Modificaciones de parches
y algunos axiomas de separación en la topología sin puntos

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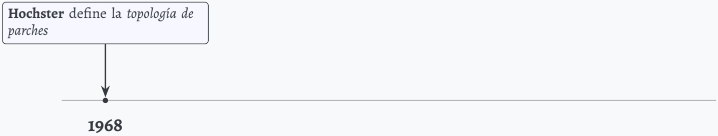
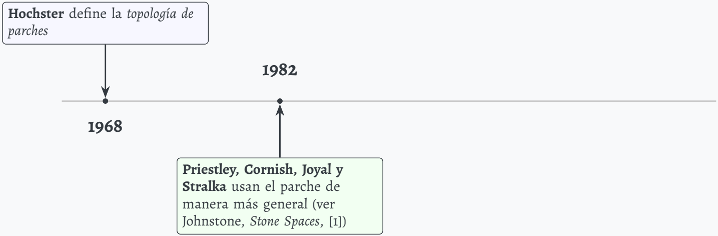
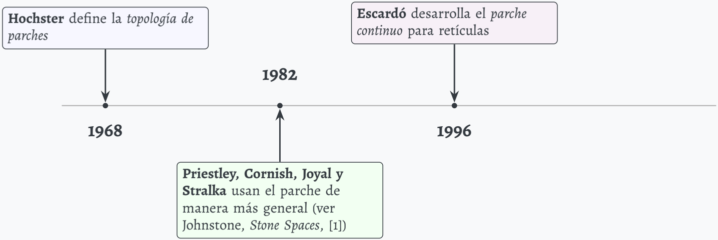
La construcción de parches



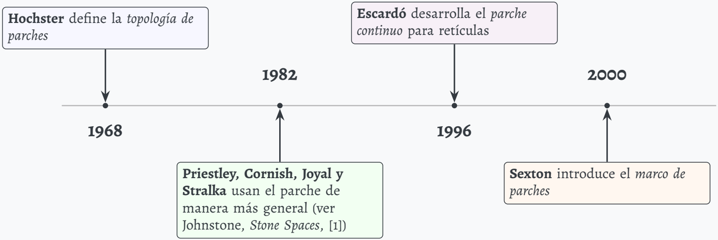
La construcción de parches



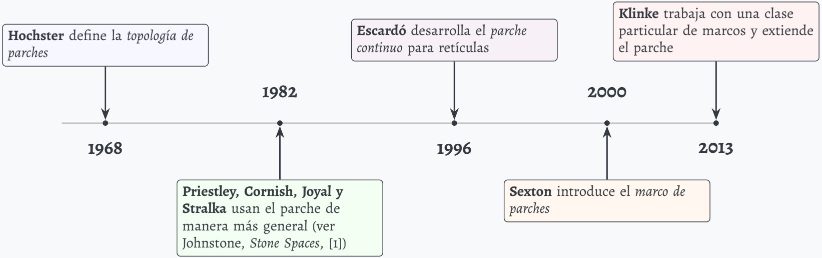
La construcción de parches



La construcción de parches



La construcción de parches



Frame theory

$$\mathsf{Frm} = \begin{cases} \mathsf{Obj}: & (A, \leqslant, \wedge, \bigvee, 1, 0) \\ \text{Arrows:} & f : A \to B \end{cases}$$

For $S \in \mathsf{Top}$,

$$(\mathcal{O}S, \subseteq, \cap, \bigcup, S, \emptyset) \in \mathsf{Frm}$$

Furthermore,

$$\begin{tikzcd} \mathsf{Top} \arrow[rr, bend left, "\mathcal{O}(\_)"] \arrow[rr, phantom, "\perp"] & & \mathsf{Frm} \arrow[ll, bend left, "\mathrm{pt}(\_)"] \end{tikzcd}$$

is an adjunction.

Packed spaces

${}^{p}S = (S, \mathcal{O}^{p}S)$, where $\mathcal{O}^{p}S$ is gerated by

$$\text{pbase} = \{U \cap Q' \mid U \in \mathcal{O}S, Q \in \mathcal{Q}S\}$$

Definition

$S \in \mathsf{Top}$ is **packed** if every compact (saturated) subset is closed

$$S \text{ is packed} \quad \Longleftrightarrow \quad {}^{p}S = S$$

$$T_2 \Rightarrow \text{packed} \Rightarrow T_1$$

Patch trivial

By *Hoffman-Mislove theorem*[1]

$$\mathrm{Pbase} = \{u_a \wedge v_F \mid a \in A, F \in A^{\wedge}\}$$

Definition

1. The **patch frame** of $A \in \mathsf{Frm}\ (PA)$, is the frame generated by Pbase
2. A is **patch trivial** if $A \simeq PA$.

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[1]**Thm:** There is a bijective correspondence between $F \in A^{\wedge}$ and $Q \in \mathcal{Q}S$

$\circ F \in A^{\wedge}$ if F is a filter in A and $\forall X \subseteq A$, with X directed, if $\bigvee X \in F$, then $a \in F$ for some $a \in X$.

$\circ$For $a \in A$, $u_a(x) = a \vee x$ and $v_a(x) = (a \succ x)$ are nuclei in A.

$\circ v_F = f^{\infty}$, where $f = \dot{\bigvee}\{v_a \mid a \in F\}$

Tidy frames

Definition [[8], Def. 8.2.1]

Let $A \in \mathsf{Frm}$, $F \in A^{\wedge}$ and $\alpha \in \mathsf{Ord}$ be. We say that:

1. F is α-tidy if for $x \in F$, $d \vee x = 1$, where

$$d = d(\alpha) = f^{\alpha}(0).$$

2. A is α-tidy if every $F \in A^{\wedge}$ is α-tidy.
3. A is **tidy** if it is α-tidy for some $\alpha \in \mathsf{Ord}$.

Proposition [[8], Lemma 8.2.2]

$$A \text{ is tidy} \iff A \text{ is patch trivial.}$$

Objectives

1. Understand tidy frames in more detail.
2. To explore the relationship with some separation axioms in Frm.
3. Provide tools to study the tidy frames.
4. Give examples.

Separation axioms in Frm

$$\begin{tikzcd}
 & (\mathbf{reg}) \arrow[dl, Rightarrow] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \\
(\mathbf{fit}) \arrow[r, dotted, no head] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & (\mathbf{sH}) \arrow[r, dotted, no head] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] \arrow[dl, dotted, no head] & (\mathbf{H}) + (\mathbf{sfit}) \arrow[d, Rightarrow] \\
(\mathbf{sfit}) \arrow[r, dotted, no head] & T_1 & (\mathbf{H}) \arrow[l, Rightarrow]
\end{tikzcd}$$

°$\forall\, a \nleq b \in A$, then

°**(reg):** $\exists\, x, y \in A$ such that $a \vee x = 1, y \nleq b$ and $x \wedge y = 0$.

°**(H):** $\exists\, c \in A$ such that $c \nleq a$ and $\neg c \leqslant b$.

°**(fit):** $\exists\, x, y \in A$ such that $x \vee a = 1, y \nleq b$ and $x \wedge y \leqslant b$.

°**(sfit):** $\exists\, c \in A$ such that $c \vee a = 1 \neq c \vee b$.

°**(sH)** and T_1 are notion some different. All this can be found in [5].

Properties of the tidy frames

This is a summary of the properties that Sexton includes in [8]

- In the spatial case ($A = \mathcal{O}S$),

$$\begin{aligned} \mathcal{O}S \text{ is 0-tidy} &\iff S = \emptyset \\ \mathcal{O}S \text{ is 1-tidy} &\iff S \text{ is } T_2 \\ \mathcal{O}S \text{ is tidy} &\iff S \text{ is packed and stacked.} \end{aligned}$$

- For $A \in \mathrm{Frm}$ arbitrary

$$\begin{aligned} A \text{ is } (\mathbf{reg}) &\Rightarrow A \text{ is tidy} \\ A \text{ is } (\mathbf{fit}) &\Rightarrow A \text{ is tidy} \\ A \text{ is tidy } &\Rightarrow A \text{ is } T_1 \end{aligned}$$

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