# THE PATCH FRAME AND ITS RELATIONS WITH SEPARATION IN POINT-FREE TOPOLOGY

ABSTRACT.

## 1. INTRODUCTION

Aquí va la introducción.

## 2. PRELIMINARIES

## 3. HAUSDORFF PROPERTIES IMPLIES PATCH TRIVIALITY

Gadgets:
$A$ the base frame
Its point space $S = \mathrm{pt}(A)$.
The compact saturated sets of $S$,

$$\mathcal{Q}(S).$$

The preframe of open filters of $A$,

$$A^\wedge.$$

The preframe of open filters of $\Omega(S)$.

$$\Omega(S)^\wedge.$$

First we recall that every open filter $F \in A^\wedge$ has three faces, that is, determines (and its determine) by :

- The compact saturated $Q \in \mathcal{Q}S$.
- $\nabla \in \Omega(S)^\wedge$.
- The compact quotient $A \to A_F$.
- The fitted nucleus $v_F$.

Hoffman-Mislove can be rephrase:

There is a bijection between compact quotients of $A$and compact saturated sets of $S$

**Definition 3.1.** A frame has $\mathrm{KC}$ if every compact quotient of $A$ is a closed one. In other words every compact sublocale is close.

Denote by $\mathcal{H}rm$ the subcategory of $Frm$ of Hausdorff frames in the sense of Johnstone and Shu.

The block structure on a frame is an important problem and its related with some separation properties of frames.

**Proposition 3.2.** *For every $A \in \mathcal{H}rm$ the interval corresponding to the block determined by a open filter $F \in A^\wedge$ is trivial, that is,*

$$[v_F, w_F] = \{*\}$$

*Proof.* We know that for all $F \in A^\wedge$ the following holds: $v_F \leq w_F$. As a contradition, suppose that exists $F \in A^\wedge$ such that $w_F \nleq v_F$, that is, exists $a \in A$ such that $w_F(a) \nleq v_F(a)$.

Note that $w_F(a) \neq 1$, otherwise

$$1 = w_F(a) = \bigwedge\{p \in M \mid a \leq p\} \leq p$$

and this is a contradition because $p \neq 1$.

Then $1 \neq w_F(a) \nleq v_F(a)$ and for the property (**H**), exists $u \in A$ such that

$$u \nleq w_F(a) \quad \text{y} \quad \neg u \nleq v_F(a) \tag{1}$$

Note that 1 is true for all $a \in A$, in particular we have that

$$i)\, u \nleq w_F(0) \quad \text{y} \quad ii)\, \neg u \nleq v_F(0). \tag{2}$$

For 2-$(i)$ is true that $u \nleq \bigwedge M$, in particular, $u \nleq p$ for all $p \in M$. Therefore, $\neg u \leq p$ and $\neg u \leq w_F(0)$. If 2-$(ii)$ is true, then $u \notin F$, in otherwise

$$u \in F \Rightarrow v_u \leq f \Rightarrow v_u(0) = \neg u \leq f(0)$$

and this is a contradition. Thus, for the Birkhoff's separation lemma, exists a completely prime filter $G$ such that $u \notin G \supseteq F$. We take

$$q = \bigvee\{y \in A \mid y \notin G\}$$

the point corresponding to $G$. Thus, $u \notin G$, $u \leq q$. If $q \notin F$, then exists $m \in M$ such that $q \leq m$. Sinse $q$ is maximum, $q = m$ or $m = 1$, but $m \neq 1$ ($1 \in F$ and $M = A \setminus F$), then $m = q \in M$. Hence $u \nleq q$ and this is a contradition. Therefore $v_F = w_F$. □

**Proposition 3.3.** *Every Hausdorff frame $A$ (in the sense of Johnstone and Shou) is tidy, that is, $A$ is patch trivial.*

*Proof.* □

## REFERENCES