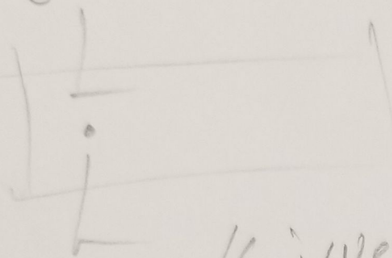
// get transform from base to wheels



$T_{bw_1}(0,0,\frac{w_b}{2})$

$T_{bw_2}(0,0,-\frac{w_b}{2})$

// inverse

adjointly: $T_{w_1b}$ & $T_{w_2b}$

$$V_{w_1} = Ad_{T_{w_1b}} V_b$$

$$V_{w_2} = Ad_{T_{w_2b}} V_b$$

$r\dot{\phi} = V_x$

$0 = V_y$

$\theta = 0$

$$T_{bw_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & +.1 \\ 0 & 0 & 1 \end{bmatrix} \quad T_{bw_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -.1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{w_1b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -.1 \\ 0 & 0 & 1 \end{bmatrix} \quad T_{w_2b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & .1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Ad_{T_{w_1b}} = \begin{bmatrix} 1 & 0 & 0 \\ -.1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad Ad_{T_{w_2b}} = \begin{bmatrix} 1 & 0 & 0 \\ .1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$r$

$$\begin{bmatrix} \dot{\theta} \\ V_{x1} \\ V_{y1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_{xb} \\ V_{yb} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -d\dot{\theta} + V_{xb} \\ V_{yb} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ V_{x2} \\ V_{y2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_{xb} \\ V_{yb} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ d\dot{\theta} + V_{xb} \\ V_{yb} \end{bmatrix}$$

@ wheel contact

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -d\dot{\theta} + V_{xb} \\ V_{yb} \end{bmatrix}, \quad \begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_2 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ d\dot{\theta} + V_{xb} \\ V_{yb} \end{bmatrix}$$

$$\dot{\phi}_1 = \frac{1}{r}\left(-\frac{d}{2}\dot{\theta} + V_{xb}\right) \quad \Big| \quad \dot{\theta}_2 = \frac{1}{r}\left(\frac{d}{2}\dot{\theta} + V_{xb}\right)$$

$$0 = V_{yb}$$

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Body frame

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \frac{1}{r} \underbrace{\overset{2\times3}{\begin{bmatrix} -d & 1 & 0 \\ d & 1 & 0 \end{bmatrix}}}_{H} \overset{3\times1}{\begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}}$$

$$= \frac{1}{r} \begin{bmatrix} -d\dot{\theta} + V_x \\ d\dot{\theta} + V_y \end{bmatrix}$$

$A^+ = (A^T \cdot A)^{-1} \cdot A^T$ ← columns lin indep

$A^+ = A^T (A^T A)$ ← rows lin indep

$$A^T = \left( 1/r \cdot \overset{3\times2}{\begin{bmatrix} -d & d \\ 1 & 1 \\ 0 & 0 \end{bmatrix}} \cdot 1/r \overset{2\times3}{\begin{bmatrix} -d & 1 & 0 \\ d & 1 & 0 \end{bmatrix}} \right)^{-1} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix} \checkmark$$

$$= 1/r^2 \cdot \begin{bmatrix} 2d^2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}^{-1}$$

det $2d^2(0) - 0$

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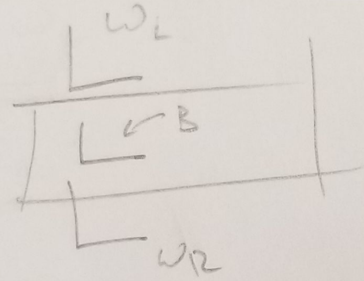
twist to wheels

$= H^{+} V_b$

$$V_b = \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$




1. get T's from wheels to body
2. get adjoints of
3. get inverse of adjoints

just invert transform & fake inverse

wheel twist

ad body to wheel

body twist

$$V_1 = A_{1b} V_b$$

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twist to wheels

$$\cancel{\begin{bmatrix} \omega \\ v_x \\ v_y \end{bmatrix} = H \begin{bmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \Delta\phi_3 \\ \Delta\phi_4 \end{bmatrix}}$$

$$\Delta\Theta = H$$

---

$2d$ = wheel_base
$(x,y)$ halfway between wheels
$\theta_1, \theta_2$ wheel angles
$\phi$ angle w world frame

$$\dot{q} = \begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \\ \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} -r/2d & r/2d \\ r/2\cos\phi & r/2\cos\phi \\ r/2\sin\phi & r/2\sin\phi \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_L \\ u_R \end{bmatrix}$$

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$\phi$ angle w ...

$$\dot{q} = \begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \\ \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} -r/2d & r/2d \\ r/2\cos\phi & r/2\cos\phi \\ r/2\sin\phi & r/2\sin\phi \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix}$$

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$$V_b = H^{\dagger} v$$

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$$\overset{V_b}{\begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix}} = \begin{bmatrix} -r/d & r/d \\ r/2 & r/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_L \\ v_R \end{bmatrix}$$

$$= \begin{bmatrix} -r/d\, v_L + r/d\, v_R \\ r/2\,(v_L + v_R) \\ 0 \end{bmatrix}$$

Body frame

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \frac{1}{r}\underbrace{\overset{2\times3}{\begin{bmatrix}-d & 1 & 0\\ d & 1 & 0\end{bmatrix}}}_{H}\overset{3\times1}{\begin{bmatrix}\dot{\theta}\\ V_x\\ V_y\end{bmatrix}}$$

$$= \frac{1}{r}\begin{bmatrix}-d\dot{\theta}+V_x\\ d\dot{\theta}+V_y\end{bmatrix}$$

$A^{+} = (A^{T}\cdot A)^{-1}\cdot A^{T}$ ← columns lin indep

$A^{+} = A^{T}\cdot(A^{T}\cdot A)$ ← rows lin indep

$$A^{T} = \left(\frac{1}{r}\cdot\overset{3\times2}{\begin{bmatrix}-d & d\\ 1 & 1\\ 0 & 0\end{bmatrix}}\cdot\frac{1}{r}\overset{2\times3}{\begin{bmatrix}-d & 1 & 0\\ d & 1 & 0\end{bmatrix}}\right)^{-1}\frac{1}{r}\begin{bmatrix}\dot{\theta}\\ V_x\\ V_y\end{bmatrix} \checkmark$$

det $2d^2(0)-0$

$$= \frac{1}{r^2}\cdot\begin{bmatrix}2d^2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 0\end{bmatrix}^{-1}$$