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Abstract

We examine the effect of short interest on corporate investment. We build a theoretical model that demonstrates that short interest incorporates negative information in addition to the information embedded in stock prices, and therefore negatively affects investment. The model predicts a stronger effect of short interest when firms are less transparent and when the level of short interest is high. Our empirical results are consistent with the predictions of the model and survive a battery of robustness tests. We conclude that managers interpret short selling activity as a negative signal about the company's prospects, and in response, reduce investment.

Keywords: Managerial Learning, Market Feedback, Short Interest.

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1 Introduction

A large literature examines the feedback effect of financial markets on the decisions of corporate managers. Since managerial attention to stock prices had been first documented in Chen et al. (2007), a number of studies have shown that managerial actions are sensitive to various aspects of information in financial markets. Previous literature has also established that corporate managers learn from institutional investors (Zhang (2023)), prices of the firm's peers (Foucault and Frésard (2014)), options and futures markets (Chen et al. (2021); Xiong and Yang (2021)), liquidity traders (Banerjee et al. (2018)), and macroeconomic announcements (Binz et al. (2022)).

We argue that there is another source of information that managers learn from - short sellers' positions. Our focus on short selling as another informational resource for corporate executives is motivated by over a decade-long push by the New York Stock Exchange (NYSE) Group Inc. and the National Investor Relations Institute (NIRI) towards more transparent disclosure of short interest data to the markets. One of the central arguments of the petitions, submitted to the SEC over these years, was the notion that corporate executives closely follow short-interest activity in their stock and find it beneficial. For example, in its 2015 letter to the SEC, the NYSE argued that "Many public companies currently utilize available short-sale data to evaluate market outlook and anticipate developments with respect to their securities (p.8)".[1] In response to these efforts, in February 2022 the SEC had finally proposed a rule to ensure a more detailed reporting of short interest. Yet, the ways in which corporate managers will be able to benefit from it, remain unclear.

In this paper, we try to shed light on this question. We argue that short interest aggregates negative signals of informed traders and provides a useful source of information to corporate managers, in addition to prices. In particular, an increase in short interest indicates a negative view of short sellers on the company's future prospects and induces managers to reduce investment. Therefore, managerial learning results in a negative relation between short interest and subsequent corporate investment.

Naturally, it leads to the following questions: Why is the negative view of short sellers not fully reflected in the stock price? Why does short interest represent an additional source of valuable information? To answer these questions and to better understand the mechanism of managerial learning, we build a theoretical model that incorporates learning from both prices and short interest. In the model, there is a price-taking risk-averse informed trader who observes a private signal about

[1] https://www.sec.gov/rules/petitions/2015/petn4-689.pdf

firm value and submits a limit order. When the execution price is above the informed trader's prior, she shorts the asset. In addition, there are liquidity traders who submit a random aggregate market order. A key assumption in the model is that the informed trader has no inventory and therefore always shorts the stock if her signal is negative. Liquidity traders, on the other hand, might or might not be endowed with inventory: a fraction of liquidity traders short if their demand is negative, while another fraction sells the stock. We argue that this assumption is likely to hold empirically. Speculators (e.g. hedge funds), endowed with private information, and seeking to profit from price declines, are most likely to short stocks. By contrast, noise traders may or may not have initial inventory and therefore their negative demand can result in either shorting or selling. For example, it is possible that an option market maker shorts the underlying stock in order to hedge her outstanding option positions. On the other hand, corporate managers, who are routinely compensated with shares sell once those shares are vested, as do individual investors continuously selling part of their 401k portfolios to generate retirement income.

We solve for the equilibrium price and short interest and show that the best estimate of the true fundamental value of the firm is positively related to price and negatively related to short interest. While price still represents an unbiased estimate of the fundamental value of the firm, an estimate based solely on the price is less precise than the one that incorporates both price and short interest. When short interest is high, it is more likely that the asset is shorted by the informed trader, who must have a negative view on the firm value (relative to the price) in order to short. Thus, observing high short interest makes the manager adjust her estimate downwards. By contrast, lower values of short interest are more likely to be driven by shorting activity of the liquidity traders, in which case the informed trader wishes to buy the asset. Therefore, lower short interest is interpreted by the manager as a positive signal. Incorporating short interest improves the accuracy of the manager's best estimate of the firm value. Therefore, a manager who learns solely from the price and ignores short interest is bound to make sub-optimal decisions.

Our model demonstrates that there is an incentive for corporate managers to learn not just from the stock price but also from short interest. If managers learn from short interest and adjust firm investment accordingly, one should expect a negative relation between short interest and investment. This is the main prediction of our model. The model also generates two additional predictions. It demonstrates that the relation between the manager's best estimate of the firm's fundamental value and short interest is non-linear: The informativeness of short interest improves at higher levels of short interest. When short interest is high, it is more likely that the informed trader is on the short side of the trade, so short interest is more informative. In addition, short interest has a stronger impact on investment when information uncertainty is high. In the model, the informed trader must have a stronger negative view on the firm with greater information uncertainty in order to initiate the short position due to her risk-aversion. Therefore, short interest becomes more informative for less transparent firms.[2] We use these theoretical predictions to design our empirical tests.

We start our empirical analysis by estimating the relation between investment and short interest in a panel regression framework. To ensure the robustness of our results, we use three measures of investment: Capital expenditures, capital expenditures combined with intangible investment as proxied for by R&D, and change in total assets. We also use three measures of short interest: Short ratio (the fraction of shares shorted out of total shares outstanding); the residual component of the short ratio, unexplained by standard firm characteristics (as in Karpoff and Lou (2010)); and days-to-cover: the short ratio scaled by turnover (Hong et al. (2015)).

Our baseline results demonstrate a significant negative association between various proxies for short interest and future investment. This relation is statistically significant and economically meaningful: An increase in short interest by one standard deviation leads to a $2\% - 26\%$ reduction in investment relative to median investment, depending on the particular measures of short interest and investment.

After establishing a negative link between short interest and investment, we proceed to testing the additional predictions of the model. First, we study how the impact of short interest on investment varies with the level of short interest. In the model, higher levels of short interest increase the odds that the informed trader is on the short side, making short interest more informative.

To examine this prediction, we modify our baseline specification by allowing short interest to have different slopes in the high (above sample median) versus low ranges of short interest values. Consistent with our hypothesis, we show that the effect of short interest on investment is negative and statistically significant at high levels of short interest, but has mixed signs and is mostly insignificant within the lower range of short interest.

Our second set of tests focuses on the firm information environment. Our model predicts a stronger effect of short interest for less transparent firms. To test this prediction empirically, we include interactions of various proxies for information transparency with short interest in our

[2] At first glance it might appear counterintuitive that the manager's incentive to learn from short interest strengthens when information uncertainty is greater, so the signals of informed traders are likely to be noisier. We discuss the intuition for this result in detail below in Section 2.

regressions. Our first proxy is analyst forecast dispersion: Higher dispersion of analyst forecasts usually indicates lower transparency. We also use two measures of stock return volatility - total and idiosyncratic - as alternative proxies for information uncertainty. Finally, we examine the role of institutional ownership: Firms with greater institutional ownership are likely to face less information uncertainty since institutions produce information.

We find strong evidence consistent with the predicted effect of information uncertainty: Firms with less information transparency exhibit stronger investment to short interest sensitivities. The effect of short interest is more pronounced among firms with greater analyst forecast dispersion, lower institutional ownership, and higher stock return volatility (both total and idiosyncratic). Taken together, this set of findings further highlights the role of managerial learning in driving the negative relation between short interest and investment.

We also perform a number of additional tests that focus on time-series, rather than cross-sectional, variation in the informativeness of short interest. First, we argue that its informativeness has likely increased over time. Alleviation of various short sale constraints throughout our sample, as well as the development of financial markets and the growing role of institutional investors contributed to greater market liquidity, better availability of shares to borrow, and easier access to shorting for informed traders. Therefore, we examine whether the effect of short interest on investment is stronger in the second half of our sample. Consistent with our conjecture, we find that the sensitivity of investment to short interest is weak and statistically insignificant in the first half of the sample, and highly significant in the second half. The stronger impact of short interest on investment in more recent years offers another layer of evidence in support of the informational role of short interest.

We also test whether the ability of short interest to predict future investment attenuates over time. Existing literature demonstrates that short sellers establish short positions one to two years prior to the realization of negative events (Karpoff and Lou (2010), Akbas et al. (2017)). If short interest conveys information about future fundamentals, it should be most valuable for investment planning in the short and medium terms. Consistent with this conjecture, we find that short interest strongly predicts investment up to two years ahead, but its impact on investment in subsequent years gradually fades away.

In an auxiliary test, we examine the effect of short interest on investment in an exogenous setting by taking advantage of a regulatory experiment - Regulation SHO (hereafter Reg SHO), which removed restrictions on short sales for a randomly selected group of firms within the Russell

3000 index. As barriers to short selling were alleviated, more opportunities arose for informed short sellers to enter the markets, likely making short interest more informative. Therefore, we interpret the enactment of Reg SHO as a positive shock to the informativeness of short interest. If managerial learning is driving the negative sensitivity of investment to short interest, then this sensitivity should strengthen following Reg SHO. Consistent with this hypothesis, we find that the negative sensitivity of investment to short interest had indeed increased following the implementation of Reg SHO.

A couple of recent papers examine potential caveats of using Reg SHO in finance research. Heath et al. (2023) criticize "reusing" natural experiments, and argue that when the same source of exogenous variation is used to test multiple null hypotheses, the probability of making at least one type one error increases. We believe that the use of Reg SHO in our setting survives the Heath et al. (2023) critique to a large extent. In particular, most of our results remain significant after applying corrections proposed by Heath et al. (2023). Another potential problem, highlighted by Boehmer et al. (2020), is the indirect, or "spillover" effect of Reg SHO. However, we believe that potential spillovers are likely to strengthen our identification given the particular design of our empirical tests. We elaborate on these issues in Section 5.5.

Furthermore, we believe that even without the Reg SHO evidence, the unique cross-sectional predictions, derived from our model, alleviate endogeneity concerns and question the plausibility of other potential channels through which short interest can impact investment. For example, it is possible that short sellers are merely good at identifying companies that are likely to perform poorly in the future, whereas managers scale down their investment projects in response to deterioration of economic conditions, rather than to short selling activity. However, this potential channel is at odds with our findings of a stronger relation between short interest and investment for more opaque firms. In all likelihood, it is easier to predict the future performance of more transparent rather than opaque firms. To further investigate this potential alternative channel, we augment our main regressions with stock returns, as in Chen et al. (2007), and find that the investment to short interest sensitivity remains negative and significant.

We also disentangle the managerial learning channel from the capital market channel whereby short interest affects future investment through its impact on stock prices and availability of external capital. If short interest signals poor future performance and inferior returns, an increase in short interest can limit the availability of external funding and therefore negatively impact investment. To look deeper into the plausibility of the capital market channel, we focus on financially constrained and equity dependent firms. Such firms would face greater difficulties in securing necessary funds for investment when firm conditions deteriorate and therefore should exhibit a stronger investment to short interest sensitivity if the capital market channel is at work. However, we find no evidence that the relation between short interest and investment is stronger among financially constrained and equity dependent firms.

Finally, we consider the role of price manipulation. Goldstein and Guembel (2008) show that short sellers can establish short interest positions with the purpose of driving stock prices down and inducing firms to cut investment in response. However, in their model, short interest affects investment through its impact on stock prices. In contrast, our model shows that stock prices and short interest have complementary roles, and both convey information about fundamental values. In a recent paper, Campello et al. (2020) further explore the idea of manipulation and show that managers fight short sellers using stock repurchases, the strategy that could be accompanied by investment cuts. To study the potential role of repurchases, we interact short interest with repurchasing activity, but find no evidence that the relation between short interest and investment is stronger for firms that repurchase more.

Our paper contributes to the literature in a variety of ways. On the theory side, we rationalize the role of short interest in managerial learning and show that it contains valuable information beyond that impounded in prices. Relying on both price and short interest results in a more precise estimate of the firm's fundamental value. On the empirical side, we document a robust negative relation between short interest and subsequent investment that is driven (at least in part) by managerial learning. We therefore extend the literature that focuses on the feedback effects of financial markets on the actions of corporate managers by demonstrating the importance of learning from short interest.[3]

Second, our paper extends the literature that examines the effect of short interest on real activity of firms, and in particular on corporate investment. Existing work shows that short interest can be linked to investment through a number of channels: financial constraints (Grullon et al. (2015)), managerial discipline (Massa, Wu, Zhang and Zhang (2015)), bear raids (Goldstein and Guembel (2008)), and share repurchases (Campello et al. (2020)). In this paper we highlight the importance of managerial learning. While we do not claim that the managerial learning channel is *the only one* that gives rise to the negative investment-to-short interest sensitivity that we document, other

[3]In addition to the literature discussed at the beginning of this section, the relevant papers include Baker et al. (2003) and Dessaint, Foucault, Frésard, and Matray (2019) (the effect of non-fundamental price movements on investment); Bird et al. (2020) (the effect of publicizing corporate filings on the investment-to-price sensitivity); Ye, Zheng, and Zhu (2023) (the effect of tick size on investment), Yan (2023) (learning by the managers of private firms), and Glode and Green (2011) (learning by hedge funds and mutual funds managers).

potential mechanisms are not able to explain the totality of our empirical results, pointing to the importance of managerial learning.

Finally, our paper adds to the literature that studies the impact of short selling activity on the informational environment of the firm and the quality of its financial reporting. Massa, Zhang and Zhang (2015), Fang et al. (2016), and Jiang et al. (2020) find that the alleviation of short selling constraints reduces accruals management and earnings manipulation. Clinch et al. (2019) and Chen et al. (2020) find a positive relation between short interest and the quality of corporate disclosures and managerial forecasts. Our paper contributes to this literature by demonstrating that short interest enhances firm transparency by aggregating private information of informed traders and revealing that information to corporate managers, facilitating learning from short interest and impacting corporate investment. Our approach allows us to gain further insights into the mechanism through which short interest impacts firm transparency.

2 Model

Our central argument is that the informational contents of the stock price and short interest are complementary. This is not a trivial claim because the market price aggregates information of the informed market participants, and in this sense, embeds all private signals. To establish the informational role of short interest beyond that of stock prices, we construct a two-period model of the market populated with privately informed and noise traders. The noise coming from the uninformed demand also gets reflected in the price. We show that relying on short interest in addition to the market price reduces that noise and produces a more precise estimate of the firm value.

2.1 The Economy

The economy features a single representative price taking, risk averse, privately informed trader who submits a price contingent claim (limit order).[4]

We also assume that there is a number of liquidity (noise) traders who submit a random aggregate market order u, which is normally distributed with zero mean and variance σ_u^2: $u \sim N\left(0, \sigma_u^2\right)$. There are two tradable assets: a single risky asset (the firm's stock) and a risk-free asset with the risk-free rate normalized to zero. The informed trader privately observes a noisy signal s about the

[4]Equivalently, we could assume that there is a finite number N of informed traders who have the same information and hence employ the same strategies in a symmetric Nash equilibrium.

fundamental value of the risky asset v: $s = v + \sigma_s \varepsilon_s$, where $\varepsilon_s \sim N(0,1)$. The fundamental value v is unobserved. To simplify exposition, we assume that the informed agent's priors on v are diffuse, i.e. all her information comes from the private signal that she observes.

Finally, there is a manager who does not trade, but uses the information available to her to construct the optimal estimator of the fundamental value. Her objective is to maximize the precision of the estimator by minimizing its variance. For simplicity, we assume that the manager does not observe any private signal, and all her information about the value of the asset comes directly from observing the market price and short interest.[5]

Given her private signal, the informed trader's best estimate of the fundamental value of the firm and its conditional variance are $\widehat{v} = E[v|s] = s$ and $\sigma^2_{v|s} = E\left[(v-\widehat{v})^2|s\right] = \sigma^2_s$. The informed trader submits limit orders and the market price is determined by the market clearing condition and the noise traders' aggregate demand.

2.2 Equilibrium

We assume that the informed trader has a CARA utility and solves the following problem:

$$\begin{aligned} U &= \max_{y(P)} E_v\left[-\exp(-\alpha w)\right], \\ w &= y(v-P), \end{aligned} \tag{1}$$

where U is the expected utility of the informed trader, $y(P)$ is her (limit) order, w and α are the wealth and risk-aversion parameters of the informed trader, and P is the execution price. In (1), the expectation is taken with respect to the realizations of the fundamental value v. We assume that the informed trader is a price taker (non-strategic), but she can submit price contingent claims (limit orders), and therefore can effectively condition on the execution price P (see, e.g., Grossman and Stiglitz (1980), Hellwig (1980), DeJong and Rindi (2009)). The certainty equivalent W for the informed trader is given by the following standard result (see, e.g., Grossman and Stieglitz (1980), p. 396, DeJong and Rindi (2009), p. 40):

$$W = y(\widehat{v} - P) - \frac{1}{2}\alpha\sigma_s^2 y^2, \tag{2}$$

[5] This assumption does not affect our results. In the Internet Appendix, we consider the case when the manager receives her own private signal $s_m = v + \sigma_m \varepsilon_m$ with $\varepsilon_m \sim N(0,1)$. This dramatically increases the complexity of the model, while keeping its predictions qualitatively unchanged.

where y is the demand for the asset by informed trader. The first term on the right hand side of (2) is the agent's expected wealth (proportional to her demand y), while the second term is the absolute risk aversion coefficient multiplied by the variance of the informed trader's wealth (proportional to y^2). The certainty equivalent decreases in that variance and therefore, the second term has a negative sign.

The first order condition for (2) yields the following optimal strategy of the informed trader:

$$y^*(P) = \frac{\widehat{v} - P}{\alpha\sigma_s^2}. \tag{3}$$

Applying the market clearing condition $y^*(P) + u = 0$, we obtain the equilibrium price as

$$P = \widehat{v} + \alpha\sigma_s^2 u. \tag{4}$$

As follows from (4), the execution price is the sum of the informed trader's private signal $\widehat{v}$ and the noise component $\alpha\sigma_s^2 u$. This additional noise component arises from the presence of the noise traders with the aggregate demand u. The magnitude of the noise is also proportional to the informed trader's risk aversion α and conditional variance σ_s^2. This happens for the following reason. If the informed trader's signal is perfect, i.e. $\sigma_s^2 = 0$ and she knows the fundamental value v precisely, she trades an infinitely large amount of the risky asset for any nonzero mispricing, causing the price to converge immediately to the fundamental value. The same logic applies when the risk aversion goes to zero, since risk neutral informed traders trade infinitely large amounts for any non-zero mispricing. This follows from the informed trader's demand (3): as the denominator goes to zero, demand becomes infinitely large unless the numerator also goes to zero.

Note that, as follows from (4), the execution price provides an unbiased estimate of the fundamental value. Namely, we can construct an estimator $\widehat{v}_P = P$ distributed as

$$\begin{aligned} \widehat{v}_P &= P \sim N\left(v, \sigma_P^2\right), \\ \sigma_P^2 &= \sigma_s^2 + \alpha^2\sigma_s^4\sigma_u^2. \end{aligned} \tag{5}$$

2.3 Short interest and its informational content

For simplicity, we assume that the informed speculator has no initial endowments. In this case, negative demand in (3) implies that the speculator wishes to short the risky asset and hence contributes to the short interest. We assume that there are two types of liquidity traders labelled as Type 1 and Type 2. Type 1 liquidity traders do not have any endowments (inventory) and hence, similar to the informed speculator, they short the asset when their demand is negative. Type 2 liquidity traders, on the contrary, have unlimited amount of inventory and never short. This assumption is important for our analysis: Because noise traders can either short or sell (depending on their inventory), while the informed agent always shorts, short interest becomes informative. As we show below, incorporating short interest (in addition to the price) improves the precision of the manager's estimate of the firm's fundamental value. We argue that this assumption is likely to hold empirically. Speculators (e.g. hedge funds), endowed with private information, and seeking to profit from a price decline, are most likely to short the stocks. By contrast, noise traders may or may not have initial inventory and therefore their negative demand can result in either shorting or selling. For example, it is possible that an option market maker shorts the underlying securities in order to hedge its outstanding option positions. On the other hand, corporate managers who are routinely compensated with shares sell once those shares are vested, as do individual investors routinely selling part of their 401k portfolios to generate retirement income.

It follows that the total liquidity demand has two components, $u = u_1 + u_2$, corresponding to the liquidity traders of type 1 and type 2, respectively. We assume that u_j, $j = 1, 2$, are i.i.d., uncorrelated, and normally distributed: $u_j \sim N(0, \sigma_{u,j})$. We introduce the following notation $\sum_{j=1,2} \sigma^2_{u,j} = \sigma^2_u$ and a parameter $k = \frac{\sigma^2_{u,2}}{\sigma^2_u} \in [0; 1]$, reflecting a relative share of liquidity traders who do not short, in the variance of total liquidity demand.

Aggregating (3), we obtain the following expression for the short interest S:

$$S = -y^* (P)\, \theta (-y^* (P)) - u_1 \theta (-u_1) = \sigma_u \widehat{S}, \tag{6}$$

where

$$\widehat{S} = \frac{1}{\sigma_u} S = \frac{1}{\alpha \sigma^2_s \sigma_u} (P - \widehat{v})\, \theta (P - \widehat{v}) + \xi_1 \theta (\xi_1), \tag{7}$$

with the notation $\xi_j = -\frac{1}{\sigma_u} u_j$, $j = 1, 2$, and $\theta(x)$ is the Theta function defined by

$$\theta(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0. \end{cases} \tag{8}$$

By construction, the short interest is non-negative, $\widehat{S} \geq 0$. We can define the true mispricing as difference between the price and the firm's fundamental value $\Delta = P - v$, and the informed trader's perceived mispricing as $z = P - \widehat{v}$. The limit orders contributing to the short interest are sell orders, i.e. those with $z \geq 0$.

Using the following notations: $\sigma_\xi = \alpha\sigma_s^2\sigma_u$ and $\sigma_{\xi,j} = \alpha\sigma_s^2\sigma_{u,j}$, $j = 1, 2$, we have $k = \frac{\sigma_{\xi,2}^2}{\sigma_\xi^2}$. It is convenient to introduce a relative perceived mispricing $t = \frac{1}{\sigma_\xi} z = \frac{1}{\sigma_\xi}\left(P - \widehat{v}\right)$. As follows from (7), $\widehat{S} = t\theta\left(t\right) + \xi_1\theta\left(\xi_1\right)$. Since the market clearing condition imposes $t + \xi = 0$, where $\xi = \xi_1 + \xi_2$, we obtain $\widehat{S} = t\theta\left(t\right) - \left(\xi - \xi_2\right)\theta\left(-\left(\xi - \xi_2\right)\right) = t\theta\left(t\right) + \left(t + \xi_2\right)\theta\left(t + \xi_2\right)$.

Proposition 1 derives the best estimate of the manager who incorporates both stock price and short interest in her .

Proposition 1 *The best estimate of the firm's fundamental value for the manager who observes both the stock price and short interest is given by*

$$\widehat{v}_M = E\left[v|P, S\right] = P - \sigma_\xi \Lambda\left(\frac{S}{\sigma_u}\right), \tag{9}$$

with

$$\begin{aligned} \Lambda\left(x\right) &= x\frac{1-\gamma\left(x\right)}{1+\gamma\left(x\right)} + \Gamma\left(x\right), \\ \Gamma\left(x\right) &= \frac{1}{\sqrt{2\pi}}\frac{\gamma\left(x\right)}{1+\gamma\left(x\right)}\frac{\left(1-2k\right)e^{-\frac{x^2}{2k}} - \left(1-k\right)e^{-\frac{x^2\left(1-2k\right)}{2k\left(1-k\right)}}}{\sqrt{k}\Phi\left(x\sqrt{\frac{1}{k}}\right)}, \end{aligned} \tag{10}$$

and

$$\gamma\left(x\right) = \frac{\frac{1}{\sqrt{1-k}}e^{-\frac{x^2}{2}\frac{k}{1-k}}\frac{\Phi\left(x\sqrt{\frac{1}{k}}\right)}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}}{1 + \frac{1}{\sqrt{k}}e^{-\frac{x^2}{2}\frac{1-k}{k}}\frac{\left(\Phi\left(x\sqrt{\frac{1}{1-k}}\right)-\frac{1}{2}\right)}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}}, \tag{11}$$

where $\Phi\left(x\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{\xi^2}{2}}d\xi$ is a standard Normal CDF.

In (9), $\sigma_\xi\Lambda\left(\widehat{S}\right)$ is the correction to the manager's best estimate $\widehat{v}_M$ due to short interest; for a manager who does not observe short interest, the best estimate is equal to the price P. Note that the mean of $\Lambda\left(\widehat{S}\right)$ with respect to the marginal distribution of $\widehat{S}$ is zero, $E\left[\Lambda\left(\widehat{S}\right)\right] = 0$, as we show formally in the Appendix. Also, we prove in the Appendix that $E\left[\widehat{S}\right] = \frac{1}{\sqrt{2\pi}}\left(1 + \sqrt{1-k}\right)$

and hence $E[S] = \frac{\sigma_u}{\sqrt{2\pi}}\left(1+\sqrt{1-k}\right)$. Thus, the expected short interest S is proportional to the noise parameter σ_u. In particular, when $k=0$, $E[S] = \frac{2}{\sqrt{2\pi}}\sigma_u \approx 0.8\sigma_u$.

As follows from Proposition 1, the sensitivity of the best estimate to short interest has the following form:

$$\frac{\partial \widehat{v}_M}{\partial S} = -\alpha \sigma_s^2 F\left(\frac{S}{\sigma_u}\right), \tag{12}$$

where $F(x) = \frac{\partial \Lambda(x)}{\partial x}$.

Note that in the limit $k \to 0$, corresponding to the case when all liquidity traders have no inventory, $\gamma(x) \to 1$ and $\Lambda(x) \to 0$. In this case short interest becomes uninformative, and the best estimate of the fundamental value reduces to the market price, $\widehat{v}_M \to P$.

The functional form of the sensitivity of the best estimate of the fundamental value to short interest, $\frac{\partial \widehat{v}_M}{\partial S}$, is a key result for our empirical analysis. This sensitivity is negative[6] because when short interest is high, it is more likely that the asset is shorted by the informed trader, who must have a negative view on the firm value (relative to the price) in order to short. Because only a fraction $1-k$ of liquidity traders short, the resulting distribution of short interest induced by liquidity traders is more narrow than that resulting from informed shorting. Lower values of short interest are more likely caused by shorting by liquidity traders, in which case the informed trader wishes to buy the asset. Therefore, lower short interest is interpreted by the manager as a positive signal, and for low values of S, $\Lambda\left(\frac{S}{\sigma_u}\right) < 0$. By contrast, high short interest values imply a greater probability of shorting by the informed trader, and for those values $\Lambda\left(\frac{S}{\sigma_u}\right) > 0$, and hence observing high short interest is interpreted by the manager as a negative signal. Prediction 1 follows.

Prediction 1. *The best estimate of the risky asset's fundamental value $\widehat{v}_M$ decreases in short interest.*

Figure 1 illustrates this prediction and displays the difference between the price and the best estimate of the manager who observes both price and short interest, $P - \widehat{v}_M$, for various values of the share of liquidity traders k who are endowed with inventory and hence do not short. As expected, the effect of short interest on the manager's best estimate is minimal when k is close to zero, and gets larger with k. The effect is economically large: For example, for $k = 0.4$, $\mu = \frac{\sigma_\xi}{P} = 0.1$ (μ can be interpreted as the average mispricing relative to the fundamental value), and the normalized value of short interest $\widehat{S} = \frac{S}{\sigma_u} = 2$, the best estimate based on both short interest and the price is

[6] In the Appendix we prove analytically that $F(x)$ is positive (and hence $\frac{\partial \widehat{v}_M}{\partial S}$ is negative) as long as the fraction of constrained liquidity traders k does not exceed a threshold value of $k_m = \frac{4}{\pi}\left(\sqrt{1+\frac{\pi}{2}}-1\right) \approx 0.768$. We also perform additional numerical analysis that demonstrates that this function is positive for higher values of k.

about 11% lower than the one based solely on price (which is equal to the price).

Second, short interest is more important when the informed agent's uncertainty about the the fundamental value, $\sigma^2_{v|s} = \sigma^2_s$, is high. The term σ^2_s in (12) comes from the functional form of optimal demand (3). The optimal demand is proportional to the perceived mispricing $\widehat{v} - P$ and is inversely proportional to $\alpha\sigma^2_s$. Therefore, the perceived mispricing is proportional to the product of short interest and a factor $\alpha\sigma^2_s$. Conditional on the level of short interest, higher information uncertainty σ^2_s indicates greater mispricing and results in a greater sensitivity of the best estimate to short interest.

Figure 2 illustrates the information uncertainty effect by providing the "pricing correction" due to short interest for various levels of the uncertainty parameter $\mu = \frac{\sigma_\xi}{P} = \frac{\alpha\sigma^2_s\sigma_u}{P}$. This result gives rise to Prediction 2.

Prediction 2. *The sensitivity of* $\widehat{v}_M$ *to short interest increases in the level of information uncertainty about the fundamental value of the firm.*

At first glance, Prediction 2 might come across as counterintuitive: If there is more noise in the informed trader's signal, resulting in a greater variance of her estimate $\sigma^2_{v|s} = \sigma^2_s$, then why would the manager put a higher weight on the (presumably noisier) signal that she extracts from short interest? The economic mechanism that explains this seeming contradiction is as follows. Higher values of σ_s indeed result in a higher variance of the price (as follows from equation 5), making the stock price less informative to the manager. However, the effect of higher uncertainty on the informativeness of short interest is different: Because the informed trader is risk-averse, the same level of informed trader's demand (that gives rise to short interest) indicates that the informed trader observes a stronger negative signal about firm value when the uncertainty is high to be willing to establish this position (mathematically, this follows from equation 3 that shows that informed demand is proportional to perceived mispricing scaled by σ^2_s). Therefore, the informativeness of short interest does not deteriorate with higher uncertainty, while the informativeness of the price does. As a result, when σ^2_s goes up, managers optimally increase the sensitivity of their estimates to short interest.

As seen in Figure 1, the relationship between the manager's optimal estimate and short interest is for the most part convex. This result has an intuitive explanation: Because short interest increases the odds that the informed trader is on the short side and receives a large negative signal, it becomes more informative and the sensitivity of the fundamental value to short interest increases at higher levels of short interest. At higher values of short interest it becomes increasingly less likely that short interest is driven by liquidity traders and hence short interest becomes a direct reflection of the informed agent's signal. At lower values of short interest, it is more likely a result of a combination of liquidity and informed shorting and is therefore less informative.

While the intuition for the convexity of the best estimate in short interest is straightforward, this result cannot be proven analytically for an arbitrary value of the share of unconstrained liquidity traders, k. In the subsequent analysis, in order to be able to obtain analytically tractable results, we therefore consider the case when the fraction of unconstrained liquidity traders is relatively small, $k << 1$. We then perform numerical analysis to demonstrate that the results are robust to arbitrary values of k. As we prove in the Appendix, for low values of k, the function $\Lambda(x)$ simplifies to the following expression:

Corollary 1 *In the limit $k << 1$, the best estimate of the risky asset's fundamental value $\widehat{v}_M$ is approximately given by*

$$\Lambda(x) \simeq x \frac{1 - \gamma_a(x)}{1 + \gamma_a(x)} - \sqrt{\frac{k}{2\pi}} \frac{\gamma_a(x)}{1 + \gamma_a(x)}, \tag{13}$$

with

$$\gamma_a(x) = \frac{1}{\sqrt{1-k}} e^{-\frac{x^2}{2}\frac{k}{1-k}}. \tag{14}$$

The asymptotic approximation given by (13) works well for higher values of k as well. To illustrate this, Figure 3 displays the function $\Lambda(x)$ using both its asymptotic expression (13) and its general form given by (10), for $k = 0.2$. As follows from this figure, even for relatively high values of k, the deviation of $\Lambda(x)$ from its asymptotic expression is minimal and only shows up for very low values of short interest.

In the Appendix we prove analytically that $\Lambda(x)$ is convex in the asymptotic case of $k << 1$. Numerical results indicate that, on average, it is also convex for arbitrary values of k. These numerical results are presented in Figure 4 that plots the average second derivative of $\Lambda(.)$ as a function of the share of unconstrained liquidity traders k. This average is evaluated making use of the conditional short interest distribution derived in close form in the Appendix. Note that, as we discuss above, $\Lambda\left(\widehat{S}\right)$ is a function of the ratio of the short interest S and the noise parameter σ_u (which is proportional to the expected value of short interest), so varying σ_u does not affect the average second derivative $\frac{\partial^2 \Lambda(\widehat{S})}{\partial \widehat{S}^2}$. Furthermore, the noise of the informed trader's signal, σ_s^2, enters the manager's best estimate, given by (9), multiplicatively, and therefore varying σ_s^2 preserves the sign of the average second derivative.

Thus, on average, $\frac{\partial^2 \langle \widehat{v}_M \rangle}{\partial S^2}$ is negative and higher values of short interest increase the sensitivity of the best estimate to short interest. This result is summarized in Prediction 3.

Prediction 3. *The sensitivity of* $\widehat{v}_M$ *with respect to the short interest increases in the magnitude of the short interest itself.*

To summarize, our model, described in this section, predicts that short interest increases the precision of the best estimate of the firm's true value (in addition to the price) and has a negative effect on the best estimate of the firm value.

3 Hypothesis Development and Discussion

In this section, we discuss the implications of learning from short interest for managerial investment decisions in more detail.

In developing our model, we assume that the manager has an information set different from that of the informed trader and to some extent relies on information in financial markets when making her investment decisions. To keep the model tractable, we do not explicitly model the investment decisions of the manager. Instead, we follow the vast literature on the feedback effects of financial markets on corporate actions, pioneered by Chen et al. (2007), that establishes that the relationship between manager's estimate of the firm's fundamentals and investment is expected to be positive.[7] According to Chen et al. (2007), "if, at a given point in time, managers decide on the level of investment attempting to maximize the expected value of the firm, they will use all information available to them at that point. This includes both the information in the stock price and other information that managers have and that has not found its way to the price yet" (p.620).

In developing our predictions, we rely on this argument to gauge the link between managerial learning from short interest as one type of information that is not yet incorporated in stock prices on the one side, and corporate investment decisions on the other side. Our model shows that short interest bears additional informational content and hence enables the manager to extract additional information beyond that impounded in the price. If managers learn from short interest, then a negative update on the firm's fundamental value and its future prospects (arising from short

[7] One can rationalize a positive relationship between firm value and investment in a variety of frameworks. For example, a greater NPV of the firm's potential investment projects would result in a higher firm value and would also lead to more investment. A similar relation would obtain in a real option framework (see, e.g. McDonald and Siegel (1986)): a positive shock to the underlying state variable leads to a higher value of the frim, while also increasing the probability of investment.

interest) optimally leads to the abandonment or scale-down of some investment projects. Therefore, managerial learning leads to a negative effect of short interest on investment. This gives rise to our main hypothesis:

Hypothesis 1: There is a negative relation between short interest and subsequent investment.

Our model predicts that the sensitivity of the best estimate of the fundamental value to short interest is higher when short interest itself is high. As short interest increases, the likelihood that the informed trader has established positions on the short side increases, hence making short interest more informative, so the sensitivity of the fundamental value to short interest also increases.

Hypothesis 2: The negative relation between short interest and subsequent investment is stronger when short interest is high.

Our model also shows that short interest provides a more valuable signal for the manager (relative to the price) when there is greater information uncertainty about the fundamental value of the firm, as measured by the precision of the informed trader's signal, $\sigma^2_{v|s}$, as well as by the level of noise trading σ^2_u. When information uncertainty is high, so is the benefit of learning the extent of mispricing from the pessimistic traders (in addition to observing the price). For a given level of short interest, higher values of $\sigma^2_{v|s}$ indicate greater mispricing perceived by the informed trader and hence result in a greater sensitivity of the best estimate to short interest. This leads to Hypothesis 3:

Hypothesis 3: The negative relation between short interest and subsequent investment is stronger when there is more information uncertainty.

We test these main hypotheses and provide additional empirical results in Section 5.

4 Data and Variables

Our primary variable of interest - *Short Interest* - is constructed from the monthly series provided by NASDAQ and NYSE, and reported in the Supplemental Short Interest File, available through Compustat. We use three alternative measures of short selling activity based on these series. We start by calculating each measure at the monthly frequency.[8] Our first measure, *Short interest scaled by shares (SI/shares)*, is the number of all open short positions on the last business day on or before the 15th of each calendar month scaled by the number of shares outstanding at the

[8]A small fraction of firms have time-series gaps in the short interest data. The data provider has advised treating missing data months as zero short interest. To this end, we identify the earliest and the latest dates reported for each firm in the short interest file and fill in all the missing months in between with zeroes.

end of the month.[9] To construct our second measure, *Abnormal short interest*, we closely follow Karpoff and Lou (2010), and extract the unexpected component of short interest. In particular, at the beginning of each month, we sort stocks independently into tercile portfolios by size, book-to-market, momentum (measured by past six month return), turnover, and institutional ownership, all measured at the end of the prior month. We then run monthly cross-sectional regressions of short interest on tercile dummy variables corresponding to those portfolios as well as industry (defined at a SIC 2-digit level) fixed-effects. We use the residuals from these cross-sectional regressions as the measure of abnormal short interest. Our third and final measure of short interest, *Days-to-cover*, is based on Hong et al. (2015). It is obtained by scaling *SI/shares* by the same month's average daily share turnover.[10] Intuitively, this measure shows the number of days it would take short sellers to cover their short positions, assuming the average trading volume. To convert the short interest data into annual frequency, for every firm, we average the monthly short interest over its fiscal year.

We obtain accounting information and equity prices from Compustat and CRSP, respectively. Since the short interest data are available since 1973 and we lag most of our explanatory variables, our final sample period is $1974 - 2018$. As is standard in the literature, we exclude financials (SIC codes $6000 - 6999$) and utilities (SIC codes $4000 - 4949$). We further exclude firms with negative sales (Compustat item SALE), and limit the analysis to firms with ordinary common shares and with value of assets of at least $1 million in 2004 dollars. Finally, we require non-missing information on total assets (Compustat item AT), fixed assets (Compustat item PPENT), capital expenditures (Compustat item CAPX) and fiscal year-end share price (Compustat item PRCC_F).

For robustness, we employ several measures of investment. Our first measure is the ratio of capital expenditures in year t to fixed assets at the end of year $(t-1)$. This measure is motivated by classical models of investment and their empirical tests, and incorporates only fixed, or tangible investment (see, for example, Fazzari et al. (1988)). Since the role of intangible capital, and in particular, technological capital, has more than doubled in the past few decades (see, e.g., Corrado and Hulten (2010)), R&D expenses have become an important component of investment (see, e.g., Peters and Taylor (2018)). To account for investment in technology, we construct an alternative measure of investment, and scale the sum of capital expenditures and R&D (Compustat item XRD)

[9] Starting from 2007, short interest data is reported twice a month: on or before the 15th and also at the end of the month. For consistency, we use the earlier monthly reporting date throughout our sample period.

[10] Following Hong et al. (2015), we adjust trading volume of NASDAQ stocks during the $2001 - 2003$ period in our calculations of turnover. See footnote 8 of their paper for more details.

by the beginning-of-the-year total assets.[11] Finally, we consider *Change in Total Assets*, defined as the percentage change in total assets between years $t-1$ and t, as our last measure of investment.

In addition to our main variables, we also use control variables that have been shown to affect corporate investment. Numerous studies have found that investment is sensitive to the Market-to-book ratio, cash flow, and size (see, e.g, Fazzari et al. (1988), Chen et al., (2007)). Therefore, we use these variables as additional controls in our investment regressions. We define Market-to-book ratio (MB) as the total book value of assets plus market cap (the product of the number of common shares outstanding (CSHO) and share price at the end of the fiscal year-end), minus book value of equity (CEQ), all divided by the total book value of assets. Cash Flow (CF) is the sum of income before extraordinary items (variable IB) and depreciation and amortization (variable DP) divided by the beginning-of-the-year total book value of assets. Size is the natural logarithm of the total book value of assets. All variables used in the analysis, with the exception of size, are winsorized at 1% and 99%.

Our model predicts that the informational content of short interest is more valuable to managers when there is more information uncertainty about the firm. In our tests, we use several proxies for information uncertainty. First, we use analyst forecast dispersion, obtained from I/B/E/S. Every month, we calculate the standard deviation of the end-of-the-year EPS forecasts across all analysts, and scale it by the mean forecast. The resulting monthly ratios are then averaged within the fiscal year to obtain annual measures of dispersion. Second, we use measures of stock return volatility − both total and idiosyncratic. Total volatility is measured as the standard deviation of daily stock returns in a given month, which is then averaged over the fiscal year. Idiosyncratic volatility is the standard deviation of the residuals from daily regression of stock returns net of the risk-free rate on the Fama-French three factors. The regressions are estimated monthly, and the resulting standard deviations of the residuals are averaged over the fiscal year. Third, we use institutional ownership, obtained from 13F files. Institutional ownership is the fraction of shares held by all institutional investors divided by total shares outstanding, and is measured using the most recent quarter prior to the fiscal year-end.

Table 1 presents the summary statistics for our main variables. The average firm in our sample has 2.78% of its shares shorted. It would take 4.7 days, on average, to cover the outstanding short positions based on the average daily volume. These numbers are similar to those in the literature (see, e.g. Hong et al. (2015)). By construction, the mean abnormal short interest is zero. The

[11]Values of zero are assigned to observations with missing R&D values.

size of an average firm in our sample, as measured by total assets, is around $3 billion. The mean firm has a market to book ratio of 1.85 and generates annual cash flows of 3.8%. The mean firm also has analyst forecast dispersion of about 19%. The total volatility of daily returns is about 2.97%, whereas the idiosyncratic volatility component is 2.56% (corresponding to 47.2% and 40.6% annualized volatility values, respectively). Around 43% of the firm's shares are held by institutional investors.

5 Empirical tests

5.1 Baseline results

As discussed in the introduction, the NYSE and SEC believe that short interest contains information that is useful to public companies and their management. Similarly, our model is premised on the assumption that informed short sellers receive a signal that reflects (albeit with noise) the fundamental value of the firm. While we remain agnostic as to what exact aspects of firm fundamentals are observed by short sellers, we start our analysis by verifying that there is indeed a relationship between short interest and future fundamentals. For this reason we focus on the firm cash flow, CF (see Section 4 for the definition), as a variable that has the first-order effect on firm valuation and regress future cash flows on each of our short interest measures. If short interest contains negative information about future fundamentals, it is likely to be reflected in lower levels of future cash flows. In our regression specification we include controls for market-to-book and firm size, both as of t-1, as well as firm and year fixed-effects. Results, reported in Table 2 confirm the validity of our assumption: All three measures of short interest are indeed negatively associated with future cash flows, and their impact is economically significant. For example, a one standard deviation increase in *SI/shares* in year t-1 reduces cash flow in year t by 14% relative to the cash flow mean. Although the effect decays over time, it remains negative and statistically significant up to 2 years into the future.

After confirming the key assumption underlying our theoretical framework, we turn to the baseline tests. Our main hypothesis (Hypothesis 1) is that managers learn about the fundamental value of the firm by observing short selling activity. Managerial learning, in turn, leads to a negative effect of short interest on investment. To test this prediction empirically, we start our analysis by regressing our measures of investment on various proxies for short interest and control variables, used in prior studies. Our baseline empirical specification has the following form:

$$I_{i,t} = \alpha + \beta SI_{i,t-1} + \delta_1 MB_{i,t-1} + \delta_2 CF_{i,t} + \delta_3 log(Assets)_{i,t-1} + \eta_i + \nu_t + \varepsilon_{i,t}, \tag{15}$$

where $I_{i,t}$ is a measure of investment of firm i in year t, $SI_{i,t-1}$ is a measure of lagged short interest, and *MB*, *CF* and *log(Assets)* are the market-to-book ratio, cash flow, and the logarithm of firm's assets, respectively.[12] We include year fixed effects ν_t to absorb potential impact of global time-varying conditions on firms' investments. We also include firm fixed effects η_i to account for potential time-invariant omitted variables. For example, high quality firms might invest more and at the same time be targeted less often by short sellers, resulting in a mechanical negative relation between short interest and investment. The inclusion of firm fixed effects addresses this concern by focusing the analysis on the within-firm variation in short selling and investment over time. To account for potential correlation of residuals, we double cluster the standard errors at the firm and year level.

Table 3 reports the results. Panels A (B,C) is based on *SI/shares* (*Days-to-cover*, *Abnormal short interest*) as a measure of short interest. As follows from Table 3, regression coefficients on all three measures of short interest are negative and highly significant in all specifications. The effect of short interest on investment is also economically large. For example, increasing short interest scaled by shares outstanding by one standard deviation reduces our measures of investment by $6\% - 26\%$ relative to their medians. The corresponding economic effect for both days-to-cover and abnormal short interest ranges between 2% and 19%.

We also find that coefficients on all three control variables are highly statistically significant. Consistent with prior studies, investment is strongly and positively associated with market-to-book and is negatively associated with size. The coefficient on cash flow is positive in the majority of our specifications (the negative cash flow coefficient in column (2) is driven by inclusion of R&D in the numerator of the investment measure). The magnitude of coefficients on market-to-book and cash flow is in line with earlier studies on investment-Q and investment-CF sensitivity (see, e.g., Baker et al. (2003) and Chen and Chen (2012)).

After documenting a negative and significant relation between short interest and investment, we next examine more closely whether this relation is driven by managerial learning, as our model

[12] As is standard in the empirical literature on investment-cash flow sensitivity, we do not lag *CF* in our regressions. To address a potential concern that short interest may affect investment through its correlation with past *CF*, we re-estimate all the equations with lagged, rather than contemporaneous cash flows, and obtain very similar results.

suggests. The model provides specific predictions about the sensitivity of investment to short interest. First, we expect a stronger effect of short interest on investment at higher levels of short interest (Hypothesis 2). We also expect the short interest signal to be more valuable when information transparency of the firm is low (Hypothesis 3). We proceed by testing these specific predictions of the model.

5.2 Non-linearity of investment-to-short interest sensitivity

Hypothesis 2 predicts that the negative sensitivity of investment to short interest, established in the previous section, is stronger for higher levels of short interest. In the model, this effect arises because high levels of short interest are more likely to be driven by informed trading.

To test this hypothesis, we use the following empirical specification:

$$\begin{aligned} I_{i,t} = \alpha + \beta_H HighSI_{i,t-1} + \beta_L LowSI_{i,t-1} + \beta_I HighInd_{i,t-1} + \delta_1 MB_{i,t-1} + \\ \delta_2 CF_{i,t} + \delta_3 log(Assets)_{i,t-1} + \eta_i + \nu_t + \varepsilon_{i,t}, \end{aligned} \tag{16}$$

where $HighSI_{i,t-1}$ equals SI if it is above its sample median in that year, and zero otherwise. Likewise, $LowSI_{i,t-1}$ equals short interest if the corresponding measure is below or equal to the sample median in a given year and equals zero otherwise. Finally, we include an indicator variable $HighInd_{i,t-1}$ that takes on the value of one if the short interest proxy is above its sample median in that year, and zero otherwise. This specification allows short interest to have different slopes and intercepts at high and low levels. Following this methodology, we split each of our short interest measures into two parts. The results from this test are presented in Table 4.

Consistent with our conjecture that short interest in more heavily shorted stocks is likely more informative, the β_H coefficients on $HighSI$ are negative and highly significant across all specifications. By contrast, the impact of short interest at low levels is weak: the β_L coefficients have mixed signs, are smaller in magnitude, and statistically insignificant in the majority of our specifications. Thus, the results in Table 4 support Hypothesis 2: consistent with the managerial learning channel, the effect of short interest on investment is stronger when short interest is high.

5.3 The effect of information uncertainty

In this subsection, we examine whether information environment of a firm impacts the sensitivity of investment to short interest. Hypothesis 3 predicts that this relation should be stronger for firms subject to greater information uncertainty. When information uncertainty is high, short interest indicates that the risk-averse informed trader perceives greater mispricing of the stock.

To test this prediction, we use several measures of information transparency. Our first measure is based on analyst forecast dispersion, constructed as described in Section 3 (given I/B/E/S data availability, our sample period in this test starts in 1977). We then average the monthly dispersion ratios over one-year period to obtain average annual dispersion. Analyst forecast dispersion as proxy for uncertainty has been used extensively in the literature. For example, Barron and Stuerke (1998) argue that forecast dispersion is a useful indicator of uncertainty about the price relevant component of firms' future earnings.

Our second proxy for information uncertainty is stock return volatility. Company valuations are less likely to be precise when stock return volatility is high. We use two measures of stock return volatility - total and idiosyncratic (see Section 3 for details about variable construction).

Finally, we examine the effect of institutional ownership. Institutions facilitate information production, so higher institutional ownership is likely associated with less information uncertainty. Furthermore, as demonstrated by Boehmer and Kelley (2009), stocks with greater institutional ownership are priced more efficiently. Thus, when institutional holdings are high, the quality of existing information about the company is also high. In this case, managers are likely to be better informed about their fair valuations and growth prospects, and hence learning from short interest becomes less important. We obtain total institutional ownership from the 13F database and use the fraction of shares owned by all institutional investors as an inverse measure of information asymmetry (given the 13F data availability, our sample period in this test starts in 1981).

To test the effect of information uncertainty on the relation between short interest and investment, for each of our information uncertainty proxies we estimate the following regression model:

$$\begin{aligned} I_{i,t} = \alpha + \beta_1 SI_{i,t-1} + \beta_2 High\_Assym_{i,t-1} \times SI_{i,t-1} + \beta_3 High\_Assym_{i,t-1} \\ + \delta_1 MB_{i,t-1} + \delta_2 CF_{i,t} + \delta_3 log(Assets)_{i,t-1} + \eta_i + \nu_t + \varepsilon_{i,t}, \end{aligned} \tag{17}$$

where $High\_Assym_{i,t-1}$ is a dummy variable set to one if the measure of information asymmetry is above the sample median in a given year, and set to zero otherwise. The coefficient of primary interest is the one on the interaction term, β_2, since it captures the effect of informational uncertainty on the sensitivity of investment to short interest.

Table 5 reports results for the effect of analyst forecast dispersion. As follows from Table 5, the high dispersion dummy *High disper* is negatively associated with subsequent investment. This finding is generally consistent with the literature that finds a negative link between analyst forecast dispersion and subsequent firm performance (see, e.g. Diether et al. (2002)). It is expected that poorly performing firms would cut their investments. More importantly for our analysis, coefficients on the interaction terms of the *High disper* dummy and each of the three measures of short interest are negative and highly statistically significant across the majority of the specifications, with the exception of the two coefficients on the interaction terms with *Days-to-cover* (where the coefficients are negative but statistically insignificant). The effect of analyst forecast dispersion on the sensitivity of investment to short interest is also economically large. For example, the coefficients of -0.183 on $SI/shares$ and -0.264 on the interaction of $SI/shares$ and the *High disper* dummy in the $CAPEX/PPE$ regression imply that this sensitivity more than doubles for high values of analyst forecast dispersion. Thus, the results in Table 5 support Hypothesis 3 – the effect of short interest on investment strengthens for more opaque firms as proxied by analyst forecast dispersion.

Table 6 reports results for information uncertainty as proxied by the total stock return volatility. Similar to the results in Table 5, firms with high total volatility tend to invest less as evidenced by the negative coefficients on the *High total vol* dummy in the investment regressions. This finding is in line with the empirical literature on the interplay between uncertainty and investment (see, e.g., Leahy and Whited (1996)). Consistent with Hypothesis 3, coefficients on the interaction terms of *High total vol* and various measures of short interest are negative and generally highly statistically significant (out of nine coefficients in various specifications, one is statistically insignificant, one is marginally significant, and the rest are highly significant). The effect of total volatility on the sensitivity of investment to short interest is also economically very strong as evidenced by the much higher magnitude of the coefficients on the interaction terms relative to the coefficients on short interest measures.

Table 7 replicates the analysis while employing an alternative volatility measure - idiosyncratic return volatility. The evidence presented in Table 7 generally mirrors that in Table 6. Coefficients on the interaction terms of the high idiosyncratic volatility dummy (*High Ivol*) and various measures of short interest are negative and generally highly statistically significant in all specifications with one exception from regressing the second investment measure (that accounts for R&D expenditures) on *Days-to-cover* as a proxy for short interest (where the coefficient on the interaction term is negative but statistically insignificant). Thus, the evidence presented in Table 7 shows the robustness of our results to this alternative volatility measure used as a proxy for information uncertainty. The results in Tables 6 and 7 are consistent with the effect of information uncertainty on managerial learning and further support Hypothesis 3.

Finally, we examine the effect of institutional ownership on the strength of the relation between short interest and investment. As we argue above, institutional investors are likely to produce information and facilitate more efficient pricing, thereby reducing information uncertainty. To study the role of institutional ownership, we interact a dummy for low versus high levels of institutional holdings with our measures of short interest and replicate our results while using institutional ownership as an (inverse) proxy for information uncertainty. The results from these tests are reported in Table 8. Because institutional ownership is likely to be inversely related to the degree of information uncertainty, we expect positive coefficients on the interaction terms of the *High Inst Hold* dummy and various short interest measures. The evidence in Table 8 corroborates this conjecture. The coefficients on the interaction terms are positive and statistically significant (with the exception of two regression specifications). The coefficients on the *High Inst Hold* dummy are generally positive as well, suggesting that firms with higher institutional ownership invest more actively.

Overall, we find strong evidence in support of Hypothesis 3. Using four different measures related to the degree of informational uncertainty (analyst forecast dispersion, total and idiosyncratic return volatility, and percentage of institutional holdings), we find an economically strong relation between the sensitivity of investment to short interest and informational uncertainty: This sensitivity is stronger for more informationally opaque and less transparent firms.

5.4 Additional effects of short interest informativeness

In this subsection, we perform two additional tests that relate the informativeness of short interest to the short interest's role in managerial learning and its effect on investment.

First, we examine whether the link between short interest and investment is stronger in the later part of our sample. Hong et al. (2015) document a positive trend in short interest over time, consistent with weakening of the barriers to short selling. This trend is present in our sample, too. For example, short interest scaled by shares grows from 0.6% in the first half of the sample (1974 to 1996) to 4.5% in the second half (1997 to 2018). Short sale constraints have been alleviated in part due to regulatory changes, such as several reductions in tick size during the 1990s, which have improved liquidity and market depth, as well as due to Reg SHO. Financial market development, particularly, the increasing role of institutional investors, has also contributed to higher market liquidity and higher availability of shares to borrow. As financial markets are becoming more conducive to short selling, more traders with negative views are able to participate in shorting a stock and hence enrich the informativeness of short interest. Thus, we expect a stronger effect of short interest on investment in the later years of our sample compared to the earlier years, as more informed traders participate in short selling.

To test this conjecture, we split our sample in half − from 1974 to 1996 and from 1997 to 2018 and repeat our tests separately for these two subsamples. The results from these tests are presented in Table 9.

As expected, the effect of short interest on investment is much stronger in the second half of our sample: The coefficients on short interest are generally higher in magnitude and highly statistically significant. By contrast, those coefficients in the first half of the sample are generally statistically insignificant. For example, the coefficient on short interest scaled by shares outstanding ($SI/shares$) in the regressions of our first investment measure, $Capex/PPE$ is -0.411 and highly statistical significant in the second half of the sample, compared to -0.156 and statistically insignificant in the first half. Coefficients on short interest in regression specifications with alternative measures of investment and short interest exhibit similar patterns. Thus, as short selling constraints weaken, short interest becomes more informationally rich and its effect on corporate investment intensifies, consistent with the managerial learning mechanism.

Second, we examine whether the effect of short interest on investment weakens for investment in a more distant future. Short interest is likely more relevant when predicting the firm's fundamental value in the relatively near future. Therefore, we expect short selling in year t to have a stronger impact on investment in the following year as opposed to several years ahead. This prediction is based on the existing literature demonstrating that short sellers are typically skillful in predicting events up to two years ahead. For example, Akbas et al. (2017) track information content of short positions. They find that most heavily shorted stocks are followed by value-decreasing events, and these events occur as late as 12 months into the future. In a similar vein, Karpoff and Lou (2010) show that short sellers establish short positions about 19 months prior to corporate misconduct. If short interest conveys information about future fundamentals, investment planning should be most valuable in the near term, but become a noisier signal for investment decisions in a more distant future.

To test this hypothesis, we re-estimate the main specification by regressing investment in years $t+1$, $t+2$, and $t+3$ on short interest. Table 10 summarizes the results. For the brevity of exposition, we only report the coefficients on short interest.

Consistent with our conjecture that short interest is most precise in the near term, we find that the impact of short interest on investment in year $t+1$ is negative and statistically significant, and the coefficient magnitudes are comparable to those in the baseline specifications (albeit generally somewhat weaker). For example, the coefficient on *Days-to-cover* in regressions of $CAPEX/PPE$ is -0.174 for investment in year $t+1$ versus -0.286 for investment in year t, both being highly statistically significant. However, the impact of short interest on investment further in the future decays. Some of the coefficients become insignificant when explaining investment in year $t+2$, and the explanatory power of investment in year $t+3$ further declines. For example, the coefficient on *Days-to-cover* declines to -0.057 and is only marginally significant in year $t+2$ and further drops to -0.005 and insignificant in year $t+3$. Overall, out of nine coefficients in regressions of investment in year $t+3$ on various measures of short interest, only three are statistically significant and two coefficients flip sign. Taken together, this pattern supports the importance of the informational content of short interest and its role in managerial learning. As the short interest signal becomes less precise, its impact on investment farther in the future declines.

5.5 Reg SHO

In this subsection, we examine the effect of an exogenous shock to the informativeness of short interest by exploiting a natural regulatory experiment - Reg SHO.

Since the 1930s, short sales could not be placed when stock prices were declining, a regulation commonly referred to as the uptick rule. Reg SHO, introduced in July 2004, had relaxed short sale constraints in a random sample of Russell 3000 stocks. In particular, Reg SHO removed the uptick rule for a random sample of 968 firms. The uptick rule did not allow short sales to be executed (i) at a lower price than the previous price, (ii) at the same price as the previous price if the preceding trade was executed at a higher price than the previous and current one. The SEC selected firms from the Russell 3000 index listed on NYSE, NASDAQ, and AMEX and ranked them independently for each stock exchange by average daily trading volume. Every third firm on these lists was then selected in the pilot group. Two years later the SEC removed restrictions for all stocks after analyzing the results from the experiment.

Multiple studies have examined the effect of Reg SHO on prices, liquidity and volatility. Boehmer et al. (2008) find an effect on short selling activity and potentially return volatility but not on prices. Alexander and Peterson (2009) document that the pilot stocks have lower price locations relative to quotes, more short trades and more short volume. Grullon et al. (2015) find that Reg SHO and the subsequent increase in short selling activity caused stock prices to fall, and the effect was stronger among small stocks.

Our empirical tests rely on a different interpretation of Reg SHO. While the majority of the existing studies interpreted Reg SHO as a shock to short interest, we argue that it had an effect on short interest *informativeness*. The uptick rule that had been in place prior to Reg SHO had inhibited short selling activity by potentially informed speculators. When the uptick rule was removed, it facilitated the flow of informed speculators to the pilot stocks, making short interest in those stocks more informative. If managerial learning is responsible for the negative relation between short interest and investment, we expect investment-to-short interest sensitivity to increase following the implementation of Reg SHO for the pilot firms relative to the control firms, due to a richer informational content of short interest among pilot firms.

Note that our test design is conceptually very different from that of Grullon et al. (2015) who document a negative effect of Reg SHO on investment. Grullon et al. (2015) interpret Reg SHO as a shock to short interest *level* and study the effect of Reg SHO on investment. By contrast, we interpret Reg SHO as a shock to short interest informativeness and examine the effect of Reg SHO on the sensitivity of investment to short interest.

To test this hypothesis, we use a difference-in-differences specification and define a dummy variable, SHO. For pilot stocks, the dummy variable equals one if the stock was in the sample of pilot stocks and was subject to Reg SHO for at least six months of its fiscal year, starting from August 2004. For non-pilot stocks, SHO equals one if the stock was in the Russell 3000 index (as of May 2004) and was subject to the repeal of Reg SHO, announced in July 2007, for at least six months of its fiscal year, otherwise SHO is set to zero. We restrict our sample to stocks included in the Russell 3000 index as of 2004 and the period before and after the announcement of Reg SHO $(2002-2008)$. We then regress investment on the three measures of short interest, the set of control variables, the SHO dummy, and the interaction term of the SHO dummy and short interest. Our regression specification is of the following form:

$$\begin{aligned} I_{i,t} = \alpha + \beta_1 SI_{i,t-1} + \beta_2 SHO_{i,t-1} \times SI_{i,t-1} + \beta_3 SHO_{i,t-1} + \delta_1 MB_{i,t-1} + \\ \delta_2 CF_{i,t} + \delta_3 log(Assets)_{i,t-1} + \eta_i + \nu_t + \varepsilon_{i,t}, \end{aligned} \tag{18}$$

As in all our empirical tests, we include time and firm fixed effects and double cluster standard errors by time and firm. We are particularly interested in the coefficient β_2 on the interaction term of $SHO_{i,t-1}$ and short interest, because it shows the incremental effect of Reg SHO on the sensitivity of investment to short interest.

The results from this analysis are presented in Table 11. As shown in this table, the coefficients on the interaction terms are negative and highly statistically significant in seven out of nine regression specifications. We interpret these results as additional evidence in favor of managerial learning from short interest that also helps us address potential endogeneity concerns. As short interest in the pilot stocks became more informative following the implementation of Reg SHO, its impact on investment had strengthened.

Certain concerns have been raised in the literature about using Reg SHO in finance research. Heath et al. (2023) critisize "reusing" natural experiments with a focus on Business Combination Laws and Reg SHO, as both regulations have received considerable attention in the literature. They argue that when the same exogenous shock is used to test multiple null hypotheses, the probability of making at least one type one error increases, and the resulting test statistics need to be corrected. They also propose specific procedures, based on Romano and Wolf (2016), to correct p-values to account for multiple hypothesis testing.

In regard to Reg SHO, Heath et al. (2020) classify its outcomes in four groups. They apply a single hypothesis testing critical value to the direct effect outcome, short interest. They group outcomes related to the price formation process as second order outcomes, outcomes related to corporate investment and disclosure decisions as third order outcomes, and outcome variables related to external parties as fourth order outcomes (see figure A1 in Heath et al. (2023)). Using the "causal chain approach", based Romano and Wolf (2016), they apply different p-value corrections to different groups. Groups further down the chain receive the largest corrections.

We believe that the use of Reg SHO in our setting survives the Heath et al. (2023) critique to a large extent. First, unlike the existing literature, we use Reg SHO as a shock to short interest informativeness, not its level. Multiple prior studies document the effect of increased short interest (following Reg SHO) on various corporate outcomes. The confounding effects of those outcomes make it more challenging to establish the exclusion restriction and isolate the direct effect of Reg SHO on a particular variable. However, the focus of our study is the effect of Reg SHO on the sensitivity of investment to short interest. It is less likely that the variables previously shown to change in response to Reg SHO would have an effect on that sensitivity – an economic channel through which such an effect might occur is unclear.[13] Second, the sensitivity of investment to short

[13]Multiple papers have examined the implications of Reg SHO for corporate finance. Grullon et al. (2015) study

interest would likely be classified as a "second order outcome" based on the Heath et al. (2023) "causal chain approach". The corresponding p-value correction leaves all our relevant regression coefficients statistically significant. Finally, even when assuming that there are twenty prior studies and using the corresponding correction suggested by Heath et al. (2023), all but one interaction coefficients still remain statistically significant. Specifically,Heath et al. (2023) argue that with twenty prior studies, the critical values for the t-statistics are in the range of 2.96 - 3.05. At this level, the only coefficient in our Reg SHO regressions that loses statistical significance is the one in the regression of *CAPEX* on *Days-to-Cover* (t-statistic of 2.83), which still remains marginally significant.

Another potential caveat of using Reg SHO has been voiced by Boehmer et al. (2020), who demonstrate the role of indirect effects or "spillovers". They argue that following the 2005 repeal of the uptick rule for the pilot stocks, short-sellers shifted their focus from control stocks to pilot stocks and therefore reduced the short aggressiveness towards control stocks. However, this potential spillover effect does not invalidate the design of our Reg SHO tests. On the contrary, we believe that it strengthens our identification mechanism. The purpose of our test is not to establish the effect of Reg SHO in the absence of potential spillovers but to use Reg SHO as a shock to short interest informativeness. Potential shift of informed short sellers from control to treatment stocks only enhances the relevance of our argument.

To summarize, while we acknowledge the potential limitations and caveats in reference to using Reg SHO in finance research, we believe that the results from our Reg SHO tests can still be interpreted as supportive of the managerial learning hypothesis, as the investment-to-short interest sensitivity has strengthened following the enactment of Reg SHO.

6 Alternative explanations

Our results in Section 4 provide strong evidence in favor of our main hypothesis about managers learning from short interest. However, other potential mechanisms might result in a negative relation between investment and short interest. First, there is a potential for reverse causality - short sellers might react to declining investment. Second, both short interest and investment might react to common economic shocks. Third, it is conceivable that short sellers are good at identifying

the effect of Reg SHO on investment and equity issuance, De Angelis et al. (2017) examine implications for executive incentive contracts, Massa et al. (2015) focus on insider trading and He and Tian (2020) look at patents, among others.

poorly performing firms whose investment naturally declines as performance deteriorates. These firms can become even more attractive to short sellers if they face difficulties in raising funds to finance investment. Finally, managers might fight short sellers by aggressively repurchasing shares, thereby diverting funds from investment projects.

Overall, we believe that the design of our empirical tests helps alleviate endogeneity concerns to a large extent. All our specifications include firm fixed effects, which account for time-invariant firm characteristics that could simultaneously affect both short interest and investment, as well as time fixed effects to control for common time-varying economic shocks.

The uniqueness of the cross-sectional predictions derived from our model further helps address endogeneity concerns. An omitted factor that explains both short interest and investment would have to pass the hurdle of simultaneously explaining the variation in sensitivity of investment to short selling activity at high versus low levels of short interest (Hypothesis 2), as well as the differences in that sensitivity among opaque versus transparent firms (Hypothesis 3). A similar logic applies to the reverse causality argument. Furthermore, a large body of literature on the investment anomaly has established a negative relation between corporate investment and subsequent returns. If firms that cut investment experience higher future returns, shorting stocks in response to investment decline should in general be an unprofitable strategy.[14]

Although not critical to the validity of our main arguments, the Reg SHO analysis further strengthens the interpretation of our results. As the informativeness of short interest had increased in the pilot stocks, the relation between investment and short interest had strengthened among those stocks. We would not see this effect if short interest and investment were merely responding to some common economic shocks.

To further enhance our conclusions about the validity of the learning channel, in the remainder of this section, we directly explore all potential economic mechanisms that can give rise to the negative investment-to-short interest sensitivity.

6.1 Predictive power of short interest

It is possible that short sellers are skillful in identifying companies that are likely to realize inferior returns in the future and therefore exhibit poor performance and possibly cut their investment. For example, Desai et al. (2002), Pownall and Simko (2005), Diether et al. (2009) and Drake, Rees and Swanson (2011) report that short sellers can correctly predict future negative abnormal returns

[14] See, e.g. Lyandres, Sun, and Zhang (2008).

and negative earning expectations. Similarly, Drake et al. (2015) document that short sellers are likely to take a short position in firms for which stock prices have yet to incorporate information about future fundamentals.

We address this explanation in a number of ways. First, we refer back to our cross-sectional findings, which point to higher sensitivity of investment to short interest among less transparent firms. If short sellers are better at forecasting future economic conditions than other market participants, their signal should predict future performance of transparent firms more precisely. This, in turn, should strengthen the association between short interest and investment. Our results show the opposite: the impact of short selling on investment is actually stronger among opaque firms. Furthermore, even if short sellers did prefer shorting opaque firms, it is not clear why managers of opaque firms would cut investment more aggressively, especially since the limited liability feature of corporation increases the benefits of investment in volatile environment. The only plausible reason for that would be financial constraints, and we address this explanation in details in the next subsection.

To further alleviate the concern of return predictability, we augment our baseline regression specifications by adding future returns as of year t to the set of our control variables. If managers merely react to inferior performance (returns) that short sellers are able to predict, then our measures of short interest should become insignificant once we include returns. However, if the signal received by short sellers is imperfectly correlated with future returns (for example, because it reflects information that takes longer to get incorporated, as well as non-fundamental information), then short interest should still remain significant. The results from these regressions are reported in Table 12. We define returns as average monthly stock returns over a fiscal year period. As follows from Table 12, the coefficients on our measures of short interest remain negative. Their magnitude is slightly lower compared to the baseline specification, suggesting that short interest does have predictive power and reflects some information about future returns. Nevertheless, the coefficients are still highly statistically and economically significant in most specifications.

For robustness, we repeat the estimation after controlling for returns in period $t + 1$, rather than t, and find consistent results. Taken together, this evidence suggests that the informational content of short interest extends beyond its potential power to predict future prices or returns and plays a direct role in affecting corporate investment.

6.2 The effect of equity dependence and financial constraints

It is possible that the negative relation between investment and short interest is driven by the company's need to raise funds in capital markets and finance its investment projects. Investors may anticipate a decline in stock prices after observing high levels of short interest and impede firms' access to capital markets, limiting future investment. Thus, even though short interest does have a direct effect on investment, the mechanism is different: managers reduce investment not because they learn about their future investment opportunities, but because access to external capital becomes more expensive.

Note that this explanation is addressed to a large extent by our analysis in the previous subsection, where we control for the effect of short interest on future price changes. Nevertheless, in this subsection we look deeper into this issue. If access to funding plays a role, it should be more relevant for firms that are financially constrained and dependent on external capital markets. Therefore, if raising new funds is harder when the levels of short interest are high, the impact of short interest on investment is likely to be stronger for more equity dependent and more financially constrained firms.

To test this hypothesis, we use two measures of dependence on capital markets/financial constraints. The first one is the equity dependence index based on Baker et al. (2003) (henceforth the BSW index). This index follows Kaplan and Zingales (1997) and is constructed as a linear combination of cash flow, dividends, and cash balances, all scaled by lagged total assets, and the book leverage ratio. Unprofitable and highly leveraged firms with low cash reserves and payouts are more reliant on external capital and hence receive a higher index score. Following BSW (2003), we do not include the Q component in the index, as it may capture growth opportunities, as well as potential mispricing of the stock (see Equation (5) of BSW (2003) for full details of the index construction). The second measure is an index of financial constraints, constructed as in Hadlock and Pierce (2010), henceforth the HP index. According to this classification, larger and more mature firms are less subject to financial constraints (see footnote 2 in Hadlock and Pierce (2010) for full details of the index construction).

Next, we define a dummy variable (high BSW index dummy) that we set equal to one if the BSW index is above its sample median in a given year, and set it to zero otherwise. We use a similar approach to introduce the high HP dummy based on the HP index. We then augment our base regression specifications by adding each of these two dummies, as well as their interactions with the measures of short interest. Table 13 presents the results for the BSW index used as a proxy for the severity of financial constraints.

As shown in Table 13, the high BSW dummy has a negative and highly statistically significant effect on investment. This result is expected - financially constrained firms have more limited access to capital markets and hence have to restrict their investments. This result is also consistent with the literature (see, e.g. Baker et al. (2003)). However, there is no evidence that the sensitivity of investment to short interest increases for high BSW index firms. The coefficients on the interaction terms of the high BSW index dummy with the measures of short interest are mostly statistically insignificant and change signs depending on the specification. Thus, contrary to the evidence that financial constraints strengthen the relation between investment and price, as documented by Baker et al. (2003), there is no such effect for the sensitivity of investment to short interest.

Table 14 replicates the analysis from Table 13 while using the HP index as a proxy for equity dependence. Again, we include a *High HP index* dummy for high HP index firms (i.e. most equity dependent firms) and interact this dummy with our measures of short interest. We are particularly interested in the coefficients on these interaction terms. Negative coefficients would indicate a stronger effect of short interest on subsequent investment for highly equity dependent firms and would support the capital market channel. However, there is no evidence of such an effect in the data. As follows from Table 14, the coefficients on the interaction terms change signs and are insignificant in most specifications (with the exception of our first proxy for short interest - short interest scaled by shares outstanding - where the coefficients on the interaction terms are negative and significant in the two specifications).

Overall, the results in Tables 13 and 14 do not provide support for the capital market channel: There is little evidence that financial constraints and equity dependence amplify the effect of short interest on investment. Thus, the evidence again points towards the managerial learning channel as the driving force behind the sensitivity of investment to short interest.

6.3 Stock price manipulation by short sellers

Finally, we examine whether short sellers strategically establish short interest positions to manipulate stock prices and induce firms to cut investment. Goldstein and Guembel (2008) show that price manipulation through short selling can distort investment even if it bears no informational content. As speculators attempt to drive the stock price down by short selling, the manager responds to the stock price decline by cancelling profitable projects. The decision to cut investment then has a negative impact on firm value and turns the temporary price decline into a self-fulfilling prophecy, allowing short sellers to cover their positions at a profit.

Overall, the manipulation channel is similar in spirit to the managerial learning channel proposed in this paper, as both mechanisms are based on learning from financial markets. However, there is also a notable conceptual difference. In Goldstein and Guembel (2008), short interest affects investment only through its ability to impact stock prices, but has no direct effect on investment, as managers learn from observing stock prices only. In contrast, in our model, both stock prices and short interest are informative and convey complementary information about firm fundamentals. Our main tests show that variation in short interest affects investment when we control for stock prices. To further ensure that the informational content of short interest goes beyond its impact on prices, we include stock returns as of $t-1$ in the set of control variables. If short interest influences the investment solely by reducing stock prices, then inclusion of this direct measure of price change should subsume the impact of short interest on investment. Contrary to this argument, we find that the impact of short interest remains negative and significant (unreported for the sake of brevity).

In a recent paper, Campello et al. (2020) extend the idea of manipulation by modelling how managers fight short sellers using stock repurchases - the strategy that is accompanied by investment cuts. Note that their paper is conceptually different from ours, as it focuses on short selling costs, rather than on variation in the levels of short interest. Admittedly, the two effects could be challenging to disentangle empirically. One implication of the model in Campello et al. (2020) is that the effect of short selling costs on investment is negative when firms repurchase their shares (because repurchases detract funds from potential investment projects). If the effect of short interest on investment is driven primarily by firms pursuing more active repurchase programs in response to intensified short selling, one expects to see a stronger sensitivity of investment to short interest for firms with higher repurchases. To examine the role of share repurchases, we define repurchases as the purchase of common and preferred stock (Compustat item PRSTKC), scaled by lagged assets. *High Repurchase* dummy is then set equal to one for firms with share repurchases above the sample median and is set equal to zero otherwise. We then augment our empirical specifications with this dummy as well as its interactions with various short interest measures. Since repurchases were rare prior to the Safe Harbor Rule of 1982, we start our sample in 1983.[15]

The results from these tests are summarized in Table 15. These results indicate that the

[15] The Safe Harbor Rule (Rule 10b-18), introduced by the SEC in 1982, outlines a set of conditions that a company engaging in share repurchase in the open market should meet in order not to trigger manipulation charges.

impact of short interest on investment weakens slightly when controlling for share repurchases. For example, in the regression of our first investment measure, *CAPEX/PPE*, on short interest scaled by shares, *SI/Shares*, the coefficient on short interest declines in magnitude from -0.321 to -0.227. However, the effect of short interest on investment remains negative and highly statistically significant in eight out of nine regression specifications. Furthermore, there is no evidence that the negative relation between short interest and investment is stronger for firms with more active share repurchase programs, since the coefficients on the interaction terms are generally positive and mostly insignificant. We therefore conclude that the effect of short interest on investment cannot be attributed solely to share repurchases. It is unlikely to be driven by firms responding to short sellers by increasing repurchases and therefore cutting investment.

7 Conclusion

In this paper, we demonstrate that managers learn from short interest when making their investment decisions. While there is ample evidence that managers learn from stock prices, we argue that short interest represents an additional source of information that is not fully impounded in prices.

To better understand the informational role of short interest in a theoretical framework, we build a model that incorporates learning from both prices and short interest. Our key assumption (which we believe holds empirically) is that speculators are more likely to short in order to bet on negative information, while noise traders can either short or sell their inventory. This assumption gives rise to additional informational content of short interest, beyond that found in the stock price. We show that in equilibrium the best estimate of the fundamental firm value is negatively related to short interest, and its sensitivity to short interest is stronger when there is more information uncertainty about the true value of the firm and when the level of short interest is high.

Empirical evidence strongly supports the predictions of the model. We demonstrate a negative and significant relation between short interest and subsequent investment. This relation is stronger for firms subject to higher information uncertainty, and for higher levels of short interest, in line with our model. The investment-to-short interest sensitivity intensifies with the informativeness of short interest (in our tests that rely on sample splits and different investment horizons), also consistent with managerial learning. To further address potential endogeneity concerns we take advantage of a regulatory experiment, Reg SHO, which removed barriers to short selling for a sample of pilot stocks and show that it had a predicted effect on the sensitivity of investment to short interest.

We examine various economic mechanisms that could confound the relation between short interest and investment and conclude that none can fully explain our results.

Overall, our paper contributes to the growing literature that examines the interplay of corporate decisions and capital markets through the lens of managerial learning. We show that managerial learning goes beyond learning from prices and that short interest presents valuable information for managerial decision making and therefore affects corporate investment.

Appendix A

Proof of Proposition 1

First we derive the p.d.f. of the fundamental value conditional on short interest and price, and then calculate the best estimate of the fundamental value $\widehat{v}_M$ as a conditional expectation in the manager's information set. Because the informed trader's estimation error of the fundamental value $v - s$ and noise traders' of types one and two demands ξ_k, $k = 1, 2$, are all i.i.d. and uncorrelated, the joint p.d.f. conditional on signal s takes the form

$$\begin{aligned} f\left(v, \xi_1, \xi_2 \mid s\right) &= C^{-1} e^{-\frac{(v-s)^2}{2\sigma_s^2}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{\xi_2^2}{2k}}, \\ C &= \sqrt{2\pi\sigma_s^2}\sqrt{2\pi\left(1-k\right)}\sqrt{2\pi k}, \end{aligned} \tag{A.1}$$

where C is a normalization constant.

With the Dirac delta function $\delta\left(x\right) = \theta'\left(x\right)$ and taking into account that market price and short interest are defined, with $\sigma_\xi t = P - s$, by market clearing condition, $t + \xi_1 + \xi_2 = 0$, and $t\theta\left(t\right) + \xi_1\theta\left(\xi_1\right) = \widehat{S} = \frac{1}{\sigma_u} S$, respectively, we obtain a conditional p.d.f. $f\left(v|P, \widehat{S}\right)$ in the form

$$\begin{aligned} f\left(v|P, \widehat{S}\right) &= B^{-1} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\frac{v-P}{\sigma_\xi}+t\right)^2}{2r}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right) + \xi_1\theta\left(\xi_1\right) - \widehat{S}\right) dt d\xi_1, \quad \text{(A.2)} \\ B\left(\widehat{S}\right) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\frac{v-P}{\sigma_\xi}+t\right)^2}{2r}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right) + \xi_1\theta\left(\xi_1\right) - \widehat{S}\right) dt d\xi_1 dv \\ &= \sigma_\xi \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{(\eta+t)^2}{2r}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right) + \xi_1\theta\left(\xi_1\right) - \widehat{S}\right) dt d\xi_1 d\eta, \end{aligned}$$

with $r = \frac{\sigma_s^2}{\sigma_\xi^2}$, and where the normalizing factor B comes from the Bayes theorem. Note that the marginal p.d.f. of the short interest $\widehat{S}$ is $g\left(\widehat{S}\right) = C^{-1} B\left(\widehat{S}\right)$, or

$$\begin{aligned} g\left(\widehat{S}\right) &= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{(\eta+t)^2}{2r}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right) + \xi_1\theta\left(\xi_1\right) - \widehat{S}\right) d\eta d\xi_1 dt}{\sqrt{2\pi r}\sqrt{2\pi\left(1-k\right)}\sqrt{2\pi k}} \quad \text{(A.3)} \\ &= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right) + \xi_1\theta\left(\xi_1\right) - \widehat{S}\right) d\xi_1 dt}{\sqrt{2\pi\left(1-k\right)}\sqrt{2\pi k}}. \end{aligned}$$

We then obtain the best estimator of the fundamental value, $\widehat{v}_M = E\left[v|P, \widehat{S}\right]$ as a conditional mean of (A.2):

$$\begin{aligned}\widehat{v}_M &= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \left(P-\sigma_\xi t\right)\delta\left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)-\widehat{S}\right) dtd\xi_1}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)-\widehat{S}\right) dtd\xi_1} \qquad \text{(A.4)}\\ &= P-\sigma_\xi E_{t,\xi_1}\left[t\right] = P-\sigma_\xi\Lambda\left(\widehat{S}\right),\end{aligned}$$

with

$$\Lambda\left(\widehat{S}\right) = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} t\delta\left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)-\widehat{S}\right) dtd\xi_1}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)-\widehat{S}\right) dtd\xi_1} = \frac{A}{B}, \qquad \text{(A.5)}$$

and

$$\begin{aligned}A &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} t\delta\left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)-\widehat{S}\right) dtd\xi_1, \qquad \text{(A.6)}\\ B &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)-\widehat{S}\right) dtd\xi_1.\end{aligned}$$

Combining (A.3) and (A.5), we obtain the expectation of $\Lambda\left(\widehat{S}\right)$: $E_{\widehat{S}}\left[\Lambda\left(\widehat{S}\right)\right] = \int_0^{+\infty}\Lambda\left(\widehat{S}\right) g\left(\widehat{S}\right) d\widehat{S}$:

$$\begin{aligned}E_{\widehat{S}}\left[\Lambda\left(\widehat{S}\right)\right] &= \frac{\int_0^{+\infty} A\left(\widehat{S}\right) d\widehat{S}}{\sqrt{2\pi\left(1-k\right)}\sqrt{2\pi k}}\\ &= \frac{\int_0^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2\sigma_{\xi_1}^2}} e^{-\frac{(t+\xi_1)^2}{2\sigma_{\xi_2}^2}} t\delta\left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)-\widehat{S}\right) dtd\xi_1 d\widehat{S}}{\sqrt{2\pi\left(1-k\right)}\sqrt{2\pi k}}\\ &= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} tdtd\xi_1}{\sqrt{2\pi\left(1-k\right)}\sqrt{2\pi k}} = 0,\end{aligned}$$

which shows that on average, the correction to the manager's best estimate due to the short interest is zero, as expected (because as we show in Section 2, the price is an unbiased estimate of the true value). Making use of (A.3), we also obtain the average value of short interest, $E_{\widehat{S}}\left[\widehat{S}\right] = \int_0^{+\infty}\widehat{S}g\left(\widehat{S}\right) d\widehat{S}$ as follows:

$$
\begin{aligned}
E_{\widehat{S}}\left[\widehat{S}\right] &= \frac{\int_0^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \widehat{S}\delta\left(t\theta(t)+\xi_1\theta(\xi_1)-\widehat{S}\right) d\xi_1 dt d\widehat{S}}{\sqrt{2\pi(1-k)}\sqrt{2\pi k}} \\
&= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \left(t\theta(t)+\xi_1\theta(\xi_1)\right) d\xi_1 dt}{\sqrt{2\pi(1-k)}\sqrt{2\pi k}} \\
&= \frac{\int_0^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} t d\xi_1 dt + \int_{-\infty}^{+\infty}\int_0^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \xi_1 d\xi_1 dt}{\sqrt{2\pi(1-k)}\sqrt{2\pi k}} \\
&= \frac{1}{\sqrt{2\pi(1-k)}}\int_0^{+\infty} e^{-\frac{\xi_1^2}{2(1-k)}}\xi_1 d\xi_1 + \frac{1}{\sqrt{2\pi}}\int_0^{+\infty} e^{-\frac{t^2}{2}} t dt \\
&= \frac{1}{\sqrt{2\pi}}\left(1+\sqrt{1-k}\right).
\end{aligned}
$$

The R^2 integration domain in (A.6) splits into two subsets: $\xi_1 \leq 0$ and $\xi_1 \geq 0$. We obtain

$$
\begin{aligned}
A &= \int_{-\infty}^{0} d\xi_1 \int_{-\infty}^{+\infty} dt e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} t\delta\left(t\theta(t)-\widehat{S}\right) \\
&\quad + \int_0^{+\infty} d\xi_1 \int_{-\infty}^{+\infty} dt e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} t\delta\left(t\theta(t)+\xi_1-\widehat{S}\right),
\end{aligned} \tag{A.7}
$$

and

$$
\begin{aligned}
B &= \int_{-\infty}^{0} d\xi_1 \int_{-\infty}^{+\infty} dt e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta(t)-\widehat{S}\right) \\
&\quad + \int_0^{+\infty} d\xi_1 \int_{-\infty}^{+\infty} dt e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta(t)+\xi_1-\widehat{S}\right).
\end{aligned} \tag{A.8}
$$

Recalling that type one ξ_1 and type two ξ_2 noise trades are the fractions of the total noise demand ξ, so that $\sigma_{\xi_1}^2 = (1-k)\,\sigma_\xi^2$ and $\sigma_{\xi_1}^2 = k\sigma_\xi^2$, where k is a fraction of the noise variance produced by the traders with unlimited inventory, we obtain

$$
e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} = e^{-\frac{t^2}{2}} e^{-\frac{(\xi_1+(1-k)t)^2}{2k(1-k)}}. \tag{A.9}
$$

Making use of this result and integrating, we obtain

$$
\begin{aligned}
A &= \widehat{S}e^{-\frac{\widehat{S}^2}{2}} \int_{-\infty}^{0} d\xi_1 e^{-\frac{\left(\xi_1+(1-k)\widehat{S}\right)^2}{2k(1-k)}} + \int_{-\infty}^{\widehat{S}} dt t e^{-\frac{t^2}{2}} e^{-\frac{\left(\widehat{S}-t\theta(t)+(1-k)t\right)^2}{2k(1-k)}} \\
&= \widehat{S}\sqrt{2\pi}\sqrt{k(1-k)}\Phi\left(\widehat{S}\sqrt{\frac{1-k}{k}}\right) e^{-\frac{\widehat{S}^2}{2}} \\
&\quad + \int_{-\infty}^{0} dt t e^{-\frac{t^2}{2}} e^{-\frac{\left(\widehat{S}+(1-k)t\right)^2}{2k(1-k)}} + \int_0^{\widehat{S}} dt t e^{-\frac{t^2}{2}} e^{-\frac{\left(\widehat{S}-kt\right)^2}{2k(1-k)}},
\end{aligned} \tag{A.10}
$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} d\eta \exp\left(-\frac{\eta^2}{2}\right)$ is a Normal c.d.f.

Similarly,

$$
\begin{aligned}
B &= e^{-\frac{\widehat{S}^2}{2}} \int_{-\infty}^{0} d\xi_1 e^{-\frac{\left(\xi_1+(1-k)\widehat{S}\right)^2}{2k(1-k)}} + \int_{-\infty}^{\widehat{S}} dt e^{-\frac{t^2}{2}} e^{-\frac{\left(\widehat{S}-t\theta(t)+(1-k)t\right)^2}{2k(1-k)}} \\
&= \sqrt{2\pi}\sqrt{k(1-k)}\Phi\left(\widehat{S}\sqrt{\frac{1-k}{k}}\right) e^{-\frac{\widehat{S}^2}{2}} \qquad \text{(A.11)} \\
&\quad + \int_{-\infty}^{0} dt e^{-\frac{t^2}{2}} e^{-\frac{\left(\widehat{S}+(1-k)t\right)^2}{2k(1-k)}} + \int_{0}^{\widehat{S}} dt e^{-\frac{t^2}{2}} e^{-\frac{\left(\widehat{S}-kt\right)^2}{2k(1-k)}}.
\end{aligned}
$$

Performing integration in (A.10) and (A.11), introducing a dimensionless variable $x = \widehat{S} = \frac{S}{\sigma_u}$ and removing common factors yields:

$$
\Lambda(x) = \frac{J_1(x) + J_2(x) + J_3(x)}{I_1(x) + I_2(x) + I_3(x)}, \qquad \text{(A.12)}
$$

with

$$
\begin{aligned}
J_1(x) &= x\left(e^{-\frac{x^2}{2}}\Phi\left(x\sqrt{\frac{1-k}{k}}\right) - \frac{1}{\sqrt{1-k}} e^{-\frac{x^2}{2(1-k)}}\Phi\left(x\sqrt{\frac{1}{k}}\right)\right), \\
J_2(x) &= \frac{1}{\sqrt{2\pi}}\left(\left(\sqrt{\frac{1-k}{k}} - \sqrt{\frac{k}{1-k}}\right) e^{-\frac{x^2}{2k(1-k)}} - \sqrt{\frac{1-k}{k}} e^{-\frac{x^2}{2k}}\right), \qquad \text{(A.13)} \\
J_3(x) &= x\frac{1}{\sqrt{k}} e^{-\frac{x^2}{2k}}\left(\Phi\left(x\sqrt{\frac{1}{1-k}}\right) - \frac{1}{2}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
I_1(x) &= e^{-\frac{x^2}{2}}\Phi\left(x\sqrt{\frac{1-k}{k}}\right), \\
I_2(x) &= \frac{1}{\sqrt{1-k}} e^{-\frac{x^2}{2(1-k)}}\Phi\left(x\sqrt{\frac{1}{k}}\right), \qquad \text{(A.14)} \\
I_3(x) &= \frac{1}{\sqrt{k}} e^{-\frac{x^2}{2k}}\left(\Phi\left(x\sqrt{\frac{1}{1-k}}\right) - \frac{1}{2}\right).
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
\Lambda(x) &= x\frac{1-\gamma(x)}{1+\gamma(x)} + \Gamma(x), \qquad \text{(A.15)} \\
\Gamma(x) &= \frac{1}{\sqrt{2\pi}} \frac{\gamma(x)}{1+\gamma(x)} \frac{(1-2k)e^{-\frac{x^2}{2k}} - (1-k)e^{-\frac{x^2(1-2k)}{2k(1-k)}}}{\sqrt{k}\Phi\left(x\sqrt{\frac{1}{k}}\right)},
\end{aligned}
$$

with

$$\gamma(x)=\frac{\frac{1}{\sqrt{1-k}}e^{-\frac{x^2}{2}\frac{k}{1-k}}\frac{\Phi\left(x\sqrt{\frac{1}{k}}\right)}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}}{1+\frac{1}{\sqrt{k}}e^{-\frac{x^2}{2}\frac{1-k}{k}}\frac{\left(\Phi\left(x\sqrt{\frac{1}{1-k}}\right)-\frac{1}{2}\right)}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}}. \tag{A.16}$$

Recall that $\sigma_\xi \Lambda\left(\frac{S}{\sigma_u}\right)$ is a negative correction to manager's optimal valuation $\widehat{v}_M$ due to the observation of short interest S. Note that Λ depends only on a dimensionless short interest parameter $x=\frac{S}{\sigma_u}$ which is proportional to the ratio of the short interest to its average value. Therefore, the dependence $\Lambda(x)$ is "universal" depending on a single parameter k, which is a fraction of unconstrained liquidity traders.

The function $\Lambda(x)$ is presented in Figure 1. As one can see, $\Lambda(x)$ is defined and monotonically increasing for $x \geq 0$. It is also convex apart from a region of very small x. One can show this analyzing (A.15) as a function of x. As it follows from Figure 4, the average $E_x[\Lambda'(x)] > 0$, and $E_x[\Lambda''(x)] > 0$. Note that $E_x[\Lambda''(x)]$ is increasing in the fraction k of unconstrained liquidity traders.

□

Proof of Corollary 1

In the limit when the fraction of the unconstrained noise traders is small, $k << 1$, we have $\Phi\left(x\sqrt{\frac{1-k}{k}}\right) \simeq \Phi\left(x\sqrt{\frac{1}{k}}\right) \to 1$, $\frac{1}{\sqrt{k}}e^{-\frac{x^2}{2}\frac{1-k}{k}} \to 0$, and (A.15) yields

$$\Lambda(x) \simeq x\frac{1-\gamma_a(x)}{1+\gamma_a(x)}-\sqrt{\frac{k}{2\pi}}\frac{\gamma_a(x)}{1+\gamma_a(x)}, \tag{A.17}$$

with

$$\gamma_a(x)=\frac{1}{\sqrt{1-k}}e^{-\frac{x^2}{2}\frac{k}{1-k}}. \tag{A.18}$$

A comparison of exact and asymptotic results is presented in Figure 1. The approximation (A.17) works remarkably well for sufficiently small fraction k, even when $k=0.2$.

□

Derivation of Equation 12

$$\frac{\partial \widehat{v}_M}{\partial S}=-\sigma_\xi\frac{\partial \mathit{\Lambda}\left(\frac{S}{\sigma_u}\right)}{\partial S}=-\frac{\alpha\sigma_s^2\sigma_u}{\sigma_u}\frac{\partial \mathit{\Lambda}(x)}{\partial x}=-\alpha\sigma_s^2 F\left(\frac{S}{\sigma_u}\right), \tag{A.19}$$

where $F(x)=\frac{\partial \mathit{\Lambda}(x)}{\partial x}$. Below we show that $\frac{\partial \mathit{\Lambda}(x)}{\partial x} \geq 0$.

□

Proof of Predictions 1 and 3

We need to prove that $\frac{\partial \Lambda(x)}{\partial x} > 0$ (prediction 1) and $\frac{\partial^2 \Lambda(x)}{\partial x^2} > 0$ (prediction 3).

Differentiating (A.15), we obtain

$$\begin{aligned}
\frac{\partial\Lambda(x)}{\partial x} &= \frac{1-\gamma(x)}{1+\gamma(x)} + 2x^2\frac{k}{1-k}\frac{\gamma(x)}{(1+\gamma(x))^2} \\
&+\sqrt{\frac{k}{2\pi}}\frac{\gamma(x)}{(1+\gamma(x))^2}\frac{1}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}\left(\gamma(x)\frac{\Phi'\left(x\sqrt{\frac{1-k}{k}}\right)}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}+\frac{\Phi'\left(x\sqrt{\frac{1}{k}}\right)}{\Phi\left(x\sqrt{\frac{1}{k}}\right)}\right) \\
&+x\frac{\gamma(x)}{(1+\gamma(x))^2}\sqrt{\frac{k}{2\pi}}\frac{k}{1-k}\frac{1}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)} \\
&+x\frac{2\gamma(x)}{(1+\gamma(x))^2}\left(\frac{\Phi'\left(x\sqrt{\frac{1-k}{k}}\right)}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}-\frac{\Phi'\left(x\sqrt{\frac{1}{k}}\right)}{\Phi\left(x\sqrt{\frac{1}{k}}\right)}\right).
\end{aligned} \tag{A.20}$$

Note that the function $\Psi(z)=\frac{\partial}{\partial z}\ln(\Phi(z))$ is monotonically increasing, $\Psi'(z)\geq 0$, and concave, $\Psi''(z)\leq 0$. Therefore, $\frac{\Phi'\left(x\sqrt{\frac{1-k}{k}}\right)}{\Phi\left(x\sqrt{\frac{1-k}{k}}\right)}\geq\frac{\Phi'\left(x\sqrt{\frac{1}{k}}\right)}{\Phi\left(x\sqrt{\frac{1}{k}}\right)}$, and the last term on the r.h.s. of (A.20) is non-negative. Next, consider the limit of small x. In the limit $x\to 0$, the first and the third terms on the r.h.s. yield

$$\begin{aligned}
\frac{\partial\Lambda(x)}{\partial x} &\simeq \frac{1}{(1+\gamma(x))^2}\left(1-\gamma^2(x)\left(1-\sqrt{\frac{k}{2\pi}}\frac{\Phi'\left(x\sqrt{\frac{1-k}{k}}\right)}{\Phi^2\left(x\sqrt{\frac{1-k}{k}}\right)}\right)\right) \\
&\simeq \frac{1}{\left(1+\frac{1}{\sqrt{1-k}}\right)^2}\left(1-\frac{1}{1-k}\left(1-\sqrt{\frac{k}{2\pi}}\frac{4}{\sqrt{2\pi}}\sqrt{\frac{1-k}{k}}\right)\right) \\
&= \frac{\frac{1}{1-k}}{\left(1+\frac{1}{\sqrt{1-k}}\right)^2}\left(\frac{4}{2\pi}\sqrt{1-k}-k\right).
\end{aligned} \tag{A.21}$$

Because $k\in[0;1]$, $\frac{\partial\Lambda(x)}{\partial x}\geq 0$ if $\frac{4}{2\pi}\sqrt{1-k}-k\geq 0$. The last inequality holds for $k\in[0;k_m]$ with $k_m=\frac{4}{\pi}\left(\sqrt{1+\frac{\pi}{2}}-1\right)\approx 0.768$. When x is not small, we have 3 additional terms (second, fourth, and fifth) that are non-negative for $x\geq 0$, and hence the derivative remains positive.

Now consider the limit of small k, $k<<1$, and sufficiently large x, $x\geq\sqrt{k}$. Differentiating (A.17) with (A.18), we obtain

$$\frac{\gamma_a'(x)}{\gamma_a(x)}=-x\frac{k}{1-k}, \tag{A.22}$$

and

$$\Lambda'(x) = \frac{1-\gamma_a(x)}{1+\gamma_a(x)} - x^2\frac{k}{1-k}\gamma_a(x)\left(-\frac{1}{1+\gamma_a(x)}\left(1+\frac{1-\gamma_a(x)}{1+\gamma_a(x)}\right)\right) + x\frac{k}{1-k}\gamma_a(x)\sqrt{\frac{k}{2\pi}}\left(\frac{1}{1+\gamma_a(x)}\left(1-\frac{\gamma_a(x)}{1+\gamma_a(x)}\right)\right) = \frac{1-\gamma_a^2(x)+2x^2\frac{k}{1-k}\gamma_a^2(x)+x\sqrt{\frac{k}{2\pi}}\frac{k}{1-k}\gamma_a(x)}{(1+\gamma_a(x))^2}. \quad (A.23)$$

The second derivative is given by

$$\Lambda''(x) = \frac{4x\frac{k}{1-k}\gamma_a^2(x)+\sqrt{\frac{k}{2\pi}}\frac{k}{1-k}\gamma_a(x)}{(1+\gamma_a(x))^2} + x\frac{k}{1-k}\frac{2\left(1-2x^2\frac{k}{1-k}\right)\gamma_a^2(x)-x\sqrt{\frac{k}{2\pi}}\frac{k}{1-k}\gamma_a(x)}{(1+\gamma_a(x))^2} + 2x\frac{k}{1-k}\frac{1-\gamma_a^2(x)+2x^2\frac{k}{1-k}\gamma_a^2(x)+x\sqrt{\frac{k}{2\pi}}\frac{k}{1-k}\gamma_a(x)}{(1+\gamma_a(x))^3}. \quad (A.24)$$

Regrouping terms yields

$$\Lambda''(x) = \frac{\sqrt{\frac{k}{2\pi}}\frac{k}{1-k}\gamma_a(x)\left(1-x^2\frac{k}{1-k}\right)}{(1+\gamma_a(x))^2} + x\frac{k}{1-k}\frac{2\left(1-2x^2\frac{k}{1-k}\right)\gamma_a^2(x)}{(1+\gamma_a(x))^2} + 2x\frac{k}{1-k}\frac{1+2x^2\frac{k}{1-k}\gamma_a^2(x)+x\sqrt{\frac{k}{2\pi}}\frac{k}{1-k}\gamma_a(x)}{(1+\gamma_a(x))^3} + 2x\frac{k}{1-k}\frac{\gamma_a^2(x)\left(2\left(1+\gamma_a(x)\right)-1\right)}{(1+\gamma_a(x))^3}. \quad (A.25)$$

It follows from (A.25), that as long as $x \leq x_m = \frac{1}{2}\sqrt{\frac{1-k}{k}}$, all terms on the right hand side of (A.25) are non-negative, and hence $\Lambda''(x) \geq 0$. Note that in the limit $k << 1$, $x_m \to +\infty$, and $x \leq x_m$ always holds. Numerical simulations confirm that on average $\Lambda(x)$ is also convex for arbitrary values of k.

□

Appendix B. Variable definitions

Variable	Definition
Short interest scaled by shares (SI/shares)	The number of all open short positions on the last business day on or before the 15th of each calendar month (Compustat – item SHORTINTS) scaled by the number of shares outstanding at the end of the month (CRSP – item SHROUT).
Abnormal short interest	The unexpected component of short interest, as in Karpoff and Lou (2010). At the beginning of each month, we sort stocks independently into tercile portfolios by size, book-to-market, momentum (measured by past six month return), turnover (all from CRSP), and institutional ownership (13F), all measured at the end of the prior month [quarter for quarterly data]. We then run monthly cross-sectional regressions of short interest on tercile dummy variables corresponding to those portfolios as well as industry (defined at a SIC 2-digit level) fixed-effects. We use the residuals from these cross-sectional regressions as the measure of abnormal short interest.
Days-to-cover	*SI/shares* scaled by the same month's average daily share turnover (CRSP). Following Hong et al. (2015), we adjust trading volume of NASDAQ stocks during the $2001-2003$ period in our calculations of turnover. See footnote 8 of their paper for more details.
CAPEX/PPE	The ratio of capital expenditures (CAPX) in year t to fixed assets (PPENT) at the end of year t-1, all from Compustat.
(Capex+R&D)/AT	The sum of capital expenditures (CAPX) and R&D (XRD) divided by the beginning-of-the-year total assets (AT), all from Compustat. Values of zero are assigned to observations with missing R&D values.
Δ*(AT)*	The percentage change in total assets (AT) between years t-1 and t (Compustat).
MB	The total book value of assets (AT) plus market cap (the product of the number of common shares outstanding (CSHO) and share price at the end of the fiscal year-end, PRCC_F), minus book value of equity (CEQ), all divided by the total book value of assets, all from Compustat.
CF	The sum of income before extraordinary items (variable IB) and depreciation and amortization (variable DP) divided by the beginning-of-the-year total book value of assets AT), all from Compustat.
log(Assets)	The natural logarithm of the total book value of assets (Compustat – item AT). No winsorizing applied.
High disper	An indicator variable that takes on a value of one if analyst dispersion is above sample median in a given year, and zero otherwise. Analyst forecast dispersion, obtained from I/B/E/S, is the monthly calculation of standard deviation of the end-of-the-year EPS forecasts across all analysts, scaled by the mean forecast. The resulting ratios are then averaged within the fiscal year to obtain annual measures of dispersion.
High total vol	An indicator variable that takes on a value of one if total return volatility is above sample median in a given year, and zero otherwise. Total volatility is measured as the standard deviation of daily stock returns in a given month, which is then averaged over the fiscal year (CRSP).

High Ivol	An indicator variable that takes on a value of one if idiosyncratic return volatility is above sample median in a given year, and zero otherwise. Idiosyncratic volatility is the standard deviation of the residuals from daily regression of stock returns net of the risk-free rate on the Fama-French three factors (CRSP). The regressions are estimated monthly, and the resulting standard deviations of the residuals are averaged over the fiscal year.
High inst hold	An indicator variable that takes on a value of one if institutional holding is above sample median in a given year, and zero otherwise. Institutional ownership is the fraction of shares held by all institutional investors, obtained from 13F divided by total shares outstanding, and is measured using the most recent quarter prior to the fiscal year-end.
SHO	An indicator variable that takes on a value of one if the stock was in the sample of pilot stocks and was subject to the Reg SHO (which started in June 2004) for at least 6 months of its fiscal year starting from year 2004 and onward, and zero otherwise. SHO also takes on a value of one if the stock was in the sample of non-pilot stocks (but part of Russell 3000 index) for at least 6 months of its fiscal year starting from year 2007 and onward, and zero otherwise.
Ret	Average monthly return during the fiscal year t (from CRSP).
High BSW	An indicator variable that takes on a value of one if BSW index is above sample median in a given year, and zero otherwise. BSW index is the index of equity dependence, based on Baker, Stein, Wurgler (2003). It is calculated as -1.002*CF_BSW - 39.368*Div- 1.315*$Cash$+ 3.139*Lev. The elements of the BSW are all from Compustat and calculated in the following way: CF_BSW=(DP+IB)/Total assets (AT) as of t-1; Div=(DVP+DVC)/ Total assets (AT) as of t-1; $Cash$=CHE/ Total assets (AT) as of t-1; Lev= (DLTT+DLC)/(DLTT+DLC+SEQ). Each element of the index is winsorized at 1% and 99% before combining into the aggregate BSW measure.
High HP index	An indicator variable that takes on a value of one if *HP index* is above sample median in a given year, and zero otherwise. Following Hadlock and Pierce (2010), *HP index* is calculated as 0.737**log(Assets)* +0.043*$log(Assets)^2$-0.040**Age*. *log(Assets)* is the total value of assets (Compustat – AT), in constant dollars of 2004, winsorized at 0 and $4,500M. *Age* is the number of years since the firm has first appeared in the Compustat database. The variable is winsorized at 0 and 37.
High repurchase	An indicator variable that equals one if the purchase of common and preferred stock (Compustat - item PRSTKC), scaled by lagged assets is above median, and zero otherwise.

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Figure 1: The effect of short interest on the best estimate of the fundamental value of the firm for different values of the fraction of unconstrained liquidity traders

This figure displays the relative difference between the price and the manager's best estimate of the fundamental value $\frac{P-E\left[v|P,\widehat{S}\right]}{P} = \mu\Lambda\left(\widehat{S}\right)$ as a function of scaled short interest $\widehat{S} = \frac{S}{\sigma_u}$, for different values of the fraction of unconstrained liquidity traders, k. $\mu = \frac{\sigma_\xi}{P} = 0.1$. The green line corresponds to $k = 0.1$, the blue line corresponds to $k = 0.2$, and the black line corresponds to $k = 0.4$.

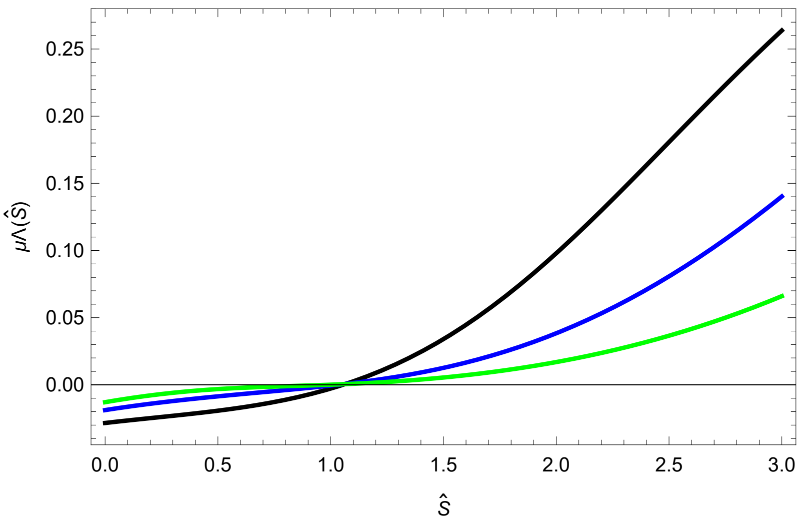

Figure 2: The effect of short interest on the best estimate for different noise levels

This figure displays the relative difference between the price and the manager's best estimate of the fundamental value $\frac{P-E\left[v|P,\widehat{S}\right]}{P} = \mu\Lambda\left(\widehat{S}\right)$ as a function of scaled short interest $\widehat{S} = \frac{S}{\sigma_u}$, for different values of the noise-to-price ratio $\mu = \frac{\sigma_\xi}{P}$. The green line corresponds to $\mu = 5\%$, the black line $\mu = 10\%$, and red line $\mu = 15\%$. The fraction of unconstrained liquidity traders $k = 0.4$.

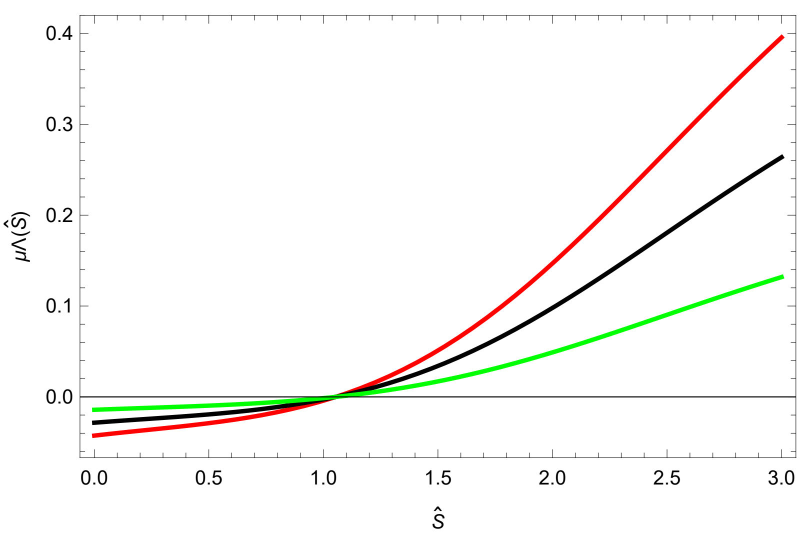

Figure 3: The effect of short interest on the best estimate, approximation

This figure displays the difference between the price and the manager's best estimate of the fundamental value $\Lambda\left(\widehat{S}\right) = P - E\left[v|P,\widehat{S}\right]$ as a function of scaled short interest $\widehat{S} = \frac{S}{\sigma_u}$, for the share of unconstrained liquidity traders $k = 0.2$. The blue line provides its exact value, given by (10), while the yellow provides its approximate value, derived in the asymptotic case of $k << 1$, and given by (13).

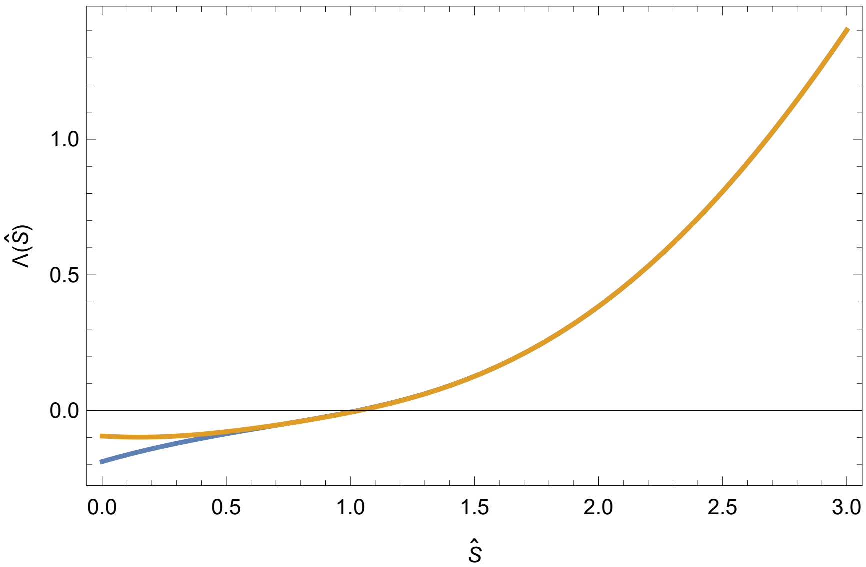

Figure 4: The average sensitivity of manager's best estimate and its elasticity w.r.t. short interest

This figure displays the average value of the second derivative of $\Lambda\left(\widehat{S}\right)$, namely, $E_{\widehat{S}}\left[\Lambda''\left(\widehat{S}\right)\right]$, where the expectation is performed w.r.t. different realizations of the short interest $\widehat{S}$, as a function of the share of unconstrained liquidity traders k.

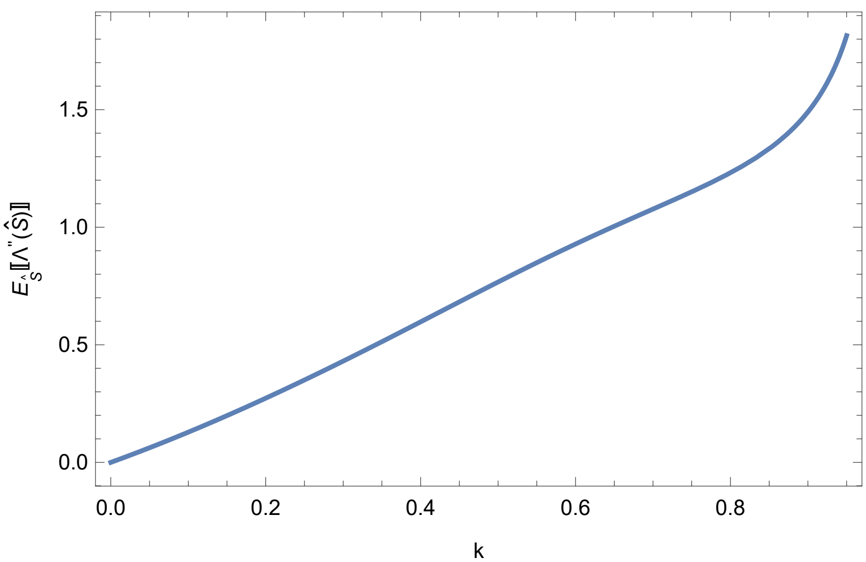

Table 1: Descriptive statistics

This table reports the distribution of short interest and investment measures, as well as control variables, over the period $1974 - 2018$. See Section 3 for sample description. *SI/Shares* is the monthly ratio of short interest (shares held short on the 15th business day of each month) scaled by total shares. *Days-to-cover* is short interest scaled by volume, constructed as described in Hong et al. (2015). *Abnormal SI* is scaled short interest (*SI/Shares*) net of expected short interest based on the benchmark from Karpoff and Lou (2010) that reflects the firm's characteristics (see Section 3 for details). All short interest measures are calculated monthly and averaged over the fiscal year period. *Capex/PPE* is capital expenditures in year t (variable CAPX) scaled by total net plant, property, and equipment (variable PPENT) at $t-1$. *(Capex+R&D)/AT* is capital and R&D (variable XRD) expenditures in year t scaled by the total book value of assets (variable AT) at $t-1$; values of zero are assigned to missing R&D values. $\Delta(AT)$ is the percentage change in the total book value of assets between years $t-1$ and t. *M/B* is market-to-book ratio, defined as the total book value of assets plus market cap (the product of the number of common shares outstanding (CSHO) and share price (PRCC_F) at the end of the fiscal year-end), minus book value of equity (CEQ), all divided by the total book value of assets. *CF* is the sum of income before extraordinary items (variable IB) and depreciation and amortization (variable DP), all divided by the total book value of assets at $t-1$. *Ret* is stock monthly returns, averaged over the fiscal year. *Analyst dispersion* is monthly analyst forecast estimates' dispersion (scaled by mean estimate), averaged over the fiscal year. *St. dev (Ret)* is the standard deviation of daily return within a month, averaged over the fiscal year. *Idiosync. vol* is the standard deviation of the residuals from daily regression of stock returns net of the risk-free rate on the Fama-French three factors. *Inst. hold* is the fraction of shares held by institutions out of the total shares outstanding, measured from the most recent quarter prior to the fiscal year-end.

	N	Mean	Median	25th Pctl	75th Pctl	St. Dev
Main dependent and independent variables						
SI/shares (%)	83,977	2.78	0.92	0.10	3.58	4.28
Days-to-cover	83,854	4.67	2.80	0.77	6.39	5.62
Abnormal SI (%)	74,486	0.00	-0.30	-1.42	0.20	3.59
Capex/PPE (%)	83,728	30.21	20.83	12.26	35.24	33.17
(Capex+R&D)/AT (%)	83,977	11.45	7.43	3.63	13.91	13.08
Δ(AT) (%)	83,977	10.58	5.37	-3.68	16.52	33.74
Control variables						
M/B	83,959	1.85	1.39	1.05	2.06	1.42
CF (%)	83,929	3.79	8.49	2.50	13.57	21.73
log(Assets)	83,977	5.84	5.77	4.35	7.25	2.04
Ret (%)	83,839	1.06	1.09	-1.14	3.26	4.37
Analyst dispersion	42,533	0.19	0.06	0.03	0.14	0.43
St. dev (Ret) (%)	83,884	2.97	2.56	1.86	3.62	1.58
Idiosync. Vol (%)	75,316	2.53	2.11	1.49	3.10	1.52
Inst. hold (%)	83,977	43.36	10.11	42.24	72.78	33.18

Table 2: Short interest and and future cash flows

This table reports estimates from regressing future cash flows on short interest proxies and control variables over the period $1974-2018$. The short interest proxies are *SI/shares* in Panel A, *Days-to-cover* in Panel B, and *Abnormal SI* in Panel C. See Section 3 and Table 1 for sample description and variables construction. All independent variables are from period $t-1$. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	CF_t	CF_{t+1}	CF_{t+2}	CF_{t+3}
Panel A: Short interest scaled by shares				
SI/shares	-0.214***	-0.126***	-0.078**	-0.039
	(0.035)	(0.032)	(0.032)	(0.037)
M/B	1.064***	1.104***	0.824***	0.556***
	(0.204)	(0.172)	(0.157)	(0.131)
log(Assets)	1.173***	-0.685**	-1.237***	-1.611***
	(0.355)	(0.281)	(0.265)	(0.240)
N	83911	75392	67828	61177
adj. R^2	0.69	0.68	0.67	0.66
Panel B: Days-to-cover				
Days-to-cover	-0.149***	-0.087***	-0.041**	-0.022
	(0.019)	(0.016)	(0.017)	(0.014)
M/B	1.057***	1.084***	0.793***	0.538***
	(0.206)	(0.175)	(0.154)	(0.129)
log(Assets)	1.056***	-0.770***	-1.300***	-1.626***
	(0.344)	(0.279)	(0.264)	(0.236)
N	83790	75278	67719	61067
adj. R^2	0.69	0.69	0.67	0.66
Panel C: Abnormal short interest				
Abnormal SI	-0.165***	-0.110***	-0.075**	-0.049
	(0.029)	(0.029)	(0.031)	(0.033)
M/B	1.346***	1.175***	0.891***	0.629***
	(0.240)	(0.198)	(0.172)	(0.133)
log(Assets)	0.605*	-0.799***	-1.298***	-1.574***
	(0.310)	(0.271)	(0.262)	(0.228)
N	74442	69180	62492	56665
adj. R^2	0.66	0.66	0.65	0.64

Table 3: Short interest and investment - baseline results

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period $1974-2018$. The short interest proxies are *SI/shares* in Panel A, *Days-to-cover* in Panel B, and *Abnormal SI* in Panel C. See Section 3 and Table 1 for sample description and variables construction. All control variables, except *CF*, are from period $t-1$. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.321***	-0.105***	-0.321***
	(0.067)	(0.018)	(0.064)
M/B	6.454***	2.115***	8.774***
	(0.273)	(0.086)	(0.382)
CF	0.152***	-0.063***	0.321***
	(0.027)	(0.014)	(0.046)
log(Assets)	-3.639***	-2.408***	-13.361***
	(0.392)	(0.185)	(0.754)
N	83664	83911	83911
adj. R^2	0.33	0.71	0.23
Panel B: Days-to-cover			
Days-to-cover	-0.286***	-0.029***	-0.181***
	(0.049)	(0.010)	(0.039)
M/B	6.464***	2.110***	8.756***
	(0.275)	(0.088)	(0.383)
CF	0.147***	-0.061***	0.321***
	(0.027)	(0.014)	(0.046)
log(Assets)	-3.791***	-2.458***	-13.495***
	(0.384)	(0.191)	(0.763)
N	83550	83790	83790
adj. R^2	0.33	0.71	0.23
Panel C: Abnormal short interest			
Abnormal SI	-0.139***	-0.075***	-0.293***
	(0.047)	(0.017)	(0.054)
M/B	5.879***	2.205***	8.506***
	(0.279)	(0.094)	(0.410)
CF	0.160***	-0.045***	0.389***
	(0.027)	(0.014)	(0.048)
log(Assets)	-3.454***	-2.195***	-11.962***
	(0.370)	(0.194)	(0.759)
N	74320	74442	74442
adj. R^2	0.31	0.69	0.21

Table 4: Short interest and investment - high versus low short interest

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period 1974−2018. The short interest (*SI*) proxies are *SI/shares* in Panel A, *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. See Section 3 and Table 1 for sample description and variables construction. Each measure of short interest is split into two measures of high versus low short interest. *HighSI* takes on the value of *SI* if it is above the sample median in a given year, and zero otherwise. *LowSI* takes on the value of *SI* if it is below or equal to the sample median in a given year, and zero otherwise. *HighInd* is an indicator variable that takes on a value of one if *SI* is above the sample median in a given year, and zero otherwise (Panel A). All control variables, except *CF*, are from the period $t-1$. For the sake of brevity, coefficients on control variables are not presented. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
	Panel A: Short interest scaled by shares		
HighSI	-0.348***	-0.126***	-0.327***
	(0.067)	(0.019)	(0.065)
LowSI	-0.024	0.089	0.308
	(0.280)	(0.057)	(0.266)
HighInd	0.815*	0.598***	0.817**
	(0.469)	(0.118)	(0.398)
Controls	Yes	Yes	Yes
N	68974	69100	69100
adj. R^2	0.328	0.691	0.229
	Panel B: Days-to-cover		
HighSI	-0.274***	-0.040***	-0.162***
	(0.046)	(0.011)	(0.041)
LowSI	0.012	0.031	0.112
	(0.192)	(0.043)	(0.128)
HighInd	0.224	0.336***	0.073
	(0.485)	(0.123)	(0.437)
Controls	Yes	Yes	Yes
N	83550	83790	83790
adj. R^2	0.33	0.71	0.23
	Panel C: Abnormal short interest		
HighSI	-0.112*	-0.099***	-0.335***
	(0.060)	(0.022)	(0.073)
LowSI	-0.236*	0.042	-0.286*
	(0.118)	(0.045)	(0.159)
HighInd	0.003	-0.110	0.452
	(0.238)	(0.103)	(0.368)
Controls	Yes	Yes	Yes
N	74320	74442	74442
adj. R^2	0.31	0.69	0.22

Table 5: Short interest and investment - interaction with analyst forecast dispersion

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period 1977 − 2018. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *High disper* is a dummy variable that takes on a value of one if *Analyst dispersion* is above sample median in a given year, and zero otherwise. See Section 3 and Table 1 for sample description and variables construction. All control variables, except *CF*, are from the period $t-1$. For the sake of brevity, coefficients on control variables are not presented. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
	Panel A: Short interest scaled by shares		
SI/shares	-0.183***	-0.069***	-0.212**
	(0.063)	(0.018)	(0.087)
SI/shares*High disper	-0.264***	-0.045*	-0.133*
	(0.077)	(0.022)	(0.071)
High disper	-2.044***	-1.119***	-2.766***
	(0.565)	(0.144)	(0.510)
Controls	Yes	Yes	Yes
N	42427	42468	42468
adj. R^2	0.39	0.76	0.25
	Panel B: Days-to-cover		
Days-to-cover (DTC)	-0.283***	-0.022	-0.112
	(0.051)	(0.015)	(0.080)
DTC*High disper	-0.150**	-0.011	-0.104
	(0.062)	(0.019)	(0.073)
High disper	-2.141***	-1.245***	-2.732***
	(0.545)	(0.146)	(0.559)
Controls	Yes	Yes	Yes
N	42422	42463	42463
adj. R^2	0.40	0.76	0.25
	Panel C: Abnormal short interest		
Abnormal SI (ASI)	0.012	-0.028	-0.106
	(0.073)	(0.019)	(0.084)
ASI*High disper	-0.279**	-0.083***	-0.173*
	(0.122)	(0.028)	(0.098)
High disper	-3.482***	-1.143***	-3.208***
	(0.373)	(0.116)	(0.481)
Controls	Yes	Yes	Yes
N	37725	37747	37747
adj. R^2	0.38	0.74	0.23

Table 6: Short interest and investment - interaction with total return volatility

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period 1974 − 2018. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *High total vol* is a dummy variable that takes on a value of one if St. Dev (Ret) is above sample median in a given year, and zero otherwise. See Section 3 and Table 1 for sample description and variables construction. All control variables, except *CF*, are from the period $t-1$. All control variables, except *CF*, are from the period $t-1$. For the sake of brevity, coefficients on control variables are not presented. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.186***	-0.054***	-0.071
	(0.052)	(0.016)	(0.066)
SI/shares*High total vol	-0.191***	-0.064***	-0.327***
	(0.065)	(0.019)	(0.075)
High total vol	-0.554	-0.635***	-2.304***
	(0.427)	(0.117)	(0.341)
Controls	Yes	Yes	Yes
N	83578	83818	83818
adj. R^2	0.33	0.71	0.23
Panel B: Days-to-cover			
Days-to-cover (DTC)	-0.162***	-0.021**	-0.041
	(0.034)	(0.010)	(0.042)
DTC*High total vol	-0.233***	-0.017	-0.269***
	(0.057)	(0.013)	(0.047)
High total vol	-0.298	-0.825***	-2.243***
	(0.460)	(0.120)	(0.355)
Controls	Yes	Yes	Yes
N	83549	83789	83789
adj. R^2	0.33	0.71	0.23
Panel C: Abnormal short interest			
Abnormal SI (ASI)	-0.039	-0.008	-0.054
	(0.037)	(0.019)	(0.066)
ASI*High total vol	-0.140*	-0.094***	-0.335***
	(0.073)	(0.021)	(0.088)
High total vol	-1.255***	-0.710***	-2.840***
	(0.292)	(0.092)	(0.338)
Controls	Yes	Yes	Yes
N	74319	74441	74441
adj. R^2	0.31	0.69	0.22

Table 7: Short interest and investment - interaction with idiosyncratic return volatility
This table reports estimates from regressions of investment measures on short interest proxies and control variables over the period 1974 − 2018. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *High Ivol* is a dummy variable that takes on a value of one if *Idiosync.vol* is above sample median in a given year, and zero otherwise. See Section 3 and Table 1 for sample description and variables construction. All control variables, except *CF*, are from the period $t-1$. For the sake of brevity, coefficients on control variables are not presented. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.166***	-0.050***	-0.051
	(0.054)	(0.016)	(0.066)
SI/shares*High Ivol	-0.266***	-0.070***	-0.349***
	(0.074)	(0.020)	(0.070)
High Ivol	-0.159	-0.780***	-2.708***
	(0.537)	(0.125)	(0.380)
Controls	Yes	Yes	Yes
N	75018	75251	75251
adj. R^2	0.34	0.72	0.23
Panel B: Days-to-cover			
Days-to-cover (DTC)	-0.140***	-0.023**	-0.035
	(0.033)	(0.010)	(0.045)
DTC*High Ivol	-0.297***	-0.015	-0.281***
	(0.064)	(0.014)	(0.053)
High Ivol	0.147	-1.022***	-2.656***
	(0.573)	(0.142)	(0.475)
Controls	Yes	Yes	Yes
N	75004	75237	75237
adj. R^2	0.34	0.72	0.23
Panel C: Abnormal short interest			
Abnormal SI (ASI)	-0.036	-0.013	-0.085
	(0.039)	(0.020)	(0.063)
ASI*High Ivol	-0.160**	-0.084***	-0.279***
	(0.068)	(0.024)	(0.075)
High Ivol	-1.465***	-0.895***	-3.284***
	(0.299)	(0.101)	(0.355)
Controls	Yes	Yes	Yes
N	66007	66123	66123
adj. R^2	0.32	0.71	0.22

Table 8: Short interest and investment - interaction with institutional holding

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period $1981-2018$. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *High inst hold* is a dummy variable that takes on a value of one if institutional holding is above sample median in a given year, and zero otherwise. See Section 3 and Table 1 for sample description and variables construction. All control variables, except *CF*, are from the period $t-1$. For the sake of brevity, coefficients on control variables are not presented. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.747***	-0.147***	-0.651***
	(0.117)	(0.030)	(0.103)
SI/shares*High inst hold	0.573***	0.041	0.442***
	(0.106)	(0.029)	(0.096)
High inst hold	0.580	1.242***	3.364***
	(0.589)	(0.182)	(0.585)
Controls	Yes	Yes	Yes
N	74865	75104	75104
adj. R^2	0.34	0.72	0.23
Panel B: Days-to-cover			
Days-to-cover (DTC)	-0.393***	-0.019	-0.248***
	(0.066)	(0.013)	(0.050)
DTC*High inst hold	0.212***	-0.032**	0.142**
	(0.056)	(0.016)	(0.062)
High inst hold	1.067*	1.422***	3.683***
	(0.570)	(0.175)	(0.604)
Controls	Yes	Yes	Yes
N	74772	75005	75005
adj. R^2	0.34	0.72	0.23
Panel C: Abnormal short interest			
Abnormal SI (ASI)	-0.432***	-0.143***	-0.603***
	(0.085)	(0.028)	(0.080)
ASI*High inst hold	0.414***	0.096***	0.454***
	(0.091)	(0.028)	(0.080)
High inst hold	2.786***	1.292***	4.395***
	(0.411)	(0.160)	(0.499)
Controls	Yes	Yes	Yes
N	64460	64574	64574
adj. R^2	0.32	0.71	0.22

Table 9: Short interest and investment - early versus late sub-period

This table reports estimates from regressing investment measures on short interest proxies and control variables over early (1974 – 1996) versus late (1997 – 2018) sample sub-period. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover* in Panel B, and *Abnormal SI (ASI)* in Panel C. See Section 3 and Table 1 for sample description and variables construction. All control variables, except *CF*, are from the period $t-1$. For the sake of brevity, coefficients on control variables are not presented. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered at firm and year levels. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	1974 – 1996			1997 – 2018		
	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)	(4) Capex/PPE	(5) (Capex+R&D)/AT	(6) Δ(AT)
	Panel A: Short interest scaled by shares					
SI/shares	-0.156	-0.086	-0.279	-0.411***	-0.110***	-0.237***
	(0.201)	(0.065)	(0.194)	(0.073)	(0.018)	(0.066)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	29976	30021	30021	53688	53890	53890
adj. R^2	0.32	0.58	0.29	0.34	0.76	0.25
	Panel B: Days-to-cover					
Days-to-cover	-0.072	-0.029	-0.119*	-0.424***	-0.030**	-0.161***
	(0.048)	(0.017)	(0.059)	(0.062)	(0.012)	(0.049)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	29929	29969	29969	53621	53821	53821
adj. R^2	0.32	0.58	0.29	0.34	0.76	0.25
	Panel C: Abnormal short interest					
Abnormal SI	-0.129	-0.078	-0.306	-0.196***	-0.090***	-0.269***
	(0.191)	(0.065)	(0.216)	(0.051)	(0.016)	(0.055)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	28417	28451	28451	45903	45991	45991
adj. R^2	0.29	0.57	0.27	0.33	0.75	0.23

Table 10: Short interest and future investment

This table reports results from regressing investment in years $t+1$, $t+2$, and $t+3$ on three measures of short interest. The sample period is $1974-2018$. The short interest measures are *SI/shares*, *Days-to-cover (DTC)*, and *Abnormal SI (ASI)*. For the sake of brevity, only coefficients on short interest measures are presented (that is, each cell in the table corresponds to a separate regression where the independent variables are the measure of short interest, as indication in the beginning of the row, and control variables). All control variables, except *CF*, are from the period $t-1$. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
	Panel A: Investment in t+1		
SI/shares	-0.323***	-0.113***	-0.358***
	(0.053)	(0.018)	(0.050)
Days-to-cover	-0.174***	-0.034***	-0.168***
	(0.039)	(0.011)	(0.037)
Abnormal SI	-0.099*	-0.054***	-0.271***
	(0.053)	(0.017)	(0.050)
	Panel B: Investment in t+2		
SI/shares	-0.199***	-0.099***	-0.252***
	(0.045)	(0.018)	(0.050)
Days-to-cover	-0.057*	-0.025**	-0.083***
	(0.028)	(0.010)	(0.027)
Abnormal SI	-0.024	-0.042**	-0.191***
	(0.045)	(0.017)	(0.054)
	Panel C: Investment in t+3		
SI/shares	-0.071	-0.074***	-0.172***
	(0.051)	(0.018)	(0.062)
Days-to-cover	-0.005	-0.005	0.012
	(0.028)	(0.009)	(0.026)
Abnormal SI	0.018	-0.030*	-0.149***
	(0.048)	(0.016)	(0.048)

Table 11: Short interest and investment - the impact of Reg SHO

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period of before and after Regulation SHO (2002 − 2008) for the sample of stocks included in Russell 3000 index (as of 2004). The short interest proxies are *SI/shares* in Panel A, *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *SHO* is a dummy variable that takes on a value of one if the stock was in the sample of pilot stocks and was subject to the Reg SHO (which started in June 2004) for at least 6 months of its fiscal year starting from year 2004 and onward, and zero otherwise. *SHO* also takes on a value of one if the stock was in the sample of non-pilot stocks (but part of Russell 3000 index) for at least 6 months of its fiscal year starting from year 2007 and onward, and zero otherwise. See Section 3 and Table 1 for sample description and variables construction. All control variables, except *CF*, are from the period $t-1$. For the sake of brevity, coefficients on control variables are not presented. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered at firm and year levels. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(Assets)
	Panel A: Short interest scaled by shares		
SI/shares	-0.138	-0.051	-0.286*
	(0.176)	(0.029)	(0.126)
SI/shares*SHO	-0.628***	-0.148***	-0.231
	(0.130)	(0.029)	(0.135)
SHO	4.201**	0.717***	1.039
	(1.336)	(0.168)	(1.508)
Controls	Yes	Yes	Yes
N	11432	11443	11443
adj. R^2	0.45	0.79	0.33
	Panel B: Days-to-cover		
Days-to-cover	-0.254*	-0.049*	-0.014
	(0.107)	(0.024)	(0.081)
Days-to-cover*SHO	-0.334**	-0.114***	-0.462***
	(0.118)	(0.029)	(0.103)
SHO	2.921*	0.662**	2.924
	(1.339)	(0.238)	(1.825)
Controls	Yes	Yes	Yes
N	11431	11442	11442
adj. R^2	0.45	0.79	0.33
	Panel C: Abnormal short interest		
Abnormal SI	-0.067	-0.036	-0.241
	(0.117)	(0.031)	(0.134)
Abnormal SI*SHO	-0.723***	-0.137***	-0.258
	(0.151)	(0.036)	(0.144)
SHO	0.821	-0.056	-0.196
	(0.954)	(0.153)	(1.727)
Controls	Yes	Yes	Yes
N	10962	10968	10968
adj. R^2	0.46	0.80	0.32

Table 12: Short interest and investment - baseline results with controls for stock return

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period $1974 - 2018$. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover* in Panel B, and *Abnormal SI* in Panel C. See Section 3 and Table 1 for sample description and variables construction. *Ret* is average monthly return during the fiscal year t. All control variables, except *CF* and *Ret*, are from the period $t-1$. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.296***	-0.092***	-0.208***
	(0.063)	(0.017)	(0.059)
Ret	0.343***	0.185***	1.624***
	(0.070)	(0.016)	(0.117)
Controls	Yes	Yes	Yes
N	83532	83773	83773
adj. R^2	0.33	0.71	0.26
Panel B: Days-to-cover			
Days-to-cover	-0.281***	-0.026***	-0.160***
	(0.048)	(0.010)	(0.037)
Ret	0.350***	0.189***	1.632***
Controls	Yes	Yes	Yes
N	83481	83721	83721
adj. R^2	0.33	0.71	0.26
Panel C: Abnormal short interest			
Abnormal SI	-0.129***	-0.069***	-0.243***
	(0.045)	(0.016)	(0.049)
Ret	0.278***	0.161***	1.454***
	(0.062)	(0.016)	(0.110)
Controls	Yes	Yes	Yes
N	74253	74375	74375
adj. R^2	0.31	0.69	0.24

Table 13: Short interest and investment - interaction with equity dependence index

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period $1974-2018$. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *BSW index* is the index of equity dependence, based on Baker, Stein, Wurgler (2003). *High BSW index* is a dummy variable that takes on a value of one if *BSW index* is above sample median in a given year, and zero otherwise. See Section 3 and Table 1 for sample description and other variables construction. All control variables, except *CF*, are from the period $t-1$. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.073	-0.103***	-0.283***
	(0.078)	(0.018)	(0.085)
SI/shares*High BSW index	-0.128	0.026	0.030
	(0.088)	(0.026)	(0.099)
High BSW index	-6.709***	-1.218***	-3.925***
	(0.420)	(0.159)	(0.461)
Controls	Yes	Yes	Yes
N	80242	80403	80403
adj. R^2	0.33	0.70	0.22
Panel B: Days-to-cover			
Days-to-cover (DTC)	-0.250***	-0.021	-0.182***
	(0.051)	(0.013)	(0.053)
DTC*High BSW index	0.119**	0.005	0.048
	(0.053)	(0.018)	(0.060)
High BSW index	-7.527***	-1.191***	-4.098***
	(0.475)	(0.150)	(0.500)
Controls	Yes	Yes	Yes
N	80141	80296	80296
adj. R^2	0.33	0.70	0.22
Panel C: Abnormal short interest			
Abnormal SI (ASI)	0.038	-0.037**	-0.235**
	(0.080)	(0.018)	(0.089)
ASI*High BSW index	-0.241**	-0.037	-0.045
	(0.103)	(0.026)	(0.104)
High BSW index	-6.356***	-1.204***	-3.687***
	(0.349)	(0.132)	(0.410)
Controls	Yes	Yes	Yes
N	73409	73522	73522
adj. R^2	0.32	0.69	0.22

Table 14: Short interest and investment - interaction with financial constraints index

This table reports estimates from regressing investment measures on short interest proxies and control variables over the period 1974 − 2018. The short interest proxies are *SI/shares* in Panel A; *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *HP index* is the index of financial constraints, constructed as in Hadlock and Pierce (2010). *High HP* is a dummy variable that takes on a value of one if *HP* is above sample median in a given year, and zero otherwise. See Section 3 and Table 1 for sample description and other variables construction. All control variables, except *CF*, are from the period $t-1$. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.121**	-0.105***	-0.186***
	(0.052)	(0.019)	(0.063)
SI/shares*High HP index	-0.466***	-0.001	-0.321***
	(0.104)	(0.027)	(0.101)
High HP index	2.418***	-0.562***	-0.172
	(0.545)	(0.189)	(0.578)
Controls	Yes	Yes	Yes
N	83664	83911	83911
adj. R^2	0.33	0.71	0.23
Panel B: Days-to-cover			
Days-to-cover (DTC)	-0.109***	-0.050***	-0.134***
	(0.033)	(0.013)	(0.047)
DTC*High HP index	-0.324***	0.039**	-0.088
	(0.067)	(0.018)	(0.062)
High HP index	2.706***	-0.748***	-0.559
	(0.591)	(0.201)	(0.587)
Controls	Yes	Yes	Yes
N	83550	83790	83790
adj. R^2	0.33	0.71	0.23
Panel C: Abnormal short interest			
Abnormal SI (ASI)	-0.117***	-0.063***	-0.221***
	(0.043)	(0.018)	(0.058)
ASI*High HP index	-0.044	-0.031	-0.174*
	(0.090)	(0.028)	(0.100)
High HP index	1.372***	-0.589***	-0.989*
	(0.456)	(0.169)	(0.575)
Controls	Yes	Yes	Yes
N	74320	74442	74442
adj. R^2	0.31	0.69	0.22

Table 15: Short interest and investment - accounting for share repurchases

This table reports estimates from regressing investment on three measures of short interest and control variables. The sample period is $1974 - 2018$. The short interest measures are *SI/shares* in Panel A; *Days-to-cover (DTC)* in Panel B, and *Abnormal SI (ASI)* in Panel C. *Repurchase* is the purchase of common and preferred stock (item PRSTKC), scaled by lagged assets. *High repurchase* is a dummy variable that equals one if *Repurchase* measure is above sample median in a given year, and equals zero otherwise. See Section 3 and Table 1 for sample description and other variables construction. All control variables, except *CF*, are from the period $t-1$. The regressions are estimated using the OLS model, and include firm and year fixed effects. Standard errors, reported in parentheses below coefficient estimates, are double clustered by firm and year. Significance at the 1%, 5%, and 10% level are indicated by ***, **, and *, respectively.

	(1) Capex/PPE	(2) (Capex+R&D)/AT	(3) Δ(AT)
Panel A: Short interest scaled by shares			
SI/shares	-0.227***	-0.107***	-0.336***
	(0.072)	(0.021)	(0.080)
SI/shares*High repurchase	0.050	0.048**	0.138*
	(0.059)	(0.020)	(0.079)
High repurchase	0.296	0.231**	0.772**
	(0.310)	(0.103)	(0.373)
Controls	Yes	Yes	Yes
N	65154	65290	65290
adj. R^2	0.34	0.71	0.23
Panel B: Days-to-cover			
Days-to-cover (DTC)	-0.217***	-0.006	-0.200***
	(0.046)	(0.012)	(0.047)
DTC*High repurchase	0.003	-0.019	0.096*
	(0.043)	(0.015)	(0.048)
High repurchase	0.422	0.495***	0.719*
	(0.323)	(0.122)	(0.370)
Controls	Yes	Yes	Yes
N	65087	65221	65221
adj. R^2	0.34	0.71	0.23
Panel C: Abnormal short interest			
Abnormal SI (ASI)	-0.178**	-0.093***	-0.336***
	(0.069)	(0.019)	(0.078)
ASI*High repurchase	0.096	0.071***	0.169**
	(0.073)	(0.019)	(0.077)
High repurchase	0.515*	0.343***	1.198 ***
	(0.264)	(0.087)	(0.304)
Controls	Yes	Yes	Yes
N	59007	59104	59104
adj. R^2	0.33	0.70	0.22

Internet Appendix to

"Short Interest and Investment"

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A version of the model with a private manager's signal

In this Section we extend our model to the case when the manager observes a private signal about the firm's value, $s_m = v + \sigma_m \epsilon_m$, where the error is normally distributed, $\epsilon_m \sim N(0,1)$ and is independent and uncorrelated with the liquidity traders' demand and the errors of informed traders' signals.

The manager's best estimate of the fundamental value of the firm is then given by Proposition 1A:

Proposition 1A *After observing her private signal s_m, short interest S, and execution price P, the manager's best estimate of the fundamental value is given by*

$$\widehat{v}_M\left(s_m, \widehat{S}, P\right) = P + (s_m - P)\frac{r}{q+r} - \sigma_\xi \frac{q}{q+r}\Lambda\left(\widehat{S}\right),$$

where $r = \frac{\sigma^2_{v|s}}{\sigma^2_\xi} = \frac{\sigma^2_P - \sigma^2_\xi}{\sigma^2_\xi}$, $q = \frac{\sigma^2_m}{\sigma^2_\xi}$, $\widehat{S} = \frac{S}{\sigma_u}$, *and*

$$\Lambda(x) = \frac{J_1(x) + J_2(x) + J_3(x) + J_4(x)}{I_1(x) + I_2(x) + I_3(x)}, \tag{IA.1}$$

with

$$\begin{aligned}
J_1(x) &= x e^{-\frac{r+q+1}{r+q}\frac{\left(x+\frac{\Delta}{r+q+1}\right)^2}{2}} \Phi\left(x\sqrt{\frac{1-k}{k}}\right), \\
J_2(x) &= -\frac{a}{\sqrt{\frac{(1-k)(k+r+q)}{r+q}}} e^{-\frac{\left(x-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} \Phi\left(\frac{a}{\sigma_a}\right), \\
J_3(x) &= \frac{x}{\sqrt{\frac{k(1-k+r+q)}{r+q}}} e^{-\frac{\left(x+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}} \left(\Phi\left(\frac{x-b}{\sigma_b}\right) - \Phi\left(\frac{-b}{\sigma_b}\right)\right), \\
J_4(x) &= -\frac{e^{-\frac{\left(x-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{a^2}{2\sigma_a^2}}}{\sqrt{2\pi\frac{1-k}{k}\frac{k+r+q}{r+q}}} + \frac{e^{-\frac{\left(x+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}}\left(e^{-\frac{(x-b)^2}{2\sigma_b^2}} - e^{-\frac{b^2}{2\sigma_b^2}}\right)}{\sqrt{2\pi\frac{k}{1-k}\frac{1-k+r+q}{r+q}}},
\end{aligned} \tag{IA.2}$$

and

$$
\begin{aligned}
I_1(x) &= e^{-\frac{r+q+1}{r+q}\frac{\left(x+\frac{\Delta}{r+q+1}\right)^2}{2}}\Phi\left(x\sqrt{\frac{1-k}{k}}\right), \\
I_2(x) &= \frac{e^{-\frac{\left(x-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}}}{\sqrt{\frac{(1-k)(k+r+q)}{r+q}}}\Phi\left(\frac{a}{\sigma_a}\right), \\
I_3(x) &= \frac{e^{-\frac{\left(x+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}}}{\sqrt{\frac{(1-k)(k+r+q)}{r+q}}}\left(\Phi\left(\frac{x-b}{\sigma_b}\right)-\Phi\left(\frac{-b}{\sigma_b}\right)\right).
\end{aligned} \tag{IA.3}
$$

With the notations $\Delta = \frac{s_m - P}{\sigma_\xi}$,

$$
\begin{aligned}
a &= \frac{(r+q)\,x+k\Delta}{k+r+q},\ \sigma_a^2 = \frac{k\,(r+q)}{k+r+q}, \\
b &= \frac{(1-k)\,(x+\Delta)}{1-k+r+q}\ \sigma_b^2 = \frac{(1-k)\,(r+q)}{1-k+r+q},
\end{aligned}
$$

and where $\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{z^2}{2}}dz$ *is a standard error function.*

Proof:

First we derive the p.d.f. of fundamental value conditional on manager's private signal, short interest and execution price, and then calculate the best estimate of the fundamental $\widehat{v}_M$ as a conditional expectation in manager's information set. Because the manager's and informed trader's estimation errors of the fundamental, $v - s_m$ and $v - s$, and noise traders' of types one and two demands ξ_k, $k = 1, 2$, are all i.i.d. and uncorrelated, their joint p.d.f. takes the form

$$
\begin{aligned}
f(v,\xi_1,\xi_2) &= C^{-1}e^{-\frac{(s_m-v)^2}{2\sigma_m^2}}e^{-\frac{(s-v)^2}{2\sigma_{v|s}^2}}e^{-\frac{\xi_1^2}{2(1-k)}}e^{-\frac{\xi_2^2}{2k}}, \\
C &= \sqrt{2\pi\sigma_m^2}\sqrt{2\pi\sigma_{v|s}^2}\sqrt{2\pi\,(1-k)}\sqrt{2\pi k},
\end{aligned} \tag{IA.4}
$$

where C is a normalization constant.

With the Dirac's delta function $\delta(x) = \theta'(x)$ and taking into account that market price and short interest do not depend on manager's private signal and are still defined, with $z = \sigma_\xi t = P - s$, by market clearing condition, $t + \xi_1 + \xi_2 = 0$, and $t\theta(t) + \xi_1\theta(\xi_1) = \widehat{S} = \frac{1}{\sigma_u}S$, respectively. Then we obtain a conditional p.d.f. $f\left(v|P,\widehat{S},s_m\right)$ in the form

$$f\left(v|P,\widehat{S},s_m\right) = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{(v-s_m)^2}{2\sigma_m^2}} e^{-\frac{\left(\frac{v-P}{\sigma_\xi}+t\right)^2}{2r}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta(t)+\xi_1\theta(\xi_1)-\widehat{S}\right) dtd\xi_1}{B}, \quad \text{(IA.5)}$$

$$B\left(P,\widehat{S},s_m\right) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{(v-s_m)^2}{2\sigma_m^2}} e^{-\frac{\left(\frac{v-P}{\sigma_\xi}+t\right)^2}{2r}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta(t)+\xi_1\theta(\xi_1)-\widehat{S}\right) dtd\xi_1 dv$$

$$= \sigma_\xi \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{(\eta-\Delta)^2}{2q}} e^{-\frac{(\eta+t)^2}{2r}} e^{-\frac{\xi_1^2}{2(1-k)}} e^{-\frac{(t+\xi_1)^2}{2k}} \delta\left(t\theta(t)+\xi_1\theta(\xi_1)-\widehat{S}\right) dtd\xi_1 d\eta,$$

with $r=\frac{\sigma_{v|s}^2}{\sigma_\xi^2}=\frac{\sigma_P^2-\sigma_\xi^2}{\sigma_\xi^2}$, $q=\frac{\sigma_m^2}{\sigma_\xi^2}$, $\eta=\frac{v-P}{\sigma_\xi}$, and $\Delta=\frac{s_m-P}{\sigma_\xi}$, and where the normalizing factor B comes from Bayes theorem. Note that the marginal p.d.f. of the short interest $\widehat{S}$ is $g\left(\widehat{S}|P,s_m\right)=C^{-1}B\left(P,\widehat{S},s_m\right)$, or, with the notation $\varphi(t,\xi_1)=t\theta(t)+\xi_1\theta(\xi_1)$ and

$F(\eta,\Delta,t,\xi_1)=e^{-\frac{\left(\eta-\frac{r\Delta-qt}{q+r}\right)^2}{2\frac{qr}{q+r}}} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}}$, we have

$$g\left(\widehat{S}|P,s_m\right) = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\eta,\Delta,t,\xi_1)\,\delta\left(\varphi(t,\xi_1)-\widehat{S}\right) d\eta d\xi_1 dt}{\sqrt{2\pi\frac{qr}{q+r}}\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} \quad \text{(IA.6)}$$

$$= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(\varphi(t,\xi_1)-\widehat{S}\right) d\xi_1 dt}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}}.$$

Making use of (IA.5), we obtain

$$f\left(v|P,\widehat{S},s_m\right) = B^{-1}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\eta,\Delta,t,\xi_1)\,\delta\left(\varphi(t,\xi_1)-\widehat{S}\right) dtd\xi_1,$$

$$B = \sigma_\xi\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\eta,\Delta,t,\xi_1)\,\delta\left(\varphi(t,\xi_1)-\widehat{S}\right) dtd\xi_1 d\eta,$$

and the best estimator of the fundamental, $\widehat{v}_M=E\left[v|P,\widehat{S}\right]$ as a conditional mean of (IA.5)

$$\widehat{v}_M = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(\varphi(t,\xi_1)-\widehat{S}\right)\left(P-\sigma_\xi\frac{qt-r\Delta}{q+r}\right) dtd\xi_1}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(\varphi(t,\xi_1)-\widehat{S}\right) dtd\xi_1}$$

$$= P-\sigma_\xi E_{t,\xi_1}\left[\frac{qt-r\Delta}{q+r}\right] = P+\sigma_\xi\Delta\frac{r}{q+r}-\sigma_\xi\frac{q}{q+r}\Lambda\left(\widehat{S}\right),$$

with $\Lambda\left(\widehat{S}\right) = E_{t,\xi_1}[t]$ given by

$$\Lambda\left(\widehat{S}\right) = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(\varphi(t,\xi_1)-\widehat{S}\right)dtd\xi_1}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(\varphi(t,\xi_1)-\widehat{S}\right)dtd\xi_1} = \frac{A}{B}, \tag{IA.7}$$

and

$$\begin{aligned} A &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(\varphi(t,\xi_1)-\widehat{S}\right)dtd\xi_1, \\ B &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(\varphi(t,\xi_1)-\widehat{S}\right)dtd\xi_1. \end{aligned} \tag{IA.8}$$

Combining (IA.7) and (IA.6), we obtain $g\left(\widehat{S}|P,s_m\right) = \frac{B\left(\widehat{S},P,s_m\right)}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}}$, and the value of $\Lambda\left(\widehat{S}\right)$ average w.r.t. the realization of short interest $\widehat{S}$, $E_{\widehat{S}}\left[\Lambda\left(\widehat{S}\right)\right] = \int_0^{+\infty}\Lambda\left(\widehat{S}\right)g\left(\widehat{S}\right)d\widehat{S}$:

$$\begin{aligned} E_{\widehat{S}}\left[\Lambda\left(\widehat{S}\right)\right] &= \frac{\int_0^{+\infty} A\left(\widehat{S}\right)d\widehat{S}}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} \\ &= \frac{\int_0^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(\varphi(t,\xi_1)-\widehat{S}\right)dtd\xi_1 d\widehat{S}}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} \\ &= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} tdtd\xi_1}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} = 0, \end{aligned}$$

which means that the correction to the manager's best estimate due to the short interest is zero on average, as it should be.

Making use of (IA.6), we also obtain the average value of short interest, $E_{\widehat{S}}\left[\widehat{S}\right]=\int_0^{+\infty}\widehat{S}g\left(\widehat{S}\right)d\widehat{S}$:

$$
\begin{aligned}
E_{\widehat{S}}\left[\widehat{S}\right] &= \frac{\int_0^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \widehat{S}\delta\left(\varphi\left(t,\xi_1\right)-\widehat{S}\right)d\xi_1 dt d\widehat{S}}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} \\
&= \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \left(t\theta\left(t\right)+\xi_1\theta\left(\xi_1\right)\right)d\xi_1 dt}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} \\
&= \frac{\int_0^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t d\xi_1 dt}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} \\
&\quad + \frac{\int_{-\infty}^{+\infty}\int_0^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \xi_1 d\xi_1 dt}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}\sqrt{2\pi\frac{k(r+q)}{k+r+q}}} \\
&= \frac{1}{\sqrt{2\pi\frac{(1-k)(k+r+q)}{1+r+q}}}\int_0^{+\infty} e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}}\xi_1 d\xi_1 \\
&\quad + \frac{e^{-\frac{1-k}{(k+r+q)(1+r+q)}\frac{\Delta^2}{2}}}{\sqrt{2\pi\frac{r+q}{1+r+q}}}\int_0^{+\infty} e^{-\frac{1+r+q}{r+q}\frac{\left(t+\frac{\Delta}{1+r+q}\right)^2}{2}} t dt.
\end{aligned}
$$

The R^2 integration domain in (IA.8) splits into two subsets: $\xi_1\leq 0$ and $\xi_1\geq 0$. We obtain

$$
\begin{aligned}
A &= \int_{-\infty}^{0}d\xi_1\int_{-\infty}^{+\infty}dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(t\theta\left(t\right)-\widehat{S}\right) \\
&\quad + \int_0^{+\infty}d\xi_1\int_{-\infty}^{+\infty}dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(t\theta\left(t\right)+\xi_1-\widehat{S}\right),
\end{aligned}
\tag{IA.9}
$$

and

$$
\begin{aligned}
B &= \int_{-\infty}^{0}d\xi_1\int_{-\infty}^{+\infty}dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(t\theta\left(t\right)-\widehat{S}\right) \\
&\quad + \int_0^{+\infty}d\xi_1\int_{-\infty}^{+\infty}dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(t\theta\left(t\right)+\xi_1-\widehat{S}\right).
\end{aligned}
\tag{IA.10}
$$

Dividing integration domain w.r.t. t into positive and negative regions yields

$$\begin{aligned} A = & \int_{-\infty}^{0} d\xi_1 \int_{-\infty}^{0} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(-\widehat{S}\right) \\ & + \int_{-\infty}^{0} d\xi_1 \int_{0}^{+\infty} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(t-\widehat{S}\right) \\ & + \int_{0}^{+\infty} d\xi_1 \int_{-\infty}^{0} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(\xi_1-\widehat{S}\right) \\ & + \int_{0}^{+\infty} d\xi_1 \int_{0}^{+\infty} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} t\delta\left(t+\xi_1-\widehat{S}\right), \end{aligned} \tag{IA.11}$$

and

$$\begin{aligned} B = & \int_{-\infty}^{0} d\xi_1 \int_{-\infty}^{0} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(-\widehat{S}\right) \\ & + \int_{-\infty}^{0} d\xi_1 \int_{0}^{+\infty} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(t-\widehat{S}\right) \\ & + \int_{0}^{+\infty} d\xi_1 \int_{-\infty}^{0} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(\xi_1-\widehat{S}\right) \\ & + \int_{0}^{+\infty} d\xi_1 \int_{0}^{+\infty} dt e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \delta\left(t+\xi_1-\widehat{S}\right). \end{aligned} \tag{IA.12}$$

Integrating w.r.t. Dirac's delta functions and taking into account that $\delta\left(-\widehat{S}\right)=0$ for any nonzero value of short interest, we arrive at

$$\begin{aligned} A = & \widehat{S} \int_{-\infty}^{0} d\xi_1 e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(\widehat{S}+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \\ & + \int_{-\infty}^{0} dt t e^{-\frac{\left(\widehat{S}-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\widehat{S}+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \\ & + \int_{0}^{\widehat{S}} d\xi_1 \left(\widehat{S}-\xi_1\right) e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(\widehat{S}-\xi_1+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}}, \end{aligned} \tag{IA.13}$$

and

$$B = \int_{-\infty}^{0} d\xi_1 e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(\widehat{S}+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}} \tag{IA.14}$$
$$+\int_{-\infty}^{0} dt e^{-\frac{\left(\widehat{S}-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(t+\frac{(r+q)\widehat{S}+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}}$$
$$+\int_{0}^{\widehat{S}} d\xi_1 e^{-\frac{\left(\xi_1-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{\left(\widehat{S}-\xi_1+\frac{(r+q)\xi_1+k\Delta}{k+r+q}\right)^2}{2\frac{k(r+q)}{k+r+q}}}.$$

Simplifying and introducing notations,

$$a = \frac{(r+q)\,\widehat{S}+k\Delta}{k+r+q},\ \sigma_a^2 = \frac{k\,(r+q)}{k+r+q},$$
$$b = \frac{(1-k)\left(\widehat{S}+\Delta\right)}{1-k+r+q}\ \sigma_b^2 = \frac{(1-k)\,(r+q)}{1-k+r+q},$$

we obtain

$$A = e^{-\frac{r+q+1}{r+q}\frac{\left(\widehat{S}+\frac{\Delta}{r+q+1}\right)^2}{2}}\sqrt{2\pi}\sqrt{k\,(1-k)}\widehat{S}\Phi\left(\widehat{S}\sqrt{\frac{1-k}{k}}\right)$$
$$+e^{-\frac{\left(\widehat{S}-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}}\left(-a\sqrt{2\pi\sigma_a^2}\Phi\left(\frac{a}{\sigma_a}\right)-\sigma_a^2 e^{-\frac{a^2}{2\sigma_a^2}}\right)$$
$$+e^{-\frac{\left(\widehat{S}+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}}\left(x\sqrt{2\pi\sigma_b^2}\left(\Phi\left(\frac{\widehat{S}-b}{\sigma_b}\right)-\Phi\left(\frac{-b}{\sigma_b}\right)\right)-\sigma_b^2\left(e^{-\frac{b^2}{2\sigma_b^2}}-e^{-\frac{(\widehat{S}-b)^2}{2\sigma_b^2}}\right)\right),$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}{}_{-\infty}^{x} d\eta \exp\left(-\frac{\eta^2}{2}\right)$ is a Normal c.d.f.

Similarly,

$$B = e^{-\frac{r+q+1}{r+q}\frac{\left(\widehat{S}+\frac{\Delta}{r+q+1}\right)^2}{2}}\sqrt{2\pi}\sqrt{k\,(1-k)}\Phi\left(\widehat{S}\sqrt{\frac{1-k}{k}}\right)$$
$$+e^{-\frac{\left(\widehat{S}-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}}\sqrt{2\pi\sigma_a^2}\Phi\left(\frac{a}{\sigma_a}\right) \tag{IA.15}$$
$$+e^{-\frac{\left(\widehat{S}+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}}\sqrt{2\pi\sigma_b^2}\left(\Phi\left(\frac{\widehat{S}-b}{\sigma_b}\right)-\Phi\left(\frac{-b}{\sigma_b}\right)\right).$$

Introducing a dimensionless variable $x = \widehat{S} = \frac{S}{\sigma_u}$ and removing common factors finally yields

$$\Lambda(x) = \frac{J_1(x)+J_2(x)+J_3(x)+J_4(x)}{I_1(x)+I_2(x)+I_3(x)}, \tag{IA.16}$$

with

$$
\begin{aligned}
J_1(x) &= x e^{-\frac{r+q+1}{r+q}\frac{\left(x+\frac{\Delta}{r+q+1}\right)^2}{2}} \Phi\left(x\sqrt{\frac{1-k}{k}}\right), \\
J_2(x) &= -\frac{a}{\sqrt{\frac{(1-k)(k+r+q)}{r+q}}} e^{-\frac{\left(x-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} \Phi\left(\frac{a}{\sigma_a}\right), \\
J_3(x) &= \frac{x}{\sqrt{\frac{k(1-k+r+q)}{r+q}}} e^{-\frac{\left(x+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}} \left(\Phi\left(\frac{x-b}{\sigma_b}\right) - \Phi\left(\frac{-b}{\sigma_b}\right)\right), \\
J_4(x) &= -\frac{e^{-\frac{\left(x-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}} e^{-\frac{a^2}{2\sigma_a^2}}}{\sqrt{2\pi\frac{1-k}{k}\frac{k+r+q}{r+q}}} + \frac{e^{-\frac{\left(x+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}} \left(e^{-\frac{(x-b)^2}{2\sigma_b^2}} - e^{-\frac{b^2}{2\sigma_b^2}}\right)}{\sqrt{2\pi\frac{k}{1-k}\frac{1-k+r+q}{r+q}}},
\end{aligned}
\tag{IA.17}
$$

and

$$
\begin{aligned}
I_1(x) &= e^{-\frac{r+q+1}{r+q}\frac{\left(x+\frac{\Delta}{r+q+1}\right)^2}{2}} \Phi\left(x\sqrt{\frac{1-k}{k}}\right), \\
I_2(x) &= \frac{e^{-\frac{\left(x-\Delta\frac{1-k}{1+r+q}\right)^2}{2\frac{(1-k)(k+r+q)}{1+r+q}}}}{\sqrt{\frac{(1-k)(k+r+q)}{r+q}}} \Phi\left(\frac{a}{\sigma_a}\right), \\
I_3(x) &= \frac{e^{-\frac{\left(x+\Delta\frac{k}{1+r+q}\right)^2}{2\frac{k(1-k+r+q)}{1+r+q}}}}{\sqrt{\frac{(1-k)(k+r+q)}{r+q}}} \left(\Phi\left(\frac{x-b}{\sigma_b}\right) - \Phi\left(\frac{-b}{\sigma_b}\right)\right).
\end{aligned}
\tag{IA.18}
$$

□

Making use of the results of Proposition 1A, we obtain the sensitivity of the best estimate to the short interest in the following way:

$$
\frac{\partial \widehat{v}_M}{\partial S} = -\frac{\sigma_\xi}{\sigma_u}\frac{q}{q+r}\Lambda'\left(\widehat{S}\right) = -\alpha\sigma_{v|s}^2 \frac{\sigma_m^2}{\sigma_m^2+\sigma_{v|s}^2}\Lambda'\left(\widehat{S}\right),
\tag{IA.19}
$$

where $\Lambda(x)$ is defined by (IA.1) in the Proposition 1A, and we took into account that $\widehat{S} = \frac{S}{\sigma_u}$ and $\sigma_\xi = \alpha\sigma_{v|s}^2\sigma_u$.

AS follows from (IA.19), the sensitivity of the manager's valuation w.r.t. the short interest is negative if $\Lambda'(x) \geq 0$, and is increasing both in the uncertainty of informed trader's $\sigma_{v|s}^2$ and manager's private signals σ_m^2. This is illustrated in Figures IA.1, IA.2, and IA.3.

Numerical results indicate that $\Lambda(x)$ is monotonically increasing in x, and is also convex apart from a narrow range of small x. Making use of the conditional distribution of the short interest $g\left(\widehat{S}|P, s_m\right)$ that we obtain in close analytic form, we show that, on average, $\Lambda(x)$ is monotonically increasing and convex for arbitrary values of $\Delta = \frac{s_m - P}{\sigma_\xi}$ and a fraction of unconstrained traders k. These numerical results are presented in Figure IA.4 and Figure IA.5 that plot the average first and second derivatives of $\mathit{\Lambda}(.)$, respectively, as a function of the share of unconstrained liquidity traders k. Note that, as we discuss above, $\mathit{\Lambda}\left(\widehat{S}\right)$ is a function of the ratio of the short interest S and the noise parameter σ_u (which is proportional to the expected value of short interest).

The function $\Lambda(x)$ for several values of relative mispricing based on manager's private signal $\Delta = \frac{s_m - P}{\sigma_\xi}$ is presented in Figure IA.1. Clearly, the manager's valuation is lower when the private signal indicates overpricing, $\Delta \leq 0$, and is higher otherwise, $\Delta \geq 0$. As one can see, $\Lambda(x)$ is defined and monotonically increasing for $x \geq 0$. It is also convex apart from a region of very small x. As it follows from Figure IA.4 and Figure IA.5, the average $E_x[\Lambda'(x)] > 0$, and $E_x[\Lambda''(x)] > 0$. Note that $E_x[\Lambda''(x)]$ is increasing in the fraction k of unconstrained liquidity traders.

Figure IA.1: The effect of short interest on the best estimate of the fundamental value of the firm

This figure displays the relative difference between the price and the manager's best estimate of the fundamental value $\frac{P-E\left[v|P,\widehat{S}\right]}{P} = \mu\Lambda\left(\widehat{S}\right)$ as a function of scaled short interest $z = \widehat{S} = \frac{S}{\sigma_u}$, for different values of $\Delta = \frac{s_m - P}{\sigma_\xi}$. $\mu = \frac{\sigma_\xi}{P} = 0.1$. The blue line corresponds to $\Delta = 0.1$, the green line corresponds to $\Delta = 0$, and the red line corresponds to $\Delta = -0.1$. Other parameters are: $q = 1, r = 1, k = 0.2$.

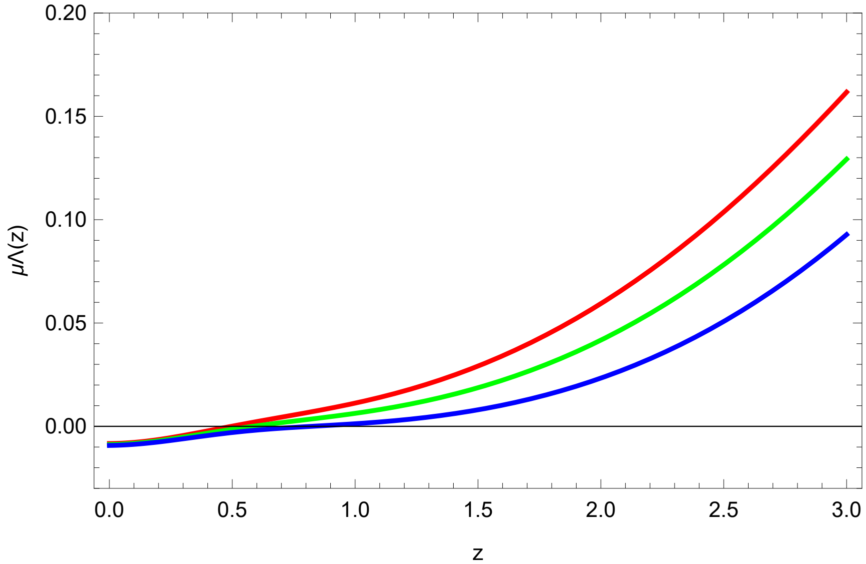

Figure IA.2: The effect of short interest on the best estimate of the fundamental value of the firm

This figure displays the relative difference between the price and the manager's best estimate of the fundamental value $\frac{P-E\left[v|P,\widehat{S}\right]}{P} = \mu\Lambda\left(\widehat{S}\right)$ as a function of scaled short interest $z = \widehat{S} = \frac{S}{\sigma_u}$, for different values of k. $\mu = \frac{\sigma_\xi}{P} = 0.1$. The green line corresponds to $k = 0.1$, the blue line corresponds to $k = 0.2$, and the black line corresponds to $k = 0.4$. Other parameters are: $\Delta = 0, r = 1, q = 1$.

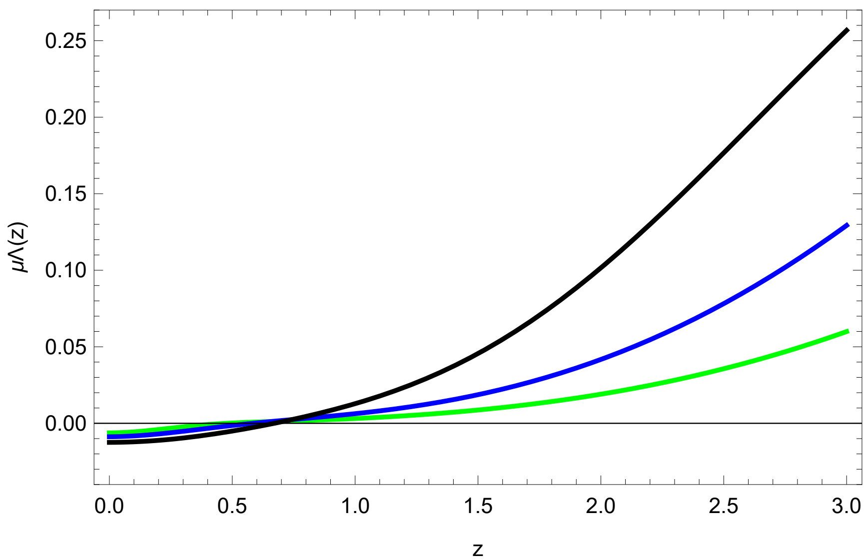

Figure IA.3: The effect of short interest on the best estimate for different noise levels

This figure displays the relative difference between the price and the manager's best estimate of the fundamental value $\frac{P-E\left[v|P,\widehat{S}\right]}{P} = \mu\Lambda\left(\widehat{S}\right)$ as a function of scaled short interest $\widehat{S} = \frac{S}{\sigma_u}$, for different values of the noise-to-price ratio $\mu = \frac{\sigma_\xi}{P}$. The green line corresponds to $\mu = 5\%$, the black line $\mu = 10\%$, and red line $\mu = 15\%$. The percentage of unconstrained liquidity traders $k = 0.4$. Other parameters are: $\Delta = 0, r = 1, q = 1$.

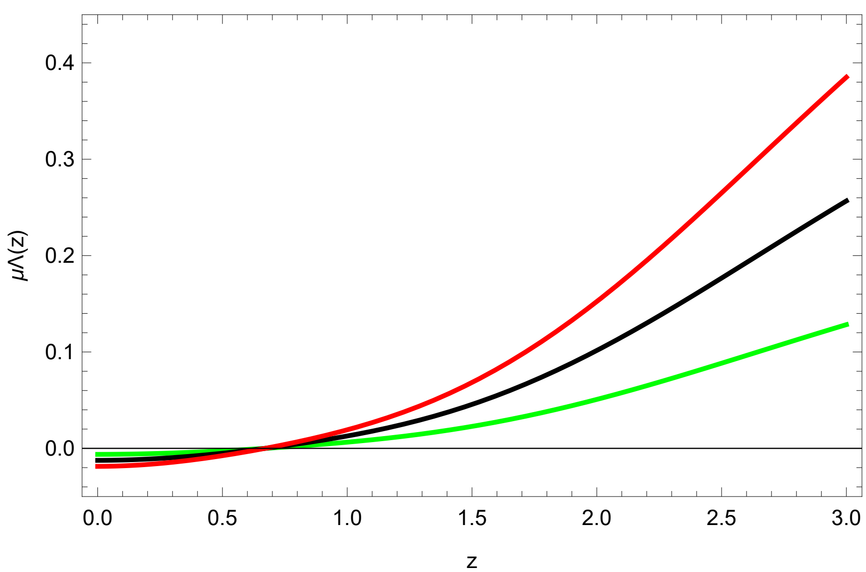

Figure IA.4: The average sensitivity of manager's best estimate w.r.t. short interest

This figure displays the average value of the first derivative of $\Lambda\left(\widehat{S}\right)$, namely, $E_{\widehat{S}}\left[\Lambda'\left(\widehat{S}\right)\right]$, where the expectation is performed w.r.t. different realizations of the short interest $\widehat{S}$, as a function of the share of unconstrained liquidity traders k. Other parameters are: $\Delta = 0, r = 1, q = 1$.

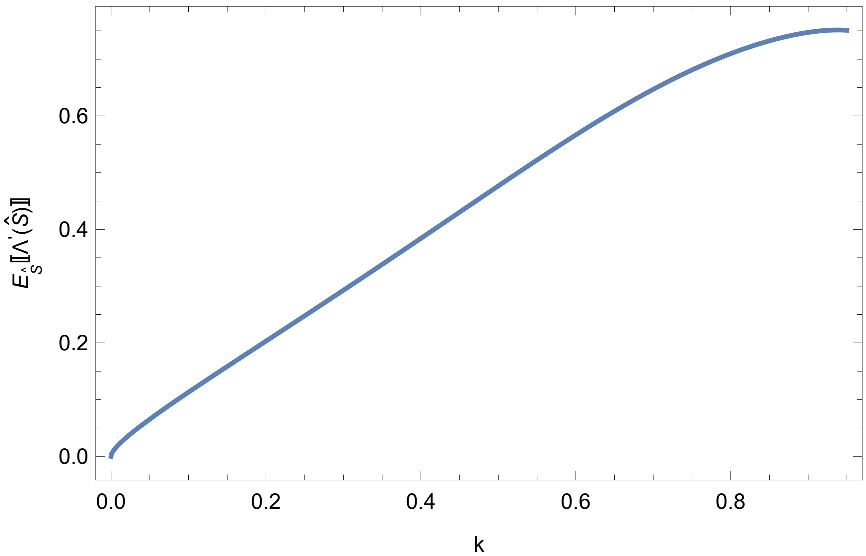

Figure IA.5: The average sensitivity of manager's best estimate elasticity w.r.t. short interest

This figure displays the average value of the second derivative of $\Lambda\left(\widehat{S}\right)$, namely, $E_{\widehat{S}}\left[\Lambda''\left(\widehat{S}\right)\right]$, where the expectation is performed w.r.t. different realizations of the short interest $\widehat{S}$, as a function of the share of unconstrained liquidity traders k. Other parameters are: $\Delta = 0, r = 1, q = 1$.