Modificaciones de parches

y algunos axiomas de separación en la topología sin puntos

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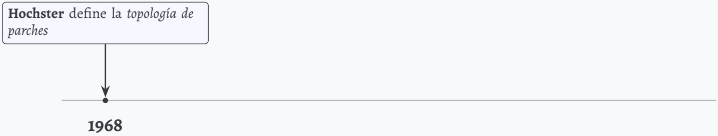
La construcción de parches



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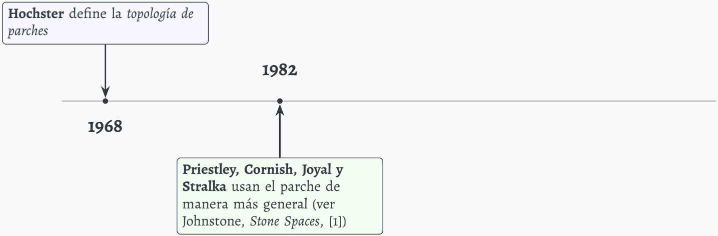

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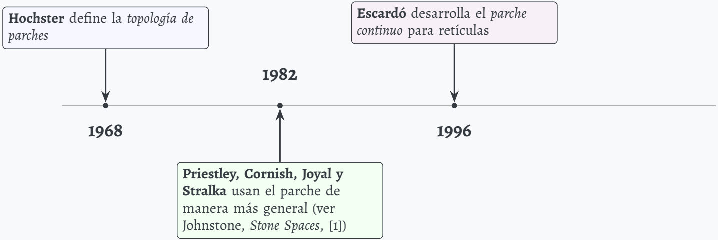

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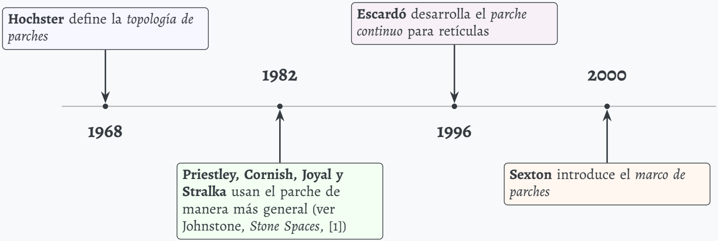

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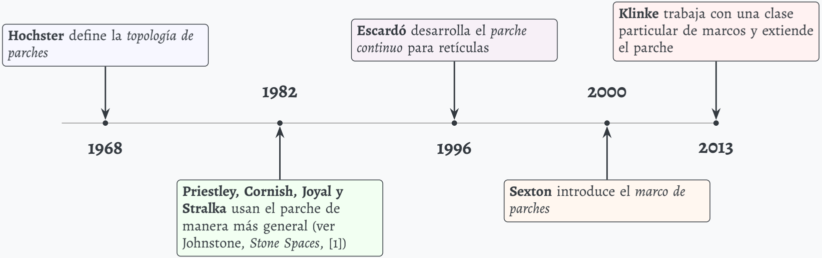

Teoría de marcos

$$\mathsf{Frm} = \begin{cases} \mathsf{Obj} : & (A, \leqslant, \wedge, \bigvee, 1, 0) \\ \text{Flechas:} & f \colon A \to B \end{cases}$$

Para $S \in \mathsf{Top}$,

$$(\mathcal{O}S, \subseteq, \cap, \bigcup, S, \emptyset) \in \mathsf{Frm}$$

Además,

$$\begin{tikzcd} \mathsf{Top} \arrow[rr, bend left, "\mathcal{O}(\_)"] \arrow[rr, phantom, "\perp"] & & \mathsf{Frm} \arrow[ll, bend left, "\mathsf{pt}(\_)"] \end{tikzcd}$$

es una adjunción.

Espacio de parches

${}^pS = (S, \mathcal{O}^pS)$, donde $\mathcal{O}^pS$ está generado por

$$\text{pbase} = \{U \cap Q' \mid U \in \mathcal{O}S, Q \in \mathcal{Q}S\}$$

Definición

$S \in \mathrm{Top}$ es **empaquetado** si todo subconjunto compacto (saturado) es cerrado

$$S \text{ es empaquetado} \quad \iff \quad {}^pS = S$$

$$T_2 \Rightarrow \text{empaquetado} \Rightarrow T_1$$

Parche trivial

Por el *Teorema de Hoffman-Mislove*[1]

$$\text{Pbase} = \{u_a \wedge v_F \mid a \in A, F \in A^{\wedge}\}$$

Definición

1. El **marco de parches** de $A \in \mathsf{Frm}$ (PA), es el marco generado por la Pbase.
2. A es **parche trivial** si $A \simeq PA$.

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[1]**Teo:** Existe una correspondencia biyectiva entre $F \in A^{\wedge}$ y $Q \in \mathcal{Q}S$

$\circ F \in A^{\wedge}$ si F es un filtro en A y $\forall X \subseteq A$, con X dirigido, si $\bigvee X \in F$, entonces $a \in F$ para algún $a \in X$.

$\circ$Para $a \in A$, $u_a(x) = a \vee x$ y $v_a(x) = (a \succ x)$ son núcleos en A.

$\circ v_F = f^{\infty}$, donde $f = \dot{\bigvee}\{v_a \mid a \in F\}$

Marcos eficientes

Definition [[8], Def. 8.2.1]

Let $A \in \mathsf{Frm}$, $F \in A^{\wedge}$ and $\alpha \in \mathsf{Ord}$ be. We say that:

1. F is α-tidy if for $x \in F$, $d \vee x = 1$, where
$$d = d(\alpha) = f^{\alpha}(0).$$
2. A is α-tidy if every $F \in A^{\wedge}$ is α-tidy.
3. A is **tidy** if it is α-tidy for some $\alpha \in \mathsf{Ord}$.

Proposition [[8], Lemma 8.2.2]

$$A \text{ is tidy} \quad \Longleftrightarrow \quad A \text{ is patch trivial.}$$

Objectives

1. Understand tidy frames in more detail.
2. To explore the relationship with some separation axioms in Frm.
3. Provide tools to study the tidy frames.
4. Give examples.

Separation axioms in Frm

$$\begin{tikzcd}[column sep=large, row sep=large]
& (\textbf{reg}) \arrow[dl, Rightarrow] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & \\
(\textbf{fit}) \arrow[r, dotted, dash] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & (\textbf{sH}) \arrow[r, dotted, dash] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] \arrow[dl, dotted, dash] & (\textbf{H}) + (\textbf{sfit}) \arrow[d, Rightarrow] \\
(\textbf{sfit}) \arrow[r, dotted, dash] & T_1 & (\textbf{H}) \arrow[l, Rightarrow]
\end{tikzcd}$$

°$\forall\, a \nleq b \in A$, then

°**(reg):** $\exists\, x, y \in A$ such that $a \vee x = 1, y \nleq b$ and $x \wedge y = 0$.

°**(H):** $\exists\, c \in A$ such that $c \nleq a$ and $\neg c \leqslant b$.

°**(fit):** $\exists\, x, y \in A$ such that $x \vee a = 1, y \nleq b$ and $x \wedge y \leqslant b$.

°**(sfit):** $\exists\, c \in A$ such that $c \vee a = 1 \neq c \vee b$.

°**(sH)** and T_1 are notion some different. All this can be found in [5].

Properties of the tidy frames

This is a summary of the properties that Sexton includes in [8]

- In the spatial case ($A = \mathcal{O}S$),

$$\begin{aligned} \mathcal{O}S \text{ is 0-tidy} &\iff S = \emptyset \\ \mathcal{O}S \text{ is 1-tidy} &\iff S \text{ is } T_2 \\ \mathcal{O}S \text{ is tidy} &\iff S \text{ is packed and stacked.} \end{aligned}$$

- For $A \in \mathrm{Frm}$ arbitrary

$$\begin{aligned} A \text{ is } (\mathbf{reg}) &\Rightarrow A \text{ is tidy} \\ A \text{ is } (\mathbf{fit}) &\Rightarrow A \text{ is tidy} \\ A \text{ is tidy } &\Rightarrow A \text{ is } T_1 \end{aligned}$$

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