

Find nondimensional form of x & y components of momentum equation:

From (1),

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

The nondimensional variables,

$$\frac{dx}{L} = \frac{U}{L} \frac{d\tilde{x}}{\partial \tilde{x}}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{U}{L} \frac{\partial}{\partial \tilde{x}} \left(\frac{\partial \tilde{u}}{\partial \tilde{x}} \right) \frac{\partial \tilde{x}}{\partial x} = \frac{U}{L^2} \frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2}$$

$$\frac{\partial v}{\partial y} = \frac{U}{L} \frac{\partial \tilde{v}}{\partial \tilde{y}}$$

$$\frac{\partial^2 v}{\partial y^2} = \frac{U}{L^2} \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}$$

$$** \Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad \text{in 2D}$$

$$\text{and } \tilde{\nabla} = \frac{\partial}{\partial \tilde{x}} \hat{i} + \frac{\partial}{\partial \tilde{y}} \hat{j} \quad \text{in 2D}$$

Plug into (1),

$$\rho \left(\left(\frac{U}{L} \right) \left(\frac{U}{L} \right) \frac{\partial \tilde{u}}{\partial \tilde{t}} + (\tilde{u}U) \left(\frac{U}{L} \frac{\partial \tilde{u}}{\partial \tilde{x}} \right) + (\tilde{v}U) \left(\frac{U}{L} \frac{\partial \tilde{u}}{\partial \tilde{y}} \right) \right) = -\frac{\partial P}{\partial \tilde{x} L} + \mu \left(\frac{U}{L^2} \frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{U}{L^2} \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \right)$$

$$\rho \frac{U}{L} \left(\frac{\partial \tilde{u}}{\partial \tilde{t}^2} + \tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} \right) = -\frac{\rho U^2}{L} \frac{\partial \tilde{P}}{\partial \tilde{x}} + \mu \frac{U}{L^2} \left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \right)$$