Discovering Optimal Gait Transitions

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***Abstract*—It is a longstanding goal in robotics to create legged machines that move with the grace and agility of their biological counterparts. Many methods are available for creating dynamic quadruped behavior, but Model-Predictive Control (MPC) and Whole-Body Impulse Control (WBIC) remains a popular option for its computational efficiency. However, current studies using MPC+WBIC have only considered operating the quadruped under a single gait mode, whereas animals in the wild switch between different gaits at will. In this study, we focus on the problem of finding optimal gait transitions.**

***Index Terms*—quadrupeds, gait transition, MPC**

I. Introduction

Gait transition is a critical aspect of locomotion for quadruped robots. It refers to the ability of the robot to smoothly switch between different gaits, such as trot and gallop, in order to adapt to changing environments and tasks or increase body speed. In this paper, we present a method that utilizes model predictive control (MPC) for gait transition in quadruped robots. We begin by introducing the MPC framework for finding the foot forces. Then we discuss how the contact sequence can be used to transition from one gait to another by running multiple MPCs in parallel. We then present experimental results, which demonstrate optimal gait selections. Finally, we discuss how we can increase MPC performance to keep the problem real-time.

The use of MPC for gait transition in quadruped robots has several advantages. MPC is a well-established control framework that allows for the optimization of control inputs over a finite horizon, taking into account the current state and constraints of the system. This makes it well-suited for gait transitions, where the robot must adapt its motion to changing environments and tasks. Even though MPC can handle nonlinear dynamics, it is important to keep the problem formulation convex to get fast and globally optimal solutions.

In our approach, we use discrete footstep sequences to model gait transitions. A gait phase refers to a specific stage of the gait cycle, such as the stance or swing phase. By fitting multiple gait transitions from one gait to another, we can choose the appropriate next gait phase based on the least cost. This allows us to smoothly and efficiently switch between different gaits while accounting for the current state and constraints of the system. In the subsequent sections, we will describe our specific implementation of the approach in detail, including the methods we used for fitting the gait transitions and for choosing the appropriate gait phase.

II. Model Predictive Control

We are using MPC to determine the foot reaction forces for the quadruped. Using a predefined contact sequence, the MPC controller finds contact forces to allow the lumped mass model to follow a defined trajectory. A gait scheduler and a footstep planner specify the contact sequence. Using a predefined contact sequence allows the MPC formulation to be convex, meaning the program can be solved quickly to a global optimum.

A. Gait Scheduler

We are re-implementing the phase-based gait scheduler from [1], which uses only two parameters to specify a gait type: the phase offset and the stance period for each foot. Because most gaits are periodic, the spacing of the swing and stance periods in one cycle is sufficient to define different gaits. For example, it would be easy to define the trot gait by specifying an offset of 0.5 to the diagonal legs and then a gait period of 0.6, refer to Fig.1 and Fig.2 to understand the footstep sequence. It is also important to specify an appropriate gait frequency, as Fig.7 shows drastic changes in tracking error as gait period increases. Moreover, the gait frequency can be easily changed by adjusting the duration of the cycle, and the gait types will remain the same even if the gait frequency changes. This makes it easy to adjust the gait frequency while maintaining the desired gait type. Figures 1, 2 show us the trot and bound gait.

B. Foot Step Planner

For the footstep planner, we re-implemented the equations presented in [1]. The equations are as follows.

$$r_i^{cmd} = P_{shoulder,i} + P_{symmetry} + P_{centrifugal} \tag{1}$$

where for each r_i^{cmd} we have:

$$P_{shoulder,i} = p_k + R_z(\psi) l_i \tag{2}$$

For equation 2 we have p_k as the body position at time k-th time step. Then $R_z(\psi)$ is the rotation matrix for the body rotation ϕ. Finally, l_i is the shoulder location with respect to the body. and below 2 are constant through all r_i^{cmd}'s

$$P_{symmetry} = \frac{t_{stance}}{2} v + k(v - v^{cmd}) \tag{3}$$

$$P_{centrifugal} = \frac{1}{2}\sqrt{\frac{h}{g}} v x w^{cmd} \tag{4}$$

For the equations, $P_{shoulder,i}$ is the shoulder location of the ith leg in the body frame. The $P_{symmetry}$ is the Raibert Heuristic [2]. This keeps the landing angle and leaving angle the same. k is given a value of 0.03.

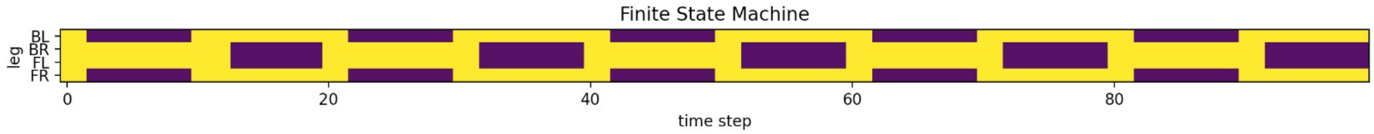

Fig. 1: Trot gait foot contacts

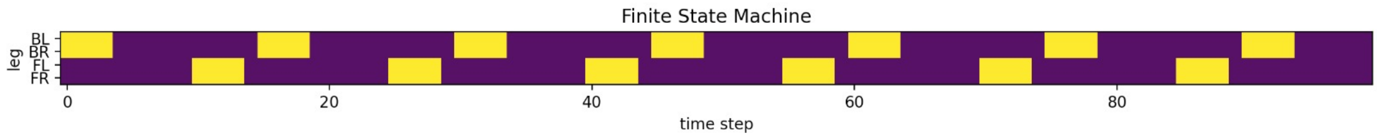

Fig. 2: Bound gait foot contacts

C. Formulation of MPC problem for foot forces

For the problem to work in real-time, it is important that the optimization program remains convex. To achieve a convex MPC, there are three simplifications that were made in [1]. These simplifications are made to make the constraints linear. The first simplification is that the roll and pitch angles are small. This simplifies the equations to:

$$\dot{\theta} \approx R_z(\psi)\omega \tag{5}$$

$$gI \approx R_z(\psi) I R_z(\psi)^T \tag{6}$$

where $\dot{\theta} = ([\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}])^T$ is the angular velocity of the body, and the terms inside being roll, pitch, yaw. $R_z(\psi)$ is the rotation matrix translating angular velocity in the global frame to the local coordinate.

The second assumption made for the MPC is that the states are close to the commanded values. This assumption allows us to create time-varying linearization using $R_z(\psi)$. This allows us to set the moment arm with the predefined one from the commanded trajectory and step locations. The moment equation simplifies to the following:

$$\frac{d}{dt}(\mathbf{I}\omega) = \mathbf{I}\dot{\omega} + \omega \times (\mathbf{I}\omega) \approx \mathbf{I}\dot{\omega} \tag{7}$$

Based on the above assumption, we retrieve the linear dynamics as:

$$x(k+1) = A_k x(k) + B_k \hat{f}(k) + \hat{g} \tag{8}$$

$$x = ([\theta^T \quad p^T \quad \omega^T \quad \dot{p}^T])^T \tag{9}$$

$$\hat{f} = ([f_1 \quad ... \quad f_n])^T \tag{10}$$

$$\hat{g} = ([0_{1\times3} \quad 0_{1\times3} \quad 0_{1\times3} \quad g^T])^T \tag{11}$$

$$A = \begin{bmatrix} 1_{3\times3} & 0_{3\times3} & R_z(\psi_k)\Delta t & 0_{3\times3} \\ 0_{3\times3} & 1_{3\times3} & 0_{3\times3} & 1_{3\times3}\Delta t \\ 0_{3\times3} & 0_{3\times3} & 1_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 1_{3\times3} \end{bmatrix} \tag{12}$$

$$B = \begin{bmatrix} 0_{3\times3} & ... & 0_{3\times3} \\ 0_{3\times3} & ... & 0_{3\times3} \\ g\mathbf{I}^{-1}[r_1] \times \Delta t & ... & g\mathbf{I}^{-1}[r_n] \times \Delta t \\ 1_{3\times3}\Delta t/m & ... & 1_{3\times3}\Delta t/m \end{bmatrix} \tag{13}$$

The cost for our QP is given by:

$$\min_{x,f}\left(\sum_{k=0}^{m}(||x(k+1) - x^{ref}(k+1)||)_Q + (||f(k)||)_R\right) \tag{14}$$

We also add the friction constraints as:

$$|f_x| \leq \mu f_z \quad |f_y| \leq \mu f_z \quad f_z \geq 0 \tag{15}$$

III. Gait Transition

To achieve gait transition from one gait to another, we use desired body speed to determine the current gait. Our objective is to find contact sequences for the quadruped when it is transitioning from one gait to another. For example, consider trot gait shown in Fig.5. The dotted line represents the point at which the transition is occurring. Assuming that we are transitioning to the bound gait, there are a set of finite foot contacts to which we can transition to. This can be observed in Fig.6. The foot contact states we can transfer to are represented by the dotted lines. Using the trot as the initial condition, we use the finite states to formulate the MPC problems. For us to choose a proper foot contact, we simulate over the horizon of at least one gait period. We then look at the average cost that is incurred by each of these transitions and choose the least one for the transition. We can write this as:

$$\gamma_{next} = \underset{\gamma \in [\gamma_i^{NextGait}]}{\arg\min} \quad (CostFunction) \tag{16}$$

$\gamma_i^{NextGait}$ is the discretized phase of the next gait.

IV. Results

Our project was split into 2 parts. The first part consisted of recreating the results from the 2019 Mini Cheetah paper to obtain contact forces from the MPC formulation. Fig.3 represents a single MPC tracking (Trot Gait) for the angular positions (First row), position error (Second row), angular velocity (Third row), and Linear velocity (Fourth row). The results for the bound gait can be seen in fig.3 as well. It is pretty clear from the results that bound is a less stable gait as it contains aerial parts where non of the feet are on the ground. This is clearly seen with the velocity and position tracking of linear z. Thus, it is important that we have included slack within our constraints for the z position. While re-implementing the MPC formulation, we noticed that

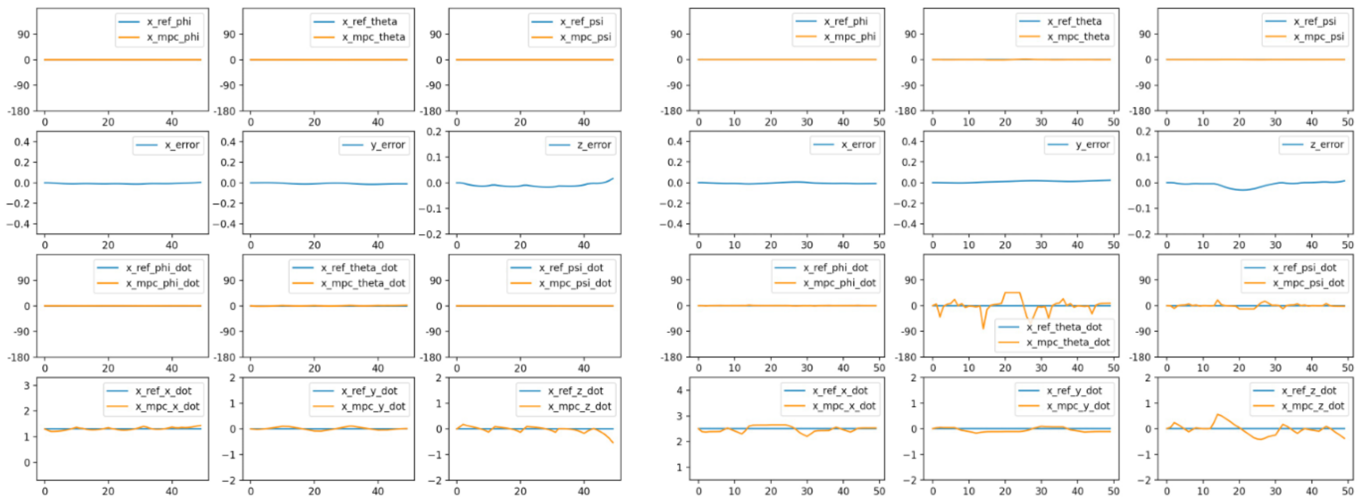

Fig. 3: Trot gait(left), Bound(Right) Single MPC tracking

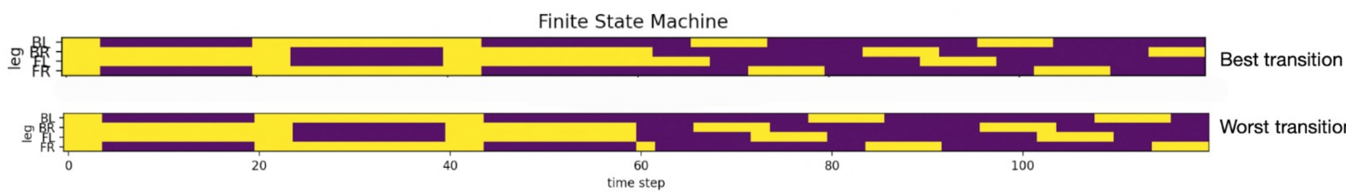

Fig. 4: Foot contact placements

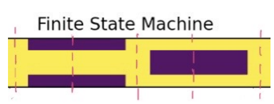

Fig. 5: Trot gait sampled

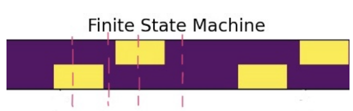

Fig. 6: Bound gait sampled

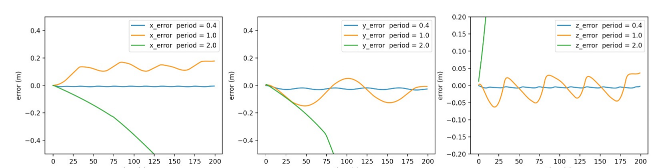

Fig. 7: Gait period varied tracking

the stability of the gait over long horizons depends a lot on the gait period. By varying the gait period, we produced position-tracking error plots seen in fig.7. From the graphs, it is obvious that as the gait period increases, the stability of the gait decreases. This follows our intuition because if the gait period is small and the speed is high the robot would try to take longer steps to move the distance. Such huge steps will cause instability within the system over time. Let us now look at the gait transition results. In Fig.4 we can look at the transition from the trot gait to the gallop gait. The top figure is the contact sequence with the lowest MPC cost, and the bottom figure represents the transition which would have the highest MPC cost. When we look at these gaits, we can infer that the transition will happen to a state in the future gait that tries to maintain the same contacts. This is intuitive, since the body would be more stable if a foot that is in contact stays in contact. If we take a look at the bottom graph with the highest cost, the two legs of the robot is jumping into the air, while another swing leg comes in contact for a brief moment. This characterizes an unstable transition as it is a jump with a short contact time, leading to high impulses. We can also observe the foot contact forces in Fig.9, where we notice that during the transition, the contact forces are higher for a brief moment as the robot attempts to accelerate to match the desired speed. These forces after transitioning match the fsm in 8.

The next important part in tackling this problem is reducing solve times for the MPC problems. At the point of gait transition, we can see that the solve time shoots up in Fig.10 due to multiple MPC runs. We can reduce the solve time at transitions limiting the horizon to a half gait period. This decision can be justified because we are only looking at the stability of the system when there is a change in foot contact. The reduction in time can be seen in Fig.11.

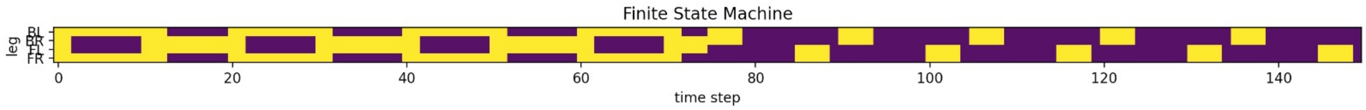

Fig. 8: Trot to Bound transition

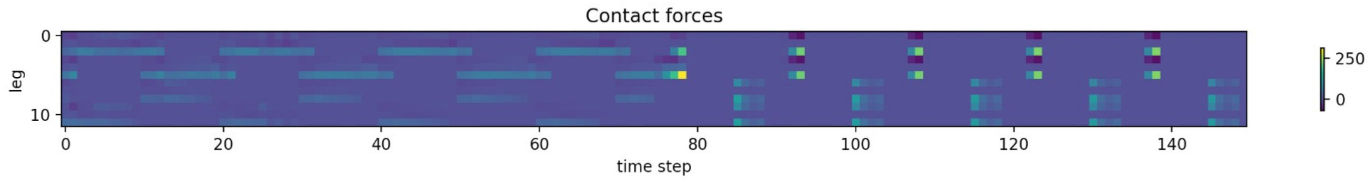

Fig. 9: Forces for the best MPC policy

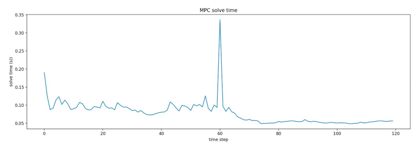

Fig. 10: Solve time for full gait period

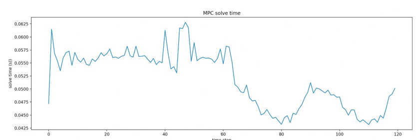

Fig. 11: Solve time for half gait period

V. Conclusion

We demonstrate the use of MPC for computing optimal gait transitions in a quadruped robot. The MPC controllers take the center of mass trajectory, contact sequences, and contact locations as input, and output optimized contact forces for a finite horizon. When a robot requires a gait transition, we determine the unique states the robot can transition to via the finite-state machine. The next state with the least MPC cost will be chosen as the transition. The code available at https://github.com/JChunX/gait-trans

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References

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