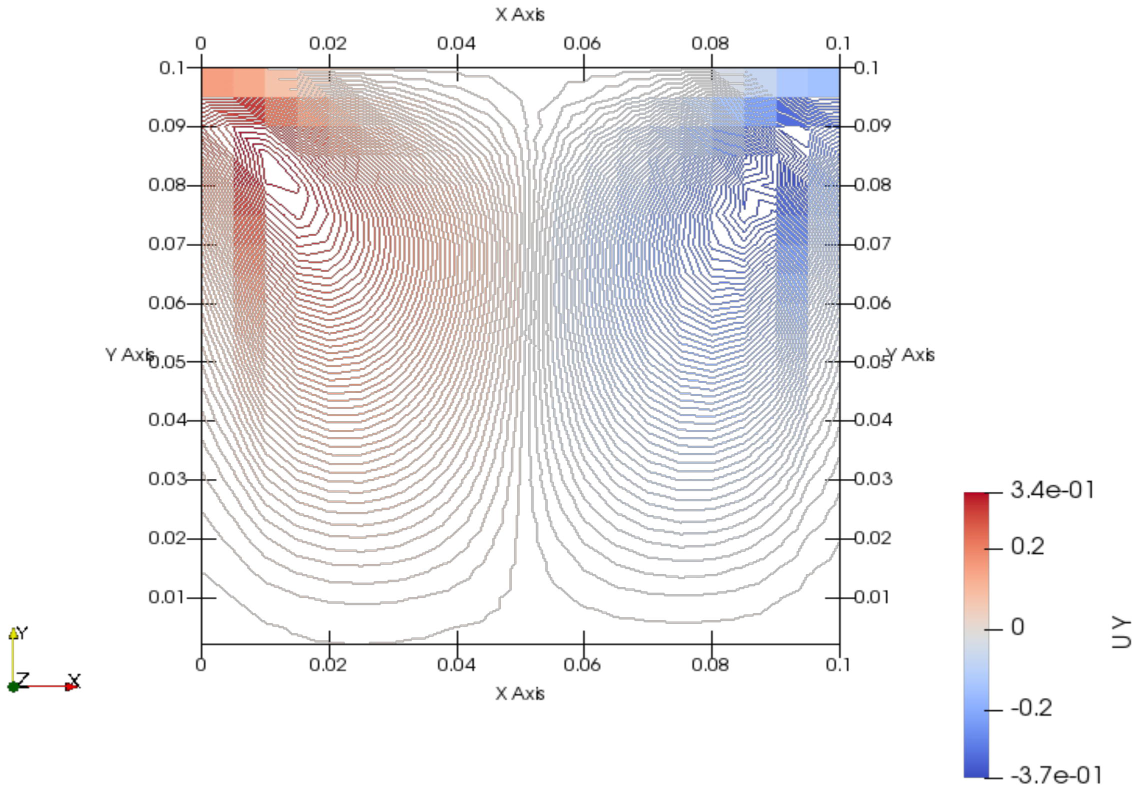
The only tunable parameter that appears in the lid-driven cavity momentum equations is the inverse of the Reynolds' number $\frac{\nu}{UL} = \frac{1}{Re}$. If this term approaches infinity, the Reynolds' number reaches zero, causing the viscous term to dominate the momentum equation. If this term approaches zero, the Reynolds'

number approaches infinity, and the viscous term goes to zero and we can make the inviscid flow assumption.
Description of the flow for Re = 10 {.chapter}

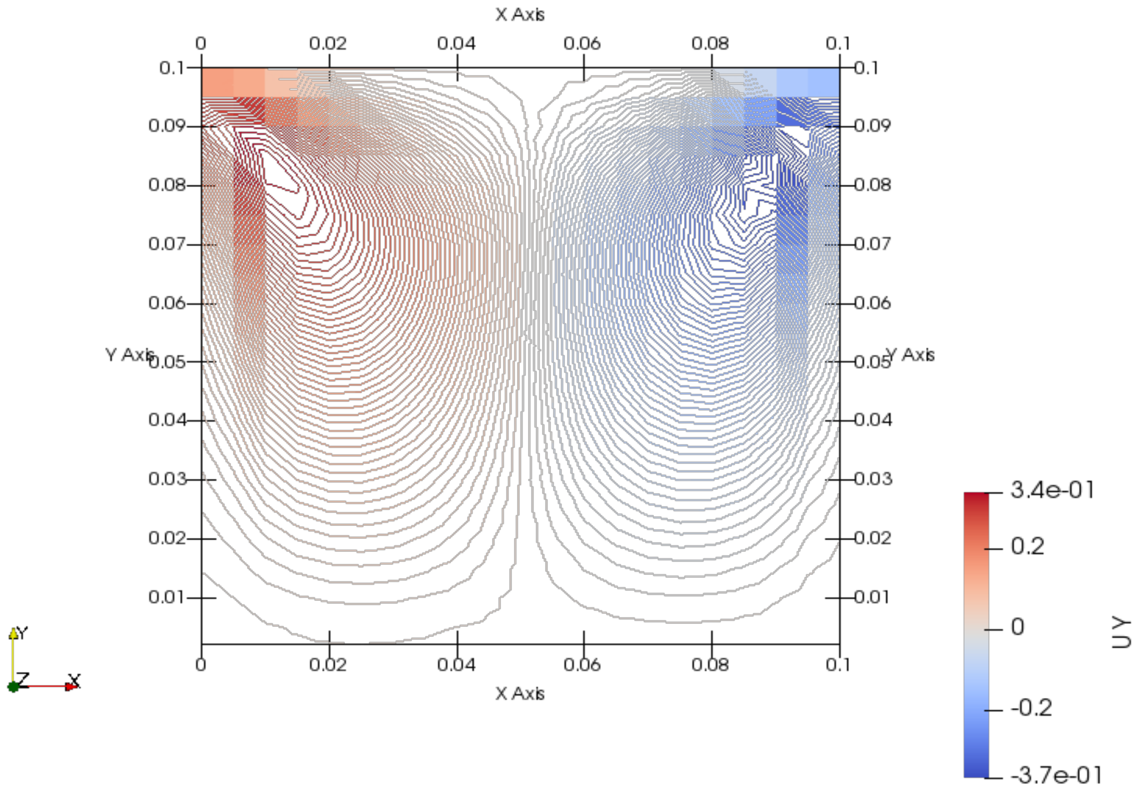
Following is the solution to the lid-driven cavity with lid velocity $U_0 = 1\frac{m}{s}$ to the right, a characteristic length $L = 0.1m$, and kinematic viscosity $\nu = \frac{\mu}{\rho} = 0.01\frac{m^2}{s}$, yielding a Reynolds number of $Re = \frac{UL}{\nu} = 10$. Note the upper plane $(y = 0.1m)$ represents the moving lid.
Please note that all output variables have been nondimensionalized to allow for easier comparision. So we report $\pi = \frac{P}{\rho}$ pressure, and $\tilde{u} = \frac{U}{U_0}$, $\tilde{v} = \frac{V}{U_0}$.
20 grid points in each axis were used.

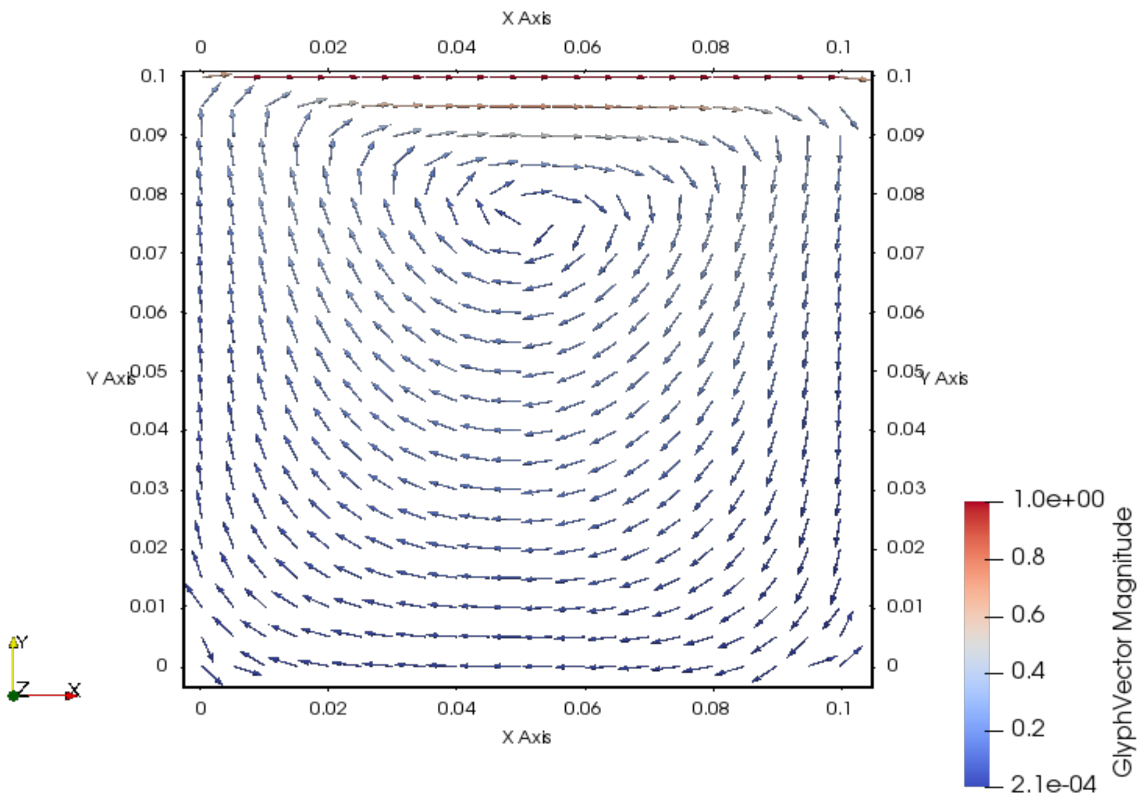
Contour plots for the X and Y components of fluid velocity, respectively:



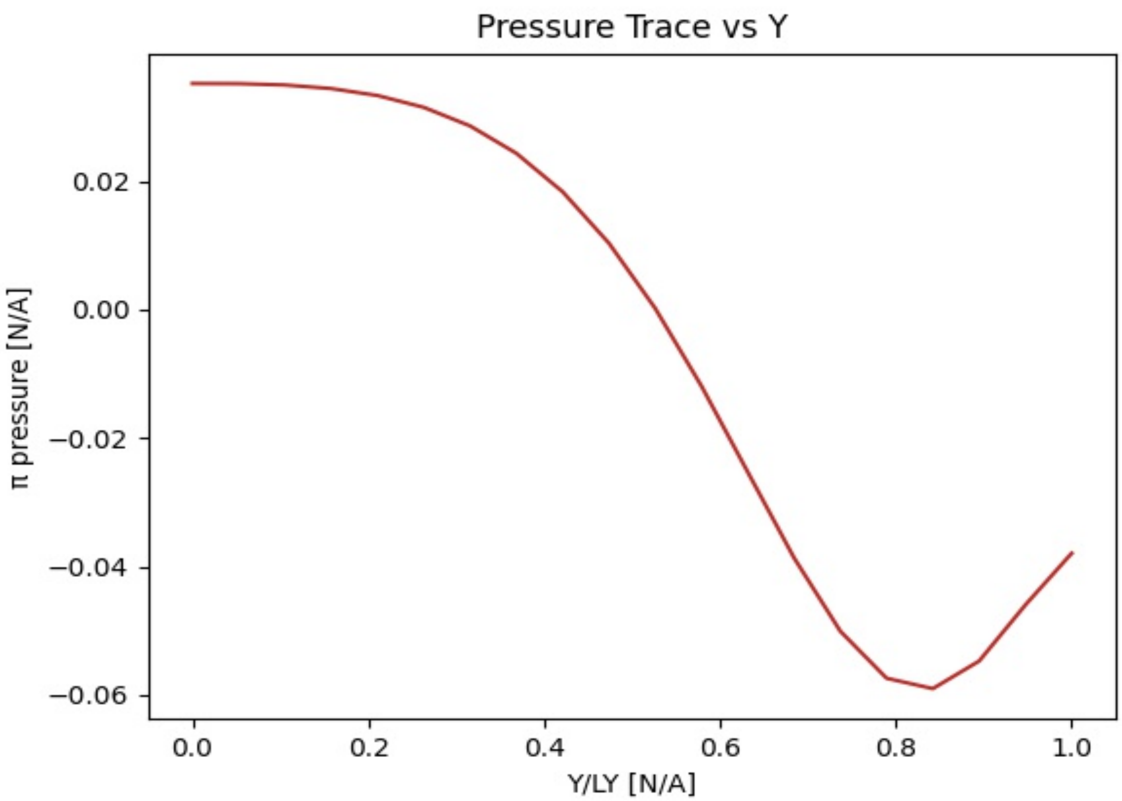


A streamline plot, and a glyph-based plot to show velocity direction:

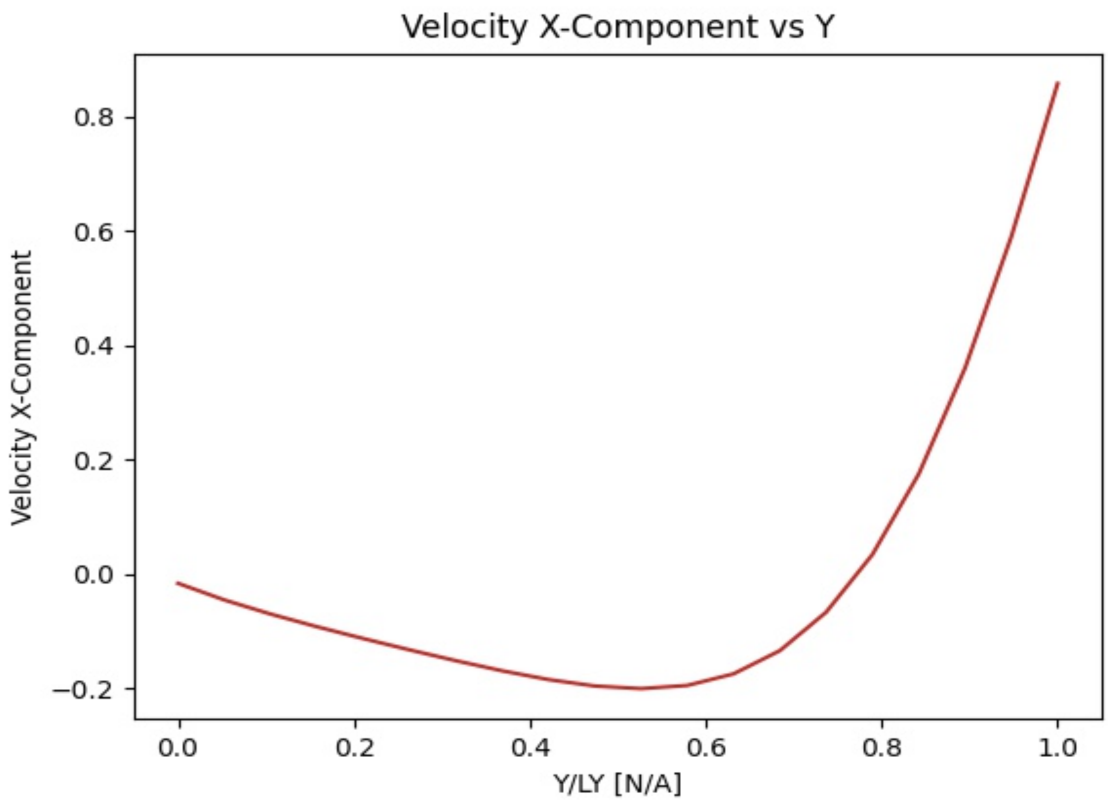

X Axis
0
0.02
0.04
0.06
0.08
0.1
Y Axis
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
3.4e-01
0.2
0
-0.2
-3.7e-01
U Y

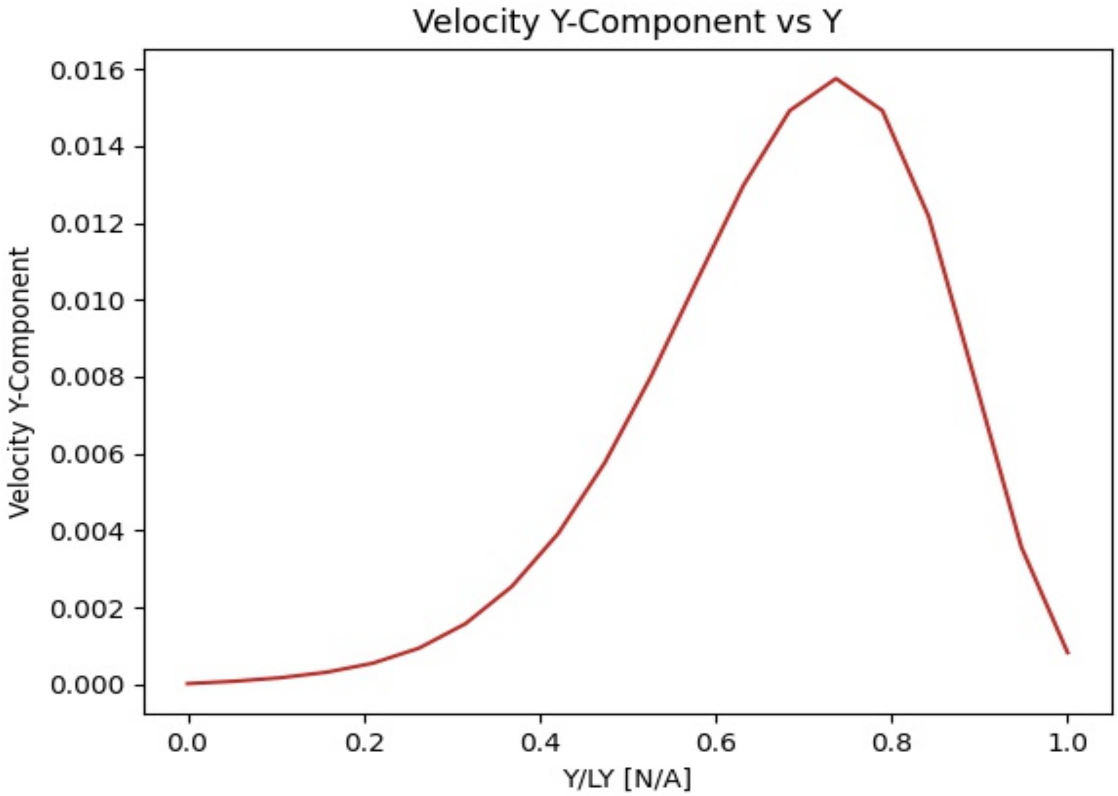


A pressure and velocity profile sampled along the midline: $\tilde{x} = 0.5$, in component form. Respectively, $\pi(\tilde{x} = 0.5, \tilde{y})$, $\tilde{u}(\tilde{x} = 0.5, \tilde{y})$, $\tilde{v}(\tilde{x} = 0.5, \tilde{y})$.



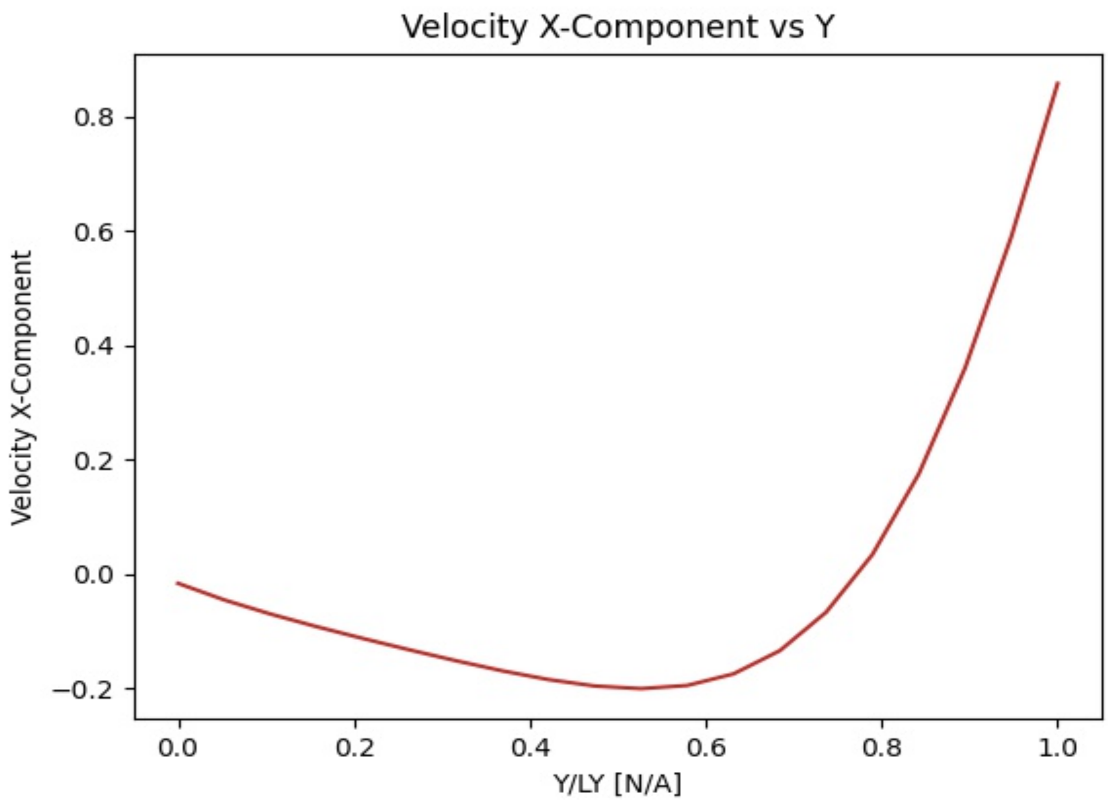
Pressure Trace vs Y
π pressure [N/A]
0.02
0.00
−0.02
−0.04
−0.06
0.0
0.2
0.4
0.6
0.8
1.0
Y/LY [N/A]

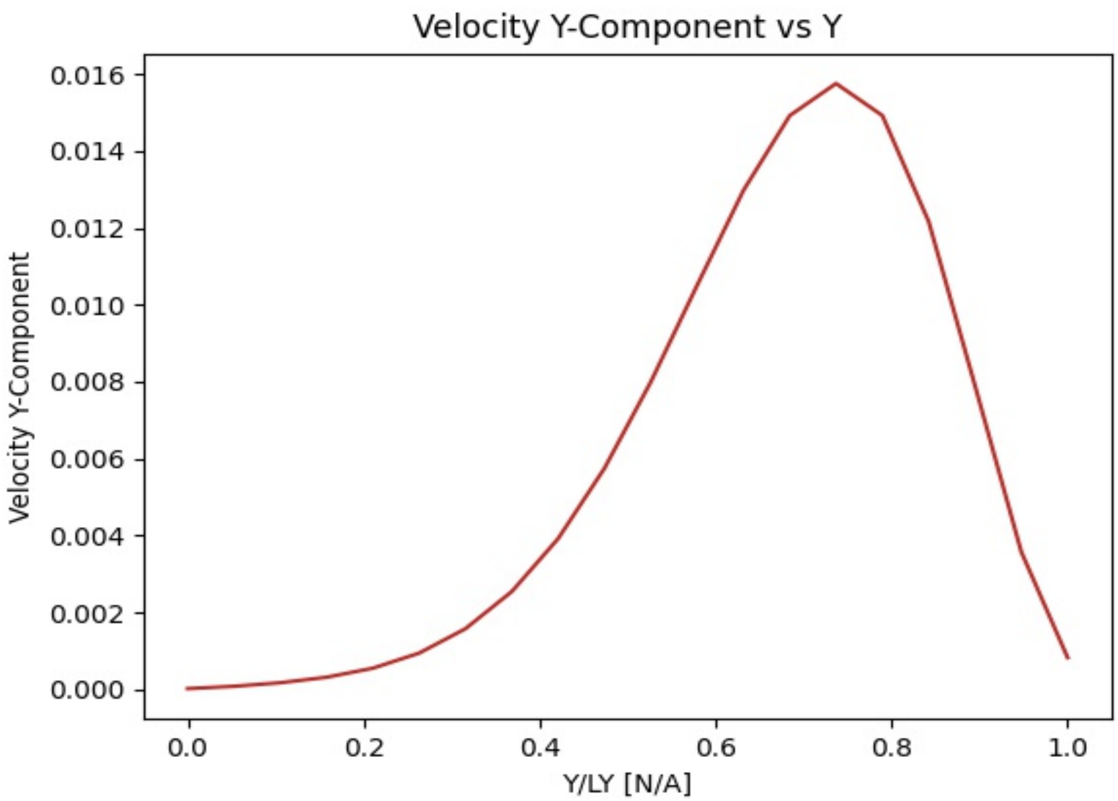

Velocity X-Component vs Y
Velocity X-Component
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
Y/LY [N/A]

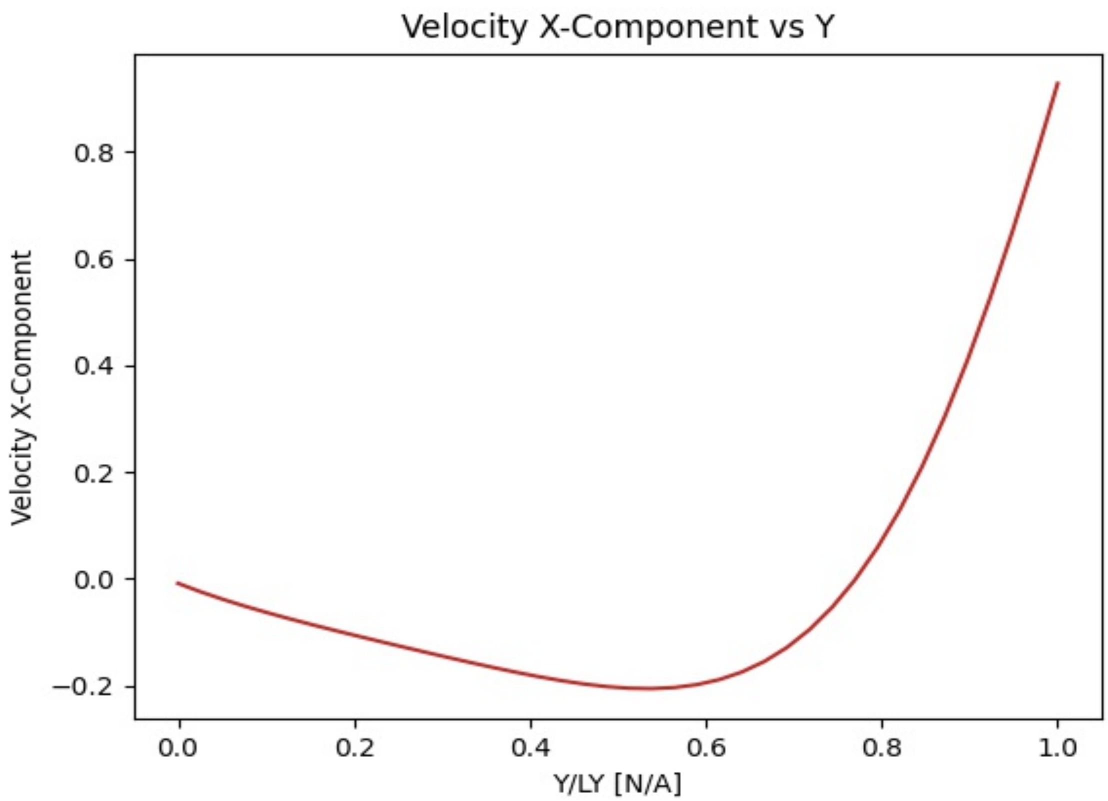


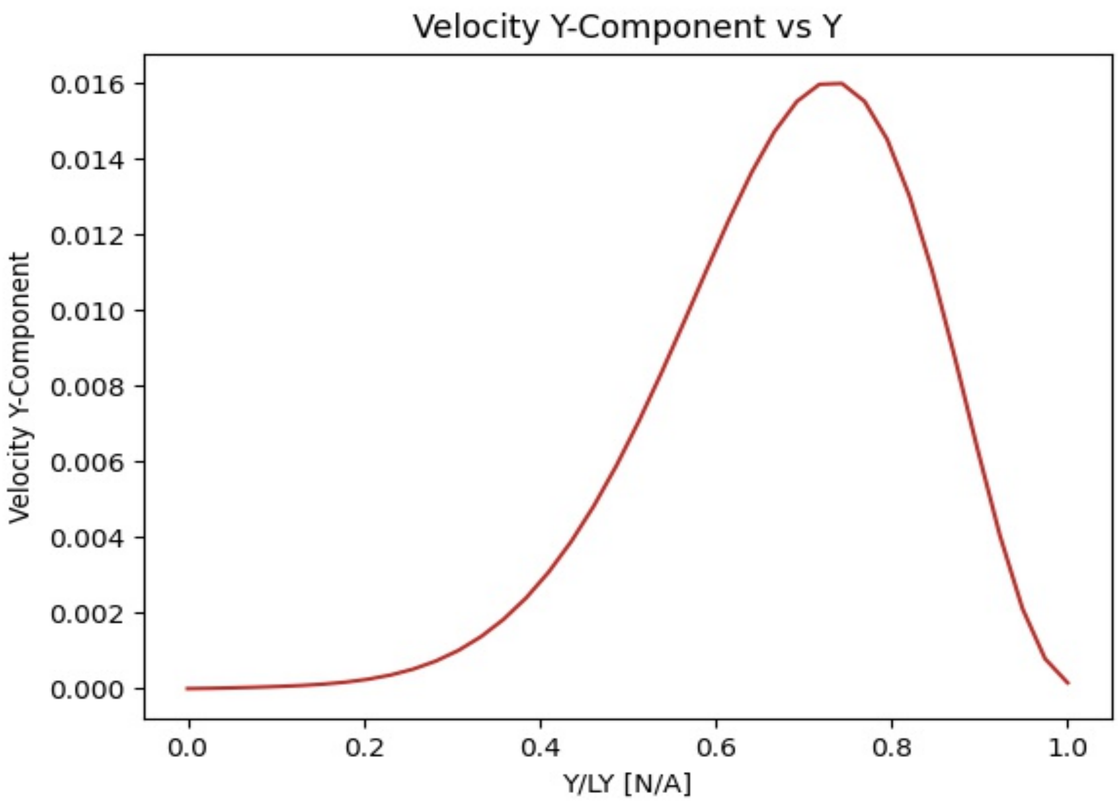
3. Refining the solution

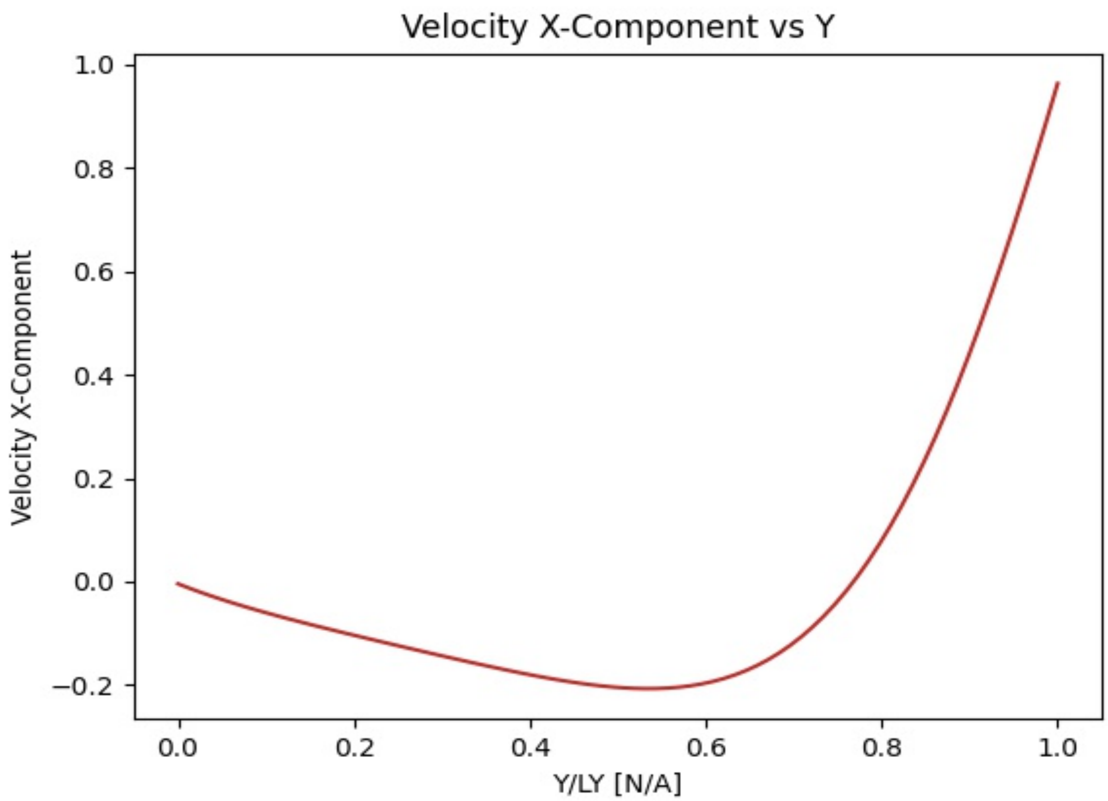
We will demonstrate the increase in image fidelity due to finer discretization. The earlier midline velocity profiles: $\tilde{u}(\tilde{x} = 0.5, \tilde{y})$, $\tilde{v}(\tilde{x} = 0.5, \tilde{y})$ are successively halved in binwidth (grid size) in all directions: x, y, and t, for comparision.

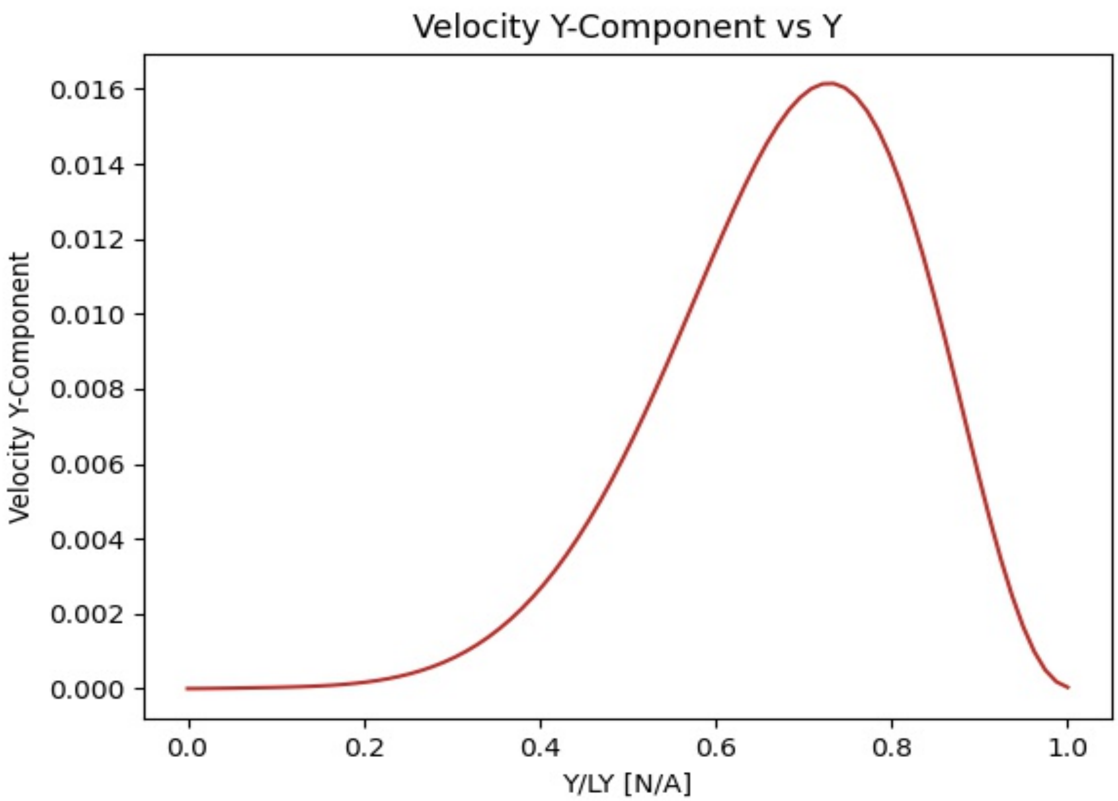

Velocity X-Component vs Y
Velocity X-Component
Y/LY [N/A]
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

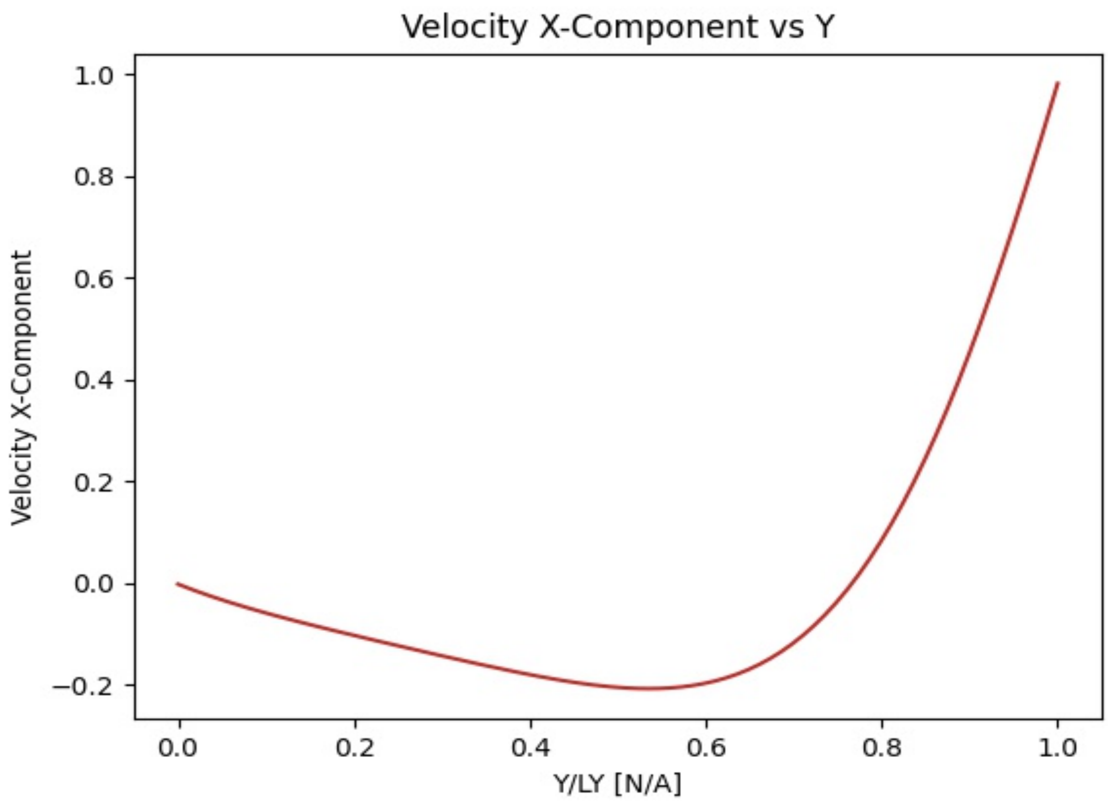

Velocity Y-Component vs Y
Velocity Y-Component
Y/LY [N/A]
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0.014
0.016
0.0
0.2
0.4
0.6
0.8
1.0


Velocity X-Component vs Y
Velocity X-Component
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
Y/LY [N/A]

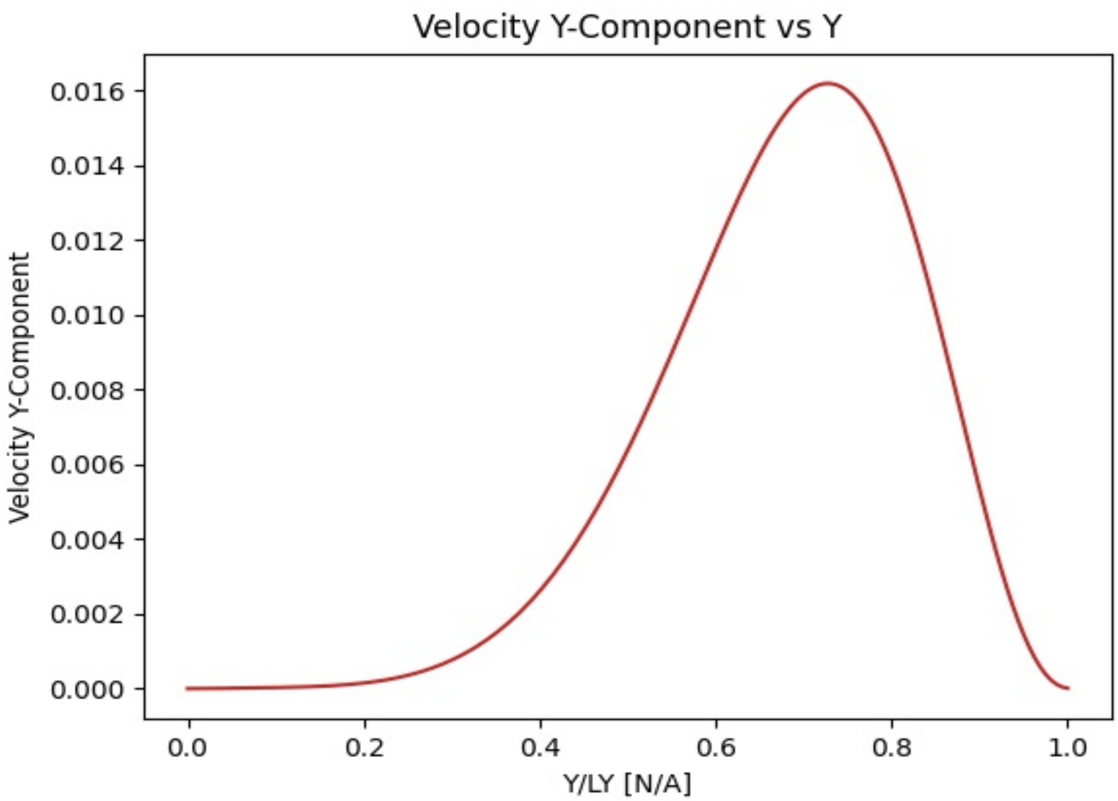

Velocity Y-Component vs Y
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
Velocity Y-Component
0.0
0.2
0.4
0.6
0.8
1.0
Y/LY [N/A]


Velocity X-Component vs Y
Velocity X-Component
Y/LY [N/A]
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

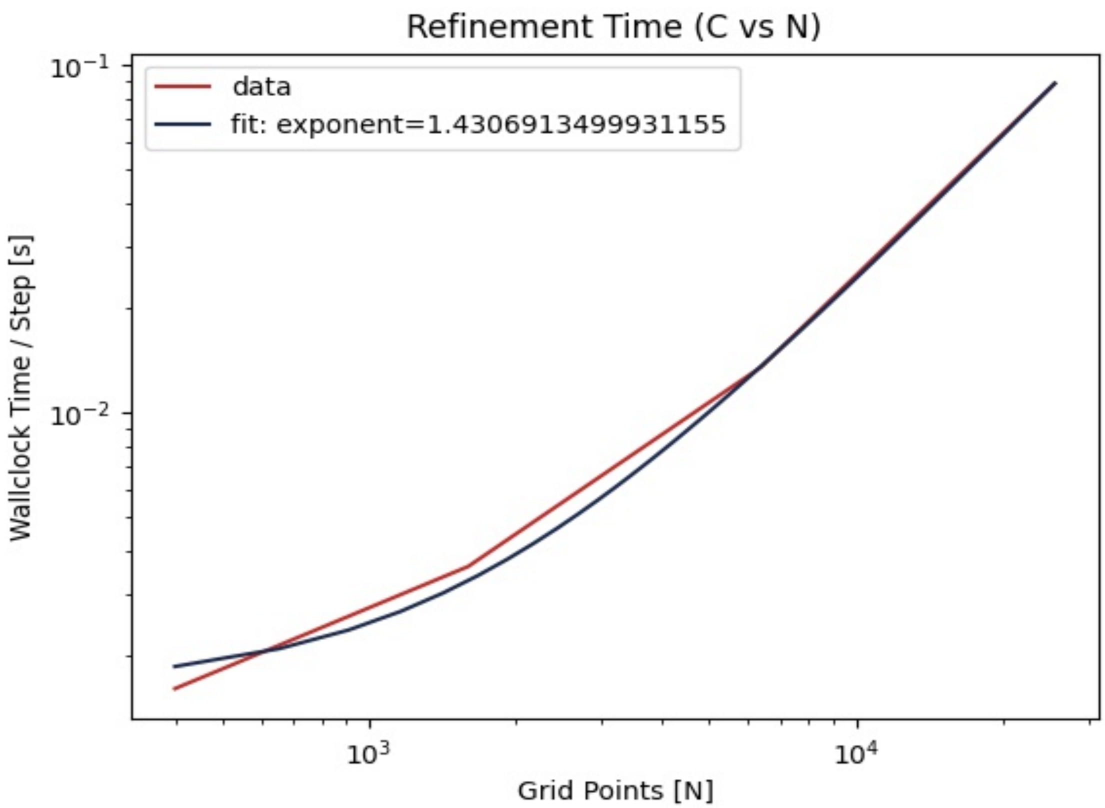

Velocity Y-Component vs Y
Velocity Y-Component
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0.0
0.2
0.4
0.6
0.8
1.0
Y/LY [N/A]



Velocity X-Component vs Y
Velocity X-Component
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
Y/LY [N/A]



As a final comparision, we plot C, the wallclock time per second versus N, the number of grid points used. As this is a 2D simulation, we expect a form similar to $C = \gamma + \beta * N^{\alpha}$, where $\alpha = 2$, due to the number of dimensions. A fit to this curve is also shown.



Interestingly, we do not get $\alpha = 2$. The decreased alpha implies a large processing overhead necessary to start the simulation, especially as the fit is less accurate with fewer grid points. We can see that the fit has a large curvature, which would be produced by a constant overhead: the γ variable, so there is some evidence for the is explanation. A similar test with a higher overall number of grid points ought to produce an alpha closer to 2. An alternative explanation would be accuracy saturation, where the solver reaches the required accuracy in fewer steps than expected, due to the fineness of the grid, causing increased performance, but this explanation is highly improbable.

Finally, we note that there are nearly imperceptible differences between the velocity profiles for grids 3 and 4, so a grid size of 80 points per axis satisfies the necessary accuracy.

4. Force on a Lid

In this section, the nondimensional stress () and the nondimensional force (F) along the lid of the cavity were investigated at different Reynolds numbers.

This was done by keeping U and L constant while only changing the dynamic viscosity.

Nondimensional stress was calculated using Equation # using the finite difference method to approximate the derivative. Nondimensional force was then calculated using Equation #.

$$t = dudy$$

$$F = \int t(x)x$$

The stress plot has a minimum at the center of the lid and increases to a maximum close to both sides of the cavity's walls. This may be due to boundary conditions at the walls.

The clear trend apparent in the nondimensional force plot is that it linearly increases as the Reynolds number increases. This is supported by the understanding that at higher Reynolds number we experience flows dominated by turbulence caused by inertial forces.

References

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Appendix

Thank you so much for reading this work!