THE PATCH FRAME AND ITS RELATIONS WITH SEPARATION IN POINT-FREE TOPOLOGY

ABSTRACT.

1. INTRODUCTION

2. PRELIMINARIES

3. HAUSDORFF PROPERTIES IMPLIES PATCH TRIVIALITY

Gadgets:
A the base frame
Its point space $S = \mathrm{pt}(A)$.
The compact saturated sets of S,

$$\mathcal{Q}(S).$$

The preframe of open filters of A,

$$A^{\wedge}.$$

The preframe of open filters of $\Omega(S)$.

$$\Omega(S)^{\wedge}.$$

First we recall that every open filter $F \in A^{\wedge}$ has three faces, that is, determines (and its determine) by :

- The compact saturated $Q \in \mathcal{Q}S$.
- $\nabla \in \Omega(S)^{\wedge}$.
- The compact quotient $A \to A_F$.
- The fitted nucleus v_F.

Hoffman-Mislove can be rephrase:

There is a bijection between compact quotients of $A$and compact saturated sets of S

Definition 3.1. A frame has KC if every compact quotient of A is a closed one. In other words every compact sublocale is close.

Denote by $\mathcal{H}rm$ the subcategory of Frm of Hausdorff frames in the sense of Johnstone and Shu.

The block structure on a frame is an important problem and its related with some separation properties of frames.

Proposition 3.2. *For every $A \in \mathcal{H}rm$ the interval corresponding to the block determined by a open filter $F \in A^\wedge$ is trivial, that is,*

$$[v_F, w_F] = \{*\}$$

Proposition 3.3. *Every Hausdorff frame A (in the sense of Johnstone and Shou) is tidy, that is, A is patch trivial.*

Proof. $\square$