

# Package ‘DiffOp’

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**Title** Nonstationary Spatial Cross-Covariance Functions Based on the  
Differential Operators Approach in 3D

**Description** An R package that implements a flexible multivariate nonstationary cross-covariance function model based on the differential operators approach defined in 3-dimensional (3D) space. A defining feature of the model is that it can accommodate nonstationarity in the variances, colocated correlations, and other spatial features of a multivariate Gaussian random field, and is particularly flexible along the vertical dimension. With the implemented cross-covariance function model, users can simulate synthetic multivariate Gaussian random fields. Inference for model parameters is done via maximum likelihood estimation. Predictions at locations with no observations can also be performed.

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**Depends** R (>= 3.5.0)

**Imports** graphics, stats, mvtnorm, pbdBASE

**Suggests** dplyr, testthat, devtools, roxygen2

**Remotes** marysalvana/pbdBASE

**License** GPL-3

**URL** <https://github.com/marysalvana/DiffOp>

**BugReports** <https://github.com/marysalvana/DiffOp/issues>

**Encoding** UTF-8

**ByteCompile** true

**LazyData** true

**RoxygenNote** 7.2.3

**Roxygen** list(markdown=TRUE)

**NeedsCompilation** yes

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bsplineBasis	<i>Compute the value of the bspline bases</i>
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### Description

Compute the value of the bspline bases

### Usage

```
bsplineBasis(
  x,
  degree,
  innerknots,
  lowknot = min(x, innerknots),
  highknot = max(x, innerknots)
)
```

### Arguments

x	A matrix of basis function values.
degree	A number indicating the degrees.
innerknots	A vector of knot locations.
lowknot	The first knot location.
highknot	The last knot location.

### Value

A matrix of basis function values.

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cov\_bi\_differential

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Compute the bivariate differential operator cross-covariance function

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### Description

`cov_bi_differential` evaluates the nonstationary spatial cross-covariance function model based on the differential operators approach in 3D of the form:

$$C_{ij}(L_1, L_2, l_1 - l_2, p_1, p_2) = K_{ij}^1 \mathcal{M}_{\nu_{ij}-1} \{h(L_1, L_2, l_1 - l_2, p_1 - p_2)^{1/2}\} \\ + K_{ij}^2 \mathcal{M}_{\nu_{ij}} \{h(L_1, L_2, l_1 - l_2, p_1 - p_2)^{1/2}\},$$

for different pairs of locations  $(L_1, l_1, p_1)$  and  $(L_2, l_2, p_2)$ , where  $L$  represents the latitude,  $l$  the longitude, and  $p$  the pressure coordinates, respectively. The forms of  $K_{ij}^1$  and  $K_{ij}^2$  can be found in Appendix of Salvana, M. L., & Jun, M. (2022) and  $h(L_1, L_2, l_1 - l_2, p_1 - p_2)$  is a distance function of the form:

$$h(L_1, L_2, l_1 - l_2, p_1 - p_2) = a_h^2 ch^2(L_1, L_2, l_1 - l_2) + a_v^2 (p_1 - p_2)^2,$$

where  $a_h$  and  $a_v$  are the scale parameters in the horizontal and vertical directions, respectively, and  $ch(L_1, L_2, l_1 - l_2)$  is the chordal distance with the following formula:

$$ch(L_1, L_2, l_1 - l_2) = 2R \left\{ \sin^2 \left( \frac{L_1 - L_2}{2} \right) + \cos L_1 \cos L_2 \sin^2 \left( \frac{l_1 - l_2}{2} \right) \right\}^{1/2}.$$

Here  $R$  is the radius of the sphere. Note that for global processes, the relevant sphere is the Earth with  $R = 6,371$  km.

### Usage

```
cov_bi_differential(location, beta, scale_horizontal, scale_vertical,
a1, b1, c1, d1, a2, b2, c2, d2, radius)
```

### Arguments

<code>location</code>	An $n \times 3$ matrix of coordinates.
<code>beta</code>	A numeric constant indicating the colocated correlation parameter.
<code>scale_horizontal</code>	A numeric constant indicating the horizontal scale parameter.
<code>scale_vertical</code>	A numeric constant indicating the vertical scale parameter.
<code>a1</code>	A numeric constant indicating the anisotropy in latitude parameter associated with variable 1.
<code>b1</code>	A numeric constant indicating the anisotropy in longitude parameter associated with variable 1.
<code>c1</code>	A numeric vector indicating the nonstationary parameters with depth associated with variable 1.
<code>d1</code>	A numeric constant indicating the variance parameter from the fully isotropic component associated with variable 1.

a2	A numeric constant indicating the anisotropy in latitude parameter associated with variable 2.
b2	A numeric constant indicating the anisotropy in longitude parameter associated with variable 2.
c2	A numeric vector indicating the nonstationary parameters with depth associated with variable 2.
d2	A numeric constant indicating the variance parameter from the fully isotropic component associated with variable 2.
radius	A numeric constant indicating the radius of the sphere.

### Value

A cross-covariance matrix of dimension  $2n \times 2n$ .

### Author(s)

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### References

Salvana, M. L., & Jun, M. (2022). 3D Bivariate Spatial Modelling of Argo Ocean Temperature and Salinity Profiles. *arXiv preprint arXiv:2210.11611*.

### Examples

```
library(dplyr)

x <- seq(0, 1, length.out = 10)
y <- seq(0, 1, length.out = 10)

loc2d <- expand.grid(x, y) %>% as.matrix()
depth <- seq(0, 1, length.out = 10)
loc3d <- cbind(rep(loc2d[, 1], each = length(depth)), rep(loc2d[, 2], each = length(depth)))

earthRadiusKm = 6371

BETA = 0.5
SCALE_HORIZONTAL = 0.03
SCALE_VERTICAL = 0.3
A1 = A2 = 0.00001
B1 = B2 = 0.00001
C1 = sin((loc3d[1:10, 3] + 0.1) * pi / 0.5)
C2 = cos((loc3d[1:10, 3] + 0.1) * pi / 0.5)
D1 = D2 = 0

cov_mat <- cov_bi_differential(location = loc3d, beta = BETA,
                              scale_horizontal = SCALE_HORIZONTAL,
                              scale_vertical = SCALE_VERTICAL,
                              a1 = A1, b1 = B1, c1 = C1, d1 = D1,
                              a2 = A2, b2 = B2, c2 = C2, d2 = D2,
                              radius = earthRadiusKm)
```

DAT

*Argo profile data in a local region***Description**

Shrimp are classified by size, 0-15 shrimp per pound, 15-20 shrimp per pound, etc. A smaller number per pound indicates larger shrimp. Nominal prices are total monthly value of brown shrimp landings within size class divided by total monthly landings within the size class.

**Usage**

DAT

**Format**

A tibble with 243 rows and 10 variables:

**Year** dbl Year price was recorded

**Month** dbl Month price was recorded. Ranges from 1-12 for January - December

0-15 dbl denoting monthly price for the 0-15/lb. size class

15-20 dbl denoting monthly price for the 15-20/lb. size class

20-25 dbl denoting monthly price for the 20-25/lb. size class

25-30 dbl denoting monthly price for the 25-30/lb. size class

30-40 dbl denoting monthly price for the 30-40/lb. size class

40-50 dbl denoting monthly price for the 40-50/lb. size class

50-67 dbl denoting monthly price for the 50-67/lb. size class

**Pieces** dbl denoting monthly price of pieces of shrimp totalling a pound

**Source**

<https://www.pnas.org/content/114/7/1512/tab-figures-data>

est\_bi\_differential

*Step 2: MLE estimation of the parameters of the bivariate differential operator cross-covariance function*

**Description**

Step 2: MLE estimation of the parameters of the bivariate differential operator cross-covariance function

**Usage**

```

est_bi_differential(
  residuals,
  location,
  init_beta,
  init_scale_horizontal,
  init_scale_vertical,
  init_scale_horizontal_fix = F,
  init_scale_vertical_fix = F,
  init_a1,
  init_b1,
  init_c1,
  init_d1 = NULL,
  init_a1_fix = F,
  init_b1_fix = F,
  init_a2,
  init_b2,
  init_c2,
  init_d2 = NULL,
  init_a2_fix = F,
  init_b2_fix = F,
  init_beta_fix = F,
  radius,
  basis1,
  nb1 = ncol(basis1),
  basis2,
  nb2 = ncol(basis2),
  splines_degree = 4,
  knots1,
  knots2,
  MAXIT = 2000,
  RERUNS = 20,
  STEPMAX = 1
)

```

**Arguments**

<code>residuals</code>	A vector of residuals
<code>location</code>	An nx3 matrix of coordinates.
<code>init_beta</code>	A number for colocated correlation parameter.
<code>init_scale_horizontal</code>	A number for the horizontal scale parameter.
<code>init_scale_vertical</code>	A number for the vertical scale parameter.
<code>init_scale_horizontal_fix</code>	An indicator whether horizontal scale parameter should be estimated.
<code>init_scale_vertical_fix</code>	An indicator whether vertical scale parameter should be estimated.
<code>init_a1</code>	A number for the anisotropy parameter in Latitude associated with variable 1.
<code>init_b1</code>	A number for the anisotropy parameter in longitude associated with variable 1.

init_c1	A number for vector for the nonstationarity parameter in depth associated with variable 1.
init_d1	A number for the variance parameter of the fully isotropic associated with variable 1.
init_a1_fix	An indicator whether a1 parameter should be estimated.
init_b1_fix	An indicator whether b1 parameter should be estimated.
init_a2	A number for the anisotropy parameter in Latitude associated with variable 2.
init_b2	A number for the anisotropy parameter in longitude associated with variable 2.
init_c2	A number for vector for the nonstationarity parameter in depth associated with variable 2.
init_d2	A number for the variance parameter of the fully isotropic associated with variable 2.
init_a2_fix	An indicator whether a1 parameter should be estimated.
init_b2_fix	An indicator whether b1 parameter should be estimated.
init_beta_fix	An indicator whether colocated correlation parameter should be estimated.
radius	A number for the radius of the sphere.
basis1	A matrix of basis function values for variable 1.
nb1	A number indicating the number of bases for variable 1.
basis2	A matrix of basis function values for variable 2.
nb2	A number indicating the number of bases for variable 2.
splines_degree	A number indicating the degree of the splines.
knots1	A vector of knot locations for variable 1.
knots2	A vector of knot locations for variable 2.
MAXIT	A number indicating the maximum number of iterations for optim.
RERUNS	A number indicating the number of times optim is re-run from previous MLE.
STEPMAX	A number indicating the stepmax of nlm.

**Value**

A vector of estimated parameter values.

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est\_bi\_differential\_wls

*Step 1: WLS estimation of the parameters of the bivariate differential operator cross-covariance function*

---

**Description**

Step 1: WLS estimation of the parameters of the bivariate differential operator cross-covariance function

**Usage**

```

est_bi_differential_wls(
  empirical_values,
  location,
  init_beta,
  init_scale_horizontal,
  init_scale_vertical,
  init_scale_horizontal_fix = F,
  init_scale_vertical_fix = F,
  init_a1,
  init_b1,
  init_c1,
  init_d1 = NULL,
  init_a1_fix = F,
  init_b1_fix = F,
  init_a2,
  init_b2,
  init_c2,
  init_d2 = NULL,
  init_a2_fix = F,
  init_b2_fix = F,
  init_beta_fix = F,
  radius,
  basis1,
  basis2,
  splines_degree = 4,
  knots1,
  knots2
)

```

**Arguments**

<code>empirical_values</code>	A matrix of empirical values
<code>location</code>	An nx3 matrix of coordinates.
<code>init_beta</code>	A number for colocated correlation parameter.
<code>init_scale_horizontal</code>	A number for the horizontal scale parameter.
<code>init_scale_vertical</code>	A number for the vertical scale parameter.
<code>init_scale_horizontal_fix</code>	An indicator whether horizontal scale parameter should be estimated.
<code>init_scale_vertical_fix</code>	An indicator whether vertical scale parameter should be estimated.
<code>init_a1</code>	A number for the anisotropy parameter in Latitude associated with variable 1.
<code>init_b1</code>	A number for the anisotropy parameter in longitude associated with variable 1.
<code>init_c1</code>	A number for vector for the nonstationarity parameter in depth associated with variable 1.
<code>init_d1</code>	A number for the variance parameter of the fully isotropic associated with variable 1.



init_a1_fix	An indicator whether a1 parameter should be estimated.
init_b1_fix	An indicator whether b1 parameter should be estimated.
init_a2	A number for the anisotropy parameter in Latitude associated with variable 2.
init_b2	A number for the anisotropy parameter in longitude associated with variable 2.
init_c2	A number for vector for the nonstationarity parameter in depth associated with variable 2.
init_d2	A number for the variance parameter of the fully isotropic associated with variable 2.
init_a2_fix	An indicator whether a2 parameter should be estimated.
init_b2_fix	An indicator whether b2 parameter should be estimated.
init_beta_fix	An indicator whether colocated correlation parameter should be estimated.
radius	A number for the radius of the sphere.
basis1	A matrix of basis function values for variable 1.
basis2	A matrix of basis function values for variable 2.
splines_degree	A number indicating the degree of the splines.
knots1	A vector of knot locations for variable 1.
knots2	A vector of knot locations for variable 2.

**Value**

A vector of estimated parameter values.

---

plot\_bi\_differential

*Plotting the bivariate differential operator cross-covariance function*

---

**Description**

Plotting the bivariate differential operator cross-covariance function

**Usage**

```
plot_bi_differential(
  location,
  est_beta,
  est_scale_horizontal,
  est_scale_vertical,
  est_a1,
  est_b1,
  est_c1,
  est_d1 = NULL,
  est_a2,
  est_b2,
  est_c2,
  est_d2 = NULL,
```

```

    radius,
    basis1,
    nb1 = ncol(basis1),
    basis2,
    nb2 = ncol(basis2),
    splines_degree = 4
)

```

### Arguments

<code>location</code>	An nx3 matrix of coordinates with latitude and longitude of the reference location and the pressure coordinate is ordered from surface to bottom.
<code>est_beta</code>	A number for colocated correlation parameter.
<code>est_scale_horizontal</code>	A number for the horizontal scale parameter.
<code>est_scale_vertical</code>	A number for the vertical scale parameter.
<code>est_a1</code>	A number for the anisotropy parameter in Latitude associated with variable 1.
<code>est_b1</code>	A number for the anisotropy parameter in longitude associated with variable 1.
<code>est_c1</code>	A number for vector for the nonstationarity parameter in depth associated with variable 1.
<code>est_d1</code>	A number for the variance parameter of the fully isotropic associated with variable 1.
<code>est_a2</code>	A number for the anisotropy parameter in Latitude associated with variable 2.
<code>est_b2</code>	A number for the anisotropy parameter in longitude associated with variable 2.
<code>est_c2</code>	A number for vector for the nonstationarity parameter in depth associated with variable 2.
<code>est_d2</code>	A number for the variance parameter of the fully isotropic associated with variable 2.
<code>radius</code>	A number for the radius of the sphere.
<code>basis1</code>	A matrix of basis function values for variable 1.
<code>nb1</code>	A number indicating the number of bases for variable 1.
<code>basis2</code>	A matrix of basis function values for variable 2.
<code>nb2</code>	A number indicating the number of bases for variable 2.
<code>splines_degree</code>	A number indicating the degree of the splines.

### Value

Figure/plots

---

```
predict_bi_differential
```

*Prediction function*

---

## Description

Prediction function

## Usage

```
predict_bi_differential(
  residuals,
  location,
  location_new,
  masked_residuals = NULL,
  est_beta,
  est_scale_horizontal,
  est_scale_vertical,
  est_a1,
  est_b1,
  est_c1,
  est_d1 = NULL,
  est_a2,
  est_b2,
  est_c2,
  est_d2 = NULL,
  radius,
  splines_degree = 4,
  knots1,
  knots2
)
```

## Arguments

<code>residuals</code>	A vector of residuals
<code>location</code>	An nx3 matrix of coordinates.
<code>location_new</code>	An nx3 matrix of coordinates where prediction is required.
<code>masked_residuals</code>	A vector of residuals removed to test prediction performance.
<code>est_beta</code>	A number for colocated correlation parameter.
<code>est_scale_horizontal</code>	A number for the horizontal scale parameter.
<code>est_scale_vertical</code>	A number for the vertical scale parameter.
<code>est_a1</code>	A number for the anisotropy parameter in Latitude associated with variable 1.
<code>est_b1</code>	A number for the anisotropy parameter in longitude associated with variable 1.
<code>est_c1</code>	A number for vector for the nonstationarity parameter in depth associated with variable 1.

<code>est_d1</code>	A number for the variance parameter of the fully isotropic associated with variable 1.
<code>est_a2</code>	A number for the anisotropy parameter in Latitude associated with variable 2.
<code>est_b2</code>	A number for the anisotropy parameter in longitude associated with variable 2.
<code>est_c2</code>	A number for vector for the nonstationarity parameter in depth associated with variable 2.
<code>est_d2</code>	A number for the variance parameter of the fully isotropic associated with variable 2.
<code>radius</code>	A number for the radius of the sphere.
<code>splines_degree</code>	A number indicating the degree of the splines.
<code>knots1</code>	A vector of knot locations for variable 1.
<code>knots2</code>	A vector of knot locations for variable 2.

**Value**

A vector of prediction values.

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