Homework #1

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Question 1

For the 3 methods of computing Riemann sums {left, right, trapezoidal} compute the corresponding discrete transfer function $H(z)$ from u to y and list all poles and zeros.

a. Left-hand Rule:

$$y[k] = y[k-1] + Tu[k-1]$$

$$Y(z) = z^{-1}Y(z) + Tz^{-1}U(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{Tz^{-1}}{1 - z^{-1}}$$

$$H(z) = \frac{T}{z-1}$$

$\therefore$ pole @ z = 1 & no zeros

b. Right-hand Rule:

$$y[k] = y[k-1] + Tu[k]$$

$$Y(z) = z^{-1}Y(z) + TU(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{T}{1 - z^{-1}}$$

$$H(z) = \frac{Tz}{z-1}$$

$\therefore$ pole @ z = 1 & zero @ z = 0

c. Trapezoidal Rule:

$$y[k] = y[k-1] + Tu[k-1] + \frac{1}{2}(u[k] - u[k-1])$$

$$y[k] = y[k-1] + \frac{1}{2}T(u[k] + u[k-1])$$

$$Y(z) = z^{-1}Y(z) + \frac{1}{2}TU(z) + \frac{1}{2}z^{-1}TU(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{1}{2}T\frac{1+z^{-1}}{1-z^{-1}}$$

$$H(z) = \frac{T}{2}\frac{z+1}{z-1}$$

$\therefore$ pole @ z = 1 & zero @ z = -1

Question 2

For the given matlab implemented LTI system, find the transfer function of the system, $H(z)$ and calculate its poles and zeros. First calculate output y analytically given input $u = ones(1, 10)$ and using z-transforms then solve numerically using the $y = H(u)$ function defined in a Matlab script. Are these the same? Is the transfer function stable? If not find bounded input sequence that gives unbounded output sequence, verify in Matlab using the given function.

$$x[k] = -x[k-2] + u[k-2]$$

$$(z) = -z^{-2}X(z) + -z^{-2}U(z)$$

$$(1 + z^{-2})X(z) = -z^{-2}U(z)$$

$$\frac{X(z)}{U(z)} = \frac{1}{z^2+1} \rightarrow X(z) = \frac{1}{z^2+1}U(z)$$

and

$$y[k] = -x[k] + x[k-1] + u[k] - u[k-1]$$

$$Y(z) = -X(z) + z^{-1}X(z) + U(z) - z^{-1}U(z) \leftarrow X(z)$$

$$Y(z) = \frac{z^2 - z}{z^2+1}U(z)$$

$$\therefore H(z) = \frac{z^2 - z}{z^2+1}$$

This has poles @ $\pm j$ and zeros @ $0, 1$. Since two of the poles exist on the unit circle, this is not a stable transfer function.
Now for the $u[k] = 1, 1, 1, 1, ...$ this can be seen as the unit step which has a z-transform of $\frac{z}{z-1}$

$$\therefore Y(z) = H(z)U(z) = \frac{z^2}{z^2+1}$$

This can be converted to the time domain via tools like wolfram alpha or partial fraction decomposition and using the Z-transform look up table to get the equation in the time domain:

$$y[k] = \frac{1}{2}((-j)^k + j^k)$$

Plugging into the matlab function:

```
function y = analyticH(k)
for it = 1:k
   y(it) =  1/2 * ( (-exp(1i*pi/2))^(it-1) + exp(1i*pi/2)^(it-1));
end
```

We get the same result as evaluating the given function of a k = 10!
However, we can find that for a bounded function such as $cos(k-1)pi/2$ we get unbounded output!