MATH 226
FINAL Review: more problems

Problem 1
(a) Find an equation of the plane that passes through the points $A(2, 1, 1)$, $B(-1, -1, 10)$, and $C(1, 3, -4)$.
(b) Find symmetric equations for the line through B that is perpendicular to the plane in part (a).
(c) A second plane passes through (2, 0, 4 and has normal vector $< 2, -4, -3 >$.
Find the angle between the planes.
(d) Find parametric equations for the line of intersection of the two planes.

Problem 2 Let C be the intersection of $x^2 + y^2 = 16$ and $x + y + z = 5$.
1) Find a parametrization of the curve.
2) Find the parametric equations of the tangent line to that curve at the point $(0, 4, 1)$.

Problem 3: Consider the curve $\mathbf{r}(t) = (\sqrt{2}t, e^t, e^{-t})$.
1) Compute $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.
2) Find the equation of the Normal plane to the curve when $t = 0$.
3) Find the equation of the Osculating plane to the curve when $t = 0$.

Problem 4 Consider $f(x, y, z) = \frac{x}{y+z}$
1) Compute the directional derivative of f at the point $(4, 1, 1)$ in the direction of the vector $(1, 2, 3)$.
2)Find the maximum value of the rate of change of f at the point $(1, 0, 1)$ and the direction in which it occurs.

Problem 5 Consider the function $f(x, y) = 1 + 2xy - x^2 - y^2$.

1) Classify all the critical points of f as local maximum, local minimum or saddle point.
2) Find the Absolute minimum and maximum values of f on the domain bounded by the circle of equation $x^2 + y^2 = 4$.

Problem 6
1) Find the volume of below the plane $2x + 5y + z = 10$ above the xy-plane within the cylinder of equation $x^2 + y^2 = 9$.
2) Find the surface area of the piece of the plane $2x + 5y + z = 10$ that lies within the cylinder of equation $x^2 + y^2 = 9$.

Problem 7

Let $F(x, y) = (4x^3y^2 - 2xy^3)\vec{i} + (2x^4y - 3x^2y^2 + 4y^3)\vec{j}$.

a) Show that $\vec{F}$ is conservative. Then find a function f such that $\vec{F} = \nabla f$.

b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve C:

$$\vec{r}(t) = (t + \sin \pi t)\vec{i} + (2t + \cos \pi t)\vec{j}, \quad 0 \le t \le 1.$$

Problem 8
Consider the surface S with upwards orientation defined as the open paraboloid $z = 4 - x^2 - y^2$ for $z \ge 0$

1) Evaluate the surface area of S.

2) If $\mathbf{F}(x, y, z) = yz\mathbf{i}+xz\mathbf{j}+xy\mathbf{k}$, use the method of your choice to evaluate $\int\int_S curl(\mathbf{F}) \cdot d\mathbf{S}$

Problem 9
Consider the surface S given by $z = cosx$ for all y. If $\vec{F} = (x^2, y^2, xz)$, show that $\int_C \vec{F}.d\vec{r} = 0$ for all closed curves C lying on the surface S.

Problem 10
Evaluate the work done by the vector field
$\vec{F} = (y + e^{\sqrt{x}}, 2x + cos(y^2))$ around the curve C, oriented clockwise, which is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Problem 11

Using the method of your choice, evaluate the flux $\displaystyle\int_S \vec{F}\cdot\vec{n}\,dS$ where

$$\vec{F}(x,y,z) = z^2x\vec{i} + (\frac{1}{3}y^3 + \tan z)\vec{j} + (x^2z + y^2)\vec{k}$$

and S is the CLOSED top half of the sphere (so includes the bottom disk) $x^2 + y^2 + z^2 = 4$ oriented inwards.

Problem 12

$$\vec{F}(x,y,z) = (x, z, -y)$$

Consider the tetrahedron with vertices $A = (0,0,0)$, $B = (0,1,0)$, $C = (1, \frac{1}{2}, 0)$ and $D = (0,0,2)$.

1) If S_1 is the closed surface oriented outwards defined as the boundary of the solid tetrahedron, Use the Divergence Theorem to compute $\int\int_{S_1} \vec{F}.dS$.

2) Evaluate $\int\int_{S_2} \vec{F}.dS$ where S_2 is the surface oriented outwards defined as the OPEN tetrahedron NOT containing the roof BCD.