Step	Algorithm: $y := Ax + y$
1a	$\{ y = \widehat{y} \qquad \}$
4	$A \rightarrow \left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$, $x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right)$, $y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$ where A_{TL} is 0×0, x_T has 0 rows, y_T has 0 rows
2	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL} x_T + \widehat{y}_T \\ \hline \widehat{y}_B \end{array} \right) \right\}$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL} x_T + \widehat{y}_T \\ \hline \widehat{y}_B \end{array} \right) \wedge m(A_{TL}) < m(A) \right\}$
5a	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c\|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array} \right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array} \right)$ where α_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row
6	$\left\{ \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) = \left(\begin{array}{lr} A_{00} x_0 + & \widehat{y}_0 \\ & \widehat{\psi}_1 \\ & \widehat{y}_2 \end{array} \right) \right\}$
8	$y_0 := \chi_1 (a_{10}^T)^T + y_0$ $\psi_{:} := a_{10}^T x_0 + \alpha_{11} \chi_1 + \psi_1$
7	$\left\{ \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right) = \left(\begin{array}{r} A_{00} x_0 + \chi_1 (a_{10}^T)^T + \widehat{y}_0 \\ a_{10}^T x_0 + \alpha_{11} \chi_1 + \widehat{\psi}_1 \\ \widehat{y}_2 \end{array} \right) \qquad \text{(Note: } (a_{10}^T)^T \chi_1 = \chi_1 (a_{10}^T)^T \text{)} \right\}$
5b	$\left(\begin{array}{c\|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc\|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$, $\left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array} \right)$, $\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array} \right)$
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	endwhile
2,3	$\left\{ \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL} x_T + \widehat{y}_T \\ \hline \widehat{y}_B \end{array} \right) \wedge \neg (m(A_{TL}) < m(A)) \right\}$
1b	$\{ y = Ax + \widehat{y} \qquad \}$

Step	Algorithm: $y := Ax + y$
1a	$\{ \quad \}$
4	where
2	$\left\{ \quad \right\}$
3	while do
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6	$\left\{ \quad \right\}$
8	
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3	while do
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5a	where
6	$\left\{ \right\}$
8	
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</table>

Algorithm: $y := Ax + y$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), \; x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), \; y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$$

where A_{TL} is 0×0, x_T has 0 rows, y_T has 0 rows

while $m(A_{TL}) < m(A)$ do

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array} \right), \; \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array} \right), \; \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array} \right)$$

where α_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row

$$\begin{aligned} y_0 &:= \chi_1 (a_{10}^T)^T + y_0 \\ \psi_1 &:= a_{10}^T x_0 + \alpha_{11} \chi_1 + \psi_1 \end{aligned}$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \; \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array} \right), \; \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array} \right)$$

endwhile

Algorithm: $y := Ax + y$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right)$$

where A_{TL} is 0×0, x_T has 0 rows, y_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ y_2 \end{array}\right)$$

where α_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row

$$y_0 := \quad \chi_1 (a_{10}^T)^T + y_0$$

$$\psi_1 := a_{10}^T x_0 + \alpha_{11}\chi_1 + \psi_1$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} A_{00} & a_{01} & A_{02} \\ a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array}\right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array}\right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ \hline x_2 \end{array}\right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array}\right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ \hline y_2 \end{array}\right)$$

endwhile