Package 'DiffOp'

May 11, 2023

Version 1.0.0

Date 2023-05-10

Title Nonstationary Spatial Cross-Covariance Functions Based on the Differential Operators Approach in 3D

Description An R package that implements a flexible multivariate nonstationary cross-covariance function model based on the differential operators approach defined in 3-dimensional (3D) space. A defining feature of the model is that it can accommodate nonstationarity in the variances, colocated correlations, and other spatial features of a multivariate Gaussian random field, and is particularly flexible along the vertical dimension. With the implemented cross-covariance function model, users can simulate synthetic multivariate Gaussian random fields. Inference for model parameters is done via maximum likelihood estimation. Predictions at locations with no observations can also be performed.

Author Mary Lai Salvana [aut, cre] (<https://orcid.org/0000-0003-4868-7713>), Mikyoung Jun [aut]

Maintainer Mary Lai Salvana <yourlainess@gmail.com>

Depends R (>= 3.5.0)

Imports graphics, stats, mvtnorm, pbdBASE

Suggests dplyr, testthat, devtools, roxygen2

Remotes marysalvana/pbdBASE

License GPL-3

URL https://github.com/marysalvana/DiffOp

BugReports https://github.com/marysalvana/DiffOp/issues

Encoding UTF-8

ByteCompile true

LazyData true

RoxygenNote 7.2.3

Roxygen list(markdown=TRUE)

NeedsCompilation yes

R topics documented:

`bsplineBasis` *Compute the value of the bspline bases*

Description

Compute the value of the bspline bases

Usage

```
bsplineBasis(
  x,
  degree,
  innerknots,
  lowknot = min(x, innerknots),
  highknot = max(x, innerknots)
)
```

Arguments

`x`	A matrix of basis function values.
`degree`	A number indicating the degrees.
`innerknots`	A vector of knot locations.
`lowknot`	The first knot location.
`highknot`	The last knot location.

Value

A matrix of basis function values.

`cov_bi_differential` *Compute the bivariate differential operator cross-covariance function*

Description

`cov_bi_differential` evaluates the nonstationary spatial cross-covariance function model based on the differential operators approach in 3D of the form:

$$C_{ij}(L_1, L_2, l_1 - l_2, p_1, p_2) = K_{ij}^1 \mathcal{M}_{\nu_{ij}-1}\{h(L_1, L_2, l_1 - l_2, p_1 - p_2)^{1/2}\}$$

$$+K_{ij}^2 \mathcal{M}_{\nu_{ij}}\{h(L_1, L_2, l_1 - l_2, p_1 - p_2)^{1/2}\},$$

for different pairs of locations (L_1, l_1, p_1) and (L_2, l_2, p_2), where L represents the latitude, l the longitude, and p the pressure coordinates, respectively. The forms of K_{ij}^1 and K_{ij}^2 can be found in Appendix of Salvana, M. L., & Jun, M. (2022) and $h(L_1, L_2, l_1 - l_2, p_1 - p_2)$ is a distance function of the form:

$$h(L_1, L_2, l_1 - l_2, p_1 - p_2) = a_h^2 ch^2(L_1, L_2, l_1 - l_2) + a_v^2 (p_1 - p_2)^2,$$

where a_h and a_v are the scale parameters in the horizontal and vertical directions, respectively, and $ch(L_1, L_2, l_1 - l_2)$ is the chordal distance with the following formula:

$$ch(L_1, L_2, l_1 - l_2) = 2R\left\{\sin^2\left(\frac{L_1 - L_2}{2}\right) + \cos L_1 \cos L_2 \sin^2\left(\frac{l_1 - l_2}{2}\right)\right\}^{1/2}.$$

Here R is the radius of the sphere. Note that for global processes, the relevant sphere is the Earth with $R = 6,371$ km.

Usage

```
cov_bi_differential(location, beta, scale_horizontal, scale_vertical,
a1, b1, c1, d1, a2, b2, c2, d2, radius)
```

Arguments

`location`	An $n \times 3$ matrix of coordinates.
`beta`	A numeric constant indicating the colocated correlation parameter.
`scale_horizontal`	A numeric constant indicating the horizontal scale parameter.
`scale_vertical`	A numeric constant indicating the vertical scale parameter.
`a1`	A numeric constant indicating the anisotropy in latitude parameter associated with variable 1.
`b1`	A numeric constant indicating the anisotropy in longitude parameter associated with variable 1.
`c1`	A numeric vector indicating the nonstationary parameters with depth associated with variable 1.
`d1`	A numeric constant indicating the variance parameter from the fully isotropic component associated with variable 1.

a2	A numeric constant indicating the anisotropy in latitude parameter associated with variable 2.
b2	A numeric constant indicating the anisotropy in longitude parameter associated with variable 2.
c2	A numeric vector indicating the nonstationary parameters with depth associated with variable 2.
d2	A numeric constant indicating the variance parameter from the fully isotropic component associated with variable 2.
radius	A numeric constant indicating the radius of the sphere.

Value

A cross-covariance matrix of dimension $2n \times 2n$.

Author(s)

Mary Lai Salvana <yourlainess@gmail.com>

References

Salvana, M. L., & Jun, M. (2022). 3D Bivariate Spatial Modelling of Argo Ocean Temperature and Salinity Profiles. *arXiv preprint arXiv:2210.11611*.

Examples

```
library(dplyr)

x <- seq(0, 1, length.out = 10)
y <- seq(0, 1, length.out = 10)

loc2d <- expand.grid(x, y) %>% as.matrix()
depth <- seq(0, 1, length.out = 10)
loc3d <- cbind(rep(loc2d[, 1], each = length(depth)), rep(loc2d[, 2], each = length(depth

earthRadiusKm = 6371

BETA = 0.5
SCALE_HORIZONTAL = 0.03
SCALE_VERTICAL = 0.3
A1 = A2 = 0.00001
B1 = B2 = 0.00001
C1 = sin((loc3d[1:10, 3] + 0.1) * pi / 0.5)
C2 = cos((loc3d[1:10, 3] + 0.1) * pi / 0.5)
D1 = D2 = 0

cov_mat <- cov_bi_differential(location = loc3d, beta = BETA,
                               scale_horizontal = SCALE_HORIZONTAL,
                               scale_vertical = SCALE_VERTICAL,
                               a1 = A1, b1 = B1, c1 = C1, d1 = D1,
                               a2 = A2, b2 = B2, c2 = C2, d2 = D2,
                               radius = earthRadiusKm)
```

DAT *Argo profile data in a local region*

Description

Shrimp are classified by size, 0-15 shrimp per pound, 15-20 shrimp per pound, etc. A smaller number per pound indicates larger shrimp. Nominal prices are total monthly value of brown shrimp andings within size class divided by total monthly landings within the size class.

Usage

```
DAT
```

Format

A tibble with 243 rows and 10 variables:

Year dbl Year price was recorded

Month dbl Month price was recorded. Ranges from 1-12 for January - December

`0-15` dbl denoting monthly price for the 0-15/lb. size class

`15-20` dbl denoting monthly price for the 15-20/lb. size class

`20-25` dbl denoting monthly price for the 20-25/lb. size class

`25-30` dbl denoting monthly price for the 25-30/lb. size class

`30-40` dbl denoting monthly price for the 30-40/lb. size class

`40-50` dbl denoting monthly price for the 40-50/lb. size class

`50-67` dbl denoting monthly price for the 50-67/lb. size class

Pieces dbl denoting monthly price of pieces of shrimp totalling a pound

Source

https://www.pnas.org/content/114/7/1512/tab-figures-data

`est_bi_differential` *Step 2: MLE estimation of the parameters of the bivariate differential operator cross-covariance function*

Description

Step 2: MLE estimation of the parameters of the bivariate differential operator cross-covariance function

Usage

```
est_bi_differential(
  residuals,
  location,
  init_beta,
  init_scale_horizontal,
  init_scale_vertical,
  init_scale_horizontal_fix = F,
  init_scale_vertical_fix = F,
  init_a1,
  init_b1,
  init_c1,
  init_d1 = NULL,
  init_a1_fix = F,
  init_b1_fix = F,
  init_a2,
  init_b2,
  init_c2,
  init_d2 = NULL,
  init_a2_fix = F,
  init_b2_fix = F,
  init_beta_fix = F,
  radius,
  basis1,
  nb1 = ncol(basis1),
  basis2,
  nb2 = ncol(basis2),
  splines_degree = 4,
  knots1,
  knots2,
  MAXIT = 2000,
  RERUNS = 20,
  STEPMAX = 1
)
```

Arguments

`residuals`	A vector of residuals
`location`	An nx3 matrix of coordinates.
`init_beta`	A number for colocated correlation parameter.
`init_scale_horizontal`	A number for the horizontal scale parameter.
`init_scale_vertical`	A number for the vertical scale parameter.
`init_scale_horizontal_fix`	An indicator whether horizontal scale parameter should be estimated.
`init_scale_vertical_fix`	An indicator whether vertical scale parameter should be estimated.
`init_a1`	A number for the anisotropy parameter in Latitude associated with variable 1.
`init_b1`	A number for the anisotropy parameter in longitude associated with variable 1.

init_c1	A number for vector for the nonstationarity parameter in depth associated with variable 1.
init_d1	A number for the variance parameter of the fully isotropic associated with variable 1.
init_a1_fix	An indicator whether a1 parameter should be estimated.
init_b1_fix	An indicator whether b1 parameter should be estimated.
init_a2	A number for the anisotropy parameter in Latitude associated with variable 2.
init_b2	A number for the anisotropy parameter in longitude associated with variable 2.
init_c2	A number for vector for the nonstationarity parameter in depth associated with variable 2.
init_d2	A number for the variance parameter of the fully isotropic associated with variable 2.
init_a2_fix	An indicator whether a1 parameter should be estimated.
init_b2_fix	An indicator whether b1 parameter should be estimated.
init_beta_fix	An indicator whether colocated correlation parameter should be estimated.
radius	A number for the radius of the sphere.
basis1	A matrix of basis function values for variable 1.
nb1	A number indicating the number of bases for variable 1.
basis2	A matrix of basis function values for variable 2.
nb2	A number indicating the number of bases for variable 2.
splines_degree	A number indicating the degree of the splines.
knots1	A vector of knot locations for variable 1.
knots2	A vector of knot locations for variable 2.
MAXIT	A number indicating the maximum number of iterations for optim.
RERUNS	A number indicating the number of times optim is re-run from previous MLE.
STEPMAX	A number indicating the stepmax of nlm.

Value

A vector of estimated parameter values.

`est_bi_differential_wls`

Step 1: WLS estimation of the parameters of the bivariate differential operator cross-covariance function

---

Description

Step 1: WLS estimation of the parameters of the bivariate differential operator cross-covariance function

Usage

```
est_bi_differential_wls(
  empirical_values,
  location,
  init_beta,
  init_scale_horizontal,
  init_scale_vertical,
  init_scale_horizontal_fix = F,
  init_scale_vertical_fix = F,
  init_a1,
  init_b1,
  init_c1,
  init_d1 = NULL,
  init_a1_fix = F,
  init_b1_fix = F,
  init_a2,
  init_b2,
  init_c2,
  init_d2 = NULL,
  init_a2_fix = F,
  init_b2_fix = F,
  init_beta_fix = F,
  radius,
  basis1,
  basis2,
  splines_degree = 4,
  knots1,
  knots2
)
```

Arguments

`empirical_values`	A matrix of empirical values
`location`	An nx3 matrix of coordinates.
`init_beta`	A number for colocated correlation parameter.
`init_scale_horizontal`	A number for the horizontal scale parameter.
`init_scale_vertical`	A number for the vertical scale parameter.
`init_scale_horizontal_fix`	An indicator whether horizontal scale parameter should be estimated.
`init_scale_vertical_fix`	An indicator whether vertical scale parameter should be estimated.
`init_a1`	A number for the anisotropy parameter in Latitude associated with variable 1.
`init_b1`	A number for the anisotropy parameter in longitude associated with variable 1.
`init_c1`	A number for vector for the nonstationarity parameter in depth associated with variable 1.
`init_d1`	A number for the variance parameter of the fully isotropic associated with variable 1.

`init_a1_fix`	An indicator whether a1 parameter should be estimated.
`init_b1_fix`	An indicator whether b1 parameter should be estimated.
`init_a2`	A number for the anisotropy parameter in Latitude associated with variable 2.
`init_b2`	A number for the anisotropy parameter in longitude associated with variable 2.
`init_c2`	A number for vector for the nonstationarity parameter in depth associated with variable 2.
`init_d2`	A number for the variance parameter of the fully isotropic associated with variable 2.
`init_a2_fix`	An indicator whether a2 parameter should be estimated.
`init_b2_fix`	An indicator whether b2 parameter should be estimated.
`init_beta_fix`	An indicator whether colocated correlation parameter should be estimated.
`radius`	A number for the radius of the sphere.
`basis1`	A matrix of basis function values for variable 1.
`basis2`	A matrix of basis function values for variable 2.
`splines_degree`	A number indicating the degree of the splines.
`knots1`	A vector of knot locations for variable 1.
`knots2`	A vector of knot locations for variable 2.

Value

A vector of estimated parameter values.

`plot_bi_differential` *Plotting the bivariate differential operator cross-covariance function*

Description

Plotting the bivariate differential operator cross-covariance function

Usage

```
plot_bi_differential(
  location,
  est_beta,
  est_scale_horizontal,
  est_scale_vertical,
  est_a1,
  est_b1,
  est_c1,
  est_d1 = NULL,
  est_a2,
  est_b2,
  est_c2,
  est_d2 = NULL,
```

```
  radius,
  basis1,
  nb1 = ncol(basis1),
  basis2,
  nb2 = ncol(basis2),
  splines_degree = 4
)
```

Arguments

location	An nx3 matrix of coordinates with latitude and longitude of the reference location and the pressure coordinate is ordered from surface to bottom.
est_beta	A number for colocated correlation parameter.
est_scale_horizontal	A number for the horizontal scale parameter.
est_scale_vertical	A number for the vertical scale parameter.
est_a1	A number for the anisotropy parameter in Latitude associated with variable 1.
est_b1	A number for the anisotropy parameter in longitude associated with variable 1.
est_c1	A number for vector for the nonstationarity parameter in depth associated with variable 1.
est_d1	A number for the variance parameter of the fully isotropic associated with variable 1.
est_a2	A number for the anisotropy parameter in Latitude associated with variable 2.
est_b2	A number for the anisotropy parameter in longitude associated with variable 2.
est_c2	A number for vector for the nonstationarity parameter in depth associated with variable 2.
est_d2	A number for the variance parameter of the fully isotropic associated with variable 2.
radius	A number for the radius of the sphere.
basis1	A matrix of basis function values for variable 1.
nb1	A number indicating the number of bases for variable 1.
basis2	A matrix of basis function values for variable 2.
nb2	A number indicating the number of bases for variable 2.
splines_degree	A number indicating the degree of the splines.

Value

Figure/plots

predict_bi_differential *Prediction function*

Description

Prediction function

Usage

```
predict_bi_differential(
  residuals,
  location,
  location_new,
  masked_residuals = NULL,
  est_beta,
  est_scale_horizontal,
  est_scale_vertical,
  est_a1,
  est_b1,
  est_c1,
  est_d1 = NULL,
  est_a2,
  est_b2,
  est_c2,
  est_d2 = NULL,
  radius,
  splines_degree = 4,
  knots1,
  knots2
)
```

Arguments

residuals	A vector of residuals
location	An nx3 matrix of coordinates.
location_new	An nx3 matrix of coordinates where prediction is required.
masked_residuals	A vector of residuals removed to test prediction performance.
est_beta	A number for colocated correlation parameter.
est_scale_horizontal	A number for the horizontal scale parameter.
est_scale_vertical	A number for the vertical scale parameter.
est_a1	A number for the anisotropy parameter in Latitude associated with variable 1.
est_b1	A number for the anisotropy parameter in longitude associated with variable 1.
est_c1	A number for vector for the nonstationarity parameter in depth associated with variable 1.

est_d1	A number for the variance parameter of the fully isotropic associated with variable 1.
est_a2	A number for the anisotropy parameter in Latitude associated with variable 2.
est_b2	A number for the anisotropy parameter in longitude associated with variable 2.
est_c2	A number for vector for the nonstationarity parameter in depth associated with variable 2.
est_d2	A number for the variance parameter of the fully isotropic associated with variable 2.
radius	A number for the radius of the sphere.
splines_degree	A number indicating the degree of the splines.
knots1	A vector of knot locations for variable 1.
knots2	A vector of knot locations for variable 2.

Value

A vector of prediction values.

Index