# Homework #4

Problems: 7.2,7.3,7.4,7.6,7.8,9.2

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In [144]:
```
# import numpy as np
import autograd.numpy as np
from autograd import grad
import pandas as pd
import sympy as sym
import math
%matplotlib notebook
from matplotlib import pyplot as plt

from IPython.display import Image
```

# Problem 7.2

In [2]:
```
df = pd.read_csv("4class_data.csv", header=None, index_col=None)
```

In [3]:
```
C = 4
N = 2
OHE = np.eye(C)

W_sym = sym.MatrixSymbol('\widetilde{w}', N+1, C).as_explicit()
xp_tilde = sym.MatrixSymbol('\widetilde{x}', N,1).as_explicit()
yp_Mat = sym.eye(4)

yp = sym.symbols('y_p')
xp_tilde = sym.MatrixSymbol('\widetilde{x_p}', N + 1, 1).as_explicit
()
w_tilde = sym.MatrixSymbol('\widetilde{w}', N + 1, 1).as_explicit()
```

In [4]:
```
cost = sym.Matrix([sym.log(1 + sym.exp(-yp * xp_tilde.T * w_tilde)[0
])])
grad_cost = cost.jacobian(w_tilde)
cost_lam = sym.lambdify([yp, xp_tilde, w_tilde], cost)
grad_cost_lam = sym.lambdify([yp, xp_tilde, w_tilde],grad_cost)
```

In [5]:
```
yps_multiclass = df.iloc[:,-1]
P = yps_multiclass.shape[0]

xp_tildes = np.concatenate((np.ones((P,1)) ,df.iloc[:,:2].to_numpy
()), axis=1).T
xp_tildes[:,0]
```

Out[5]:
```
array([1.     , 0.12558, 0.75948])
```

In [6]:
```
def get_w_OvA(c):
    class_indexs = df.index[df.iloc[:,-1] == c]
    OvA_yps = pd.DataFrame(np.zeros(P) -1)
    OvA_yps.iloc[class_indexs] =1
    return np.squeeze(OvA_yps.to_numpy())
```

In [7]:
```
def cost_func(w_tilde, yps):
    cost_sum = 0
    for i in range(P):
        cost_sum += cost_lam(yps[i], xp_tildes[:,i], w_tilde)  * 1/P
    ans = np.squeeze(cost_sum,axis=1)
    return ans
```

In [8]:
```
get_w_OvA(2).shape
```

Out[8]:
```
(40,)
```

In [9]:
```
def grad_cost_func(w_tilde, yps):
    grad_cost_sum = np.zeros(N + 1)
    for i in range(P):
        grad_cost_sum += np.squeeze(grad_cost_lam(yps[i], xp_tildes
[:,i], w_tilde),axis=0) * 1/P
    return grad_cost_sum
```

In [10]:
```
def grad_desc(w_last, yps):
    eps = 10e-5
    max_its = 50000
    cost_vals = np.zeros(max_its)
    num_its = 0
    cost_vals[0] = cost_func(w_last, yps)
    grad_last = grad_cost_func(w_last, yps)
    num_its += 1

    while(np.linalg.norm(grad_last) > eps):
        w_next = w_last - .1 *grad_last
        num_its += 1
        if (num_its > max_its):
            print("hit max its")
            break;

        w_last = w_next
#         cost_vals[num_its] = cost_func(w_last, yps)
        grad_last = grad_cost_func(w_last, yps)
#         print(np.linalg.norm(grad_last))


    return w_last, cost_vals, num_its
```

In [11]:
```
W = np.ones((N+1, C))
yp = get_w_OvA(3)
for i in range(4):
    yps = get_w_OvA(i+1)
    W[:,i], costs, its = grad_desc(W[:,i], yps)
```

```
hit max its
hit max its
```

In [12]:
```
def confusion_matrix_gen(w_tilde, yps):
    confusion_matrix = pd.DataFrame(np.zeros((2,2)), index=["a_bad",
"a_good"], columns=["p_bad", "p_good"]).astype('int')
    for i in range(P):
        xp_tilde = xp_tildes[:,i]
        yp = yps[i]
        #correctly predicted
        if np.sign(np.matmul(xp_tilde.T, w_tilde)) == np.sign(yp):
            #pg, ag
            if (yp > 0):
                confusion_matrix.loc["a_good", "p_good"] += 1
            else:
                confusion_matrix.loc["a_bad", "p_bad"] += 1

        else:
            if (yp > 0):
                confusion_matrix.loc["a_good", "p_bad"] += 1
            else:
                confusion_matrix.loc["a_bad", "p_good"] += 1

    return confusion_matrix
```

In [13]:
```
c = 0
confusion_matrix = confusion_matrix_gen(W[:,c],get_w_OvA(c+1))
confusion_matrix
```

Out[13]:

| | p_bad | p_good |
|---|---|---|
| **a_bad** | 28 | 2 |
| **a_good** | 2 | 8 |

In [14]:
```
x_tildes = xp_tildes
class_list = np.zeros(P)
for pt in range(P):
    max_dist = 0
    for c in range(C):
        dist = np.matmul(x_tildes[:,pt].T, W[:,c])
        if (dist > max_dist):
            class_list[pt] = c
```

In [ ]:

In [15]:
```
def number_misclass(class_list):
    misclass = 0
    for c in range(C):
        for i in class_list[10*c:10*(c+1)]:
            if i != c:
#                print("misclass: {} as: {}".format())
                misclass+=1
    return misclass
```

In [16]:
```
number_misclass(class_list)
```

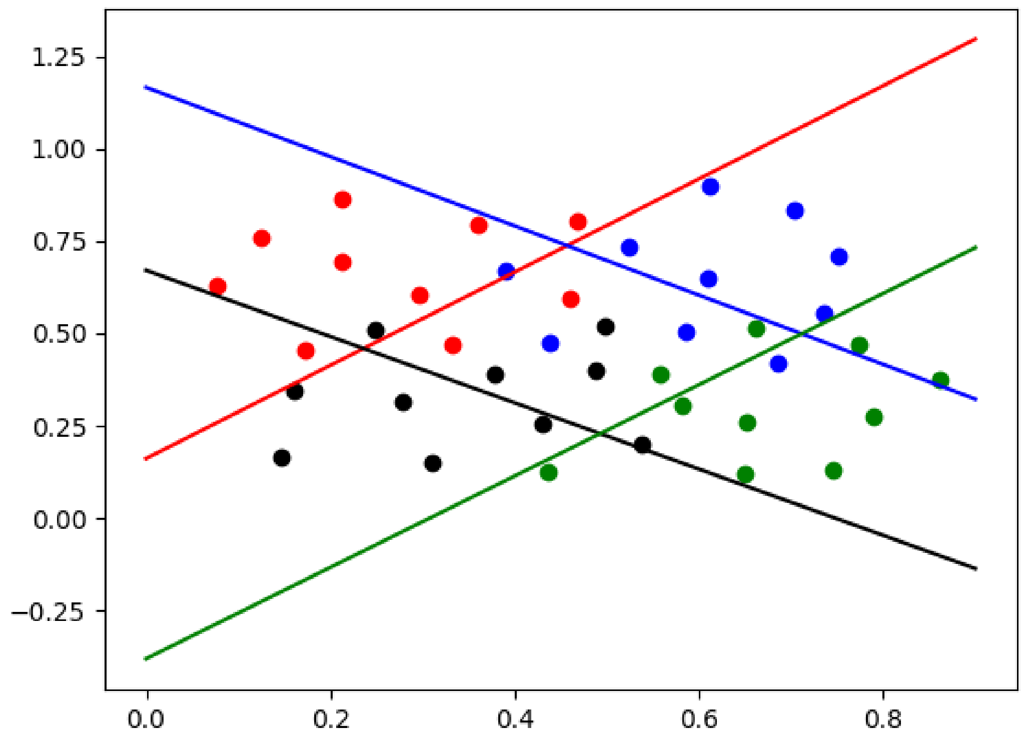
Out[16]: 13

Therefore OvA classifier misclasified 13 elemnts

In [17]:
```
x, y = sym.symbols('x y')
x_sym = sym.Matrix([1, x, y])
xvals = np.linspace(0,.9,100)
yvals = np.zeros((4,100))
for i in range(C):
    jarm = sym.Matrix(x_sym.T * W[:,i])
    getyvals = sym.lambdify([x], -1/jarm[2].args[0]* (jarm[1] + jarm[
0]))
    yvals[i,:] = getyvals(xvals)
```

In [18]:
```
plt.figure()
plt.scatter(xp_tildes[1,:10],xp_tildes[2,:10], color='r')
plt.scatter(xp_tildes[1,10:20],xp_tildes[2,10:20], color='b')
plt.scatter(xp_tildes[1,20:30],xp_tildes[2,20:30], color='k')
plt.scatter(xp_tildes[1,30:40],xp_tildes[2,30:40], color='g')
plt.plot(xvals,yvals[0,:], color='r')
plt.plot(xvals,yvals[1,:], color='b')
plt.plot(xvals,yvals[2,:], color='k')
plt.plot(xvals,yvals[3,:], color='g')

plt.show()
```




In [ ]:

In [ ]:

In [ ]:

## Problem 7.3

In [96]:
```
df = pd.read_csv("3class_data.csv", header=None, index_col=None)
N = df.shape[0] -1
P = df.shape[1]
C = 3

W = np.random.rand(N+1,C)
W_yp = np.eye(C)

x_tildes = np.concatenate((np.ones((P,1)).T, df.iloc[:2, :].to_numpy
()), axis=0)

x_p_tilde = sym.MatrixSymbol('\widetilde{x_p}', N+1, 1).as_explicit()
w = sym.MatrixSymbol('w', N+1, 1).as_explicit()
```

In [97]:
```
W
```

Out[97]:
```
array([[0.51773733, 0.2812935 , 0.75149829],
       [0.71741216, 0.24327819, 0.01467621],
       [0.67168055, 0.35068683, 0.44381238]])
```

In [122]:
```
def softmax_cost(W):
    cost_sum = 0
    for p in range(P):
        inner_sum = 0
        for c in range(C):
            inner_sum += np.exp(np.matmul(x_tildes[:,p].T, W[:,c]))
        w_yp = np.max(np.matmul(x_tildes[:,p].T,W))
        cost_sum += np.log(inner_sum) \
            - np.max(np.matmul(x_tildes[:,p].T, W))
    return cost_sum/P
```

In [123]:
```
grad_softmax = grad(softmax_cost)
softmax_cost(W)
grad_softmax(W)
```

Out[123]:
```
array([[-0.50637818,  0.22811864,  0.27825954],
       [-0.2497459 ,  0.10505614,  0.14468976],
       [-0.26151507,  0.11429539,  0.14721968]])
```

In [124]:
```
np.log(np.exp(np.matmul(x_tildes[:,0].T, W[:,0])))

p = 27
np.matmul(x_tildes[:,p].T, W_yp[:, math.floor(p/10)])
P
np.max(np.matmul(x_tildes[:,0].T,W))
```

Out[124]:
```
1.1051077595744068
```

In [125]:
```
def grad_desc(w_last):
    eps = 10e-5
    num_its = 0
    grad_last = grad_softmax(w_last)[:,0]
    num_its += 1

    while(np.linalg.norm(grad_last) > eps):
        w_next = w_last - .1 *grad_last
        num_its += 1
        if (num_its > 10000):
            print("hit max its")
            break;

        w_last = w_next
#         cost_vals[num_its] = cost(w_last)
        grad_last = grad_softmax(w_last)[:,0]
#         print(np.linalg.norm(grad_last))


    return w_last, num_its
```

In [126]:
```
w,its = grad_desc(W)
```

```
hit max its
```

In [152]:
```
W_eval = w
```

In [151]:
```
np.mean(np.matmul(x_tildes.T, W_eval).argmax(axis=1))
```

Out[151]:
```
0.0
```

Therefore the perceptron multiclass classifier has achieved zero misclassicifications

# Problem 7.4 & 7.6

```
In [145]: Image("p12.jpg")
```

Out[145]:

Homework #4

7.4 Show that multiclass Perceptron

Cost: $g(W) = \frac{1}{P}\sum_{p=1}^{P}\left[\left(\max_{j=0,\dots,C-1} \tilde{x}_p^T w_j\right) - \tilde{x}_p^T w_{y_p}\right]$

reduces to: $g(w) = \frac{1}{P}\sum_{p=1}^{P} \max\left(0, -y_p \tilde{x}_p^T w\right)$

For $C=2$

$$g(W) = \frac{1}{P}\sum_{p=1}^{P}\left[\left[\max_{j=0,1} x_p^T w_j\right] - x_p^T w_{y_p}\right]$$

$$= \frac{1}{P}\sum_{p=1}^{P}\left[\max_{j=0,1} x_p^T\left(w_j - w_{y_p}\right)\right]$$

∴ if $j = y_p$ it evaluates to 0

& if $j \neq y_p$ you get $-y_p x_p^T w_j$

this is equivalent to $= \frac{1}{P}\sum_{p=1}^{P} \max\left(0, -y_p x_p^T w_j\right)$

7.6 Show that multiclass softmax reduces to 2 class softmax for $C=2$

$$g(w) = \sum_{p=1}^{P} \log\left(1+e^{-y_p \tilde{x}_p^T w}\right)$$

$$g(W) = \frac{1}{P}\sum_{p=1}^{P}\left[\log\left(\sum_{j=0}^{C-1} e^{\tilde{x}_p^T w_j}\right) - \tilde{x}_p^T w_{y_p}\right]$$

$$= \frac{1}{P}\sum_{p=1}^{P}\left[\log\left(e^{\tilde{x}_p^T w_1} + e^{\tilde{x}_p^T w_2}\right) - \tilde{x}_p^T w_{y_p}\right]$$

$$= \frac{1}{P}\sum_{p=1}^{P}\left[\log\left(e^{\tilde{x}_p^T w_1} + e^{\tilde{x}_p^T w_2}\right) - \log\left(e^{\tilde{x}_p^T w_{y_p}}\right)\right]$$

$$= \frac{1}{P}\sum_{p=1}^{P}\log\left(\frac{e^{\tilde{x}_p^T w_1} + e^{\tilde{x}_p^T w_2}}{e^{\tilde{x}_p^T w_{y_p}}}\right) = \boxed{\frac{1}{P}\sum_{p=1}^{P}\log\left(1+e^{-y_p \tilde{x}_p^T w}\right)}$$

## Problem 7.8

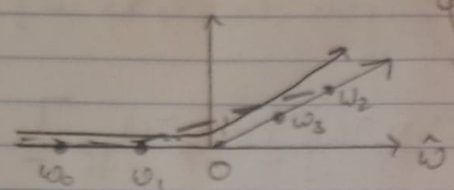
```
In [146]: Image("p13.jpg")
```

Out[146]:

7.8 Show multiclass Perceptron & Softmax costs are convex

Use the zero order definition of convexity:

$$g(\lambda \vec{w}_1 + (1-\lambda)\vec{w}_2) \leq \lambda g(\vec{w}_1) + (1-\lambda) g(\vec{w}_2)$$




percept:

Because $g(\vec{w})$ is a linear function everywhere but zero all pts on either linear section by definition obey the zero order definition of complexity. Straddle pts as well it can be seen that the pt on the 'cone' lies below that of the zero order connection satisfying the criterion

So to is all of this true with the softmax case due to the fact that its an approximation of the max function

```
In [ ]:
```