# Homework #1

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ME 374 - Digital Control

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## Question 1

**For system given in the block diagram, find the transfer function from $R(z)$ to $Y(z)$ in terms of $D(z), G_1(z), G_2(z)$, and $K$**

- Equation for $Y(z)$:

$$Y(z) = KG_2(z)E_2(z) = KG_2(z)[U(z) - Y(z)]$$

- Solving for $Y(z)$:

$$Y(z) = \frac{KG_2(z)}{1 + KG_2(z)}U(z)$$

- Equation for $U(z)$:

$$U(z) = D(z)E_1(z) = D(z)[R(z) - X(z)]$$

- Equation for $X(z)$:

$$X(z) = KG_1(z)E_2(z)S = KG_1(z)[U(z) - Y(z)]$$

- Solve for $U(z)$ subbing in $X(z)$:

$$U(z) = D(z)[R(z) - KG_1(z)[U(z) - Y(z)]]$$

$$U(z) = \frac{D(z)R(z) + KD(z)G_1(z)Y(z)}{1 + KD(z)G_1(z)}$$

- Solve for $Y(z)$ subbing in $U(z)$:

$$Y(z) = \frac{KG_2(z)}{1 + KG_2(z)} \cdot \frac{D(z)R(z) + KD(z)G_1(z)Y(z)}{1 + KD(z)G_1(z)}$$

$$Y(z) = \frac{KG_2(z)D(z)}{1 + KD(z)G_1(z) + KG_2(z)}R(z)$$

- $\therefore for H(z)$ such that $Y(z) = H(z)R(z)$

$$H(z) = \frac{KG_2(z)D(z)}{1 + KD(z)G_1(z) + KG_2(z)}$$

# Question 2

**For the given regulator control system with unit step input disturbance, plant transfer function, constant zero valued reference signal, and sampling rate of T=0.1s. For a controller $D(z) = 2$ what is the steady state value of the sampled output? What would the steady state value be if $D(z) = 2 + \frac{0.2z}{z-1}$?**

Using the fact that the equation for error of our system, due to the ZOH and the step distrubrance is $E(z) = \frac{-G(z)}{1+D(z)G(z}\frac{z}{z-1}$ and that the FVT is evulated as $(z-1)E(z)|_{z=1}$ we see that we simply have to evaluate the negative of E(z) to get our solution.

I used the following matlab script to do so:

```
%ZOH Equivalent of P
T = 0.1;
s = tf('s');
z = tf('z', T);
P = 3/(s + 2);
G = c2d(P,T,'zoh');
% D = 2;
D = 2 + (0.2 * z)/(z-1);

E = -G / (1 + D * G) %* (z /(z-1))
evalfr(E,1)
```

The results of which are as follows:

| | $T = 0.1$; $D = 2$ | $T = 1$; $D = 2$ | $T = 0.1$; $D = 2 + \frac{0.2z}{z-1}$ | $T = 1$; $D = 2 + \frac{0.2z}{z-1}$ |
|---|---|---|---|---|
| $y[\infty]$ | 0.375 | Unst | 0 | Unst |

Table 1: Steady State Value Depending on Digital Controller

The responses that sampled with $T = 1s$ can be seen to be unstable because they have poles that exist outside the unit circle in the $z$-plane as can be seen in the figures below:

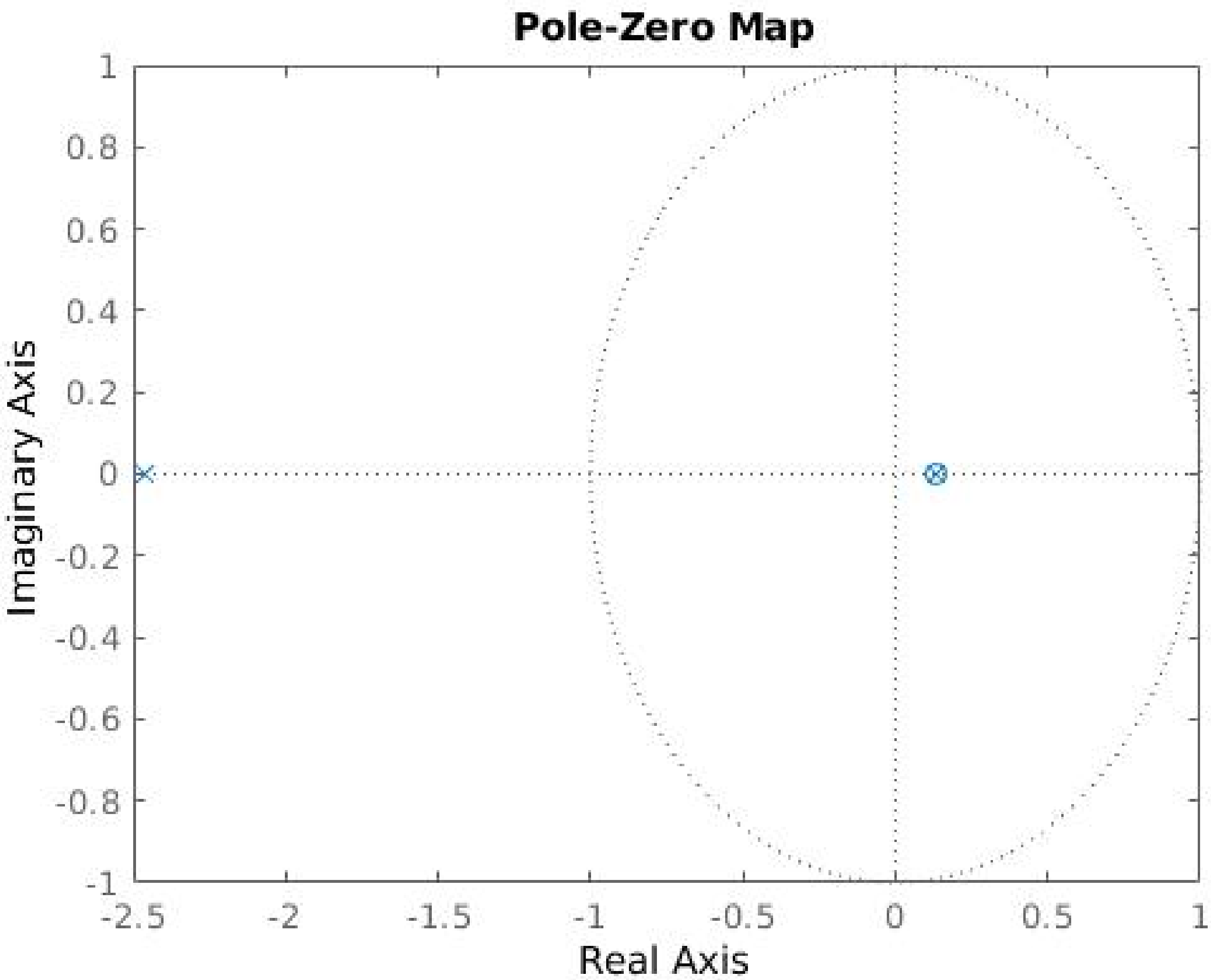


Figure 1: Unstable System w T = 1s

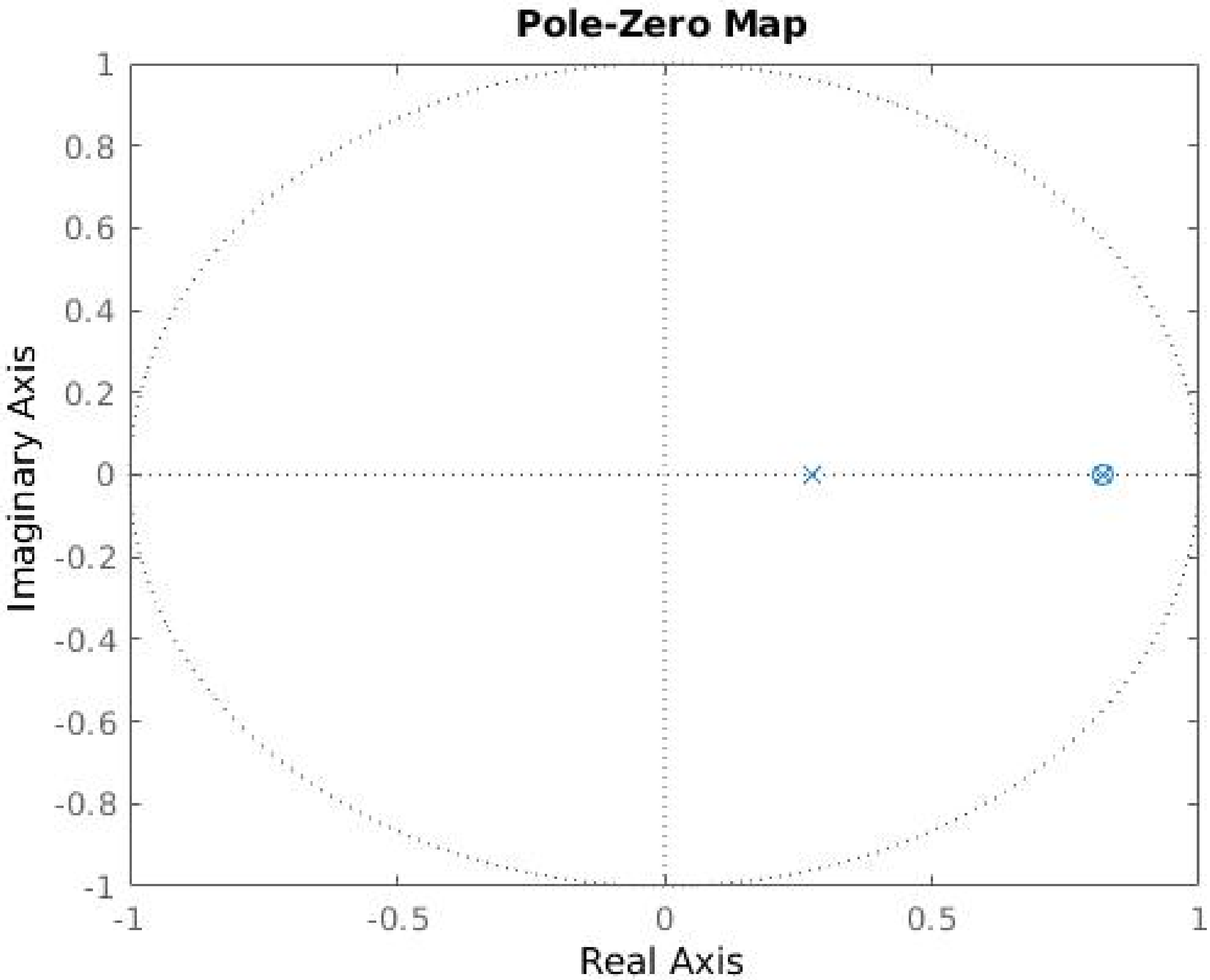


Figure 2: Stable System w T = 0.1s